AN INTERACTIVE PROCEDURE FOR AGGREGATE PRODUCTION PLANNING

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Abstract

Minimizing production cost over the planning period is usually assumed to be the objective of aggregate planning. However, other issues of strategic type may be even more important. Smoothing employment levels, driving down inventory levels or meeting high level of service usually are also considered. Thus, aggregate planning problem constitutes a multiple criteria decision making problem. In the paper a new approach for production aggregate planning problem is proposed. The procedure combines linear programming, simulation, and interactive approach. Linear programming models are used to generate initial solutions. In order to check how the fluctuations in demand will affect the results obtained under each of these solutions simulation experiments are performed. Finally, an interactive procedure is used for identifying the final solution of the problem.

Key words: *Aggregate production planning, Production risk, Simulation, Multiple criteria analysis, Interactive approach*

1. INTRODUCTION

The objective of any manufacturing system is to deliver products in right quantities, on time and at the appropriate cost. A rough taxonomy of decisions affecting the production system involves three categories: strategic, tactical, and operational. Strategic planning decisions are mostly focused on the development of resources to satisfy customers' requirements. The objective of the tactical decisions is the most effective use of these resources (Bitran and Tirupati, 1993). Finally, operational decisions are concerned with operational and scheduling problems and require disaggregation of information generated on higher levels.

In this paper, tactical production planning is considered. It deals with determining and timing of production for the intermediate future (from 3 to 18 months). Terms "aggregate production planning" or "sales and operations planning" are often used to describe this part of planning activities. The word “aggregate” means, that plans are prepared on a product family basis. Product families are defined as groupings of products that share common manufacturing facilities and setup times. Decisions made in the development of an aggregate plan include determining the best way of meeting forecasted demand by adjusting production rates, labor levels, inventory levels, overtime work, subcontracting rates, and other controllable variables (Heizer and Render, 2004).

Various optimization techniques are employed in aggregate planning. Linear programming, mixed integer programming, and dynamic programming are used most often. The objective is typically to find the lowest-cost-plan (Vollmann, Berry, Whybark and Jacobs, 2005). However, other issues may be even more important. Smoothing employment levels, driving down inventory levels or meeting high level of service are usually under managers’ consideration. As a result, aggregate planning constitutes a multiple criteria decision making problem.

Although mathematical programming approaches for aggregate planning are already substantially sophisticated, they still do not reflect the situation of most firms sufficiently. The main problem is that they ignore uncertainties inherent in any managerial decisions. In fact, the future demand can be only roughly estimated. Production costs vary due to fluctuations in raw material prices. Finally, the volume of production that company would be able to subcontract cannot be precisely evaluated. As a result, uncertainty and risk have to be taken into account while constructing a production plan.

In order to solve a multiple criteria problem, single-criterion evaluations must be aggregated. Roy (1985) identified three main aggregating concepts: a concept with a single synthetic criterion, outranking concept, and dialog concept with “trial-and-error” iterations. The last idea, also known as “interactive approach”, is often used for solving real-world problems.

Initially, interactive approach was used for solving decision making problems under certainty. In uncertain context interactive methods are mainly used for solving multiobjective linear programming problems (Novak and Ragsdale, 2003; Urli and Nadeau, 2004). On the other hand, interactive procedures are also proposed for discrete problems, where the number of feasible solutions is moderate (Nowak, 2004; Nowak, 2006).

In this paper an interactive procedure for aggregate production planning is proposed. The problem is formulated as a multiple criteria decision making problem. Instead of cost minimization, three other criteria are considered: minimization of inventory, minimization of production volume outsourced to a subcontractor, and minimization of fluctuations in production rate. The procedure combines linear programming, simulation, and interactive approach. Linear programming models are used to generate initial solutions. In order to check how the fluctuations in demand affect the results obtained under each of these solutions simulation experiments are performed. Finally, an interactive procedure is used for identifying the final solution of the problem.

2. MATHEMATICAL PROGRAMMING APPROACHES FOR AGGREGATE PRODUCTION PLANNING

None organization can operate without a planning system. Usually companies make several plans at different levels of aggregation, using different planning horizons (Thomas and McClain, 1993). Tactical plans should be harmonized with company's long-term goals and work within the resources allocated by earlier strategic decisions. They are also the starting point for short-term production scheduling. Decisions made on this level involve a medium-range planning horizon (typically one year), and aggregation of items into product families (Bitran and Tirupati, 1993).

Some examples of questions that the aggregate plan should answer are as follows (Waters, 2002):

- Should the production rates be constant, or should they be adjusted to match the demand requirements in successive planning periods?
- Should subcontractors be used to overcome capacity shortages in some periods?
- Should work-force levels be adjusted by hiring or laying off employees?
- Is back ordering a viable alternative?

