

APPLICATION OF THE DEMPSTER-SHAFER THEORY IN CONCEPTUAL DESIGN OF THE MACHINING CENTRES

Mirko Djapic, Ljubomir Lukic

Original scientific paper

In the development phase, the designer makes decisions under conditions which are characterized with some degree of uncertainty, contradiction and ignorance. These conditions appear more often in the early phases of the design process. Following this, in this paper is presented the way of using evidential systems which represent a form of expert systems in which knowledge is represented and processed by the function of the Belief Function Theory (Dempster-Shafer theory). The paper is focused on the conceptual design of machine tools, i.e. determination of the best structure of machining centre in the phase of conceptual design for a predefined group of parts that will be machined on them.

Keywords: *Conceptual Design, Dempster-Shafer Theory, Design Theory, PERA Knowledge*

Primjena Dempster-Shafer teorije u konceptijskom projektiranju obradnih centara

Izvorni znanstveni članak

Projektant tijekom razvoja donosi odluke u uvjetima punim neodređenosti, kontradiktornosti i neznanja. Ti uvjeti su više izraženi u ranim fazama procesa projektiranja. Slijedeći to, u ovom članku predstavlja se način uporabe dokaznih sustava koji predstavljaju oblik ekspertnih sustava u kojima se znanje predstavlja i procesira uporabom Teorije funkcije uvjerenja (Dempster-Shafer teorija). Članak je fokusiran na konceptijsko projektiranje alatnih strojeva tj. određivanje najbolje konstrukcije obradnog centra u fazi konceptijskog projektiranja za prethodno definiranu skupinu dijelova koja će na njemu biti obradivana.

Ključne riječi: *Dempster-Shafer teorija, konceptijsko projektiranje, PERA znanje, teorija projektiranja*

1 Introduction

The designing process is a set of activities and their related resources, through which a team of designers develops or selects means for achieving a certain goal under strictly defined conditions and restrictions. The transformations are carried out by designer's work, using his knowledge, tools, working methods and other available resources. One aspect here that should be emphasized as important is that engineering design is a very creative human activity in which decisions are being made, information and knowledge are being processed, and moreover in which you learn, acquire new knowledge and experiences. And finally, through this activity the products, processes and systems are being made as a result of previously expressed needs.

The generally accepted opinion of researchers and practitioners involved in improving engineering design is that those are still not well studied and understood activities, either in education, research or everyday practice [1]. As confirmation of this, there has been mentioned over-reliance on experience during work or work without proper tools which would guide designers throughout entire process. During development the designer makes decisions in conditions of full uncertainty, contradiction, and not having enough knowledge about the matter. Are those decisions right and to which extent? How much do they affect customer's satisfaction? These are just some of the issues that designers face every day. Their openness is higher with development process being in early conceptual stages.

For example, during the conceptual design stage more than ten versions are to be generated for the best solution [2]. The best solution means the lowest cost of product design and manufacturing maintaining required product performance. It requires an introduction of new methods

and tools which enable improvement of designers' knowledge and skills in the process of decision making [3].

Following these requests, in this paper is presented the way of using evidential systems [4] which are developed based on belief function theory (Dempster-Shafer's theory) [4, 5, 1] in conceptual designing. The main objectives are to improve designer's capability in decision-making process in the early stages of design. It is important to note that designer during the design process is moving inside uncertainty space and that uncertainty is greater in initial stages. Part of previously mentioned objectives can be illustrated as an effort made to reduce this uncertainty space. This applies to the development of methods that would allow designer to see the effects of decision immediately after making one, i.e. whether it is optimal or not.

As additional to the above mentioned the paper gives an approach to using new tools in the conceptual phase of the design process. We describe the use of these tools through determination of the best structure of machining centres for a predefined group of parts that will be machined on them.

2 Theory of belief functions

Making conclusions (reasoning) about certain situation from the real world often takes place in difficult circumstances with insufficient knowledge, not clearly defined criteria and mutual antagonism. Information about evidence can come from different resources: based on a person's experience, from signals recorded by appropriate sensors, from the contents (the context) of published papers and so on. Such evidence is rarely clearly delimited; it is often incomplete, ambiguous in its meaning and full of flaws.

