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MEASUREMENT OF FILTERS' EFFICIENCIES AND APPLICATION OF NNSFDI METHOD

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The study aims to propose the neural network filter based on NNSFDI method as an alternative filter in economics; namely, Baxter-King, Hodrick-Presscott, Christiano and Fitzgerald and Kalman filters. In this paper, it was used two different data which consist of the annual unemployment rates for 1923 – 2008 periods and the monthly inflation rates for 1964:02 – 2009:07 periods. The performance of the new method proposed and the main stream filters were, in particular, evaluated based on the annual and monthly data. The empirical findings suggest that the newly proposed NNFSDI model provedid better forecast results compared to Kalman, HP, BK and CF filters for different data sets when evaluated in the light of different error criteria such as MSE, RMSE and MAPE

Keywords:

Baxter-King filter

Hodrick-Prescott filter

Christiano and Fitzgerald filter

Kalman filter

NNSFDI filter

Adaptive threshold algorithm

JEL:
C5
C165
C82

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I. INTRODUCTION

Filters cover a wide range of the research area of economics. Several filters used in econometric analysis allow the building of multivariate economic models, and also allow the model parameters of the data to be adjusted: such as the filters with unobserved components (UC). Other filters are mechanical transformations of the original data and some of prominent examples of these filters are The Baxter-King filter (BK filter, Baxter and King, 1999), Hodrick-Prescott filter (HP Filter, Hodrick and Prescott, 1997), Christiano and Fitzgerald filter (CF Filter, Christiano and Fitzgerald, 2003) and Kalman filter (1960). These filters do not show the same specifications. The HP filter (Hodrick and Prescott, 1997), is a two-sided moving average which turns into a one-sided filter at the beginning and at the end of the observation period. The Rotemberg filter, the Christiano-Fitzgerald filter and the BK filter are univariate filters.

In this study, the Kalman filter, the HP filter, the BK filter and the CF filter that are widely used in related literature will be investigated. As an alternative, the study will suggest the Neural Network Filter Sensor Fault Detection and Identification (NNSFDI) model and its effects on the neural network.

II. THEORY

Filters can be assigned into four topics: mainly, HP filters, frequency filters (under subtitle BK and CF), state space filters (Kalman) and NNFSDI.

A. Hodrick-Prescott filter

The univariate HP (1980) filter was introduced to estimate trends from data and now it is widely used to estimate and predict business cycles and trends especially in time series.

The HP (1980) filter decomposes a time series into two components; the first is a long-term trend component and the second, a stationary cycle. The HP cyclical component x_j^{HP} is seen as the difference between the original s_j data and the trend component.

Filter;

$$\min_{\{s_j\}} \sum_{j=0}^{N-1} \left[\left(y_j - s_j\right)^2 + \lambda \left(s_{j+1} - 2s_j + s_{j-1}\right)^2 \right] \tag{1}$$

The penalty parameter λ controls the smoothness of the series as λ=∞, *t* approaches a linear trend. By Hodrick Prescott (1997), λ is proposed as 1600 for quarterly data and 100 for annual data. For the monthly series, λ is proposed as 14400. (Concerning the monthly data, the E-views use the default value at 14400, while the Dolado et al. (1993) reasoning would lead to λ =4800.) Apel et al. (1996)); Backus and Kehoe (1992), Giorno et al. (1995) or the European Central Bank (2000) use the value λ =100 for quarterly and annual data.

This minimizes the sum of the norm of the cyclical component $\left\| x_j^{HP} \right\| = \left\| y_j - s_j \right\|$ and the weighted norm of the variable component $\left\| \left(1-L\right)\left(1-L^{-1}\right) s_j \right\|$, where *L* is the lag operator $Lx_j \equiv x_{j-1}$. The amount of trend deficiency will be defined by the (λ>0) coefficient (Iacobucci and Noullez, 2005).

King and Rebello (1993) state that the HP filter can be written as $(1-L)^4 H(L)$. $|HP(w)|^2$ like the squared gain corresponding to the HP cyclical filter in which $HP(w)$ is the Fourier transform of $(1-L)^4 H(L)$ at *w* frequency. If the filter is applied to the level of series y_t, the spectrum of the cyclical component is

$$f_{y^c}(w) = |HP(w)|^2 \, |1-\mathbf{exp}(-iw)|^{-2} \, f_{\varepsilon}(w) \tag{2}$$

$(1-\mathbf{exp}(-iw))$ is the Fourier transform of *(1-L)* and $f_{\varepsilon}(w)$ is the spectrum of $f_{\varepsilon}(w)\ L\ f_{\varepsilon}(w)$. The stationary process $|1-\mathbf{exp}(-iw)|^{-2}$ is not defined for *w=0* $|1-\mathbf{exp}(-iw)|^{-2} f_{\varepsilon}(w)$ and is often named the pseudo-spectrum of y_t (Gourrieroux and Monfort; 1995, Guay and St-amant; 2005).

According to Iacabucci and Noulles (2005), quarterly data higher than approximately 10 years have disadvantages because of the wide band pass.

B. Frequency filter

The Baxter-King and Chisristiano-Fitzgerald filters will be inspected with regards to the frequency filter.

A. Baxter-King filter

Baxter and King (1999) used a finite moving average approximation of the ideal band-pass filter dependent upon Burns and Mitchells' (1946) Business cycle definition. The BK filter is designed to pass through a component, of series with fluctuations in a band of 32 quarters, because it removes higher and lower frequencies (Guay and St-amont, 2005).

