

A COMPARISON OF BASIC AND EXTENDED MARKOWITZ MODEL ON CROATIAN CAPITAL MARKET

Bruna Škarica

Faculty of Economics and Business – Zagreb, Croatia
Trg J. F. Kennedyja 6, 10000 Zagreb, Croatia
E-mail: skarica.bruna@gmail.com

Zrinka Lukač

Faculty of Economics and Business - Zagreb
Trg J. F. Kennedyja 6, 10000 Zagreb, Croatia
E-mail: zlukac@efzg.hr

Abstract

Markowitz' mean - variance model for portfolio selection, first introduced in H.M. Markowitz' 1952 article, is one of the best known models in finance. However, the Markowitz model is based on many assumptions about financial markets and investors, which do not coincide with the real world. One of these assumptions is that there are no taxes or transaction costs, when in reality all financial products are subject to both taxes and transaction costs – such as brokerage fees. In this paper, we consider an extension of the standard portfolio problem which includes transaction costs that arise when constructing an investment portfolio. Finally, we compare both the extension of the Markowitz' model, including transaction costs, and the basic model on the example of the Croatian capital market.

Key words: *portfolio optimization, Markowitz model, expected return and risk, transaction costs*

1. INTRODUCTION

1.1. Modern portfolio theory

Constructing a portfolio of investments is one of the most significant financial decisions facing both individual and institutional investors. Modern portfolio theory has gained widespread acceptance as a practical tool for portfolio construction. It has been used by investors to choose a portfolio which, given the level of investors' risk aversion, offers them an acceptable balance between risk and return. Harry M. Markowitz is often called the father of the modern portfolio theory – his original book and article (1952) on the subject clearly depicted, for the first time, modern portfolio theory. His book *Portfolio Selection: Efficient Diversification of Investments* (1959) was filled with insights and

suggestions that anticipated many of the subsequent developments in the field (Elton and Gruber, 1997).

Markowitz assumes that investors are rational and risk averse, meaning that they will always expect greater returns when investing in riskier assets. One of the main concepts of the classical Markowitz model is that a rational investor will always choose a portfolio lying on the efficient frontier, where the efficient portfolio is defined as a portfolio with the greatest expected return among those portfolios that are equally risky, or the least risky portfolio among portfolios offering the same returns. Expected future returns are based on historical data and risk is measured by the variance of those historical returns.

The important contribution of the theory Markowitz developed was that assets cannot be selected only on their individual characteristics – the covariance among individual investments is very important as it allows investors to construct a diversified portfolio that offers same expected returns and less risk than a portfolio constructed while ignoring the interactions between securities in the portfolio.

Markowitz's ideas were widely accepted in financial theory and in 1990 he was awarded Nobel prize for his contribution to the field of financial economics. However, the classical Markowitz model has often been criticized because of its assumptions about asset-holding behaviour of individuals and financial markets. It can be said that even though the model is the most important contribution to modern portfolio theory, its use in practice has often been questioned (Michaud 1989; Konno and Yamazaki 1991; Black and Litterman 1992).

G.A. Pogue (1970) listed the following assumptions about investor preferences and prior subjective beliefs regarding security returns made in the classical Markowitz model:

- The investor attempts to maximize his expected utility of terminal wealth. Here terminal wealth is considered to be identical to the market value of the investor's portfolio at the end of his planning horizon.
- The investors planning horizon consists of a single period. The investment strategy involves selection of an optimal portfolio at the beginning of the period which will be held unchanged to the terminal date.
- The investor is assumed to be risk averse. The investor's marginal utility of wealth is assumed to be everywhere non-negative and a decreasing function of wealth.

Furthermore, it is assumed that the return distribution of assets is normal, the markets are efficient and hence there are no taxes nor transaction costs. However, the inclusion of transaction costs is an essential element of any realistic portfolio optimization. Pogue (1970) extends the classical Markowitz model so that it includes transaction costs, taxes as well as short sale.