Dempster-Shafer belief function theory provides powerful tools for mathematical presentation of the subjective (opposite of what probability theory is based on) uncertainty while it relies mainly on the possibility of explicit definition of ignorance [5]. This theory is intuitively adapted formalism for reasoning below uncertainty limit. It, actually, represents the generalization of Bayesian theory of conditional probability. As such, it provides formally consistent method for interpretation and connection of evidence, which inside itself carries some degree of uncertainty, and in addition, provides getting meaningful answers to posed questions using only partial evidence. Complete records can be used only in necessary cases.

2.1 The basic concepts of belief functions

Model of the belief function consists of variables, their values and the evidence, which supports the value of variables. Variables represent specific questions regarding the aspect of the problem under consideration. Given questions are answered using data originating from various sources, i.e. from context of published papers, from measurement data, from expert opinions, etc. Fully integrated support to the sought answer is called evidence.

Evidence can be represented by belief functions, which are defined as follows:

Definition 1. [6] Let Θ be a finite nonempty set called the frame of discernment, or simply the frame. Mapping $Bel: 2^\Theta \rightarrow [0,1]$ is called the (unnormalized) belief function if and only if a basic belief assignment (bba) $m: 2^\Theta \rightarrow [0,1]$ exists, such that:

$$\sum_{A \subset \Theta} m(A) = 1 \tag{1}$$

$$Bel(A) = \sum_{B \subset A, B \neq \emptyset} m(B) \tag{2}$$

$$Bel(\emptyset) = 0 \tag{3}$$

Expression $m(A)$ can be viewed as the measure of belief which corresponds to subset A and takes values from this set.

Condition (1) means that one's entire belief, supported by evidence, can take the maximum value 1, and condition (3) refers to the fact that one's belief, corresponding to an empty set, must be equal to 0.

Value $Bel(A)$ represents the overall belief corresponding to the set A and all of its subsets.

Each subset A such that $m(A) > 0$ is called a focal element.

The empty belief function is the function which satisfies $m(\Theta) = 1$, and $m(A) = 0$ for all subsets of $A \neq \Theta$. This function represents total ignorance about the problem under consideration.

2.2 Dempster rule of combining belief functions

Let the several independent belief functions be given on the same recognition frame but with different bodies of evidence. The Dempster's combination rule (Fig. 1) (4, 5) produces new belief function which represents effect re-

sulting from the connection of the different bodies of evidence.

Let us assume that the belief functions Bel_1 and Bel_2 are created on Θ frame. Let $A_1, \dots, A_k, k < 2^{|\Theta|}$ be the focal elements of function Bel_1 with corresponding m - values $m_1(A_i)$ for $i=1, \dots, k$; and let $B_1, \dots, B_j, j < 2^{|\Theta|}$ be focal elements of function Bel_2 with corresponding m - values $m_2(B_i)$ for $i=1, \dots, j$.

Combination of these two functions is denoted as $Bel_1 \oplus Bel_2$ and its focal elements are C_1, \dots, C_m with corresponding m - values $m_3(C_k)$ for $k=1, \dots, m$, created in the following way:

$$m_3(C_k) = K \left[\sum_{\substack{i,j \\ A_i \cap B_j = C_k}} m_1(A_i) m_2(B_j) \right], \tag{4}$$

where K represents a normalization factor

$$K = \left[1 - \sum_{\substack{i,j \\ A_i \cap B_j = \emptyset}} m_1(A_i) m_2(B_j) \right]^{-1}. \tag{5}$$

The normalization factor K is greater than 1 whenever Bel_1 and Bel_2 contain a part of mass of some belief that corresponds to the subjective probability for the decoupled (contradictory) subsets of Θ . In fact, K represents the conflict measure of the two belief functions. Whenever two or more functions are combined, the combination rule is associative and commutative. In general, $Bel \oplus Bel = Bel$. Combination of a certain number of belief functions $Bel_1 \oplus \dots \oplus Bel_n$ is denoted as $\oplus \{Bel_1, \dots, Bel_n\}$.

