DEA-BASED INVESTMENT STRATEGY AND ITS APPLICATION IN THE CROATIAN STOCK MARKET

Margareta Gardijan
Faculty of Economics and Business, University of Zagreb
Trg. J. F. Kennedyja 6, 10000 Zagreb
E-mail: mgardijan@efzg.hr

Vedran Kojić
Faculty of Economics and Business, University of Zagreb
Trg. J. F. Kennedyja 6, 10000 Zagreb
E-mail: vkojic@efzg.hr

Abstract

This paper describes the DEA-based investment strategy for constructing of a stock portfolio in the Croatian stock market. The relative efficiency of the DMUs, which are in this case the selected stocks from Zagreb Stock Exchange, is obtained from the output oriented CCR and BCC models. The set of inputs consists of risk measures, namely return variance, Value at Risk (VaR) and beta coefficient (β), while monthly return represents an output. Following the „efficiency scores“, obtained from the models, we construct a portfolio of DEA-efficient stocks (DEA-portfolio). This portfolio can be modified over time according to changes of the DMU's efficiency scores. By comparing the returns of the DEA-portfolio and the market return during the given time period, the applicability of the investment strategy based on a DEA methodology, as a strategy for achieving superior returns, is estimated.

Key words: *Data envelopment analysis, Investment strategy, Stock portfolio, Efficiency*

1. INTRODUCTION

Stock portfolio investment strategies usually follow the fundamental concept of a modern portfolio theory based on balancing the expected return and the return variance of the portfolio. Therefore, for investors who want to maximize their return on investment, the problem is finding the stock portfolio which will achieve the highest possible return for a given level of risk. As a possible solution of that problem, in this paper we examine the investment strategy based on Data Envelopment Analysis (DEA). Generally, DEA is methodology which connects operational research, mathematics and economy. The first DEA models were developed by Charnes, Cooper and Rhodes in 1978, and by Banker, Charnes and Cooper in 1984. Since then, DEA has been used as a very powerful management tool, as well as a technique for solving various types of multi-criteria decision-making problems.

However, applications of DEA in estimating efficiency of securities appeared just a decade ago (Powers and McMullen, 2002; Lopes et al, 2008; Chen, 2008; Pätäri and Leivo 2010). According to our knowledge, this paper is the first application of DEA technique in the Croatian stock market. Our work follows the methodology of constructing a DEA efficient portfolio presented in paper *DEA investment strategy in the Brazilian stock market* by (Lopes et al, 2002).

Next section describes general terms in DEA and software which wasused in this paper. Section 3 presents data and methodology of research, while results are presented in section 4. The final section gives possible guidelines for further research.

2. THE BACKGROUND OF DATA ENVELOPMENT ANALYSIS

The DEA methodology uses mathematical programming to process empirical data on inputs and outputs of a given group of decision making units (DMUs). As a result, each DMU is assigned a value within interval (0,1]. Value 1 represents relatively efficient DMU, while the DMU with value less than 1 is deemed inefficient. In this way, the efficiency of each DMU is evaluated with respect to other DMUs. The subgroup of relatively efficient DMUs serves as a basis for the determination of the efficiency frontier, and for the establishment of goals for the inefficient DMUs (Lopes et al, 2002).

Basic models within the DEA, named after their founders, are Charnes – Cooper – Rhodes (CCR) model (1978), and Banker – Charnes – Cooper (BCC) model (1984). The CCR model assumes constant returns to scale (CRS), while the BCC assumes variable returns to scale (VRS). Both models can be either input or output oriented regarding whether the inefficient units aim to maximize their outputs or minimize their inputs.

In this section we describe the CCR model according to (Cooper et al, 2007). CCR model is based on the assumption of constant returns to scale. Let us consider n DMUs: DMU_1, DMU_2, ..., DMU_n. All DMUs use m inputs and convert them to s outputs. Let the input and output data for DMU_j be $(x_{1j}, x_{2j}, ..., x_{mj})$ and $(y_{1j}, y_{2j}, ..., y_{sj})$, respectively. For each DMU_o, $o \in \{1, 2, ..., n\}$, a *virtual input* and *virtual output* is formed by (yet unknown) weights (v_i) and (u_r): $v_1 x_{1o} + ... + v_m x_{mo}$ and $u_1 y_{1o} + ... + u_s y_{so}$, respectively. Now, the idea is to determine the weight, using linear programming so as to maximize the ratio

