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DAMAGE IDENTIFICATION OF BRIDGES FROM VIBRATION FREQUENCIES

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Preliminary notes
This paper outlines the research aimed at developing an effective and reliable monitoring method for structural damage identification. A computational procedure for a direct iteration technique based on the non-linear perturbation theory is proposed to identify structural damage, when information about only natural frequencies for the damaged structure is required. The presented damage identification technique is applied to six concrete girder bridges of different ages in Croatia. It is found that the proposed approach is quite sensitive to the quality of measured natural frequencies for structural damage identification due to the ill-conditioned system of governing equations.

Keywords: *bridges, monitoring, structural damage identification, vibration frequencies*

Utvrđivanje oštećenja mostova na osnovi vlastitih frekvencija

Prethodno priopćenje
U ovom radu je prikazano istraživanje usmjereno na razvoj učinkovite i pouzdane metode za monitoring konstrukcija sa svrhom utvrđivanja njihova oštećenja. Predložen je numerički postupak za tehniku direktne iteracije zasnovan na teoriji nelinearne perturbacije u kojem se koriste samo izmjerene vlastite frekvencije oštećene konstrukcije. Predložena metoda utvrđivanja oštećenja primijenjena je na šest betonskih grednih mostova različite starosti u Hrvatskoj. Uvidjelo se da je predloženi pristup za utvrđivanje oštećenja prilično osjetljiv na kvalitetu izmjerenih vlastitih frekvencija zbog loše-uvjetovanog sustava osnovnih jednadžbi.

Ključne riječi: *identifikacija oštećenja, monitoring, mostovi, vlastite frekvencije*

1 Introduction

For condition monitoring and detecting degradation in structures there are a number of non-destructive techniques available or being developed. Most of the non-destructive evaluation techniques that are in use today are *local* inspection techniques, such as mechanical, radiographic, electromagnetic, magnetic and ultrasonic. For evaluation of complex structures, such as bridges, methods of vibration testing offer the opportunity for *global* inspection techniques that may be able to detect critical local failures. These methods, however, do not produce quantitative damage information that can be used to design a repair or assess the safety of the damaged structure. These shortcomings can be overcome when vibration measurements are used with the system identification algorithms. The overall dynamic response provides a measure of the condition of the structure. Existence of damage in a structure leads to the changes in the vibration response and dynamic characteristics of the structure such as the natural frequencies, mode shapes and the modal dampings. Therefore, the changes in dynamic characteristics of a structure can be used in turn to detect, locate and quantify the structural damage if damage occurs.

In the literature, a variety of structural damage identification methods can be found depending on the type of experimental data used to detect, locate and quantify structural damage. They include the changes in modal data [1], the strain energy [2], the flexibility matrix [3], the residual forces [4] and the frequency response function [5]. Modal frequencies and mode shapes are the most popular parameters used in the identification. Furthermore, since the natural frequencies are rather easy to measure with a relatively high level of accuracy, the methods based on the measurements of natural frequencies are potentially attractive. The authors [1] proposed a novel perturbation-based approach using the exact relationship between the changes of structural parameters and the changes of modal parameters in order to avoid the insufficiency of the first-order sensitivity analysis. For damage detection, the first-order approximation may be inaccurate since a large change of structural parameters due to damage might need to be detected.

In this paper the direct iteration technique based on the non-linear perturbation theory is utilized to identify structural damage, when only natural frequencies for the damaged structure are required. The effectiveness of the proposed numerical procedure was already demonstrated by a numerical example of the real concrete girder bridge with simulated damage and also through laboratory testing of a simply supported reinforced concrete beam subjected to various levels of static load [6]. Here, the application of the proposed technique to structural inspection by vibration frequencies monitoring is described in the following.

2 Non-linear perturbation theory

The governing equation for the structural dynamic system in finite element representation can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{f}(t), \tag{1}$$

where the matrices $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ represent the discretised mass or inertia, damping, and stiffness distribution; $\ddot{\boldsymbol{x}}, \dot{\boldsymbol{x}}$ and $\boldsymbol{x}$ are the acceleration, velocity, and displacement vectors of the degrees of freedom (DOFs) being modelled, and $\boldsymbol{f}(t)$ is the external forcing function vector.

Neglecting the damping terms, and the homogeneous solutions to equation (1) leads to the eigenvalue problem, i.e. the characteristic equation for the original (undamaged) structural system

$$(\boldsymbol{K} - \lambda_i \boldsymbol{M})\boldsymbol{\phi}_i = 0, \tag{2}$$

where the i^{th} eigenvalue λ_i is the square of the circular natural frequency, ω_i, and the eigenvector, $\boldsymbol{\phi}_i$, is the corresponding natural mode shape of the system.

For the modified (damaged) structural system, the characteristic equation can be expressed as

$$(\boldsymbol{K}^* - \lambda_i^* \boldsymbol{M}^*)\boldsymbol{\phi}_i^* = 0, \tag{3}$$

where quantities with a superscript * indicate those associated with the modified structural system.