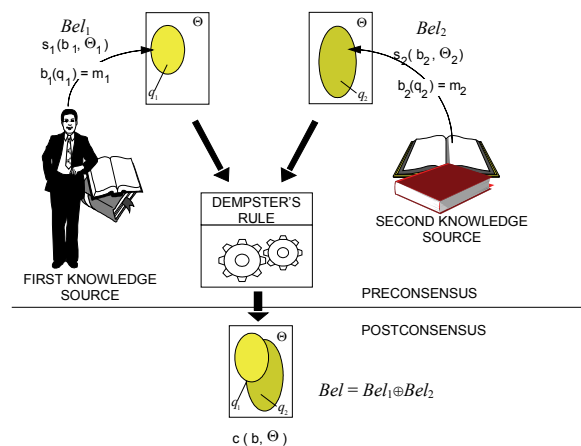


Figure 1 Graphics illustration using Dempster's rule of belief function combining [1]

2.3 What are the evidential systems?

Valuation Based Systems - VBS is an abstract framework proposed by Shenoy [4] for representing and reasoning on the basis of uncertainty. It allows representation of uncertain knowledge in various domains, including Bayes' probability theory, Dempster-Shafer's theory of evidence [5] which is based on belief functions and

Zadeh-Dubais-Prad theory of possibility. Graphically presented VBS is called valuation network.

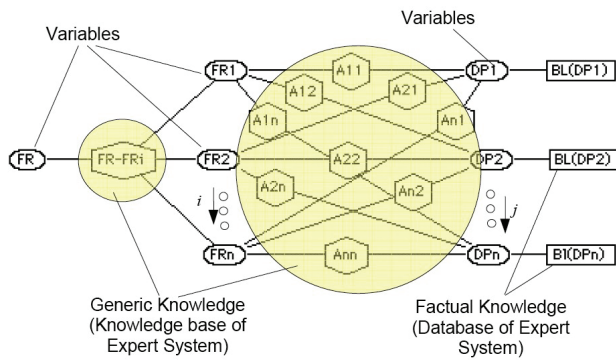


Figure 2 The concept of evidential networks

VBS consists of a set of variables and a set of valuations that are defined on the subsets of these variables. The set of all variables is denoted by U and represents a space covered with the problem which is under consideration. Each variable represents a relevant aspect of the problem. For each variable X_i will be used $\mathcal{O}X_i$ to denote the set of possible values of variables called the frame of X_i . For a subset A ($|A| > 1$) of U , set of valuations that are defined over $\mathcal{O}A$ represents the relationship between variables in A . Frame $\mathcal{O}A$ is a direct (Cartesian) product of all $\mathcal{O}X_i$ for X_i in A . The elements $\mathcal{O}A$ are called configurations of A .

Knowledge presented in this type of valuations is called generic or general knowledge (Fig. 2), which can be represented as a knowledge base in expert systems.

The VBS also defines valuations on individual variables, which represents the so-called factual knowledge, and it constitutes database in expert systems (Fig. 2). For a problem, general-generic knowledge defines an expert. During reasoning process that knowledge will not be modified. Factual knowledge will vary in accordance with condition of the problem currently being under consideration. The VBS treats in the same way these two kinds of knowledge.

The VBS systems suited for processing uncertain knowledge described by functions of belief function theory are called Evidential Reasoning Systems or Evidential Systems, and valuation networks are now called evidential networks (EN) (Fig. 2).

The objective of reasoning based on the evidence is an assessment of a hypothesis, in case when the actual evidence is given (the facts). This can be accomplished by evaluating valuation networks in two steps [4]:

- Combining all belief functions in evidential network, resulting in a so-called global belief function;
- Marginalization of global belief functions in the framework of each individual variable or subsets of variables produces marginalized values for each variable or subset of variables.

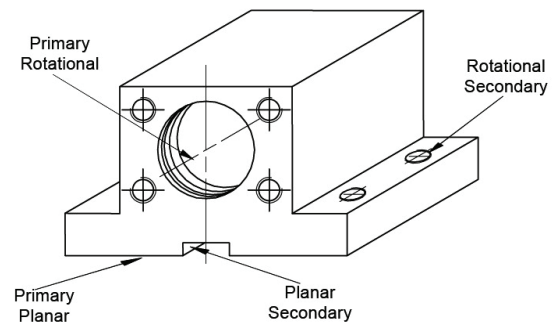
Easy way of understanding the reasoning process and its graphical interpretation is the condition on which depends whether and how fast these systems will be applied in solving everyday problems. As a software support to the VBS systems application, several software tools have been developed. For evidential systems the very known

are: McEvidence, Pulcinella and DELIEF. McEvidence is an application that was developed for reasoning under conditions of uncertainty (Figs. 5, 7 and 8). Using this system the user can create a graphical network of variables, their relationship and to bring in any records related to the variables. When all available input records that reflect current system status or process under analysis are being entered, evaluation of network can start.