The method proposed by Baxter and King relies on the use of a symmetric finite odd-order $M = 2K + 1$ moving average so that:

$$\begin{aligned} v_j &= \sum_{n=-K}^{K} g_n u_{j-n} \\ &= g_0 u_j + \sum_{n=1}^{K} h_n (u_{j-n} + u_{j+n}) \end{aligned} \tag{3}$$

The set of $\{h_j^{BK}\}$ is obtained by truncating the ideal filter coefficients at M under the constraint of the correct amplitude at $v = 0$, and $H(0) = 0$ for band-pass and high-pass filters and $H(0) = 1$ for low-pass filters. The BK filter coefficients have to solve the following optimization problem (Iacobucci and Noullez, 2005),

$$\left.\begin{array}{l} \min\limits_{\{h_n^{BK}\}_{n=-K,\ldots,K}} \int_{-(2\Delta t)^{-1}}^{(2\Delta t)^{-1}} d\nu \left| \sum_{n=-K}^{K} \left(h_n^{BK} - h_n^{ideal} \right) e^{-i2\pi n\nu\Delta t} \right|^2 \\ s.t. \qquad \sum_{n=-K}^{K} h_n^{BK} = \frac{H(0)}{\Delta t}. \end{array}\right\} \tag{4}$$

The solution of the constrained problem involves shifting all ideal coefficients by the same constant quantity

$$h_j^{BK} = h_j^{ideal} + \frac{H(0) - \Delta t \sum_{n=-K}^{K} h_n^{ideal}}{M\Delta t} \tag{5}$$

The frequency response of the BK filter with $K = 16$ selecting the band [2,8] year is shown. Since the length of the BK filter is always $M = 2K + 1$ and does not depend on N, Iacobucci and Noullez (2005) show how the N-length filters improve with growing values of N, while the BK filter remains unchanged.

B. The Christiano-Fitzgerald filter

The Christiano-Fitzgerald filter is a band pass filter that was built on the same principles as the Baxter and King (BK) filter. These filters formulate the de-trending and smoothing problems in the frequency domain. However, the granularity and finiteness of time series in real life do not allow perfect frequency filtering (Nilsson and Gyomai, 2007).

Christiano and Fitzgerald built a filter using two new ingredients: (i) Take into account the assumed spectral density of the original data; (ii) drop the conditions of stagnancy and symmetry on the filter coefficients. If the exact spectral density of the original data $U^{exact}(\nu)$ is known beforehand, the set of coefficients $\left\{ g_j \right\}$ is given by the solution of the optimization problem (Iacobucci and Noullez ,2005).

$$\min_{\{g_j\}} \int_{-(2\Delta t)^{-1}}^{(2\Delta t)^{-1}} d\nu \left| \sum_{\{j\}} \left(g_j - g_j^{ideal} \right) e^{-i2\pi j\nu\Delta t} \right|^2 \left| U^{exact}(\nu) \right|^2 \tag{6}$$

which is equivalent to the minimization

$$\left| v_j - v_j^{ideal} \right|^2 = \int_{-(2\Delta t)^{-1}}^{(2\Delta t)^{-1}} d\nu \left| H(\nu) - H^{ideal}(\nu) \right|^2 \left| U^{exact}(\nu) \right|^2 \tag{7}$$

of the discrepancy between the ideally filtered data and the effectively filtered ones. According to different types of optimization problems, which give rise to different filters, the set of indexes *{j}* can be constant symmetric $j = -K, \ldots, K$, constant asymmetric $j = -K, \ldots, K'$ or even a time-varying general one such as $j = -(n-j), \ldots, j-1$. If $\left| U^{exact}(\nu) \right|^2$ which is chosen as independent of frequency (white noise, referred to as IID case). The solution is simply found by truncating the ideal filter coefficients. If u^j has one unit root and $\left| U^{exact}(\nu) \right|^2$ goes like ν^{-2} for small frequencies but tends to a constant at large frequencies, it is shown that the optimal coefficients are again obtained by truncating the ideal ones. In the end, the Christiano-Fitzgerald filter is obtained by taking the power spectral density $\left|U^{exact}(v)\right|^2 \infty v^{-2}$ for all frequencies. The coefficients can be obtained explicitly and are given by truncating the ideal filters and then adjusting only g_{-K} and $g_{K'}$. The filtering operation is discussed in Iacobucci and Noullez (2005), where the g_j's are the ideal filter coefficients (2), $\hat{g} = g_0 / 2$, and we define $g_{\{0,j\}} = g_0 + g_1 + ... + g_j$ to simplify the notation.

Asymmetric C-F filter is time-varying. Moreover, at each time, the coefficients are asymmetric with respect to past and future data. The effect of asymmetry is that the CF filter response is complex, thus it has a nonzero phase. The CF filter frequency response and its standardized phase is (Iacobucci and Noullez, 2005):

$$\Phi_{CF} \wedge (v,t) = \frac{\Phi_{CF}(v,t)}{2\pi v} = \frac{1}{2\pi v} \arctan\left(\frac{\Im H_{CF}(v,t)}{\Re H_{CF}(v,t)}\right) \tag{8}$$

The Christiano and Fitzgerald (2003) band-pass filter may produce filtered components for all variables that are covariance stationary. This is of importance in order to avoid spurious correlations in the sense of Granger and Newbold (1974).

If the study compares Christiano and Fitzgerald (2003) and Hodrick-Prescott(1997) filters, it can be shown that they separate sharpness and covariance stagnancies.

C. Kalman filter

Before the Kalman (1960), Sewerling had developed an algorithm in 1958. Kalman filter estimation is a linear Gaussian optimal filter. Unscented Kalman filter (Julier and Uhlmann, 1997), Central Difference Kalman Filter (Norgaard et al., 2000) and Ensemble Kalman Filter (Burgers et al., 1998) are some of the studies which have interesting results for nonlinear applications.