1.2. Transaction costs

Transaction costs are often considered as comprising of two parts: direct or explicit costs, such as brokerage fee and implicit costs. Implicit costs represent indirect trading costs, the major one being the price impact of the trade. Unlike explicit costs where there are typically visible accounting charges, there is no such reporting of implicit costs. As a result, there is considerable disagreement over how best to measure implicit trading costs (Domowitz, Glen and Madhavan, 2002). The importance of both implicit and direct transaction costs within mean – variance portfolio management has been examined, for example, in the work of Mitchell and Braun (2002) and Olsson (2005). Since ignoring transaction costs can lead to suboptimal solutions of the portfolio optimization problem, it is interesting to consider an extension of the classical Markowitz model in which transaction costs are incurred when rebalancing a portfolio.

2. CLASSICAL MARKOWITZ MODEL

In the classic Markowitz model, the investor considers the mean return and the variance of the returns of a portfolio in the selection of the optimal portfolio. If the distribution of the returns of the portfolio is normal, it is possible to obtain the mean and the variance and use these two parameters as measures of the expected return and the risk of a portfolio. However, one of the main criticism of the Markowitz model is the assumption of the normal distribution of the returns – empirical research have shown that the distributions of the returns of most assets are asimetric (Fama 1965).

In the standard formulation of the portfolio problem, a portfolio with n number of securities is considered. Each of the securities in the portfolio has its expected return $E(R_i)$, $i \in \{1, 2, \dots, N\}$ and variance of historical returns – which is used to measure the risk of the securities. Furthermore, since π_i denotes the weight of the i^{th} security in the portfolio it can be concluded that $\sum_{i=1}^N \pi_i = 1$. Also, if short selling is not allowed, the weights, π_i are restricted to be nonnegative, $\pi_i \geq 0$. Since the Zagreb stock exchange, on which we will perform our analsis is yet to allow short selling, we will analyze the classic and the extended Markowtiz model without including the possibility of short selling.

The expected return of a security is based on historical data, and in this analysis we use monthly closing prices for each security to compute monthly returns. The expected future returns are the mean monthly returns of each security. The expected return of a portfolio can then be obtained as the weighted mean of the expected return of each security in the portfolio. The variance of a portfolio is not a weighted mean of variances of historical returns of the securities, and to obtain it, it is necessary

to compute the covariance matrix. The covariance matrix S has asset variances along the main diagonal and all other elements are covariances between each pair of assets.

$$S = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2N} \\ \vdots & & & \vdots \\ \sigma_{N1} & \sigma_{N2} & \dots & \sigma_{NN} \end{bmatrix} \quad (1)$$

Finally, one standard formulation of the portfolio problem which generates a set of efficient portfolios – efficient frontier – is to maximize the expected return of a portfolio specifying the maximum portfolio risk – measured by its variance.

$$\max E(R_\pi) = \pi^T \cdot E(R) \quad (2)$$

subject to:

$$\sigma_\pi^2 = \pi^T \cdot S \cdot \pi \leq s^2 \quad (3)$$

$$e^T \pi = 1 \quad (4)$$

$$\pi_i \geq 0 \text{ za } i \in \{1, 2, \dots, N\} \quad (5)$$

$E(R)$ is a column vector of expected returns, S is a covariance matrix and e denotes a vector of all ones.

3. EXTENDED MARKOWITZ MODEL WITH TRANSACTION COSTS

What we consider next is an extension of the basic portfolio optimization problem in which transaction costs must be paid when rebalancing a portfolio. That is, we consider an investor who already possesses a portfolio and makes transactions to change it into a new, efficient portfolio. The transaction costs which we take into consideration are the direct proportional transaction costs, such as brokerage fees.

We can assume that u_i and v_i and the amounts bought and sold of the i^{th} security. If the amount invested in the initial portfolio is $\bar{\pi}$, the amount invested in the new portfolio (after rebalancing) is:

$$\pi = \bar{\pi} + u - v \quad (6)$$

If the total amount invested in a portfolio is, for example, 1 HRK, the amount invested in each security is equal to its weight in the portfolio.

As we assume proportional transaction costs - c_{B_i} and c_{S_i} – costs of buying and selling of one unit of i^{th} security, we can denote π_0 as the total amount spent to cover the transaction cost and define it as:

$$\pi_0 = c_B^T u + c_S^T v \quad (7)$$

As previously stated, π denotes the matrix which elements are the weights of the each security in the portfolio; c_B and c_S are the matrices showing the costs of buying and selling of one unit of each security in the portfolio. Matrices u and v show the amount of each security bought and sold, respectively.

