

# INTEGRATED PROCEDURE FOR FLATNESS MEASUREMENTS OF TECHNICAL SURFACES

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Original scientific paper

In accordance with the applicable standards, this paper describes the demands for flatness of measurement tables. The paper has elaborated methods of measurement and a more detailed method of measuring flatness deviation with the use of an electronic level. Mathematical model has been described and an integrated computer model for processing, axonometric and topographic representation of the results of measuring deviations from flatness in the process of measuring with electronic level has been developed. The established mathematical model and developed software solution have been experimentally validated.

**Keywords:** deviation from flatness, electronic level, integrated computer model

## Integrirani postupak ispitivanja ravnosti tehničkih površina

Izvorni znanstveni članak

U okviru rada, sukladno važećim normama, opisani su zahtjevi koji se postavljaju na ravnost mjernih ploča. Navedene su metode mjerenja te detaljno razrađena metoda mjerenja odstupanja od ravnosti elektronskom libelom. Opisan je matematički model te razvijen integrirani računalni model za obradu, aksonometrijski i topografski prikaz rezultata mjerenja odstupanja od ravnosti u postupku mjerenja elektronskom libelom. Postavljeni matematički model i razvijeno programsko rješenje eksperimentalno su validirani.

**Ključne riječi:** elektronska libela, integrirani računalni model, odstupanje od ravnosti

## 1 Introduction

Requirements for measuring flatness deviation of technical surfaces most often occur in the domain of the parts and in checking the geometric accuracy of machine tools. The technical area is a complex three-dimensional geometric construction whose deviation from flatness is expressed with scalar quantity.

Complete understanding of the condition of the test surface requires quality graphic display which includes a complex mathematical model.

Flatness is the surface condition in which all points are stated in one plane. Permissible deviation from flatness is determined by the distance between the two planes that are far enough one from another for permissible deviation  $T_{ra}$  (Fig. 1).

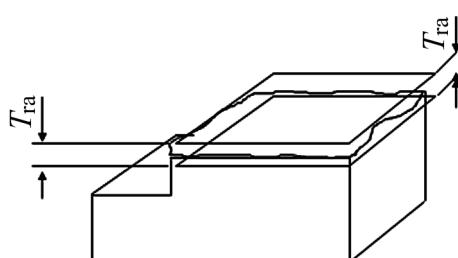


Figure 1 Displaying flatness deviation

Requirements placed on the plate flatness measurements for tables made of cast iron and natural granite are given in standards ISO 8512-1 and ISO 8512 - 2. Tolerances according to the degree of accuracy are presented in Tab. 1.

**Table 1** Tolerances according to the degree of accuracy for plates made of granite and cast iron

Size of plate / mm	Tolerance on deviation from flatness overall for plates of grade / $\mu\text{m}$			
	0	1	2	3
160 × 100	3	6	12	25
250 × 160	3,5	7	14	27
400 × 250	4	8	16	32
630 × 400	5	10	20	39
1000 × 630	6	12	24	49
1600 × 1000	8	16	33	66
2000 × 1000	9,5	19	38	75
2500 × 1600	11,5	23	46	92
250 × 250	3,5	7	15	30
400 × 400	4,5	9	17	34
630 × 630	5	10	21	42
1000 × 1000	7	14	28	56

## 2 Test methods for measuring flatness deviation of technical surfaces

Flatness measurement is essentially based on the measurement of straightness of measuring lines on that surface. Network of lines on the measuring surface allows the required geometric surface features to be found.

Before starting measurements it is necessary to develop a measurement plan establishing measurement methods, the choice of the type and the length of the measurement bases and the schedule and sequence of measurement by measuring directions.

Commonly there are two types of measurement schedules and routes in application, one rectangular or "Grid" and the other, diagonal type or "Union Jack" (Fig. 2, and Fig. 3). There is a large number of methods which can be used for measuring flatness of technical surfaces and they are primarily divided with respect to the measuring equipment used: a measurement method with lineal and comparator, a coordinate measuring machine,

an autocollimator, a laser measuring system and an electronic level.