$$\frac{\text{virtual output}}{\text{virtual input}}.$$

Precisely, we solve next fractional programming problem for DMU_o:

$$(FP_o) \qquad \max_{v_1,\dots,v_m,u_1,\dots,u_s} \theta = \frac{u_1 y_{1o} + u_2 y_{2o} + \dots + u_s y_{so}}{v_1 x_{1o} + v_2 x_{2o} + \dots + v_m x_{mo}},$$

$$\begin{aligned} \text{s. t.} \quad & \frac{u_1 y_{1j} + u_2 y_{2j} + \dots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \dots + v_m x_{mj}} \le 1, \quad (j = 1,2,\dots,n) \\ & v_1, v_2, \dots, v_m \ge 0, \\ & u_1, u_2, \dots, u_s \ge 0. \end{aligned} \tag{1}$$

It can be proved that fractional problem (1), presented above, is equivalent to the following linear problem[1]:

$$(LP_o) \qquad \max_{\mu_1,\dots,\mu_m,v_1,\dots,v_s} \theta = \mu_1 y_{1o} + \mu_2 y_{2o} + \dots + \mu_s y_{so},$$

$$\begin{aligned} \text{s. t.} \quad & v_1 x_{1o} + v_2 x_{2o} + \dots + v_m x_{mo} = 1, \\ & \mu_1 y_{1j} + \mu_2 y_{2j} + \dots + \mu_s y_{sj} \le v_1 x_{1j} + v_2 x_{2j} + \dots + v_m x_{mj}, \ (j = 1,2,\dots,n) \\ & v_1, v_2, \dots, v_m \ge 0, \\ & \mu_1, \mu_2, \dots, \mu_s \ge 0. \end{aligned} \tag{2}$$

Finally, the CCR-efficiency was obtained as follows[2]:

1. DMU_o is CCR-efficient if $\theta^* = 1$ and there exists at least one optimal (v^*,u^*), with $v^* > 0$ and $u^* > 0$.
2. Otherwise, DMU_o is CCR-inefficient.

For instance, Figure 1 depicts efficient frontier for the CCR model in case of one input and one output for $n = 7$ DMUs, where DMU_1 and DMU_3 are CCR-efficient.

The second most popular model is the BCC model. In comparison with the CCR model, the BCC model is used in situations when returns to scale are variable, i.e. in the cases when proportional increasing (decreasing) of inputs results in non proportional increasing (decreasing) outputs. “It can be shown that the BCC – efficiency is easier to achieve than the CCR – efficiency. Furthermore, an amount of the BCC – efficiency for the particular DMU is always equal or greater than the

[1] See Cooper et al. (2007), p. 24.
[2] Ibid.

corresponding amount of CCR – efficiency."[3] Efficient frontier for the BCC model is shown in Figure 2 (DMUs are same as those in Figure 1).

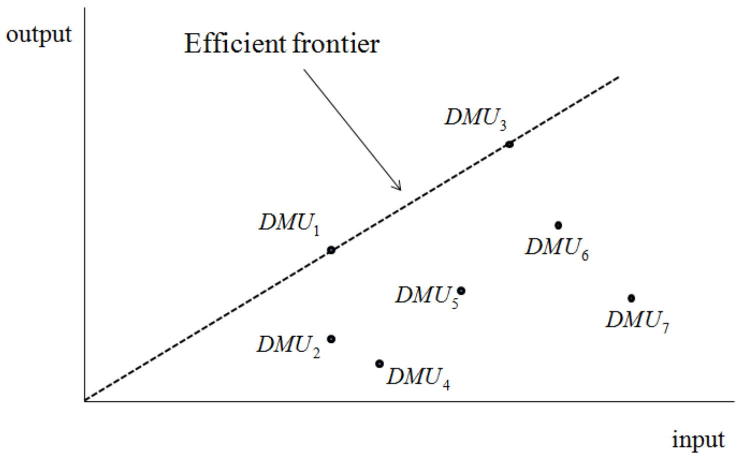

Figure 1: Efficient frontier - CCR model.