The Kalman filter analysis has the main assumption of known covariance matrix and state variables. Thus, the main aim is to estimate transfer functions of two norms from the system noise to the estimation error with a minimum error variance. This was named as H_2 filter by Jwo and Pai (2004) and the study of the process model and measurement model discrete-time form are represented as

$$y_t = b_t + N_t \alpha_t + \varepsilon_t \tag{9}$$

$$\alpha_{t+1} = k_t + T_t \alpha_t + v_t \tag{10}$$

α_t is an *mx1* unobserved state variables, α_t, N_t, k_t and T_t are conformable vectors and matrices, ε_t and v_t zero meant vectors that have Gaussion disturbances. ε_t and v_t are assumed to be serially independent disturbance vectors with contemporaneous variance structure:

$$\Omega_t = \mathbf{var}\begin{bmatrix} \varepsilon_t \\ v_t \end{bmatrix} = \begin{bmatrix} H_t & O_t \\ O_t' & Q_t \end{bmatrix} \quad (11)$$

In the matrix above i, is a symmetric variance matrix with dimensions of nxn, Q_t is an *mxm* symmetric variance matrix, O_t is an n*xm* matrix of covariance system matrices and vectors $\Xi_t \equiv \{b_t, \alpha_t, N_t, T_t, H_t, Q_t, O_t\}$ based on observable explanatory variables x_t and unobservable parameters θ the conditional distribution of the state vector α_t given information available at time *(see detailed information Harvey (1989), Hamilton (1994a; 1994b), Koopman, Shephard and Doornik (1999)).*

The mean and variance matrix of the conditional distribution is,

$$\begin{aligned} \alpha_{t/s} &= E_s(\alpha_t) \\ P_{t/s} &= E_s\left[(\alpha_t - k_{t/s})(\alpha_t - k_{t/s})'\right] \end{aligned} \quad (12)$$

which is obtained by setting s=t-1, the one-step ahead mean $a_{t/t-1}$ and one-step ahead variance $P_{t/t-1}$ of the states α_t. With Gaussian error assumption $a_{t/t-1}$ is also minimum mean square error estimator of α_t. If the normality assumption is dropped, MSE of $a_{t/t-1}$ is calculated as $P_{t/t-1}$. $a_{t/t-1}$ which is the minimum mean square linear estimator of α_t; when it was given the one-step state conditional mean formed the (linear) minimum MSE one step ahead estimate y_t

$$\tilde{y}_t = y_{t/t-1} \equiv E_{t-1}(y_t) = E(y_t \mid \alpha_{t/t-1}) = b_t + N_{\alpha_{t/t-1}} \quad (13)$$

$\tilde{\varepsilon}_t = \varepsilon_{t/t-1} \equiv y_t - \tilde{y}_{t/t-1}$ is the one-step ahead prediction error and

$$F_t = F_{t/t-1} = \mathbf{var}(\varepsilon_{t/t-1}) = N_t P_{t/t-1} N_t + H_t \quad (14)$$

is the prediction error variance (Engle and Watson;1987 Harvey; 1987: 1989).

Kalman filter can be used for computing one-step ahead estimates of the state and the associated mean square error matrix, $\{a_{t/t-1}, P_{t/t-1}\}$, the contemporaneous of filtered state mean and variance, $\{a_t, P_t\}$, and the one-step ahead prediction, prediction error, and $\{y_{t/t-1}, \varepsilon_{t/t-1}, F_{t/t-1}\}$ prediction error variance $E[.]$ represent expectation, and superscript 't' denotes matrix transpose, Q_k is the process noise covariance matrix, R_k is the measurement noise covariance matrix, and $N_k = v^{F\Delta t}$ is the state transition matrix (Engle and Watson;1987 Harvey; 1987: 1989).

Kalman filter technique's popularity is because of the formation of the confidence intervals that surveys the ambiguity of the estimations of unobserved variables.

D. Neural network filter

Neural Networks (NNs) are the biggest challengers to conventional time series especially time series forecasting methods. A variety of NN modelling are available, multilayer perceptrons (MLP) with backproapagation learning are the most employed NNs in time series studies. Lai and Wong (2001) contributed to the non-linear time series modeling methodology by making use of single-layer neural network; further, modeling of NN models for estimation and prediction for time series have important contributions governed by Weigend, Rumelhart and Huberman (1991), Weigend and Gershenfeld (1993), Hutchinson, Lo and Poggio(1994) and Refenes, Burgess, and Bentz, (1997) which contributed to financial analysis and stock market returns estimation, to pattern recognition and optimization.

In this paper, we will use Neural network filter, Sensor Fault detection and identification (SFDI) techniques to improve the conventional filters used in econometric analysis. State estimator or observation Sensor Fault detection and identification (SFDI) techniques used dependent upon partly Kalman but in this condition, it is known that these models may perform poorly in presence of significant nonlinearities and uncertainties. As a alternative method, SFDI based on NN (NNSFDI) have been proposed and developed in recent years (Qi et al. 2007b).

NNFSDI does not require modeling and has a high potential to handle nonlinearities. NNSFDI method is discussed mostly in the engineering literature and was developed by Napolitano et al. (1998); Napolitano et al. (2000); Jakubek and Strasser (2002); Perhinschi et al. (2006) and Qi et al. (2007b) in recent decade.