During evaluation process first the global belief function is generated by applying combining operation and then afterwards the marginalized values of all variables are calculated.

3 Using evidential systems in conceptual design

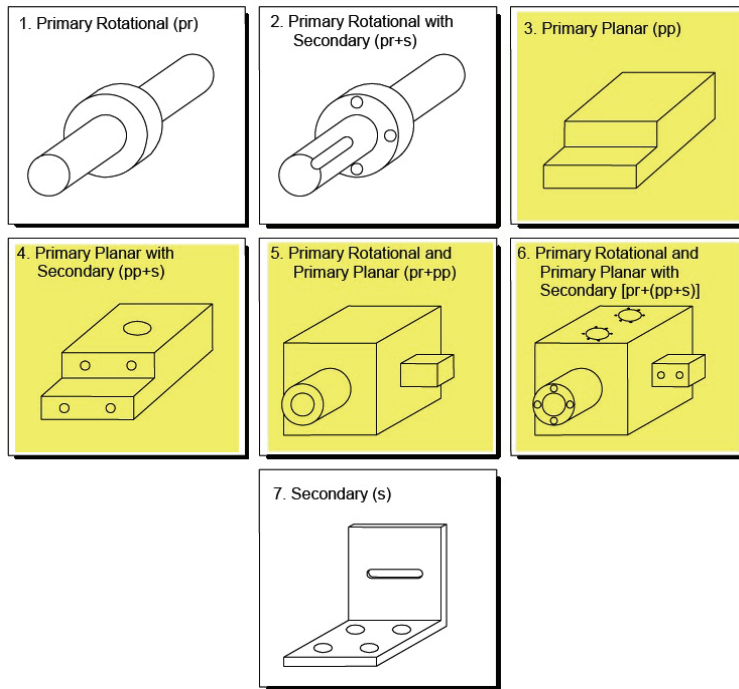
This point has two explicit goals. The first one concerns the presentation of ability of evidential networks to absorb knowledge that was generated for years in manufacturing engineering field. The second one refers to extension of previously generated evidential networks and their usage as auxiliary tool in decision-making process. The realization of these goals will be achieved through an example that relates to the choice of machine tool concept (machining centre) for processing a hypothetical group of parts.



Figures 3 Different classes of surfaces [7, 8]

British association of production engineers (PERA) has done a review of requests for machining parts in industry [7, 8]. Statistical data related to processing parameters are presented for seven categories of parts. The categorization has been done based on the type of surfaces that are processed (Fig. 3).

The machined surface types are primary rotational, primary planar, rotational secondary, and planar secondary, as shown in Fig. 3. Primary surfaces give the parts their general outline shape. Secondary surfaces, such as slots, groves, tapped holes, are machined out of primary surface. The division of surfaces into primary and secondary has not been done according to the functional significance or according to the complexity of machining. The classification of all parts into seven categories has been done according to combination of these surfaces and it is shown in Fig. 4. According to this and corresponding statistics in Tab. 1, we get the percentage of use of various machine tools, or types of mechanical operations for machining each category of parts. These percentage relations are kept also for total number of operations, and for final machining. That way in first category of parts 82 % of the work represents turning and 18 % grinding, we will also assume that 82 % of the total area is finished by turning operations and that 82 % of total number of operations are turning operations.



Figures 4 Categorization of parts (Adapted from [7, 8])

Table 1 The percentage share from total assumed work for each operation of machining divided into categories of parts (Adapted from [7] and [8])

	Parts Category						
	1 (pr)	2 (pr+s)	3 (pp)	4 (pp+s)	5 (pr+pp)	6 [pr+(pp+s)]	7 (s)
Turning	82	54.6			43	23.4	
Milling			80	44	35	20.2	5.2
Drilling		27.4		32		32.3	67.5
Threading		10.5		13		12.7	27.2
Grinding	18	7.5	20	11	22	11.2	

In the remaining part of this paper, it is shown how to apply this generated knowledge into conceptual design of machine tools. Belief function theory together with evidential systems, or evidential networks, made it possible to present this knowledge in an appropriate form, and later to use it as an aid in decision making process.

The initial goal is to implement an expert system that will provide help in conceptual design stage to a designer who needs to choose a concept of machine tools structures. The application will be presented for a practical example in which a designer got an assignment to choose the right kinematical structure of CNC machine tools for multi-axis processing for a set of parts from the third, fourth, fifth and sixth category of parts from Fig. 4.