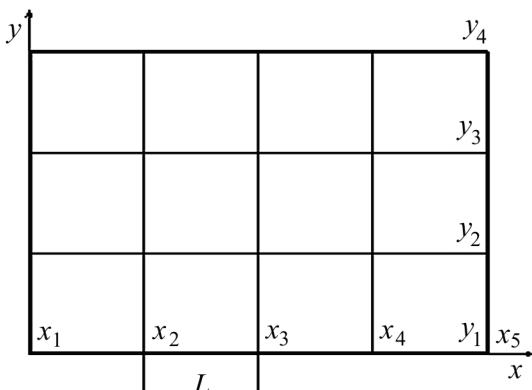


Figure 2 Rectangular or "Grid" schedule

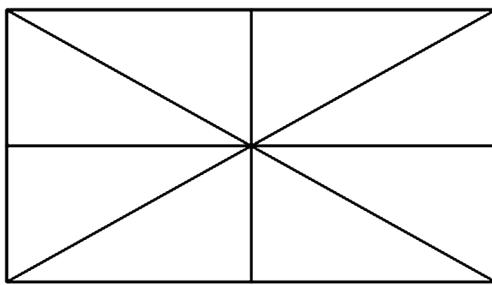


Figure 3 Diagonal or "Union Jack" schedule

Level is a measuring device that indicates the slope of the measurement surface with respect to the horizontal plane. In respect to autocollimator and laser measuring system, level has its own reference to the horizontal plane which gives the advantage to measuring flatness (Fig. 4).

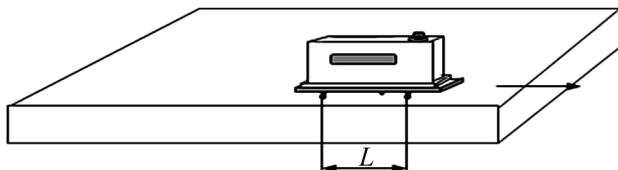


Figure 4 Level on the measuring plate

Vertical deviation is calculated using the known step and the measured slope.

### 3 Flatness measurement with electronic level

The mathematical model and integrated computer model for processing and axonometric and topographic display of measurement results of flatness deviation in the process of measuring flatness by using the electronic level has been developed in the Laboratory for the precise measurements of the length at the Faculty of Mechanical Engineering and Naval Architecture (LFSB).

Rectangular distribution of measurement lines is used in this model.

In relation to methods and programs used by the laboratory in previous studies, the advantage of this solution is in immediate correlation of measurement data and in presentation of numerical and graphical results.

It is important to emphasize multiple, optimized association developed by computer model and also the requirements imposed on the flatness of technical surface.

The program is substantially complete, operationally optimized by the duration of pod calculation and the duration of the user engagement program.

Presented software solution is already in use at the Faculty of Mechanical Engineering and Naval Architecture as a part of the course "Theory and technique of measurement" and also for the needs of industry to process and analyze the measurement results of flatness deviation of technical surfaces.

Additional and real validation of the software solutions will be done by participation in the international comparison of flatness measurement.

In defining the reference plane, developed mathematical model involves applying the least squares method, as given by expressions from (1) to (10). Graphical presentation of the reference plane is given in Fig. 5.

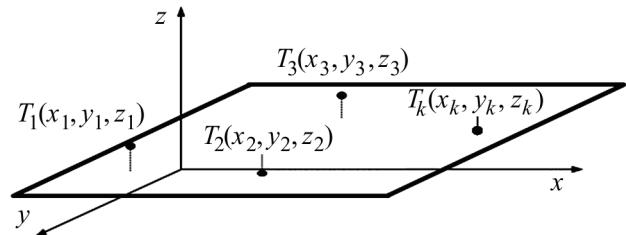


Figure 5 Mathematical referent plane

General equation of the plane:

$$Ax_i + By_i + Cz_i + D = 0. \quad (1)$$

Where the parameters of equation are given by expression (2):

$$a = -\frac{A}{C}, \quad b = -\frac{B}{C}, \quad c = -\frac{D}{C}. \quad (2)$$

The equation of the reference plane is given by expression (3):

$$z_i = ax_i + by_i + c. \quad (3)$$

On the condition that function  $S$  is the function of the sum of squares of distances between the points of these planes and that it tends to minimum, expressions (4) and (5) follow:

$$S = \sum_{i=1}^k (z_i - ax_i - by_i - c)^2, \quad (4)$$

$$\frac{\partial S}{\partial a} = 0, \quad \frac{\partial S}{\partial b} = 0, \quad \frac{\partial S}{\partial c} = 0. \quad (5)$$

