

## MODELLING OF ONLINE GROUP DISCOUNTS

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### Abstract

Web pages for group discounts have become very popular in the past few years. In this paper we concentrate on the group discounts for the service industry in which a quality of the service plays an important role in retaining customers which in return affects business profitability. We present a model of the group discount offer from a merchant's point view. A merchant decides about the size of the discount offered, having in mind quality of the service offered which is affected by the number of customers who use the service. Finally, we derive the first order optimality conditions.

**Key words:** *Group shopping, Group discount, Voucher*

### 1. INTRODUCTION

Volume discounts exist since the foundation of the trade. Internet era yielded a new version of these discounts – online group discounts. It all started in December 2008. when the web page *Groupon.com* was launched. It was the first web page offering these kind of deals. Soon after Groupon's success this model was followed by many similar web pages more or less successfully worldwide. In Croatia's market *kolektiva.hr* was the ice breaker in the beginning of 2010. Today there are approximately ten other participants in our market among which the most active ones are: *kupime.hr*, *ponudadana.hr*, *cronojaje.hr* and *megapopust.hr*. All these web pages (foreign and domestic) offer discounts of 50-90% on certain products and services. Their role is to serve as mediators between merchants and customers. The discount size and content of an offer is negotiated between merchant and personnel of the web page. Once they agree the offer is published online and sent to mailboxes of the web page subscribers. Offers are available (active) from one day to several days. During that period mail

subscribers and web page visitors are able to purchase vouchers. An offer is considered valid if the pre-planned number of purchased vouchers has been reached. An upper bound to the number of vouchers sold can also exist. Web pages usually charge 50% or less of the voucher value for the service if the offer becomes valid. Number of offers that do not become valid is negligible. Popularity of the group discounts among customers is not surprising because of the large discounts. However what are the benefits of the merchants involved? A merchant engaging in such a deal has more than one benefit. First of all, this is a powerful marketing tool because thousands of people check the group discount web pages on a daily basis or they get new offers directly to their mailboxes, so a merchant will be heard of. Furthermore, in this way people that already know about a particular merchant but haven't purchased any of his products before because they are price sensitive will use his offer and maybe become a regular customer. Therefore, it looks like a win-win-win situation for all the parties involved. In this paper we model profitability of a group shopping offer from the perspective of a merchant.

The rest of the paper is organized as follows. In section 2 we give short review of the previous research. In section 3 we present our model of online group discounts. Finally, in section 4 we give some conclusions.

## **2. LITERATURE REVIEW**

Although online group shopping is an interesting marketing phenomenon there hasn't been much scientific work done concerning it, especially from the theoretical point of view. One reason for this is probably the fact that it has appeared recently. Our paper will try to partially fill this gap.

Dholakia (2010) conducted a survey among the companies that used Groupon for their promotion. He found that 2/3 of the promotions were profitable and 1/3 unprofitable. Unprofitable promotions had significantly lower rates of users spending more than voucher value and lower repurchase rates at full prices than profitable ones. He also examined drivers of profitable Groupon promotions and surprisingly (or not so surprisingly) discovered that employee satisfaction is the most important. We'll try to imbed this discovery in our model.

Arabshahi (2010) in his paper tried to reveal principles of the underlying Groupon's business model. He states that Groupon is a fascinating company which has devised a way to monetize large number of email subscribers in a self-reinforcing virtuous cycle. He sees Groupon essentially as a price discrimination service – it detects price-sensitive consumers and prevents (most of) the existing full-price paying customers from taking advantage of the offer by their natural separation based on price-

sensitivity (Groupon subscriber or not Groupon subscriber) and geography. We will also assume this natural separation of the costumers.

The most interesting paper for us has been the work done by Edelman et al. (2011). They use framework of Bils (1989). Bils proposes a way of describing consumers who have a subjective value of a merchant's good, but do not know whether their product suits them or not. This dilemma is solved by purchasing. If they determine that product is a fit they will be willing to pay more for it in the future. Bils (1989) observes the infinite horizon while Edelman et al. have two periods: first period has reduced prices and the second has full prices. Building on the work of Bils (1989), Edelman et al. (2011) in their paper examine the mechanisms of price discrimination and advertising, the two driving forces which contribute to the benefit of the merchant, and provide a simple model which considers a single firm and a continuum of consumers who have the opportunity to buy at most one voucher for their products. They also present two extensions of the model, namely the one in which consumers have the possibility to purchase multiple vouchers and the one in which a firm might re-optimize its prices. Our model builds upon Bils' simple framework, but expands in the different direction than the work of Edelman et al.