The PERA knowledge for previous group of parts from Tab. 1 is presented using evidential network (Fig. 5) with our subjective probability that this data are reliably being 0,9. Elements of the evidential network from A_{11} to A_{43} are presented with belief functions ("Input belief of A_{11} " to "Input belief of A_{43} ").

Let us assume that in the mentioned example the designer has at his disposal modules of the main rotating movements, modules of linear and rotating auxiliary movements of horizontal and vertical machining centres (Fig. 6).

If we analyse how appropriate are particular structures of machining centres for certain types of machining over prismatic work pieces (e.g. milling, turning and drilling) the following can be concluded.

The horizontal machining centres are most suitable for face milling process, less suitable for turning and the least suitable for drilling over prismatic work pieces. The supporting structure of vertical machining centres is most suitable for drilling operations, less suitable for turning and least suitable for milling lateral surfaces of prismatic work pieces. If these expertise conclusions we describe with functions of belief functions theory and include into evidential network, we will get the knowledge base for underlying example (Fig. 7). This knowledge does not change during analysis. In evidential network defined as this one it is possible now to enter the factual knowledge (Fig. 8) which is related to machining of actual group of parts from the class three, four, five and six from Fig. 4. Let us assume that a group has 70 % parts of class three, and 10 % of class four, five and six each. This information is presented by using belief function Bel_1 (Input belief of Bl_1), Bel_2 (Input belief of Bl_2), Bel_3 (Input belief of Bl_3) and Bel_4 (Input belief of Bl_4) (Fig. 8).

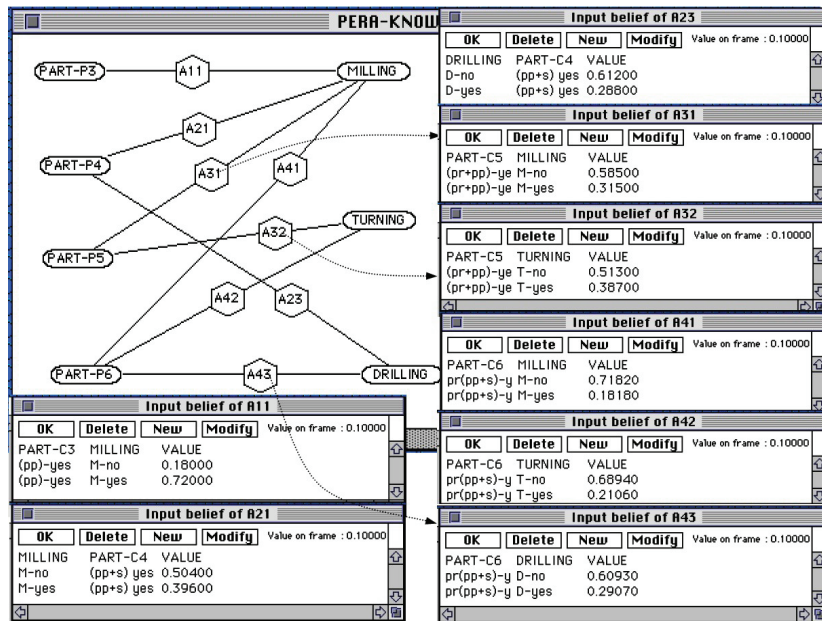


Figure 5 Knowledge from Tab. 1 presented as evidential network (for category of parts 3, 4, 5, 6 and machining operations: milling, turning and drilling)

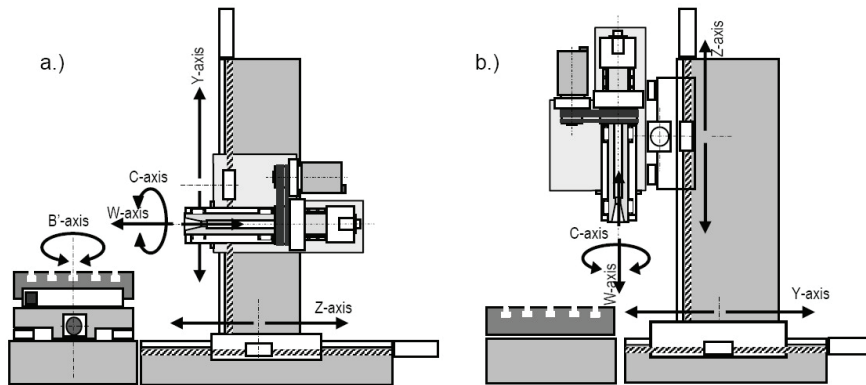


Figure 6 The concept of horizontal (a) and vertical machining center (b) [9]

