METAHEURISTICS FOR OPTIMIZING SAFETY STOCK IN MULTI STAGE INVENTORY SYSTEM

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Abstract

Managing the right level of inventory is critical in order to achieve the targeted level of customer service, but it also carries significant cost in supply chain. In majority of cases companies define safety stock on the most downstream level, i.e. the finished product level, using different analytical methods. Safety stock on upstream level, however, usually covers only those problems which companies face on that particular level (uncertainty of delivery, issues in production, etc.).

This paper looks into optimizing safety stock in a pharmaceutical supply considering the three stages inventory system. The problem is defined as a single criterion mixed integer programming problem. The objective is to minimize the inventory cost while the service level is predetermined. In order to coordinate inventories at all echelons, the variable representing the so-called service time is introduced. Because of the problem dimensions, metaheuristics based on genetic algorithm and simulated annealing are constructed and compared, using real data from a Croatian pharmaceutical company. The computational results are presented evidencing improvements in minimizing inventory costs.

Key words: *Multi stage inventory system, Inventory cost, Service level, Pharmaceutical industry, Mixed integer programming, Metaheuristics*

1. INTRODUCTION

In order to be competitive on the market, firms have to coordinate their supply chains in an efficient and effective way. One of the problems is to handle the uncertainty in demand. To be sure that the demand will be satisfied and in this way keep the service level at a high performance, firm should take care of safety stock. In the literature there are many works about how to determine safety stock at the level of finished products. Very few works consider the methods and techniques for handling safety stock at the level of intermediate products and raw material.

In this paper we consider the real problem from a Croatian pharmaceutical company. The inventory system in the considered supply chain is divided in three stages. The first stage is the stage of raw materials, including active pharmaceutical ingredients.. The second one is the stage of intermediate products which are tablets, capsules, vials etc. that should be packed. The third level is the level of finished products which are blisters of tablets packed in the boxes of various dimensions for different markets with the leaflets written in different languages.

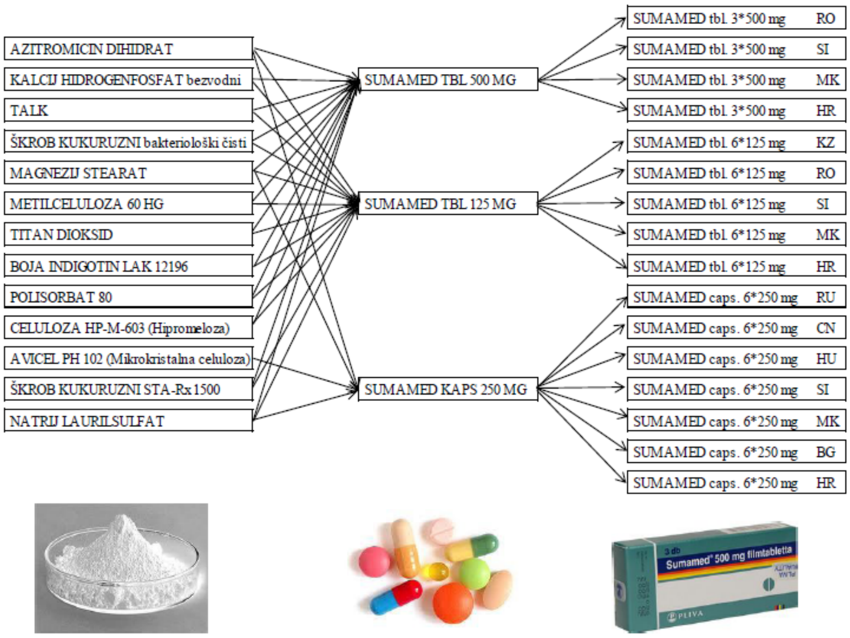

Figure 1: ***Materials at different stages of inventories***

The additional problem that has to be taken into the consideration in this Croatian case study is the fact that the products are produced in batches so we have to include that into model too. For small dimensions the problem can be solved even in Excel Solver, but for larger dimensions, metaheuristics should be created.

In Section 1 we give an introduction into the problem. In Section 2 we are describing the known results from the literature. The problem is modeled as a mathematical programming model in Section 3. In Section 4 we are talking about two ways of solving the problem. One of them is based on the genetic algorithm, while the second one is based on simulated annealing, both of them contained in MATLAB as procedures where the user should predetermine input parameters as described in the mentioned section. At the end of Section 4 computational results are presented. In Section 5 we give some conclusions and directions for future work.