### **3. MODEL OF ONLINE GROUP DISCOUNTS**

In our paper, we are primarily interested in modelling a voucher group discounts for merchants coming from the service industry (like restaurants, bistros, beauty salons, gyms etc.), where employee satisfaction plays an important role in maintaining the overall quality of service, which in return affects the profitability of business in the long run. We build our model from a point of view of a merchant who wants to give his offer through some of the group discount web pages. The merchant is facing the decision about the amount of the discount offered as well as about the maximum number of the vouchers offered. In the service industry it is very important to avoid the trap of having too many customers in a short period of time, especially those who do not pay the full price of the service. Large number of customers can decrease employee satisfaction and therefore deteriorate the overall quality of the service. For example, customers who use discounts for the restaurant service are not very popular among the employees because they create a lot of extra work and at the same time are generally less inclined to give tips. Furthermore, depending on the discount, these customers tend not to spend more than the voucher value and are less likely to return, which makes them unpopular from the point of view of the restaurant owner as well.

The assumptions of our model are as follows. We consider a merchant who places an offer for one product at some group discount web page in which he states the discount. A known percentage of a

voucher price is paid to a group discount provider as a fee for its services. Each customer can buy one voucher only. We consider two time periods. The first one consist of a period of time during which it is possible to buy and use vouchers (it ends with the time expiration of the vouchers), and the second one consists of a period of time which immediately follows and during which all customers pay the full price. The time horizon is divided in two periods in order to study the effect of this kind of advertising. This effect is reflected in number of customers willing to use the service and pay the full price after the expiration of the discounts.

### 3.1. Modelling customers' behavior

In modeling the customers' behavior we take into account that both advertising and price discrimination in the form of discounts increase a customer's subjective perception of value of the product. As Arabshahi (2010) states: "Value increases not only when a product is desired more but also when the price is lowered... Advertising does it by raising a consumer's subjective perception of the utility of the product. Price discrimination does it by lowering the price." However, one should be aware that price discrimination can also lower the value of the product if not conducted properly. If the price is too low, it can be perceived as a sign of low quality, thus lowering the perceived valued of the product. On the other hand, customers often perceive high price as a signal of high quality.

In creating the model, we will follow the setup of Bills (1989). We assume that the full price  $p$  of the product is known in advance and that the merchant doesn't change the price during the time horizon. Customers are divided in two groups. The first group consists of group discount users who are generally price sensitive. The second group consists of users unaware of group discount offer, i.e. customers who, if using the service, pay the full price. The target of the group discounts is the first group. Let us denote the first group of customers by  $D$  ( $D$  stands for discounts) and the second group by  $N$  ( $N$  stands for no discounts). Each member of a group has its personal valuation of a product  $v$ . Let  $Q_D(v)$  denote the number of customers from group  $D$  whose valuation of the product is at least  $v$ . In the same manner, let  $Q_N(v)$  denote the number of customers form group  $N$  whose valuation of the product is at least  $v$ . We assume that both  $Q_D(v)$  and  $Q_N(v)$  are given. However, consumers who haven't had the opportunity to try the product yet cannot be sure if the product is a fit. It might happen that they won't like the food, the service etc. The only way to find out the quality of the product is to try it. Therefore, for each customer (no matter which group he belongs to) we assume that he will find the product a fit with probability  $r$ , i.e. the probability that he won't like it is  $(1 - r)$ . A customer from group  $N$  will buy the product in the first period if at least one of the following conditions is satisfied:

$$r \cdot v \geq p \quad (1)$$

or

$$r \cdot v + r \cdot \delta \cdot v \geq p + r \cdot \delta \cdot p \quad (2)$$

Here,  $\delta$  reflects a customers' common rate of time discount. The condition (1) states that a customer will buy the product in the first time period if his expected benefit in the first period is greater than the cost of the product. We assume that utility of a non fit product is 0. Since  $r$  is the probability that the product is a fit and  $v$  is the valuation of a customer, the expected benefit is equal to  $r \cdot v$ . Condition (2) states that a customer will buy the product in the first period if the expected benefit over both periods is greater than the expected cost. If a customer bought the product in the first period and it was a fit, he will continue buying it in the second period. A customer who did not buy in the first period will do so in the second period if  $r \cdot v \cdot \delta \geq p \cdot \delta$ , which reduces to (1). Therefore, only customers who buy in the first period can buy in the second period.