E. NNFSDI model

Qi et al. (2007b) made use of the three-layer BP network structure, Neural network with three layer BP network are used to approximate the nonlinear continuous functions. In this paper, we will use the MLP structure. The feed-forward neural network known as the MLP is an important class of neural networks (Coakley and Brown, 2000); MLP consists of a set of sensory units that constitute the input layer, one or more hidden layers and an output layer. Each neuron is included in a nonlinear activation function.

The activation function *f* is taken to be logistic,

$$f(u) = 1/\left(1+\exp^{-u}\right) \tag{15}$$

where *u*= included local field of neuron *i*, where $u \in (-\infty, \infty)$, and $f(u) \in (0,1)$ or hyperbolic tangent functions with the form

$$f(u) = \tanh(au) = \frac{(e^{au} - e^{-au})}{(e^{au} + e^{-au})} \text{ where } u \in (-\infty, \infty) \text{ and } f(u) \in (0,1) \tag{16}$$

The back propagation algorithm is used in the same way as the learning algorithms in MLP trained by the gradient descent algorithm traditional approach. The weights are adjusted to minimize the squared difference between the outputs with an observation. The squared error is propagated backward through the NN and so it is used to adjust the weights and biases (Venables and Ripley, 1999; Bildirici and Ersin, 2009).

The training of neural networks in MLP is traditionally based on minimization of the cost function. Suppose a set of training samples are available, the problem can be characterized as choosing the weights of a given network such that the mean squared error (MSE) is minimized,

$$MSE = \frac{1}{N}\sum_{p=1}^{N}\left(d_p - y_p\right)^2 \quad (17)$$

where N denotes the total number of patterns contained in the training set; d_p and y_p represent the desired and actual output of the pth output neuron, respectively

I NNFSDI Parameters

Using online learning NN estimators, the SFDI problem can be approached by introducing a main set of 'm' NNs (MNNs) and a set of 'n' decentralized NNs (DNNs).

The MNN and DNN are computed as the following

$$d\hat{1}_{MNN-i}(k), d\hat{2}_{MNN-i}(k) \text{ and } d\hat{3}_{MNN-i}(k) \quad (18)$$

and

$$d\hat{1}_{MNN-i,DNN}(k), d\hat{2}_{MNN-i,DNN}(k) \text{ and } d\hat{3}_{MNN-i,DNN}(k) \quad (19)$$

The estimation error norm (MEEN) parameter which is gained by the MNN estimates and the actual measurements and the MNN and DNN estimation error norm (MDEEN) which is computed by MNN and DNN estimations are shown below (Qi et al., 2007a).

$$MEEN-i(k) = \sum_{g=1}^{n}\left\| dg_{MNN-i}(k) - d\hat{g}_{MNN-i}(k)\right\|_2 \quad (20)$$

where n is the number of DDNNs in MNN–i.

$$MDEEN-i(k) = \sum_{g=1}^{n}\left\| d\hat{g}_{MNN-i,DNN}(k) - d\hat{g}_{MNN-i}(k)\right\|_2 \quad (21)$$

where n is the number of DDNNs in MNN–i.

The third parameter is the DNN estimation error norm (DEEN) which is computed from MNN estimates and the actual measurements and the last parameter are the fault detection error summation (FDES) which is computed by MEEN and MDEEN, the μm and υm are weight coefficients for fault detection(Qi et al., 2007a)

$$DEEN_{MNN-i,dg}(k) = \left\| dg(k) - d\hat{g}_{MNN-i,DNN}(k)\right\|_2 \quad \text{where g=1,2,...,n} \quad (22)$$

is the number of DNNs in MNN-i.

$$FDES(k) = \sum_{m=1}^{n}\mu_m MEEN-m(k) + \sum_{m=1}^{n}\upsilon_m MDEEN-m(k) \quad (23)$$

where n is the number of MEEN in MDEEN and

$$\sum_{m=1}^{n}\mu_m = 1 \quad \sum_{m=1}^{n}\upsilon_m = 1 \quad (24)$$

The increase in unemployment rate leads to a large value of FDES between the measurements and the MNN estimates, in particular, there are large values of MEEN or MDEEN (Qi et al., 2007a).

$$FDES(k) \geq \min(MEEN - i_{threshold}, MDEEN - i_{h-threshold}). \tag{25}$$

II Threshold Algorithm

The thresholds $MEEN - i_{threshold}$, and $MDEEN - i_{threshold}$ are based on the adaptive threshold algorithm (Qi et al., 2007a),

$$Thresh_{Adaptive} = aver(Thresh) + \alpha\, dev(Thresh) + \beta\, roc(Thresh) + bias(Thresh) \tag{26}$$

where aver (Thresh) is the average value of Thresh, dev (Thresh) is the standard deviation of Thresh, roc (Thresh) is the rate of change of Thresh, bias (Thresh) is the bias of Thresh which is computed based on the above equations, α is a deviation bound factor and β is a rate of change bound factor.

III. DATA AND ECONOMETRIC RESULTS

A. Data

This study is based on the analysis of two different annual and monthly data. The Turkish unemployment data used in this study for the period of 1923 – 2008 is obtained from The Central Bank of Republic Turkey (CBRT), EVDS Electronic Data Distribution System and Bulutay (1995). The data corresponding to 1923 – 1988 periods of the study is obtained from Bulutay (1995); whereas the 1989 – 2008 periods are gathered from the EVDS. The data sets show similar data generating processes and two sets seem to fit in terms analysis and the usage of two different data sets does not create any problems. E-views 7, Rats and Matlab programmes were used.

The monthly inflation data from 1964:02 - 2009:07 is obtained from CBRT.

B. Econometric results

Initially HP and Frequency filter results for both data sets are presented. Secondly, Kalman Filter and NNSFDI results are given. At this stage filter performances of monthly and annual data were continued with analyzing forecast results (*E-Views, Rats and Matlab software packages are used in obtaining the results*).