From equation (5) the next expressions follow:

$$\frac{\partial S}{\partial a} = \sum_{i=1}^k (-2x_i z_i + 2ax_i^2 + 2cx_i + 2bx_i y_i) = -2 \sum_{i=1}^k x_i (z_i - ax_i - by_i - c) = 0,$$

$$\frac{\partial S}{\partial b} = \sum_{i=1}^k (-2y_i z_i + 2ax_i y_i + 2by_i^2 + 2cy_i) = -2 \sum_{i=1}^k y_i (z_i - ax_i - by_i - c) = 0,$$

$$\frac{\partial S}{\partial c} = \sum_{i=1}^k (-2z_i + 2ax_i + 2by_i + 2c) = -2 \sum_{i=1}^k (z_i - ax_i - by_i - c) = 0.$$

The coefficients of the referent plane are given by expressions (6), (7) and (8):

$$a = \frac{k \sum_{i=1}^k x_i z_i \sum_{i=1}^k y_i^2 + \sum_{i=1}^k x_i y_i \sum_{i=1}^k y_i \sum_{i=1}^k z_i + \sum_{i=1}^k x_i \sum_{i=1}^k y_i z_i \sum_{i=1}^k y_i - \sum_{i=1}^k z_i \sum_{i=1}^k y_i^2 \sum_{i=1}^k x_i - \sum_{i=1}^k y_i \sum_{i=1}^k y_i \sum_{i=1}^k x_i z_i - k \sum_{i=1}^k y_i z_i \sum_{i=1}^k x_i y_i}{k \sum_{i=1}^k x_i^2 \sum_{i=1}^k y_i^2 + \sum_{i=1}^k x_i y_i \sum_{i=1}^k y_i \sum_{i=1}^k x_i + \sum_{i=1}^k x_i \sum_{i=1}^k x_i y_i \sum_{i=1}^k y_i - \sum_{i=1}^k x_i \sum_{i=1}^k y_i^2 \sum_{i=1}^k x_i - \sum_{i=1}^k y_i \sum_{i=1}^k y_i \sum_{i=1}^k x_i^2 - k \sum_{i=1}^k x_i y_i \sum_{i=1}^k x_i y_i}, \quad (6)$$

$$b = \frac{k \sum_{i=1}^k x_i^2 \sum_{i=1}^k y_i z_i + \sum_{i=1}^k x_i z_i \sum_{i=1}^k y_i \sum_{i=1}^k x_i + \sum_{i=1}^k x_i \sum_{i=1}^k x_i y_i \sum_{i=1}^k z_i - \sum_{i=1}^k x_i \sum_{i=1}^k y_i z_i \sum_{i=1}^k x_i - \sum_{i=1}^k x_i^2 \sum_{i=1}^k y_i \sum_{i=1}^k z_i - k \sum_{i=1}^k x_i y_i \sum_{i=1}^k x_i z_i}{k \sum_{i=1}^k x_i^2 \sum_{i=1}^k y_i^2 + \sum_{i=1}^k x_i y_i \sum_{i=1}^k y_i \sum_{i=1}^k x_i + \sum_{i=1}^k x_i \sum_{i=1}^k x_i y_i \sum_{i=1}^k y_i - \sum_{i=1}^k x_i \sum_{i=1}^k y_i^2 \sum_{i=1}^k x_i - \sum_{i=1}^k y_i \sum_{i=1}^k y_i \sum_{i=1}^k x_i^2 - k \sum_{i=1}^k x_i y_i \sum_{i=1}^k x_i y_i}, \quad (7)$$