When considering valuations for the customers from group  $D$ , we have to take into account the percentage of discount  $(1 - \alpha)$ . For example, if merchant gives a 30% discount, then  $\alpha = 0.7$ . Now, in a similar manner as for customers from group  $N$ , a customer from group  $D$  buys the product in the first period if at least one of the following conditions is satisfied:

$$r \cdot v \geq \alpha \cdot p \quad (3)$$

or

$$r \cdot v + r \cdot \delta \cdot v \geq \alpha \cdot p + r \cdot \delta \cdot p \quad (4)$$

Here, when determining the cost, we have taken into account the price discount factor  $(1 - \alpha)$ . The discount on the price is given in first period only. In the second period a customer pays the full price. A customer from group  $D$  who did not buy the product in the first period will buy it in the second period if  $r \cdot v \cdot \delta \geq p \cdot \delta$ , i.e. if (1) holds. If the customer did not buy in the first period, then (4) does not hold. This in combination with (1) gives  $\alpha > 1$ , which is impossible, so only customers from group  $D$  who buy in the first period can buy in the second period as well. Therefore, customers from both groups can buy in the second period if and only if they bought the product in the first period and found it a fit. At the same time, if they bought the product in the first period, it does not necessarily mean that they will continue buy it in the second period as well.

Let us take another look at conditions (1) and (2). From them it follows that a customer from group  $N$  will buy a product in the first period if

$$v \geq \min \left\{ \frac{p}{r}, \frac{1 + \delta \cdot r}{r + \delta \cdot r} p \right\} \quad (5)$$

It is easy to see that if  $\delta > 0$  (which is true by assumption) the costumers from group  $N$  with valuations at least

$$v(p) = \frac{1 + \delta \cdot r}{r + \delta \cdot r} p \quad (6)$$

will purchase. He will continue to buy in the second period if the product is a fit.

Similarly, for customers from group  $D$ , from inequalities (3) and (4) it follows that a customer will buy the product in the first period if

$$v \geq \min \left\{ \frac{\alpha \cdot p}{r}, \frac{\alpha + \delta \cdot r}{r + \delta \cdot r} p \right\} \quad (7)$$

Here we have to consider two cases:  $\alpha \geq r$  and  $\alpha < r$ .

If  $\alpha \geq r$ , then a customer will purchase in the first period if his valuations is at least

$$v(p) = \frac{\alpha + \delta \cdot r}{r + \delta \cdot r} p \quad (8)$$

Furthermore, he will continue to purchase in the second period if the product is a fit.

On the other hand, if  $\alpha < r$ , then a customer will purchase in the first period if his valuations is at least

$$v(p) = \frac{\alpha \cdot p}{r} \quad (9)$$

In this case it might happen that the customer does not continue to buy in the second period, although the product is a fit (if  $v < p$ ). All customers for which  $v \geq p$  will return.

### 3.2. Group discount model

Now we are ready to introduce the whole model. Besides the functions, parameters and variables already introduced, we introduce

- $(1 - \beta)$  - the portion of a voucher value that the group discount service retains (for example, if group discount service retains 30% of voucher value, then  $\beta = 0.7$ )

- $c$  - the marginal cost of the product
- $v_N$  - threshold valuation of customers from group  $N$
- $v_D(\alpha)$  - threshold valuation of customers from group  $D$  depending on the price discount  $\alpha$
- $Q_N(v)$  - the number of customers from group  $N$  whose valuation is at least  $v$
- $Q_D(v)$  - the number of customers from group  $D$  whose valuation is at least  $v$

Since  $p$  is fixed at some level  $p = p^*$  and known in advance, we denote the threshold valuation for customers from group  $N$  as

$$v_N = \frac{1 + \delta \cdot r}{r + \delta \cdot r} p^* \quad (10)$$

Furthermore, we denote the threshold valuation for the costumers from group  $D$  as

$$v_D(\alpha) = \begin{cases} \frac{\alpha + \delta \cdot r}{r + \delta \cdot r} p^* & \text{if } \alpha \geq r \\ \frac{\alpha \cdot p^*}{r} & \text{if } \alpha < r \end{cases} \quad (11)$$

Note that  $v_D$  is a function of a price discount  $\alpha$ , while  $v_N$  is a uniquely determined by  $p^*$ .

Now we formulate the merchant's profit function with respect to price discount factor  $\alpha$  as follows. The number of customers from group  $N$  who are buying product in the first period is  $Q_N(v_N)$ , while the number of customers from group  $D$  is  $Q_D(v_D(\alpha))$ . The common probability for both groups that a product is a fit is  $r$ . In our model we want to capture the fact that the increased number of users, especially those who use big discounts and are bad tippers, results in deterioration of service and in this way affects overall consumers' valuation of the product in a negative manner. Therefore, if a product price does not change, then the number of returning customers in the second period who already used the service in the first period will not be  $r \cdot Q_N(v_N)$  and  $r \cdot Q_D(v_D(\alpha))$ , but slightly smaller. This number will be affected by the overall number of customers who used the service in the first period. Therefore, we introduce the factor