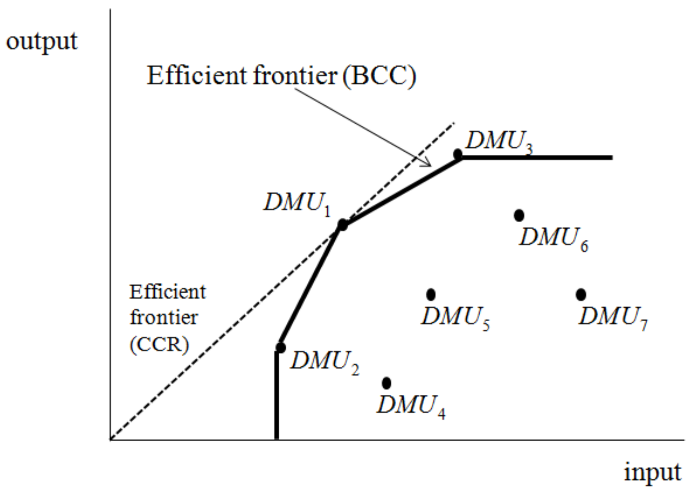

Figure 2: Efficient frontier – BCC model.

Various computer programs are used for solving many DEA models. In this paper, the DEAP Version 2.1. was used.[4]

[3] Rabar and Blažević (2011), p. 34.
[4] Free version of DEAP 2.1. can be downloaded from: http://www.uq.edu.au/economics/cepa/deap.php [Accessed 9/24/12].

3. DATA AND METHODOLOGY

The DEA approach was used to analyse the relative efficiency of chosen stocks[5] from the Zagreb Stock Exchange. To be chosen in the sample, the stocks should have been included in the official Zagreb Stock Exchange share index (CROBEX) at least once during the period of analysis or they should have been among the most actively traded stocks in any month during the analysis. However, few stocks were selected randomly in order to evaluate whether they could be scored as efficient despite the fact that they are not best quality stocks. In the end, 78 stocks were selected in the sample.

The output oriented models were chosen as the most appropriate for the purpose of the analysis. Expected monthly return was taken as an output that should be maximised and return variance, Value at Risk (95%) and beta coefficient were taken as inputs, based on which relative efficiency is to be estimated.

Collecting data started with calculating monthly returns (with dividends) as a natural logarithm of the first to last price ratio of each month in the period of 1.1.2004 – 30.6.2012. Dividends were included in the return with the assumption that the ex-dividend date was on the last day in the month of the ex-dividend date.[6]

The expected return of a stock for a certain month was calculated as a last 52-month return average. Therefore, the first expected return was calculated for May 2008 as an average of monthly returns from 1.1.2004 – 30.4.2008. However, expected return for stocks that started listing after 1.1.2004, was calculated as an average monthly return for the period since their issue.

Value at Risk (VaR) for a certain stock was calculated for given opening prices at the beginning of the month, expected monthly return and at 95% level of confidence. Lastly, beta coefficient was calculated for each month, using formula:

$$\beta = \frac{\sigma_{j,M}}{\sigma_M^2}, \tag{3}$$

where $\sigma_{j,M}$ is a covariance between stock's and market return and σ_M^2 is the variance of market return in the past 52 months, and market return is the return of the CROBEX. Since the chosen DEA models do not work with negative values, all values were re-scaled, following the procedure presented in (Lopes et al, 2008).

[5] It is more appropriate to think of stocks as the assessment units, rather than the decision making units (DMU).
[6] Dividend data was provided from the Central Depository and Clearing Company, Inc.

Using re-scaled values of 1 output and 3 inputs, DEAP software calculated relative efficiency scores of stocks with BCC and CCR model, for each month in the period of May 2008 - July 2012. Moreover, DEA software was run 3 times for each model, once including all inputs in the analysis, and after that excluding either beta or VaR as an input in order to estimate the impact of different input values on the final result.

The basic idea of this investment strategy is that DEA can find efficient stocks and that the portfolio of these stocks would have greater return than the market portfolio. Given the calculated expected values for certain month, all stocks that were scored as relatively efficient are selected in the portfolio with equal proportions. For example, if DEA model scores certain stocks as efficient based on the expected values for May 2008, that same stocks is included in the portfolio for May 2008. In addition, if the stock that is already in the portfolio is not scored as efficient given the input values for the next month, it is excluded from the portfolio in the next month. In general, the monthly DEA portfolio through time is consisted only of efficient stocks for current month. This idea was followed for each month from May 2008 until June 2012, and portfolios could be restructured each month in the period of analysis.

Initially, 6 portfolios were created following the results of both CCR and BCC models which processed data with different inputs. To evaluate the performance of the DEA-portfolios, their monthly returns were compared to the return of the market portfolio, i.e. the monthly return of the CROBEX.