I. HP AND FREQUENCY FILTER RESULTS

Filter results for annual unemployment data between 1923 – 2008 periods are given below;

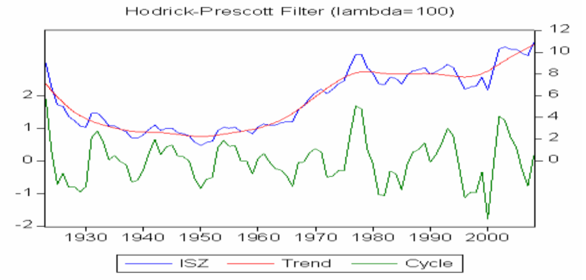

FIGURE 1.a

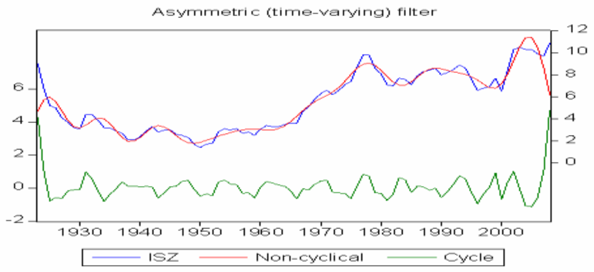

FIGURE 1.b

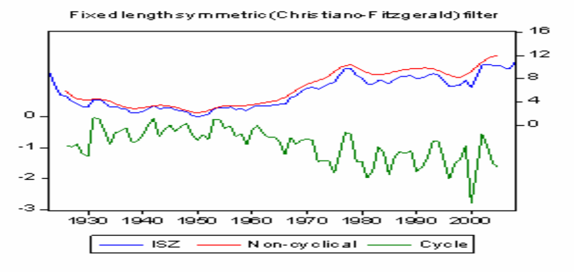

FIGURE 1.c

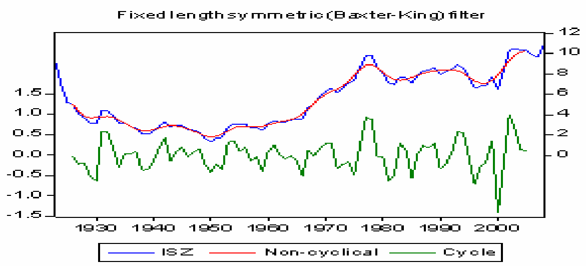

FIGURE 1.d

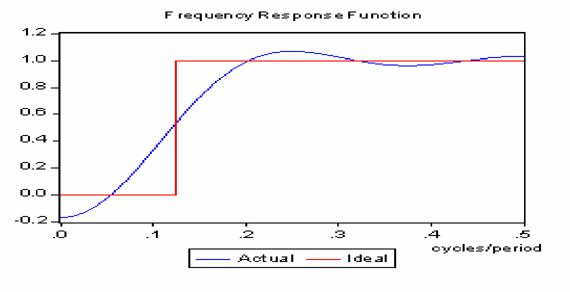

FIGURE 1.e

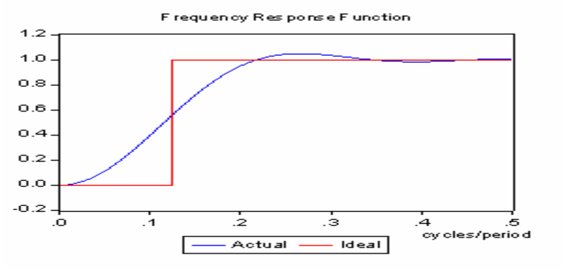

FIGURE 1.f

FIGURE 1. UNEMPLOYMENT FILTER RESULTS

Source : Author's calculation

The figures were obtained for 85 points of annual data and a pass-band of [2, 8] year. Figures 1 presents gain functions of the ideal filter, the Hodrick-Prescott filter, the Baxter-King filter, and the Christiano-Fitzgerald random-walk filter. The gain function for a specific filter determines the weight that it assigns to a given frequency when the raw series is filtered. In Figure 1.e, and 1.d, the rectangle is the ideal band-pass filter that passes through all frequencies with periods of 2 - 8 years by assigning a gain of 1. The ideal filter picks cycles per the period between .025 and .125. In Figures 1.e and 1.f the Baxter King filter provides the closest approximation to the ideal one. All filters are high-pass filters because the shortest possible cycle has 2 periods.

In the gain functions for the Baxter-King filter, other frequencies get a weight below 1, when it should be equal either to 0 or 1.