2. OVERVIEW OF THE LITERATURE

As we have already mentioned there is no to large body of literature considering the determination of optimal safety stock at different stages in a supply chain taking into account the production in batches, uncertainty in demand and keeping the service level as high as possible. Most of them are considering the demand and determine safety stock for finished goods only, or if they consider multi stage inventory system, they do not provide an effective methodology that can address complex multi-constrained supply chains (Boulaksil, Fransoo, van Halm, 2009). In the paper (Boulaksil, Fransoo, van Halm, 2009) the problem of determining safety stocks is addressed by a simulation based approach, where the simulation studies are based on solving the supply chain planning problem (formulated as a mathematical programming model) in a rolling horizon setting. For a small dimension problem a dynamic programming algorithm is proposed in (Inderfurth, 1991), both, for a serial and divergent system using risk pooling. Graves (1996) and Graves and Willems (2000) discuss the so-called guaranteed-service model for setting safety stocks in a multi-stage setting to cover demand uncertainties. They develop a model for positioning safety stocks in a supply chain where each stage is controlled by a base-stock policy, assuming an upper bound for the (customer) demand level. So they make a restrictive assumption about the demand process reducing the complexity of the problem. There are also many papers considering multi stage inventory systems from the period 1975 – 1985, but for small dimensions and with many restrictions. From literature which describes practical usage of such algorithms it is visible that complex situation in different type of the industry in reality require additional adjustment. Also, the difference from the problems considered in the literature is the production in batches which makes problem more complex.

3. MATHEMATICAL FORMULATION OF THE PROBLEM

3.1. Set and indeces

- M – set of first level materials (raw materials);
- I – set of second level materials (intermediate products);
- G – set of third level materials (finished products);
- $S(i)$ – set of direct successors of material i
- $P(i)$ – set of direct predecessors of material i

3.2. Parameters

- N – total number of materials
- w_{ij} - the quantity of material i for the production of one unit of material j
- λ_i - processing lead time for material i
- c_i - cost of holding stock for material i (defined as percentage of the stock value)
- k_i – safety coefficient for material i (z-value of required service level)
- μ_i – average demand level
- σ_i – standard deviation for material i
- ρ_{ij} - correlation between demands for materials i and j
- d_i – annual demand plan for material i
- $COGS_i$ - standard price for material i
- b_i – batch size for intermediate product i, ($i \in I$)
- $r_j = \begin{cases} 1, \text{ if } SS_j \geq w_{jk} \cdot b_k, \text{ for } k \in S(j), \\ 0, \text{ otherwise} \end{cases}$

where SS_j is the safety stock of intermediate material j.

Also, the standard deviation for material i from sets I or M is calculated according to (Cashon, Terwiesch, 2012)

$$\sigma_i = \sqrt{\sum_{j \in S_G(i)} \sum_{k \in S_G(i)} w_{i,j} \cdot w_{i,k} \, Cov_{jk}}$$

or

$$\sigma_i = \sqrt{\sum_{j \in S_G(i)} \sum_{k \in S_G(i)} w_{i,j} \cdot w_{i,k} \cdot \sigma_j^2 \cdot \sigma_k^2 \cdot \rho_{ij}}$$

The parameters that are not used directly in the mathematical model are used for the calculation of the parameters used directly in the model.

3.3. Decision variables

In order to be able to connect safety stock from different level of supply chain, we introduce here variable so called service time. Service time represents part of lead time at each level which is not covered by safety stock. On that way we ensure that safety stock on downstream level is lower if we have sufficient safety stock on upstream level and vice versa. Let S_i be the service time for material *i*.

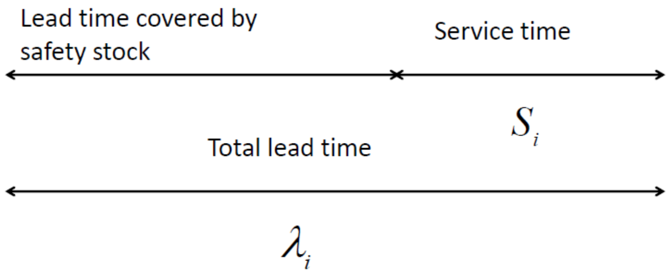

Figure 2: Service time

3.4. Objective

The objective is to minimize the total safety stock cost consisting of first level safety stock cost, second level safety stock cost and third level safety stock cost. For the construction of the objective function we use the formula determining safety stock for finished products, $SS = k \cdot \sigma \cdot \sqrt{\lambda}$ (Cashon, Terwiesch, 2012), and transform it to fit the first and second level of the multi stage inventory system taking into the consideration the service time.