With evaluation of evidential network this information changes states at all nodes including the output node MT_MC which is in charge of choosing the concept of machining centre for multi-axis machining of spatially complex surfaces. This state is expressed by output belief functions MT_MC (Output belief of MT_MC). In this case, for selected structure of group of parts the concept of horizontal machining centre with rotating table (B-axis) has basic belief assignment (bba) $m(\{HMC\})=0,51827$, concept of vertical machining centre with machining in several clamping stages has $m(\{VMC\})=0,32908$ and ignorance about which concept is better $m(\{HMC,VMC\})=0,15295$. From this it can be concluded that for the assumed group of parts more suitable is the concept of horizontal machining centre with CNC control of supporting movements in direction of three linear axis and one rotating axis which is in the same time the axis of horizontal rotating table on which the work piece is placed and that way it places the surface that is being machined perpendicular to the axis of the main spindle.

Tab. 2 gives data that shows how basic belief assignment (bba) related to conceptual solutions of machine is

changed depending on the percentage content of particular groups of parts. That way, for group which consist of 20 % parts of third category, 20 % of fourth category, 50 % of fifth category and 10 % of sixth category, both machine concepts are pretty much suitable. bba's m are: $m(\{HMC\}) = 0,41961$, $m(\{VMC\}) = 0,41407$ and $m(\{HMC,VMC\}) = 0,16644$. For group of parts consisting only of parts of the third category more suitable is the concept of horizontal machining centre [$m(\{HMC\}) = 0,53708$, $m(\{VMC\}) = 0,13435$ and $m(\{HMC,VMC\}) = 0,33073$] as for the group of parts made only from parts of the fourth category [$m(\{HMC\}) = 0,48320$, $m(\{VMC\}) = 0,31602$ and $m(\{HMC,VMC\}) = 0,20323$].

For group of parts made only from parts of the fifth or the sixth category more suitable is vertical conception of machining centre for the fifth category of parts [$m(\{HMC\}) = 0,27746$; $m(\{VMC\}) = 0,51536$; $m(\{HMC,VMC\}) = 0,20923$], for sixth category of parts [$m(\{HMC\}) = 0,32130$; $m(\{VMC\}) = 0,52401$; $m(\{HMC,VMC\}) = 0,15496$].

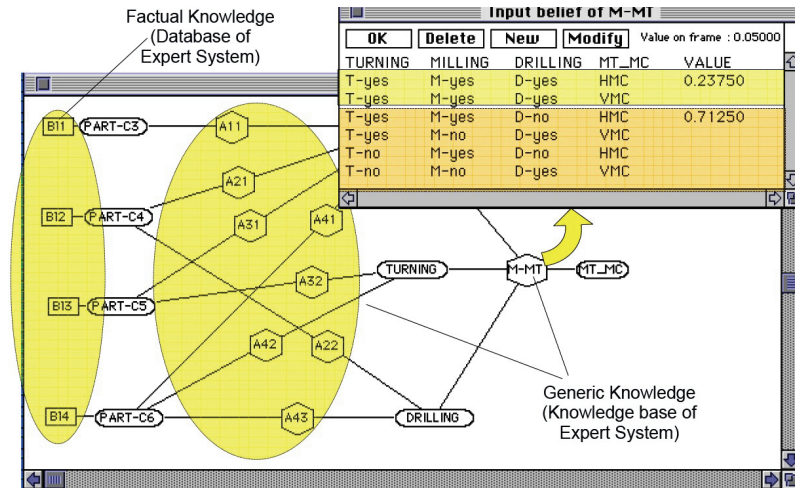


Figure 7 The knowledge base $Bel[A_{11} \text{ to } A_{41}+(M-MT)]$ and database ($Bel_1 - Bel_4$) in choosing the concept of machining centre