$$c = \frac{\sum_{i=1}^k x_i^2 \sum_{i=1}^k y_i^2 \sum_{i=1}^k z_i + \sum_{i=1}^k x_i y_i \sum_{i=1}^k y_i z_i \sum_{i=1}^k x_i + \sum_{i=1}^k x_i z_i \sum_{i=1}^k x_i y_i \sum_{i=1}^k y_i - \sum_{i=1}^k x_i \sum_{i=1}^k y_i^2 \sum_{i=1}^k x_i z_i - \sum_{i=1}^k y_i \sum_{i=1}^k y_i z_i \sum_{i=1}^k x_i^2 - \sum_{i=1}^k z_i \sum_{i=1}^k x_i y_i \sum_{i=1}^k x_i y_i}{k \sum_{i=1}^k x_i^2 \sum_{i=1}^k y_i^2 + \sum_{i=1}^k x_i y_i \sum_{i=1}^k y_i \sum_{i=1}^k x_i + \sum_{i=1}^k x_i \sum_{i=1}^k x_i y_i \sum_{i=1}^k y_i - \sum_{i=1}^k x_i \sum_{i=1}^k y_i^2 \sum_{i=1}^k x_i - \sum_{i=1}^k y_i \sum_{i=1}^k y_i \sum_{i=1}^k x_i^2 - k \sum_{i=1}^k x_i y_i \sum_{i=1}^k x_i y_i}, \quad (8)$$

where are:

$x_i, y_i, z_i$  – coordinates of point  $i$

$k$  – total number of points.

The flatness deviation is determined by the sum of the distances of the two most distant points of the plane obtained by the method of least squares.

The distance of the point  $T(x_T, y_T, z_T)$  from plane by the method of least squares is calculated by expression (9),

$$\delta_i = \frac{ax_T + by_T - z_T + c}{-\operatorname{sign}\sqrt{a^2 + b^2 + 1}}, \quad (9)$$

and flatness deviation by expression (10):

$$\Delta = \delta_{\max} + |\delta_{\min}|. \quad (10)$$

#### 4 Experimental part

As a part of this work, measurements of flatness deviations of granite plate dimensions  $1600 \times 1400$  mm with a measurement step of 165 mm are performed. The measured values of points along the  $x$ -and  $y$ -axes are given in Tab. 2 and Tab. 3.

**Table 2** The measured values of points on the  $x$  axis, mm/m

Direction/Steep / mm	0	165	330	495	660	825	990	1155	1320	1485
$x_1$	0	9,386	9,384	9,379	9,375	9,363	9,351	9,349	9,350	9,340
$x_2$	0	9,380	9,383	9,389	9,371	9,360	9,354	9,350	9,349	9,343
$x_3$	0	9,383	9,376	9,373	9,367	9,359	9,352	9,349	9,345	9,349
$x_4$	0	9,372	9,373	9,375	9,369	9,366	9,358	9,343	9,340	9,361
$x_5$	0	9,365	9,372	9,370	9,368	9,362	9,355	9,341	9,353	9,372
$x_6$	0	9,371	9,378	9,374	9,370	9,364	9,350	9,339	9,356	9,380
$x_7$	0	9,372	9,373	9,381	9,371	9,363	9,347	9,338	9,359	9,394
$x_8$	0	9,370	9,368	9,379	9,375	9,361	9,353	9,345	9,358	9,391
$x_9$	0	9,371	9,380	9,382	9,381	9,362	9,369	9,358	9,362	9,379