Thus, the safety stock cost for the first level is $c_i COGS_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\lambda_i - S_i}$, for the second level $c_i COGS_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\max_{j \in P(i)}\{ r_j \cdot S_j \} + \lambda_i - S_i}$ and for the third level $c_i COGS_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\max_{j \in P(i)}\{ S_j \} + \lambda_i}$. Thus, the objective is

$$\min \left(\begin{array}{l} \sum_{i \in M} c_i \cdot COGS_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\lambda_i - S_i} + \\ \sum_{i \in I} c_i \cdot COGS_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\max_{j \in P(i)}\left\{ r_j \cdot S_j \right\} + \lambda_i - S_i} + \\ \sum_{i \in G} c_i \cdot COGS_i \cdot k_i \cdot \sigma_i \cdot \sqrt{\max_{j \in P(i)}\left\{ S_j \right\} + \lambda_i} \end{array} \right)$$

3.5. Constraints

For the first level, the service time must be less or equal to the total lead processing time (the objective function for the first level is determined). For intermediate products (the second level), in order the objective function for the second level is determined, the service time must be less than processing time plus the maximum service time of considered material predecessor. For the third level (finished products) we define the service time equal to zero because we want to be able to satisfy the demand immediately upon the order is posted.

$$\begin{array}{ll} 0 \le S_i \le \lambda_i, & \forall i \in M, \\ 0 \le S_i \le \max_{j \in P(i)}\left\{ S_j \right\} + \lambda_i, & \forall i \in I, \\ S_i = 0, & \forall i \in G, \end{array}$$

4. METAHEURISTICS

To solve the problem we use high-level language and interactive environment for numerical computation, visualization, and programming, MATLAB. MATLAB offers a certain number of metaheuristics. We use two of them based on genetic algorithm and simulated annealing. For a specific problem from the Croatian pharmaceutical company metaheuristic based on simulated annealing gives better results regarding CPU time and objective function value. The input data where information about all the products such as lead times, prices and safety stock costs. Also we need data about demand changes for last twelve months in order to compute standard deviations for all the products and correlations between every two finished products (the calculated data about demand changes are presented in Badurina, 2012). For every finished product we need the percentage of a certain intermediate product and raw materials participating in the finished product. The code of the program can be found in (Badurina, 2012).

In the real case problem from the Croatian pharmaceutical company we consider 545 finished products, 162 intermediate products and 316 raw materials. The calculations of the data needed for the metaheuristic take four hours. The service level is fixed at the value of 97% which corresponds to z-value of 1.88. In order to obtain a good objective function value the metaheuristic takes about twenty minutes.

The frequencies of service times for raw materials are given in the following figure:

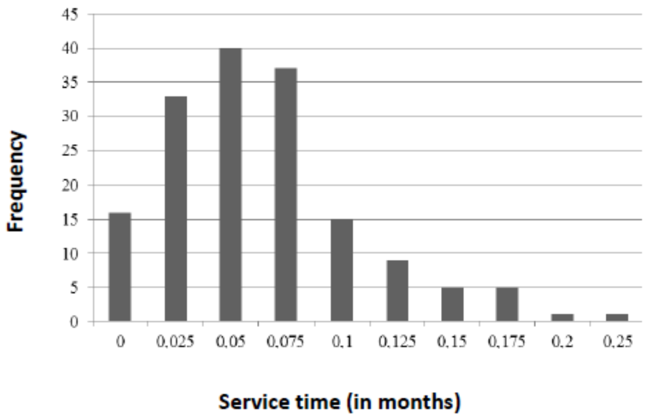

Figure 2:. Frequencies of service times for raw materials

We can notice that the service times are very small what means that the safety stock for raw materials should be very high. The similar situation is for intermediate products.

The value of the objective function is 38.51 mil kn. Thus, this is the safety stock cost in order to assure the service level of 97%. The structure of the safety stock is as follows, 71% of safety stock are finished products, 11% intermediate products and 18% are raw materials. There is no finished product having safety stock only on the third level. For all finished products there is a combination of safety stocks on at least two levels.

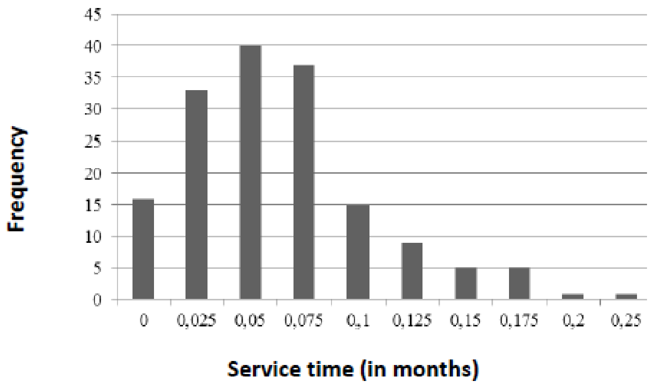

Figure 3: Frequencies of service times for intermediate products

Since in the considered Croatian pharmaceutical company the safety stocks are determined intuitively, our result is much precise and correspond to desired customer service level. Even safety stock at the level of finished products calculated using formula $SS = k \cdot \sigma \cdot \sqrt{\lambda}$ is better than the present situation in the company. So we compare our results with the situation when only the safety stock at the level of finished products is considered. In this case the safety stock cost is 60.35 mil kn what means that we successes to decrease the total cost for 36.19%.