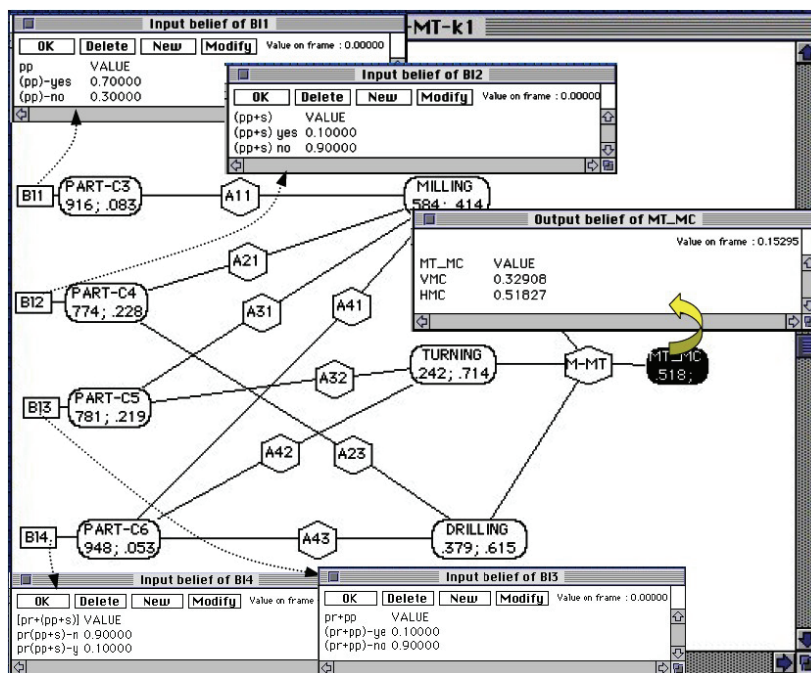


Figure 8 Evidential network with input and output beliefs (First row from Tab. 2)

Table 2 Distribution of beliefs for various concepts of machine centres (MT_MC)

Part Group No.	Third, Fourth, Fifth and Sixth Category Part Percentage Including in the Group (Input Belief Functions)												Belief Distribution (Output Belief Functions)		
	Bel_1 (pp)			Bel_2 (pp+s)			Bel_3 (pr+pp)			Bel_4 [pr+(pp+s)]			Bel (MT_MC)		
	$m(pp)$	$m(\sim pp)$	$m(pp, \sim pp)$	$m(pp+s)$	$m[\sim(pp+s)]$	$m[(pp+s), \sim(pp+s)]$	$m(pr+pp)$	$m(\sim pr+pp)$	$m[(pr+pp), \sim(pr+pp)]$	$m[pr+(pp+s)]$	$m\{\sim[pr+(pp+s)]\}$	$m\{[pr+(pp+s)], \sim[pr+(pp+s)]\}$	$m_{(HMC)}$	$m_{(VMC)}$	$m_{(HMC,VMC)}$
1	0,7	0,3	0,0	0,1	0,9	0,0	0,1	0,9	0,0	0,1	0,9	0,0	0,51827	0,32908	0,15295
2	0,2	0,8	0,0	0,2	0,8	0,0	0,5	0,5	0,0	0,1	0,9	0,0	0,41961	0,41407	0,16644
3	0,05	0,95	0,0	0,05	0,95	0,0	0,35	0,65	0,0	0,55	0,45	0,0	0,31944	0,51378	0,16687
4	0,15	0,85	0,0	0,35	0,65	0,0	0,2	0,8	0,0	0,3	0,7	0,0	0,40844	0,42167	0,17004
5	0,0	1,0	0,0	0,0	1,0	0,0	0,2	0,8	0,0	0,8	0,2	0,0	0,24178	0,61076	0,14754
6	1,0	0,0	0,0	0,0	1,0	0,0	0,0	1,0	0,0	0,0	1,0	0,0	0,53708	0,13435	0,33073
7	0,0	1,0	0,0	1,0	0,0	0,0	0,0	1,0	0,0	0,0	1,0	0,0	0,48320	0,31602	0,20323
8	0,0	1,0	0,0	0,0	1,0	0,0	1,0	0,0	0,0	0,0	1,0	0,0	0,27746	0,51536	0,20923
9	0,0	1,0	0,0	0,0	1,0	0,0	0,0	1,0	0,0	1,0	0,0	0,0	0,32130	0,52401	0,15496