4. RESULTS

When results were conducted, it was noticed that the BCC model estimates more stocks as efficient, and that stocks estimated as efficient with the CCR model were always in a BCC portfolio as well[7]. Moreover, more or less the same stocks from the sample were always selected as efficient, so despite the fact that it was possible to restructure a portfolio each month, in certain months there was no restructuring. The mode and the average of number of stocks in the portfolios differed and this is presented in the table 1.

By comparing the average return of the portfolios to market return for each month, the number of months when DEA-portfolios had greater return is obtained. The results are presented in table 2.

[7] If a DMU is CCR efficient, it is also BCC efficient (see Ahn et al. (1998)).

Table 1: The descriptive statistics of number of stocks in each portfolio.

	MODE	AVERAGE	MAX	MIN
CRS portfolio	8	8,40	6	11
VRS portfolio	9	11,6	9	15
CRS portfolio without beta	4	4,34	4	5
VRS portfolio without beta	6	7,36	5	13
CRS portfolio without VaR	4	3,5	5	2
VRS portfolio without VaR	6	5,7	8	3

Source: authors.

Table 2: The final score of DEA portfolios compared to market return.

CRS portfolio vs. market portfolio	22:28
VRS portfolio vs. market portfolio	21:29
CRS portfolio without beta vs. market portfolio	28:22
VRS portfolio without beta vs. market portfolio	22:28
CRS portfolio without VaR vs. market portfolio	25:25
VRS portfolio without VaR vs. market portfolio	23:27

Source: authors.

Obviously, just one DEA portfolio was more successful than the market portfolio for the period of analysis. However, all DEA portfolios, except one portfolio, had greater return than the market portfolio, and, what is even more significant, all DEA portfolios that included VaR as input had a greater average return than other portfolios in the period of May 2008 – June 2012. The results are shown in table 3.

Table 3: The average return of portfolios.

	Average return	Return variance
Return of the market portfolio	-1,44%	0,99%
CRS portfolio	-0,801%	0,33%
VRS portfolio	-0,851%	0,3%
CRS portfolio without beta	-0,16%	0,59%
VRS portfolio without beta	-0,560%	0,56%
CRS portfolio without VaR	-1,397%	0,49%
VRS portfolio without VaR	-1,750%	0,51%

Source: authors

It is obvious that DEA-portfolio based on a CCR model's results had significantly the greatest return for the period concerned.

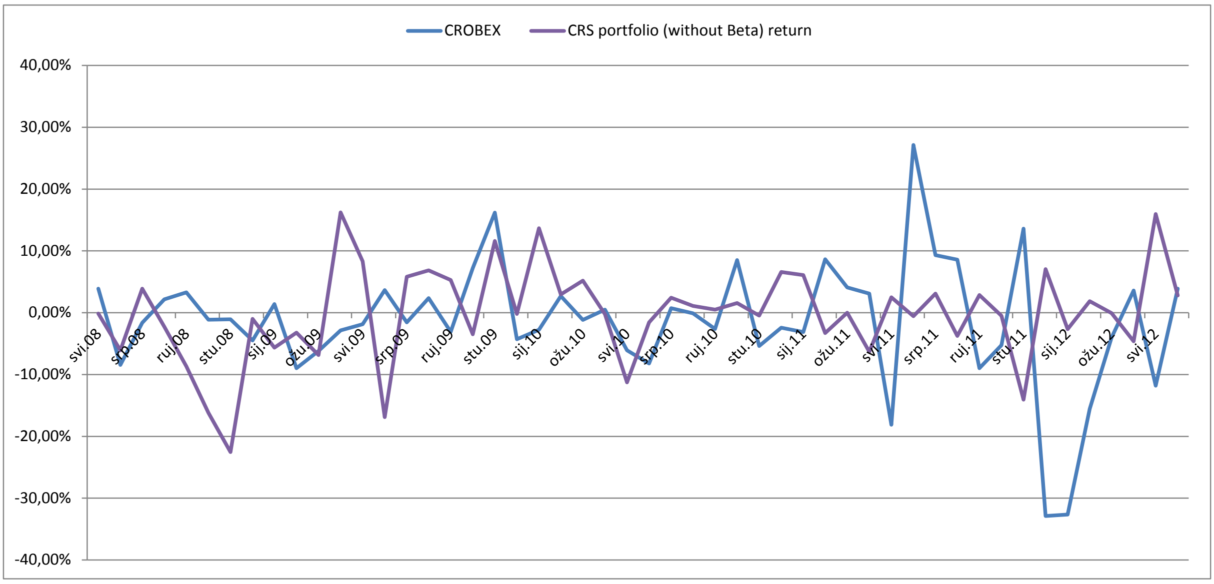

Figure 3: Monthly returns of CROBEX and CRS portfolio.