**Table 3** The measured values of points on the  $y$  axis, mm/m

Direction/Steep / mm	0	165	330	495	660	825	990	1155	1320
$y_1$	0	10,275	10,265	10,288	10,280	10,250	10,253	10,254	10,243
$y_2$	0	10,274	10,269	10,278	10,250	10,256	10,256	10,249	10,240
$y_3$	0	10,275	10,266	10,277	10,276	10,265	10,250	10,269	10,243
$y_4$	0	10,274	10,273	10,276	10,269	10,260	10,259	10,253	10,254
$y_5$	0	10,276	10,262	10,263	10,269	10,261	10,265	10,254	10,260
$y_6$	0	10,283	10,275	10,270	10,262	10,263	10,265	10,255	10,256
$y_7$	0	10,278	10,270	10,271	10,260	10,254	10,253	10,270	10,254
$y_8$	0	10,273	10,267	10,274	10,255	10,250	10,251	10,277	10,268
$y_9$	0	10,290	10,275	10,275	10,258	10,252	10,252	10,273	10,270
$y_{10}$	0	10,287	10,283	10,286	10,272	10,268	10,269	10,268	10,253

By using the developed software solution, flatness deviation of  $\Delta = 19,9 \mu\text{m}$ ;  $U = 3 \mu\text{m}$ ;  $k = 2$ ;  $P = 95\%$  is determined. The equation of the referent plane is given by expression (11). Graphical presentation of the referent

plane and the plane deviation from flatness is given by Figs. 6 and 7.

$$z_i = 0,006876 + 0,01026 \cdot x_i + 0,009367 \cdot y_i. \quad (11)$$

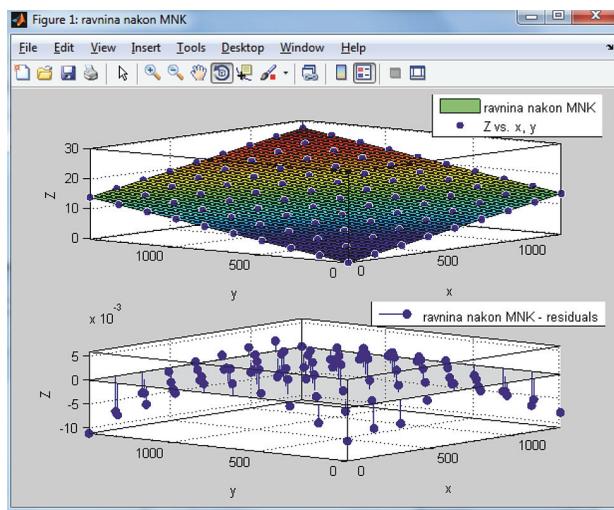


Figure 6 Referent plane

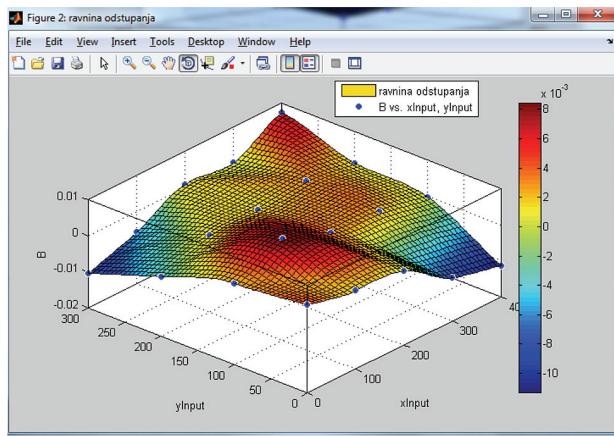


Figure 7 Deviation plane

The results of the developed integrated test procedure for flatness measurements of technical surfaces have been experimentally validated in comparison with the results achieved by the methods and programs used by the laboratory in a previous work. Additional validation of software solutions will be implemented through the international comparison measurement, in the process of flatness measurements of technical surfaces.

## 5 Conclusion

The technical area is a complex three-dimensional geometric construction whose deviation from flatness is expressed with scalar quantity. Complete understanding of the condition of the test surface requires quality graphic display which includes a complex mathematical model.

As a part of this work, an integrated computer model for processing axonometric and topographic representation of the results of measuring deviations from flatness has been developed. Presented software solution is already in use at the Faculty of Mechanical Engineering and Naval Architecture as a part of the course "Theory and technique of measurement" and also for the needs of industry to process and analyze the flatness deviation measurement results of technical surfaces.

The advantage of this software solution in relation to methods and programs used by the laboratory in a previous work is in close correlation with measurement data and in presenting numerical and graphical results.

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