4 Conclusion

In order to improve the ability of designer in decision-making process regarding optimal kinematics structure, the Dempster-Shafer belief function theory can be successfully applied or evidential systems that are developed on its basis in conceptual modular design of machining centres. This is particularly important in the early stages of defining the support structure, kinematics and modules of the main and auxiliary movements of machining centres. In these stages the designer makes decisions on geometrical, energetic, technological, statistical and kinematics characteristics under conditions when there is no reliable knowledge of what parts, from what materials and what technological processes will be used for processing the work pieces at machining centre during exploitation in their life time. The developed model allows the designer to have already defined the main and optimal functional characteristics of machining centre in the phase of conceptual design, and with those characteristics he enters into calculations and dimensioning of functional modules and components, which by integration into a single machining system enable cost-effective and productive processing of prismatic parts in optimal technological conditions.

5 References

- [1] Djapic, M. Evidential Systems in Product and Process Development, Intelligent Manufacturing Series of Monographs No. 9, LOLA Institute, Belgrade, 2005 (In Serbian).
- [2] Tally, D. N. Methodology for conceptual design of robust and opportunistic systems, (PhD Dissertations), Georgia Institute of Technology, 2008.
- [3] Bumberg, E. Principles of Rapid Machine Design, (PhD Dissertations), Massachusetts Institute of Technology, 2000.
- [4] Shenoy, P.P. Valuation-Based Systems: A framework for managing uncertainty in expert systems, John Wiley & Sons, New York, 1992.
- [5] Shafer, G. A Mathematical Theory of Evidence, Princeton University Press, 1976.
- [6] Xu, H., Smets, P. Some Strategies for Explanations in Evidential Reasoning. // IEEE Transactions on Systems Man and Cybernetics. 26, 5(1996), pp. 599 – 607.
- [7] PERA. Survey of Machining Requirements in Industry. Published and Produced by PERA, Melton, UK, 1969.
- [8] Radovanovic, P. Approximate Cost Estimating for Machined Components, (MSc). University of Rhode Island, 1989.
- [9] Lukic, Lj. Flexible Manufacturing Systems, Monograph. University of Kragujevac, Faculty of Mechanical Engineering Kraljevo, 2008 (In Serbian).
- [10] Mandić, V.; Ćosić, P. Integrated product and process development in collaborative virtual engineering environment, Technical Gazette, 18, 2(2011), pp. 368-378.

Authors' addresses

Assoc. Prof. Mirko Djapic
University of Kragujevac
Faculty of Mechanical Engineering Kraljevo
Dositejeva 19, 36000 Kraljevo, Serbia
E-mail: mdjapic@yahoo.com; djapic.m@mfv.kg.ac.rs

Assoc. Prof. Ljubomir Lukic
University of Kragujevac,
Faculty of Mechanical Engineering Kraljevo,
Dositejeva 19, 36000 Kraljevo, Serbia
E-mail: ljubomir.lukic@abs.rs ; lukic.lj@mfv.kg.ac.rs

Symbols and Abbreviations

HMC – Horizontal Machining Centre

VMC – Vertical Machining Centre

Parts Category:

- (pr) - Primary rotational parts
- (pr+s) - Primary rotational with secondary parts
- (pp) - Primary planar parts
- (pp+s) - Primary planar with secondary parts
- (pr+pp) - Primary rotational and primary planar parts
- [pr+(pp+s)] - Primary rotational and primary planar with secondary parts

Variable **PART-C3** – Percent parts category 3 (pp) in hole group of parts

Variable **PART-C4** – Percent parts category 4 (pp+s) in hole group of parts

Variable **PART-C5** – Percent parts category 5 (pr+pp) in hole group of parts

Variable **PART-C6** – Percent parts category 6 [pr+(pp+s)] in hole group of parts

Joint variables A_{ij} – Connects parts category i and machining operation j (How mach machining operation j are included in machining parts category i)

Joint variable **M-MT** – Connection of machining operation and structure of machine tools

Variable **MT_MC** – Structure of machining centres (HMC or VMC)

bba – basic belief assignment m – [$m(A)$ can be viewed as the measure of belief which corresponds to subset A and takes values from this set].

Bel – Belief Functions, for example $Bel_1(pp)$:

- $Bel_1(pp)$ - Input belief function for (pp) parts
- $m(pp)$ - bba (Our belief about percent of parts (pp) divided by 100 in hole group of parts).
- $m(\sim pp)$ - bba (Our belief about percent of non-parts (pp) divided by 100 in hole group of parts).