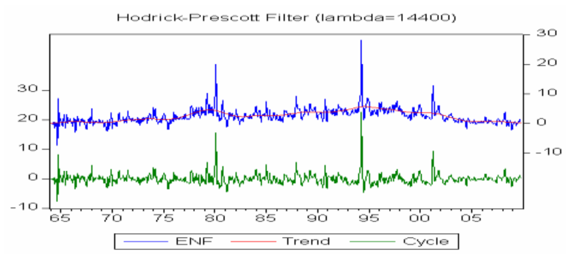

FIGURE 2.a

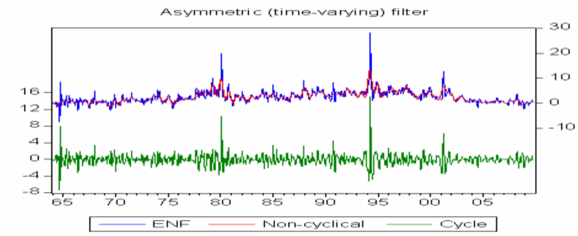

FIGURE 2.b

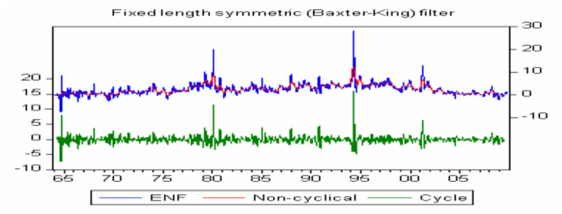

FIGURE 2.c

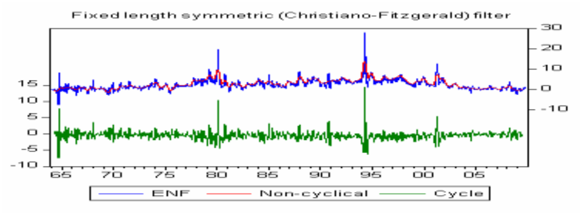

FIGURE 2.d

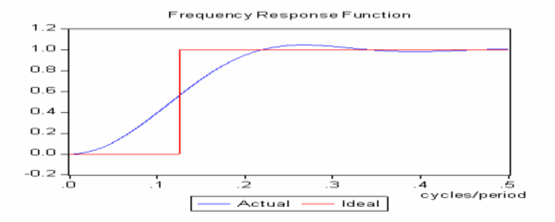

FIGURE 2.e

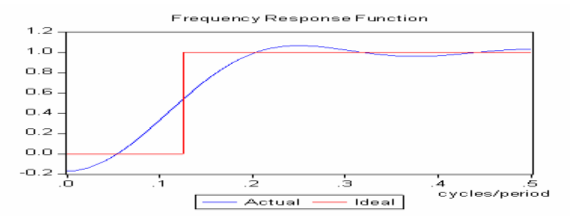

FIGURE 2.f

FIGURE 2. FILTER RESULTS FOR MONTHLY DATA

Source : Author's calculation

In terms of the BK filter, the poor performance at high frequencies stems from the increased discontinuity at the edges of the filter, once the spurious nonzero amplitude in the origin H (0) has been transferred there. Of course, if the input signal has very small components at high frequencies, as is indeed the case of a real random walk, their leaking in the pass-band is irrelevant.

When CF 2-8 is determined and delay is assumed as 3, the results are as shown. Variances in delay and/or K value change the results completely. The arbitrary determinations are a real problem. The main advantage of the HP method is that less data is needed as applied in the linear trend method. Another advantage of this method is that the trend component gives a chance to change during the estimation period as a reverse for linear trend method.

But, this method has some disadvantages. In the filter determination of λ the value is disputatious. Hodrick Prescott's (1997) suggestion is suitable for USA data; but it is not accepted as an international standard. As pointed out by Wynne and Koo (1997), the parameter does not have an intuitive interpretation for the user, and its choice is considered perhaps the main weakness of the HP method (Dolado et al., 1993; Maravall and del Río, 2001).

Another disadvantage of this method is that the HP filter technique is sensitive in relation to the data at the end of the estimation period (Arora ve Bhundia, 2003). Because the HP filter is symmetric and two-sided, the later and passed values are included in the function in the middle of the estimation period. But, in the beginning and at the end of the estimation period no values for the past and future are met. Thus, when the estimation period is widened the estimation results may vary.

Because Baxter-King is symmetric, it needs an equal amount of values of the past and future terms for every weighted moving average and thus results in the lack of observation both initially and finally. The same condition is prevailing for the symmetric CF filter. Because BK and CF filter results disregard the initial and final data, the sudden rise of unemployment in 2001 was obtained but was not filtered well. Due to the effects of the crisis, and since the breaking occurs at the last part of the period, successful results could not be observed. As these filters were used, the breaks occurred at the beginning and last part of the set could not be filtered as it is seen in our sample. This is because the period 1923 - 2008 contained high unemployment rates in 1923 - 1926 and high unemployment rates in 2002 - 2008 and that because of the 2001 crisis's effects resulted in an observation loss.

More successful results are observed in the monthly data. The mechanical structure of the method may cause misleading periodic movements. Because this can affect the potential output, the structural breakings or important political changes are not considered. As in all two-sided filter techniques, there is a term-end side problem. This problem occurs especially when estimation is used for forecast and different estimations can be observed when new data is added (Serju, 2006). The term-end observations are observed to solve this problem. However, because the estimations are dependent to the present output (Dupasquier at al., 1999) this solution shows important limitations. Another solution for advocacy problems of end of term is to estimate forward and then to filter. These methods do not give successful results for the variables. All three filter methods, instead of considering the structural breakings, are proved to flatten the series.

II. KALMAN FILTER RESULTS

According to the data set used, the Kalman filter gives two different estimations (filtered and flattened) of the unobservable variables; in other words, state variables. Filtered estimation, in period t, using all the data up to period t, estimates the state variables and is also named as one-sided. On the other hand, the flattened estimation is named as two-sided and uses all the data in the system to the period. The two-sided estimations use more data and have less standard derivations than the one-sided estimations.

After a state space show of model and determination of initial values, using the Kalman filter method, the most possible values of all the parameters in the model are observed and the data observed for 1923-2008 and 1964:1 - 2008:07 is provided in Table 1.