This means that the total amount invested in the new, efficient portfolio will be $1 - \pi_0$ or

$$e^T \pi = 1 - c_B^T u - c_S^T v \quad (8)$$

To obtain the additional constraint in the expanded Markowitz model which includes direct transaction costs, we exploit the fact that $e^T \bar{\pi} = 1$ and that $\pi = \bar{\pi} + u - v$ and we multiply the latter expression by the matrix e (consisting of all ones), which gives:

$$e^T \pi = e^T \bar{\pi} + e^T u - e^T v \quad (9)$$

$$1 - c_B^T u - c_S^T v = 1 + e^T u - e^T v \quad (10)$$

and we finally obtain the constraint:

$$(c_B + e)^T u + (c_S - e)^T v = 0. \quad (11)$$

In the extended Markowitz model we also maximize the expected return of the resulting portfolio subject to meeting a standard deviation that is not greater than the greatest acceptable level of risk, denoted s . The model is:

$$\max E(R_\pi) = \pi^T \cdot E(R) \quad (12)$$

subject to

$$\sigma_\pi^2 = \pi^T \cdot S \cdot \pi \leq s^2 \quad (13)$$

$$\pi - u + v = \bar{\pi} \quad (14)$$

$$(c_B + e)^T u + (c_S - e)^T v = 0 \quad (15)$$

$$u, v, \pi \geq 0 \quad (16)$$

4. THE CLASSIC AND THE EXTENDED MARKOWITZ MODEL ON THE EXAMPLE OF CROATIAN STOCK EXCHANGE

4.1. Methods and data used

In our analysis, we have used six securities listed on the Zagreb Stock Exchange, 5 of which are included in the official index of the Zagreb Stock Exchange – CROBEX. CROBEX is the official Zagreb Stock Exchange share index. The Zagreb Stock Exchange started publishing it on September 1, 1997. The base date of the index is July 1, 1997 and the base value is set on 1000. Stocks which were traded on more than 90% of the available trading days in the previous six month period are qualified for CROBEX selection. In the selection process stocks are ranked by the following two criteria:

- free float market capitalization
- orderbook turnover in preceding 6 month period.

The free float market capitalization in percent and the turnover in percent are each given a weighting of 50% and so called market share is calculated. Stocks are ranked by the market share and the top 25 stocks are selected to be included into the CROBEX index. We chose the following six securities: PODR-R-A (Podravka d.d.), KRAS-R-A (Kraš d.d.), KOEI-R-A (Končar - elektroindustrija d.d.), ATPL-R-A (Atlantska plovidba d.d.), LEDO-R-A (Ledo d.d.) and PBZ-R-A (Privredna banka Zagreb - dioničko društvo); the latter is no longer included in the index. The securities were chosen because it was possible to obtain their monthly returns for the past 8 years – using the formula for logarithmic returns, we computed a series of 104 monthly returns for each of the 6 securities (30.01.2004. – 31.08.2012.).

The efficient frontier in both the classical and the extended model is found using the Microsoft Excel Solver application – the minimum standard deviation was varied, and we were given the highest expected returns for the lowest level of risk (measured by standard deviation) and the weights of each security in the efficient portfolio. In the expanded model, Solver found solutions to the optimization problem by changing weights and the amounts spent on buying and selling each security u_i and v_i). Even though this extended model allows transaction costs to be different for each security as well as for buying and selling securities, in this analysis we used the same proportional transaction cost for each security and also the costs of buying were the same as the costs of selling a security – 1% of the amount bought or sold. Also, although this model allows for a single security to be bought and sold when rebalancing a portfolio to obtain a new, efficient one, it is more rational to assume that an investor will at a certain point sell some securities and acquire others – which was the approach we

used (in the rebalancing of the portfolio, the amount of 3 securities in the portfolio was reduced and the other 3 increased).