A good plan balances the conflicting objectives of minimizing production cost, maximizing customer service, minimizing inventory investments, maintaining a stable workforce. Several options can be used to absorb demand fluctuations, including changing inventory levels, subcontracting, varying production rates through overtime and idle time.

Quantitative approaches used in aggregate production planning include, among others, linear programming (Shapiro, 1993), mixed integer programming (Vollmann, Berry, Whybark and Jacobs, 2005) and dynamic programming (Trzaskalik, 1990).

Below, an example of a linear programming model for aggregate production planning problem is presented. The basic assumptions are as follows:

- The forecasts of the demand for the next T periods have been prepared – for each month probability distribution of the demand is available.
- The model should provide a plan that minimizes the particular objective assuming that in each month the demand is equal to the mean of the probability distribution representing it's fluctuations.
- The company would like to maintain a high level of the customer service. Therefore, the final inventory in month t should be high enough to guarantee a high probability of meeting the demand in month $t + 1$.

- The premier objective is to minimize total cost, which includes production cost, inventory cost and the cost of the idle time. As the model assumes that shortages are not allowed, the cost of delays in deliveries is not taken into account.
- The production is measured in production hours required for its completion.

The notation used in the model is as follows:

c_R – the cost per labor-hour of regular time production,

c_O – the cost per labor-hour of overtime production,

c_S – the cost per labor-hour of subcontractor production,

c_I – the cost per month of carrying one labor-hour of work,

c_U – the cost per labor-hour of idle regular time production,

x_t – the regular time production hours scheduled in month t,

o_t – the overtime time production hours scheduled in month t,

s_t – the subcontractor time production hours scheduled in month t,

i_t – the number of working hours stored in inventory at the end of month t,

u_t – the number of idle time regular production hours in month t,

d_t – the expected demand in month t (hours of production),

b_t – the highest demand the company should to be able to satisfy in month t (hours of production),

m_{t1} – the maximum number of regular time hours in month t,

m_{t2} – the maximum number of overtime hours in month t,

m_{t3} – the maximum number of subcontractor hours in month t,

r_t – the reduction in the number of production hours scheduled in month t compared to the number of production hours scheduled in month t – 1,

g_t – the increase in the number of production hours scheduled in month t compared to the number of production hours scheduled in month t – 1,

a_1 – initial inventory,

T – the number of months in the planning horizon,

The production plan should satisfy the following constraints:

$$i_{t-1} + x_t + o_t + s_t - i_t = d_t \qquad \text{for } t = 1, ..., T \qquad (1)$$

$$i_{t-1} + x_t + o_t + s_t \geq b_t \qquad \text{for } t = 1, ..., T \qquad (2)$$

$$x_t + u_t = m_{1t} \qquad \text{for } t = 1, ..., T \qquad (3)$$

$$x_t + o_t + s_t + r_t - g_t = x_{t-1} + o_{t-1} + s_{t-1} \qquad \text{for } t = 2, ..., T \qquad (4)$$

$$i_0 = a_1,\ r_1 = 0,\ g_1 = 0 \qquad (5)$$

$$0 \leq x_t \leq m_{t1},\ 0 \leq o_t \leq m_{t2},\ 0 \leq s_t \leq m_{t3} \qquad \text{for } t = 1, 2, ..., T \qquad (6)$$

$$r_t \geq 0,\ g_t \geq 0,\ u_t \geq 0 \qquad \text{for } t = 1, 2, ..., T \tag{7}$$

In this paper we minimize four conflicting criteria:

f_1 – the total cost:

$$\sum_{t=1}^{T} \left(c_R x_t + c_O o_t + c_S s_t + c_I i_t + c_U u_t \right) \tag{8}$$

f_2 – the total number of the overtime production hours:

$$\sum_{t=1}^{T} o_t \tag{9}$$

f_3 – the total number of the subcontractor production hours:

$$\sum_{t=1}^{T} s_t \tag{10}$$

f_4 – the total fluctuations in production rates:

$$\sum_{t=1}^{T} \left(r_t + s_t \right) \tag{11}$$

Linear programming is a good tool if the company is able to prepare precise forecasts of the future demand. Unfortunately, most companies operate in uncertain environment. If the demand cannot be predicted with high precision, another tools should be used to analyze the results of the production plan. This applies in particular to criterion f_1, as the estimates of the other criteria arise directly from the production plan. Simulation is a good tool for such analysis. In our procedure we use it for determining probability distributions of the total cost. It allows also to estimate the overall customer service level. In our simulation model we will consider the case of delayed deliveries. We will assume, that the cost the company has to cover in the case of delays is equal to c_D per one production hour.