The cumulative return of the CRS portfolio without beta as a risk measure, which was the portfolio with the best performance, is -8,15% (Figure 4).

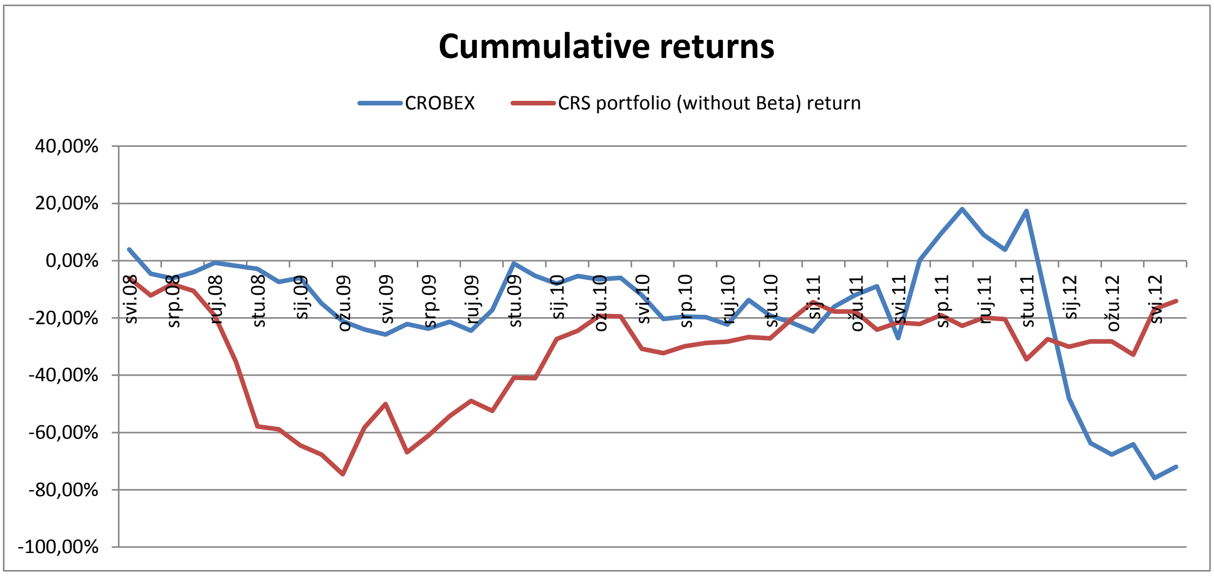

Figure 4: The cumulative monthly returns of CROBEX and CRS portfolio (without beta).

5. CONCLUSION

Following some acknowledged works, the DEA methodology was applied to select some stocks from Zagreb Stock Exchange into a portfolio that might be superior to market portfolio. Although the results were not as it was hoped they would be, all but one DEA portfolio showed to have greater average monthly return than the market. In addition, all DEA portfolios that include Value at Risk as input have better average monthly return than the others. Moreover, in comparison to market portfolio, one of those portfolios that had considered VaR actually had superior return in more than 50% of the months included in the analysis. Therefore, this investment strategy showed to be rather efficient for the analyzed period on the Croatian stock market. This also led to a conclusion that including VaR as an input for measuring the relative efficiency of stocks using the DEA methodology was worthwhile in this case, although, to our knowledge, it wasn't observed as an input variable in any of previous works on this specific topic. Although our portfolios showed to have greater return than the market, in general, this strategy did not show as successful as it had shown in other papers that tested the strategy on other capital markets, and the reasons for this could be numerous.

Nevertheless, this research needs some additional work. Appling the same analysis on data in some better times on the domestic capital market would possibly give profitable results. Changing and adding more inputs and outputs, rearranging and increasing the sample, as well as introducing multiplier bounds and other additional restrictions in model are some of considerations for future research. Moreover, in this research the transaction costs were ignored, so it would be necessary to implement those costs in model, and consequently, find an investment strategy which would include longer investment periods that would reduce those costs. Finding efficient stocks in a longer time period could be also done by using other DEA models.

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