4.2. The results and the interpretation

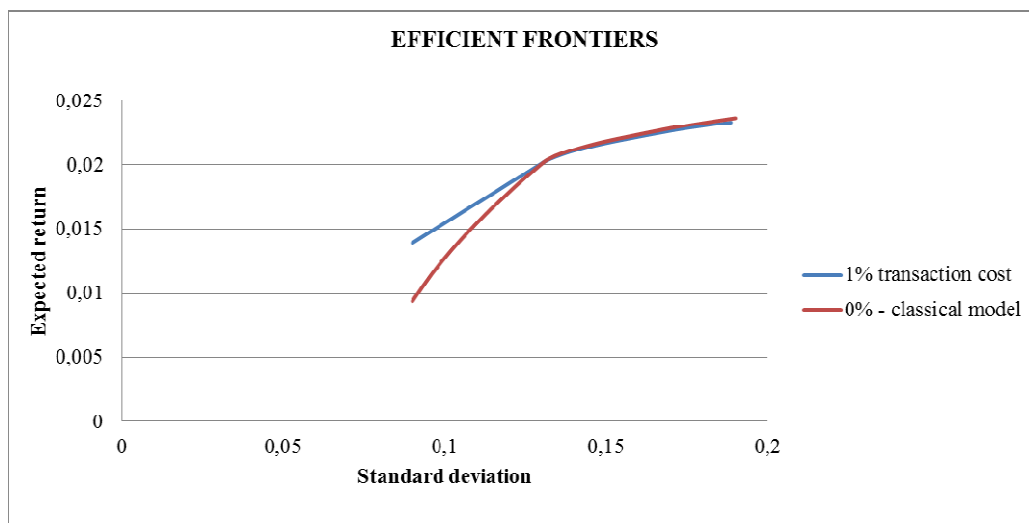


Figure 1: Efficient frontiers on the Zagreb Stock exchange retrieved by applying the classical and the extended Markowitz model on the ZSE

It was our initial hypothesis that the inclusion of transaction costs when rebalancing a portfolio in the Markowitz model will reduce the range of investment choice. But, it is visible here that the inclusion of transaction costs led to portfolios equally risky but more profitable than the portfolios on the original (0% transaction costs) efficient frontier – this is the case for the lower levels of risk as measured by standard deviation, whereas for the higher standard deviations – or the level of risk accepted by the investors, our initial hypothesis proves to be correct.

Considering that the brokerage fees are in average usually even lower than 1% of the value of the trade made, we have analyzed the same data using an assumption of 0,30% transaction cost for all transactions made. It is interesting to see that the resulting efficient frontier is almost the exact match of the efficient frontier that was a result of the model using 1% transaction cost.

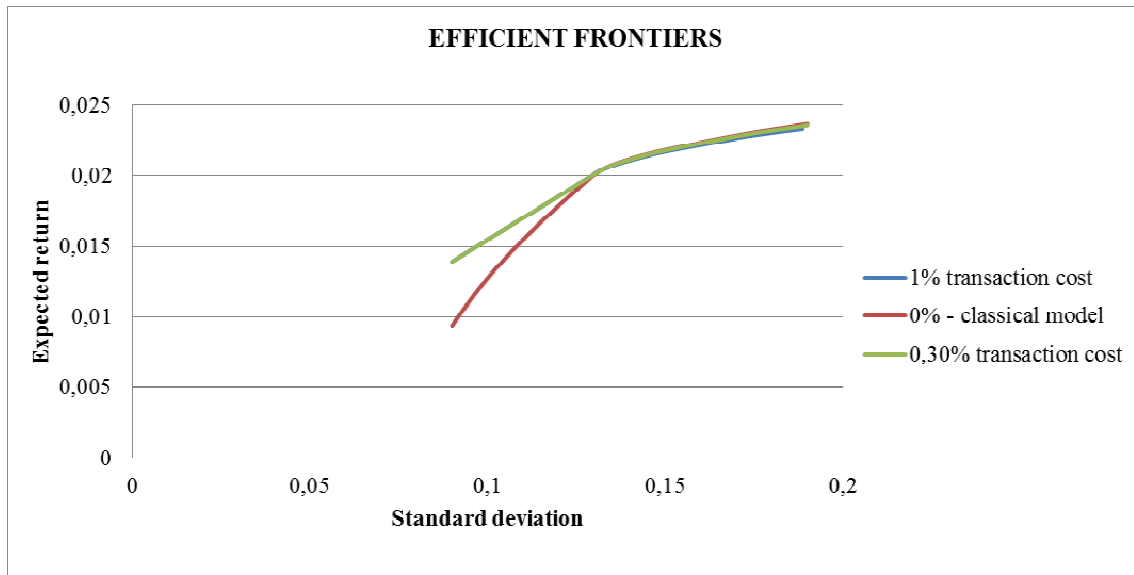


Figure 2: Efficient frontiers on the Zagreb Stock exchange retrieved by applying the classical and the extended Markowitz model on the ZSE – including 0,30% transaction cost