TABLE 1. KALMAN FILTER RESULTS

Unemployment				
	Final State	RMSE	Z-Statistic	Prob.
TREND	11.14874	0.672496	16.57814	0.0000
KAPPA	0.248740	0.262931	0.946028	0.3441
GAP	0.000011	1000.000	0.000000	1.0000
Log likelihood	102.207	Akaike info criterion		2.446677
Parameters	3	Schwarz criterion		2.532293
Diffuse priors	3	Hannan-Quinn criter.		2.481133
Inflation				
	Final State	RMSE	Z-Statistic	Prob.
TREND	0.7845	0.7030	1.1158	0.2645
KAPPA	0,00042	0.014047	-0.02969	0.9763
GAP	0.000011	1335.011	0.00199	1.0000
Log likelihood	1013.75	Akaike info criterion		3.738086
Parameters	3	Schwarz criterion		3.761794
Diffuse Priors	3	Hannan-Quinn criter.		3.747355

Source: Research results

It is obvious that the results observed from both annual and monthly data do not have the expected success. It is determined that in many studies the Kalman filter is not successful in annual data, but is generally successful in monthly data.

The study will try to raise and compare the effectiveness of Kalman filter results by using an adaptive threshold algorithm and neural-network (NN) model based approach for Sensor fault detection and identification (NNSFDI).

III. SENSOR FAULT DETECTION AND IDENTIFICATION (NNSFDI) RESULTS

Since expected results could not be reached from the widely used Kalman filters in economics and the data is filtered by using the Threshold algorithm, NNFSDI. This model which uses one threshold does not give realistic results because unemployment and inflation ratios at different periods have more than one threshold. This paper used an adaptive threshold algorithm to find a neural-network (NN) model based approach for Sensor fault detection and identification (NNSFDI). The thresholds present in the SFDI scheme that are imposed on "Thresh" are $\bar{\chi} = \frac{1}{85}\sum_{i=85}^{85} X(n-i)$ for unemployment and $\bar{\chi} = \frac{1}{544}\sum_{i=544}^{544} X(n-i)$ for inflation.

Some studies on the sensor fault detection based on an NN model propose a constant threshold. The NNSFDI process is based on comparing relevant parameters against thresholds. As noted above, because of using a threshold of unemployment rate and inflation rate the study used the NNSFDI model.

TABLE 2. NNSFDI MODEL

Unemployment results				Inflation results			
	α	**β**	**ε**		**α**	**β**	**ε**
FDES	1,3236	1,8414	0,9809	**FDES**	0,0177	1,5341	1,7678
MEEN-1	1,3231	1,8286	0,9402	**MEEN-1**	0,0175	1,5325	1,7489
MEEN-2	1,3107	1,8091	0,9371	**MEEN-2**	0,0175	1,5321	1,7308
MEEN-3	1,2991	1,7874	0,9207	**MEEN-3**	0,0175	1,5316	1,7210
MEEN-4	1,2875	1,7642	0,9066	**MEEN-4**	0,0175	1,5311	1,7156
MDEEN-1	0,0196	0,0479	0,0616	**MDEEN-1**	0,0002136	0,0024	0,0414
MDEEN-2	0,0191	0,0459	0,0593	**MDEEN-2**	0,0002104	0,0023	0,0391
MDEEN-3	0,0181	0,0431	0,0593	**MDEEN-3**	0,0002076	0,0022	0,0376
MDEEN-4	0,0170	0,0399	0,0453	**MDEEN-4**	0,0002055	0,0021	0,0367
DEEN-1	1,3233	1,8275	0,9455	**DEEN-1**	0,0175	1,5325	1,7486
DEEN-2	1,3108	1,8081	0,9416	**DEEN-2**	0,0175	1,5321	1,7304
DEEN-3	1,2992	1,7866	0,9240	**DEEN-3**	0,0175	1,5316	1,7205
DEEN-4	1,2876	1,1763	0,9081	**DEEN-4**	0,0175	1,5311	1,7149

Source: Research results

For the NNSFDI model structures, the study applied a three stage learning hybridization consisting of algorithm co-operation as discussed to overcome a local minimal problem and to obtain good forecasting power. The study of NNSFDI filter was obtained by applying BP and CGD learning algorithms. The model estimation results are reported in Table 2 (NNSFDI table). As a result, the difference of the models is that they aim to improve forecasting.

A learning system of the type BP100, CG20, CG36 is a BP algorithm with 100 epochs without early stopping optimized with “large” steps: a CGD algorithm with 20 epochs with smaller steps, and lastly, a third stage of CGD or BP with a small learning rate, this time with early stopping depending on which provides better results.

C. Comparison of forecast results

All analyzed filters maintain their RMSE as the forecast horizon increases, which provide an important result for the study. The results show that NNSFDI model prove better forecasts for both in-sample and out-of-sample sets. In related literature, there is an ongoing debate regarding the forecasting ability of the filter. With the use of the approach, there is an important improvement in filter based models which cannot be ignored.

At this stage of the study, appropriate steps of independent variables as well as the optimum number of hidden units are selected by estimating a larger set of NNSFDI models and comparing them by the calculated RMSE to avoid over-fitting. The MSE was taken as the loss function in the analysis and was compared to the forecast performance of the models in accordance with different classes of model selection criteria (MAE, RMSE, and MAPE). The most important difference between the HP, CF, CF (a), and NNSFDI is that the unemployment and inflation of NNSFDI filter estimates move more closely with the actual level of unemployment and inflation.