$$k(\alpha) = \frac{Q_D(v_D(\alpha))}{Q_N(v_N) + Q_D(v_D(\alpha))} \quad (12)$$

which denotes the proportion of discount users in overall consumer population. Note that  $k$  is a function of price discount  $\alpha$ . The higher the  $\alpha$ , the lower the proportion of discount users in overall consumer population, and vice versa. The higher the proportion of discount users in the overall consumer population, the poorer the service. Therefore, the choice of  $\alpha$  affects the quality of service provided. Moreover, we have to take into account that different groups of customers have different quality and price sensitivity. Let  $\gamma_N$  denotes the sensitivity factor for group  $N$  and  $\gamma_D$  sensitivity factor for group  $D$ . It is reasonable to assume that  $\gamma_N > \gamma_D$ , i.e. that full paying customers are more sensitive to the quality of the product. Furthermore, let us denote

$$f_N(\alpha) = (1 + \gamma_N \cdot k(\alpha)) \cdot v_N \quad (13)$$

$$f_D(\alpha) = \begin{cases} (1 + \gamma_D \cdot k(\alpha)) \cdot v_D(\alpha) & \text{if } \alpha \geq r \\ (1 + \gamma_D \cdot k(\alpha)) \cdot p^* & \text{if } \alpha < r \end{cases} \quad (14)$$

Now we can state the merchant's profit function as

$$\Pi(\alpha) = \{Q_N(v_N) + \alpha \cdot \beta \cdot Q_D(v_D(\alpha)) + r \cdot \delta \cdot [Q_N(f_N(\alpha)) + Q_D(f_D(\alpha))]\} \cdot (p^* - c) \quad (15)$$

Note that although the functional form of  $\Pi(\alpha)$  is different for  $\alpha \geq r$  and  $\alpha < r$ , it is a continuous function. The difference between these two cases is in the term which describes the number of customers from group  $D$ , who after buying the product in the first period, continue to buy the product in the second period as well. It is due to the fact that for  $\alpha < r$  a certain number of customers from group  $D$  who buy the product in the first period and find the product a fit don't continue to buy in the second period, while for  $\alpha \geq r$  all customers who find the product a fit in the first period continue to buy in the second period. This is reflected in function  $f_D(\alpha)$ .

Now we can state the first order optimality conditions as follows:

$$0 = \frac{d\Pi}{d\alpha} = \beta \cdot \left[ Q_D(v_D(\alpha)) + \alpha \cdot \frac{\partial Q_D}{\partial v_D}(v_D(\alpha)) \cdot v_D'(\alpha) \right] \cdot (p^* - c) + r \cdot \delta \cdot \left[ \frac{\partial Q_N}{\partial f_N}(f_N(\alpha)) \cdot f_N'(\alpha) + \frac{\partial Q_D}{\partial f_D}(f_D(\alpha)) \cdot f_D'(\alpha) \right] \cdot (p^* - c) \quad (16)$$

Here

$$f'_N(\alpha) = \gamma_N \cdot k'(\alpha) \cdot v_N \quad (17)$$

$$f'_D(\alpha) = \begin{cases} \gamma_D \cdot k'(\alpha) \cdot v_D(\alpha) + (1 + \gamma_D \cdot k(\alpha)) \cdot v'_D(\alpha) & \text{if } \alpha \geq r \\ \gamma_D \cdot k'(\alpha) \cdot p^* & \text{if } \alpha < r \end{cases} \quad (18)$$

and

$$v'_D(\alpha) = \begin{cases} \frac{p^*}{r + \delta \cdot r} & \text{if } \alpha \geq r \\ \frac{p^*}{r} & \text{if } \alpha < r \end{cases} \quad (19)$$

Stationary points are derived from condition (16). However, the exact formula for the stationary points will depend on the exact choice of functions  $Q_N$  and  $Q_D$ .

#### 4. CONCLUSION

Recently web pages specialized for group discount offers have become a very popular among customers and merchants offering their product. These web pages offer a significant discount on products and services, thus appealing to the great number of customers. Merchants offering the products also find it attractive primarily because of price discrimination and advertising affects. In this paper we have concentrated on the service industry in which quality of service plays an important role in success of the business. We have created a simple model of group discount offer from the merchant's point of view. The assumption of the model is that the base price of the product is already given, so a merchant has to decide about the amount of the discount that he is going to offer. The customers are naturally divided into two groups, one of which is price sensitive and is targeted by the offer. However, the amount of the discount affects the number of the customers of the service, which in return affects the quality of the service. In the end, we have modeled a merchant's profit function and derived the first order optimality conditions. In our future research, we plan to extend the model so as to include the limit on maximum number of vouchers offered, try to estimate customer life time value and determine equilibrium price in time.

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