3. THE PROCEDURE FOR AGGREGATE PRODUCTION PLANNING

The solutions of single-criterion linear programming problems with criteria functions specified above provide solutions minimizing the objectives under consideration assuming that in each period sales are equal to the expected demand. In this paper, however, we consider the situation of uncertain demand. In such case, we cannot estimate the values of the criterion f_1 precisely. As we assume here that the plan is implemented regardless of the actual demand, values of the other criteria arise directly from the production plan.

The technique that we propose combines linear programming, simulation and interactive approach. First, four single-criterion line programming problems are solved. Next simulation is used to analyze how good are these plans with respect to criterion f_1. Finally, interactive procedure is used to determine the production plan satisfying decision maker's requirements. Initially four solutions are proposed to the decision maker. If he/she accepts any of proposals, the procedure ends. Otherwise the decision maker is asked to indicate maximal acceptable values of criteria f_2 – f_4. The new proposal is identified by solving linear programming problem in which the total cost is minimized under additional constraints on the values of the other criteria. Again the simulation model is used to analyze the cost of new production plan and the results are presented to the decision maker.

The following steps are performed to complete the procedure:

1. Solve four linear programming problems with objective functions given by (8) – (11) and constraints given by (1) – (7).
2. Perform simulation runs to evaluate the total cost of production plans determined in step 1.
3. Present the solutions generated in step 1 to the decision maker and ask him/her whether he/she accepts any of them as a final solution. If the answer is *ÝES* – end the procedure.
4. Ask the decision maker to specify maximal acceptable values of criteria f_2 – f_4.
5. Generate a new proposal for the decision maker solving the linear programming problem in which the total cost is minimized under additional constraints on the values of the other criteria. If the problem is infeasible – notify the decision maker and go back to step 4.
6. Perform simulation runs to evaluate the total cost of production plan determined in step 5.
7. Present the new proposal to the decision maker. If he/she is satisfied with the proposal – end the procedure, otherwise – ask the decision maker to redefine his/her requirements – go back to step 4.

Comments:

Step 1: As alternate solutions of linear problems may exist, we recommend to use hierarchical approach. Once the minimum value of criterion f_k is identified, other criteria are minimized preserving the minimal value of f_k.

Step 2: For each solution generated in step 1, a series of simulation runs is performed taking into account probability distributions of the demand. The results are used to construct probability distributions describing how good are the solutions with respect to the criterion f_1.

Step 4: We assume here, that f_1 is the most important criterion. In order to consider decision maker's preferences on the values of other criteria additional constraints are defined specifying maximum values of them.

Step 5: If the decision maker's requirements on the criteria $f_2 - f_4$ are to strong, the problem may be infeasible. In this case the decision maker is asked to redefine constraints on criteria other than cost.

Step 7: The new solution meets the decision maker's requirements on the criteria $f_2 - f_4$. However, the increase of the cost may be too high for the decision maker. Therefore, it is proposed to allow the decision maker to change the constraints defined previously.

4. NUMERICAL EXAMPLE

To illustrate applicability of the procedure let us consider following example. A company prepares an aggregate plan. Time horizon is 6 months. Basic data are as follows:

$c_R = 1{,}00$, $c_O = 1{,}50$, $c_S = 1{,}70$, $c_I = 0{,}30$, $c_U = 0{,}50$, $c_D = 5{,}00$.

$m_{t\,1} = 900$, $m_{t\,2} = 100$, $m_{t\,3} = 300$ for $t = 1, \ldots, 6$.

Probability distributions of the demand are presented in table 1.

Table 1: Probability distributions of the demand

Month 1		Month 2		Month 3		Month 4		Month 5		Month 6	
Demand	Prob.	Demand	Prob.	Demand	Prob.	Demand	Prob.	Demand	Prob.	Demand	Prob.
620	0,05	800	0,05	1020	0,05	900	0,05	740	0,05	620	0,05
640	0,1	820	0,05	1040	0,1	920	0,05	760	0,05	640	0,1
660	0,25	840	0,1	1060	0,1	940	0,1	780	0,1	660	0,1
680	0,2	860	0,25	1080	0,3	960	0,25	800	0,3	680	0,3
700	0,15	880	0,25	1100	0,25	980	0,25	820	0,25	700	0,25
720	0,1	900	0,15	1120	0,1	1000	0,15	940	0,15	720	0,1
740	0,1	920	0,1	1140	0,05	1020	0,1	960	0,05	740	0,05
760	0,05	940	0,05	1160	0,05	1040	0,05	980	0,05	760	0,05

Means of the demand are as follows:

$d_1 = 685$, $d_2 = 874$, $d_3 = 1087$, $d_4 = 974$, $d_5 = 836$, $d_6 = 687$.