TABLE 3. FORECAST ACCURACY OF THE MODELS (IN SAMPLE)

Yearly Results					
	Baxter-King	C-F	C-F(a)	HP	NNSFDI
MAE	0,2797	0,4490	0,4597	1,3992	0,267
RMSE	0,3582	0,5465	0,7215	1,7028	0,348
MAPE	99,0866	155,57	106,998	33,540	98,99
Monthly Results					
	Baxter-King	C-F	C-F(a)	HP	NNSFDI
MAE	0,8962	0,9349	0,9900	1,2653	0,85
RMSE	1,4367	1,4579	1,5387	1,5739	1,39
MAPE	105,704	314,266	100,273	85,059	100,356

Source: Research results

In order to compare the forecasts of the NNSFDI models, the study reported the one step ahead RMSE comparison ratios in Table 3 and multi step ahead forecast comparisons in Table 3. Note that, if the calculated RMSE ratio is less than 1, the NN type models show forecast improvement. In sample forecast, NNSFDI gives the best result. That is followed by symmetric CF, BK and asymmetric CF. The worst results can be seen in relation to the HP filter. Monthly and annual results have the same structure.

RMSE, results obtained are too low in BK, CF and CF (a); these low results were not expected. Due to these unexpected results, the model was solved once more, but the same results were achieved yet again. When MAPE results and RMSE results are evaluated together, it is observed that the results differ from each others.

The results of NNSFDI in annual data were obtained as expected. Using monthly data, RMSE results obtained from BK, CF and CF (a) are closer results to those expected. RMSE results are lower in NNSFDI models.

Monthly results have better performances than the annual results. When out-of-sample results are evaluated, the study differs for monthly and annual periods. For annual data, it is chosen t+2 and t+4 and t+5 and t+7 for monthly data.

On the other hand, it was recognized that, forecasting power increased with increasing out-of-sample forecast horizons. In order to analyze the out-of-sample performance of the models, it was compared with yearly and monthly modeling specifications by the ratios of their RMSE values of out-of-sample forecasts in Table 4. A value less than 1 is interpreted as an improvement in out-of-sample power.

TABLE 4 . FORECAST COMPARISONS OF THE YEARLY AND MONTHLY MODELS (OUT-OF-SAMPLE) NNSFDI

Forecast comparisons of the yearly models (out-of-sample) NNSFDI					
	Baxter-King	C-F	C-F(a)	HP	NNSFDI
2					
MAE	0,2797	0,4530	0,4620	1,2502	0,27
RMSE	0,3582	0,5500	0,7190	1,6501	0,3499
MAPE	99,0860	157,1566	126,5790	22,4696	99,007
4					
MAE	0,2800	0,4610	0,4880	3,1319	0,2799
RMSE	0,3580	0,5530	0,7440	3,5706	0,3489
MAPE	99,4330	160,6810	150,6635	74,0063	99,4298

Forecast comparisons of the monthly models (out-of-sample) NNSFDI					
	Baxter-King	C-F	C-F(a)	HP	NNSFDI
5					
MAE	0,9200	0,9700	0,9900	1,9599	0,89
RMSE	1,5090	1,5350	1,5380	2,3970	1,437
MAPE	99,8500	327,9300	100,2732	79,2240	105,7037
7					
MAE	0,9210	0,9690	0,9910	1,2501	0,896
RMSE	1,5050	1,5320	1,5390	1,5090	1,437
MAPE	99,8200	329,4600	100,1822	93,5437	105,704

Source: Research results

If out-of-sample results are evaluated with caution, BK, CF and CF (a) results in t+5 and t+7 gave nearly equal results in terms of RMSE and MAE. For example: RMSE results obtained for in t+7 has a lower RMSE value in t+5 for HP. An interesting point is that the in-sample and the out-of-sample results are close to each other.

For two data sets, HP has shown to provide the worst performance in out-of-sample results and in-sample results.

IV. CONCLUSIONS

The NNFSDI model was suggested as an alternative model to the Kalman, HP, BK and CF filters and was analyzed for different data sets. In the study, we calculated MAE, RMSE and MAPE values for different filters. Further, the effectiveness of the NNFSDI model was compared to the effectiveness of the filters analyzed in the study.

In terms of the in-sample forecast accuracy, NNFSDI has given the best results, followed by symmetric CF and BK filters. Than by the asymmetric CF filter and finally by the HP filter. According to the empirical findings discussed in the study, the results obtained for the monthly data provide better performances than those obtained for the annual series. In terms of the empirical findings obtained for the out-of sample analysis, the best results are for t+2 and t+4 periods for the annual data and t+5 and t+7 for the monthly data.

Furthermore, the results calculated for the in-sample and out-of sample has led us to conclude in a similar fashion since the findings are very close. Sample and out-of-sample results were close to each other. These results imply that filters are not reliable. The main cause of this unreliability is the method of forecast, in which the data has been deleted from the beginning and from the end of the time series.

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MJERENJE EFIKASNOSTI FILTARA I PRIMJENA NNSFDI METODE

Sažetak: Cilj rada je prezentirati filtar neuralne mreže zasnovan na NNSFDI metodi kao alternativni filtar u odnosu na često korištene filtre u ekonomiji: naime, Baxter-King, Hodrick-Prescott, Christiano i Fitzgerald i Kalman filtre. U ovom radu su korišteni različiti podaci koji uključuju godišnje stope nezaposlenosti za period od 1923 – 2008 i mjesečne stope inflacije za period od 1964:02 – 2009:07. Ponašanje nove metode i osnovnih filtara je procijenjeno na osnovu godišnjih i mjesečnih podataka. Empirijski dobiveni rezultati upućuju na to da je novo prezentirani NNFSDI model dao bolja predviđanja rezultata u usporedbi s Kalman, HP, BK i CF filtrima za različite skupove podataka pri procjenama s obzirom na različite kriterije greške kao što su MSE, RMSE i MAPE.

Ključne riječi: Baxter-King filtar, Hodrick-Prescott filtar, Christiano i Fitzgerald filtar, Kalman filtar, NNSFDI model, prilagodljivi algoritam praga