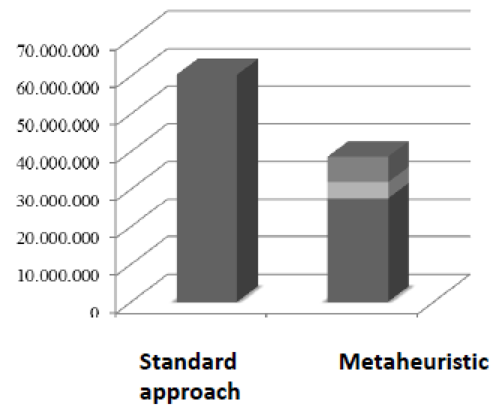

Figure 4: Comparison of standard approach and metaheuristic

The savings can be the result of the metaheuristic, but also of moving the stocks on the upstream levels and of correlations between finished products. Because of this, we calculate the safety stock for the case of zero service times. The results are presented at the following figure:

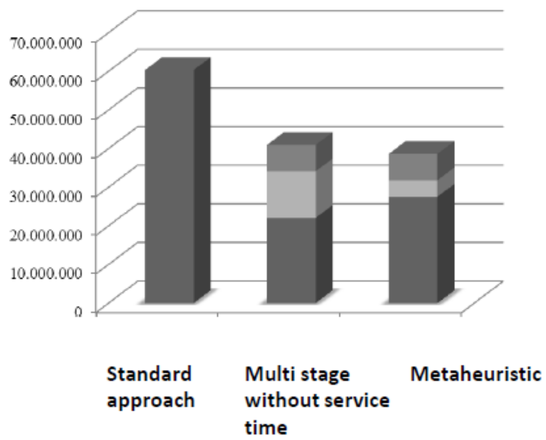

Figure 5: ***Comparison of standard approach, multi stage without service time and metaheuristic***

5. CONCLUSIONS AND FUTURE WORK

Standard ERP systems do not have appropriate modules for optimizing safety stock in a firm. One of the more complicated problems is the determination of safety stocks especially if we consider multi stage inventory system. The standard approach offers formulas for the level of finished products taking into account the processing lead time. Keeping safety stock at the level of starting materials is less expensive, as one material can be used in many different products, so correlation between demand change of finish products decreases the need for safety stock on the level of starting materials.

We coded our procedure in MATLAB using its determined metaheuristic based on simulated annealing because it gives better results than metaheuristic based on genetic algorithm. With our metaheuristic we obtain better results than the standard approach taking into account only safety stock on the level of finished products and also better than the multi stage approach without service time. We implemented our metaheuristic on real data from Croatian pharmaceutical company determining the service level at 97%. The total safety stock cost decreases for about 36%.

First of all, we intend to test our model for the problems of larger dimensions since in this work we implemented the created decision support system for the real case of Croatian pharmaceutical company and compared the results to standard procedures. Also, as a future work we intend to model the problem as a multicriteria programming problem where the second objective function will be the service level and the objective will be its maximization. We will also consider the warehouse space and its minimization. Since the production is performed in batches we will define a mixed integer programming problem and propose a metaheuristic using some already predetermined metaheuristic contained in MATLAB because we think they give good results.

REFERENCES

Badurina, G., (2012), *Models for inventory optimization in multi stage supply chain,* PhD thesis, Faculty of Economics Split.

Cashon, G. and Terwiesch, C., (2012), *Matching supply with demand: An introduction to operations management,* McGraw Hill.

Boulaksil, Y., Fransoo, J.C. and van Halm, E.N.G., (2009), "Setting safety stocks in multi-stage inventory systems under rolling horizon mathematical programming models", *OR Spectrum*, Vol. 31, pp. 121-140.

Graves, S.C., (1996), "A multiechelon inventory model with fixed replenishment intervals", *Management Science*, Vol. 42, pp. 1-18.

Graves S.C. and Willems, S.P. (2000), „Optimizing strategic safety stock placement in supply chains", *Manufacturing and Service Operations Management*, Vol. 2(1), pp. 68-83.

Inderfurth, K. (1991), „Safety stock optimization inmulti-stage inventory systems", *International Journal of Production Economics*, Vol. 24, pp. 103-113.