5. CONCLUSION

In this paper we have constructed a model for portfolio selection which includes transaction costs covered when rebalancing a portfolio. After applying our newly – constructed model, as well as the basic Markowitz model, on the Zagreb Stock Exchange, we were faced with unexpected results. Although we expected that the inclusion of transaction costs would decrease the returns for every level of risk for each portfolio on the efficient frontier, the opposite happened. The explanation for this can be found in the data we have worked with and the general characteristics of the Zagreb Stock Exchange, especially in the past couple of years.

The expected returns for all six securities chosen to construct the portfolio were positive, but very small, with a large variance – and the same goes for the portfolio – investors are expected to take great risks for relatively small gains. If we look at the historical data, the low expected returns and great variations from the mean historical return can be attributed to Croatian financial markets' great fall in the prices of all assets during 2008 and 2009 especially, during the global financial crisis. Even as we tried to apply the extended model to historical data excluding the years in which the prices have fallen the most, their average were still either very small and positive or, dominantly, negative. Also, it was not possible to work with the larger sample of daily returns because it was hard to find periods in which all the securities were traded with on the same days. Also, it is important to take into account the character of the Zagreb Stock Exchange – compared with big markets, which are usually the subject of this kind of financial research, Croatian capital market is small and still developing. Since

financial models are used to model markets when they are stable, and the application of Markowitz model in practice has often been questioned, perhaps the counter – intuitive portfolios on the efficient frontier can serve as indicators of instability on the financial markets. Finally, since the assumption of normality of the distribution of the assets return is not satisfied, further work should include a Markowitz model without the aforementioned assumption, as well as the inclusion of the implicit transaction costs in the extended model.

REFERENCES

- Aljinović, Z., Marasović, B., Šego, B. (2011), *Financijsko modeliranje*, Ekonomski fakultet u Splitu, Split
- Black, F., Litterman, R. (1992), “Global Portfolio Optimization”, *Financial Analysts Journal*, Vol. 48, pp. 28 – 43.
- Domowitz, I., Glen, J., Madhavan, A. (2001), “Liquidity, Volatility and Equity Trading Costs Across Countries and Over Time”, *International Finance*, Vol. 4, pp. 221 – 255.
- Elton, E. J., Gruber, M. J. (1997), “Modern Portfolio Theory, 1950 to date” *Journal of Banking & Finance*, 21, pp. 1743 – 1759.
- Fama, E. F. (1965), “The Behavior of Stock – Market Prices”, *The Journal of Business*, Vol. 38, pp. 34-105.
- Konno, H., Yamazaki, H. (1991), “Mean-Absolute Deviation Portfolio Optimization Model and Its Applications to Tokyo Stock Market” *Management Science*, vol. 37(5), p. 519 - 531.
- Markowitz, H. (1959), *Portfolio Selection: Efficient Diversification of Investments*, Wiley, New York
- Markowitz, H. (1952), “Portfolio selection” *Journal of Finance*, 7, pp. 77 - 91.
- Michaud, R. (1989), “The Markowitz Optimization Enigma: Is 'Optimized' Optimal?”, *Financial Analysts Journal*, Vol. 45, pp. 31 – 42.
- Mitchell, J. E., Braun, S. (2002), “Rebalancing an Investment Portfolio in the Presence of Transaction Costs”, Rensselaer Poltechnic Institute, 2002.
- Pogue, G. A., (1970), “An Extension of the Markowitz Portfolio Selection Model to Include Variable Transactions' Costs, Short Sales, Leverage Policies and Taxes” *Journal of Finance*, vol. 25(5), pp. 1005 - 1027.
- Olsson, R. (2005), “Portfolio management under transaction costs: Model development and Swedish evidence”, Umea University
- Orsag, S. (2003), *Vrijednosni papiri*, Revicon, Sarajevo
- Šošić I. (2006), *Primijenjena statistika*, Školska knjiga, Zagreb
- www.zse.hr [Accessed 30/08/2012]