The company assumes that the initial stock in each month should be enough to provide 95% probability of meeting the demand. Thus, the highest demand that the company should be able to satisfy is as follows:

$b_1 = 740$, $b_2 = 920$, $b_3 = 1140$, $b_4 = 1020$, $b_5 = 960$, $b_6 = 740$.

The initial inventory is $i_0 = 0$.

The procedure operates as follows.

Iteration 1

Step 1: Four single criterion linear programming problems with constraints given by (1)-(7) are solved. Results are presented in table 2.

Table 2: Solutions of linear programming problems solved in iteration 1.

Month	Solution 1			Solution 2			Solution 3			Solution 4		
	x_t	o_t	s_t	x_t	o_t	s_t	x_t	o_t	s_t	x_t	o_t	s_t
1	800	0	0	800	0	0	800	100	0	800	100	17
2	800	5	0	800	0	5	800	100	0	800	100	17
3	800	100	194	800	0	294	800	100	40	800	100	17
4	800	100	67	800	0	167	800	100	40	800	100	17
5	800	100	14	800	0	114	800	100	0	800	100	17
6	616	0	0	616	0	0	616	0	0	800	100	17

Step 2: Simulation experiments are performed to evaluate production plans determined in step 1 with respect to criterion f_1..

Step 3: The solutions identified in step 1 are presented to the decision maker.

Table 3: Initial solutions proposed to the decision maker.

Solution	f_1 (mean)	f_2	f_3	f_4	Service level
1	6027,5	305	275	772	99,06%
2	6080,5	0	580	772	99,09%
3	6061,2	500	80	364	99,33%
4	6303,4	600	102	0	99,69%

The decision maker says, that he is not fully satisfied with any of the proposals.

Step 4: The decision maker specifies the maximum acceptable values of criteria: $f_2 \le 300$, $f_3 \le 300$, $f_4 \le 50$.

Step 5: The new linear programming problem is formulated. Objective function f_1 is minimized under the constraints (1)-(7) and additionally:

$$\sum_{t=1}^{T} o_t \le 300 \tag{12}$$

$$\sum_{t=1}^{T} s_t \le 300 \tag{13}$$

$$\sum_{t=1}^{T} (r_t + s_t) \le 50 \tag{14}$$

As the problem is infeasible, the decision maker is asked to redefine his requirements.

Step 4: The decision maker decides to weaken the constraint on the value of the criterion f_4 – the maximum value of this criterion is changed to 150.

Step 5: The linear programming problem with new constrained is solved. The solution is presented in table 4.

Table 4: The solution no. 5 proposed to the decision maker.

Month	Solution 5		
	x_t	o_t	s_t
1	800	100	16,5
2	800	100	16,5
3	800	0	116,5
4	800	100	16,5
5	800	0	114
6	800	100	16,5

Step 6: Simulation runs are performed to evaluate the total cost of production plan defined by solution no. 5.

Step 7: The results obtained for the solution no. 5 are presented to the decision maker (table 5).

Table 5: The results obtained for the solution no. 5.

Solution	f_1 (mean)	f_2	f_3	f_4	Service level
5	6104,6	300	280	150	99,65%

According to the decision maker, the total cost is too high. Therefore, the procedure goes back to step 4.

Step 4: The decision maker decides to weaken the constraint on the value of the criterion f_4 – the maximum value of this criterion is changed to 400.

Step 5: The linear programming problem with new constrained is solved. The solution is presented in table 6.

Step 6: Simulation runs are performed to evaluate the total cost of production plan defined by solution no. 6.

Step 7: The results obtained for the solution no. 6 are presented to the decision maker (table 7).

Table 6: The solution no.6 proposed to the decision maker.

Month	Solution 6		
	x_t	o_t	s_t
1	800	0	84
2	800	84	0
3	800	16	132
4	800	100	50
5	800	100	14
6	616	0	0

Table 7: The results obtained for the solution no. 6.

Solution	f_1 (mean)	f_2	f_3	f_4	Service level
6	6065,9	300	280	400	99,19%

The decision maker decides to accept current solution.

4. CONCLUSIONS

Aggregate production planning provides a key communication links for top management to coordinate the various planning activities in a company. From a manufacturing perspective, it provides the basis to focus the detailed production resources to achieve the firm's strategic objectives.

Various mathematical programming formulations are proposed for aggregate production planning. The objective is typically to find the lowest-cost plan. However, other criteria are also taken into account by managers, including for example minimizing fluctuations in production runs or minimizing the volume of production outsourced to subcontractors. As a result, the aggregate planning can be considered as a multiple criteria decision making problem.

In this paper the interactive procedure for aggregate production planning was proposed. It was assumed that cost minimization is the most important objective. The dialog with the decision maker is conducted in order to find a solution that is acceptable with respect to all criteria. The idea of the procedure is quite simple and can be easy implemented.

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