The stiffness and mass matrices for the modified structural system can be expressed as

$$\boldsymbol{K}^* = \boldsymbol{K} + \Delta\boldsymbol{K}, \tag{4}$$

$$\boldsymbol{M}^* = \boldsymbol{M} + \Delta\boldsymbol{M}. \tag{5}$$

The eigenvalue and the corresponding eigenvector for the modified structural system can be expressed as

$$\lambda_i^* = \lambda_i + \Delta\lambda_i, \tag{6}$$

$$\boldsymbol{\phi}_i^* = \boldsymbol{\phi}_i + \Delta\boldsymbol{\phi}_i. \tag{7}$$

Upon substitution of equations (4) ÷ (7), the characteristic equation for the modified structural system, equation (3) can be rewritten as

$$\begin{aligned}&[\Delta\boldsymbol{K} - (\lambda_i + \Delta\lambda_i)\Delta\boldsymbol{M}]\{\boldsymbol{\phi}_i + \Delta\boldsymbol{\phi}_i\} + \\ &[\boldsymbol{K} - (\lambda_i + \Delta\lambda_i)\boldsymbol{M}]\{\boldsymbol{\phi}_i + \Delta\boldsymbol{\phi}_i\} = 0.\end{aligned} \tag{8}$$

Premultiplying equation (8) by $\boldsymbol{\phi}_k^{\mathrm{T}}$ and postmultiplying by $\boldsymbol{\phi}_k$, and also premultiplying equation (8) by $\boldsymbol{\phi}_i^{\mathrm{T}}$, assuming that the mode shapes of the original and the modified structural system are mass normalised, leads to

$$\boldsymbol{\phi}_i^{\mathrm{T}}[\Delta\boldsymbol{K} - (\lambda_i + \Delta\lambda_i)\Delta\boldsymbol{M}]\{\boldsymbol{\phi}_i + \Delta\boldsymbol{\phi}_i\} - \Delta\lambda_i = 0, \tag{9}$$

$$\Delta\boldsymbol{\phi}_i = \sum_{k=1,k\neq i}^{N} C_{ik}\boldsymbol{\phi}_k. \tag{10}$$

where the mode participation factor C_{ik} is defined as

$$C_{ik} = \frac{\boldsymbol{\phi}_k^{\mathrm{T}}[\Delta\boldsymbol{K} - (\lambda_i + \Delta\lambda_i)\Delta\boldsymbol{M}]\{\boldsymbol{\phi}_i + \Delta\boldsymbol{\phi}_i\}}{(\lambda_i + \Delta\lambda_i) - \lambda_k}. \tag{11}$$

It is found from equation (10) that the modification of an eigenvector of a structural system can be expressed as the linear combination of the original eigenvectors except the corresponding original one.

Since the effects of damage in a structure on stiffness can be represented by reducing its Young's modulus in most cases, the change of structural stiffness matrix can be expressed in the form

$$\Delta\boldsymbol{K} = \sum_{j=1}^{NXE} \alpha_j \boldsymbol{K}_j, \tag{12}$$

where NXE is the total number of structural elements, $\boldsymbol{K}_j$ is the contribution of element j to the global stiffness matrix, α_j is the damage parameter for the j^{th} element and ranges from –1 to 0. A 0 (zero) parameter value corresponds to the undamaged state and –1 represents complete loss of the structural element stiffness.

It will be postulated that the mass distribution of the system remains either unchanged or is changed by only a known quantity. This is a reasonable assumption because most structural damage for engineering structures will result in stiffness losses instead of complete separation or breakage with a loss of mass. Here, it is assumed that there is no change in the structural mass matrix, i.e.,

$$\Delta\boldsymbol{M} = 0. \tag{13}$$

In order to determine damage parameter α_j, governing equations can be developed depending on the information about only eigenvalue λ_i^* available.

A set of governing equations associated with the damage parameter α_j and the mode participation factor C_{ik} have to be developed since the eigenvectors for the damaged structure are not available.

Using equations (10), (12) and (13), equations (9) and (11) can be rewritten as

$$\sum_{j=1}^{NXE} \phi_i^{\mathrm{T}} \boldsymbol{K}_j \phi_i \alpha_j + \sum_{j=1}^{NXE} \sum_{l=1,l\neq i}^{N} \phi_i^{\mathrm{T}} \boldsymbol{K}_j \phi_l C_{il} \alpha_j - \Delta\lambda_i = 0, \tag{14}$$

$$C_{ik} = \frac{\sum_{j=1}^{NXE} \phi_k^{\mathrm{T}} \boldsymbol{K}_j \phi_i \alpha_j + \sum_{j=1}^{NXE} \sum_{l=1,l\neq i,k}^{N} \phi_k^{\mathrm{T}} \boldsymbol{K}_j \phi_l \alpha_j C_{il}}{\lambda_i^* - \lambda_k - \sum_{j=1}^{NXE} \phi_k^{\mathrm{T}} \boldsymbol{K}_j \phi_k \alpha_j}. \tag{15}$$

When information about only natural frequencies for the damaged structure is available, the governing equations associated with the damage parameter α_j and the mode participation factor C_{ik}, equations (14) and (15) can be rewritten as

$$\sum_{j=1}^{NXE} a_{iji} \alpha_j + \sum_{j=1}^{NXE} \sum_{l=1,l\neq i}^{NM} a_{ijl} C_{il} \alpha_j - \Delta\lambda_i = 0, \tag{16}$$

$$C_{ik} = \frac{\sum_{j=1}^{NXE} a_{kji} \alpha_j + \sum_{j=1}^{NXE} \sum_{l=1,l\neq i,k}^{NM} a_{kjl} \alpha_j C_{il}}{\lambda_i^* - \lambda_k - \sum_{j=1}^{NXE} a_{kjk} \alpha_j}, \tag{17}$$

where a_{iji}, a_{ijl}, a_{kji} and a_{kjl} are the eigenmode-stiffness sensitivity coefficients, which can be defined in a general form as

$$a_{kjl} = \boldsymbol{\phi}_k^{\mathrm{T}} \boldsymbol{K}_j \boldsymbol{\phi}_l. \tag{18}$$

It can be seen from equation (10) that when k is large enough the terms with subscripts greater than k can be neglected. Therefore, N can be suitably replaced by NM, denoting the number of the original eigenvectors available.

The computational procedure for the direct iteration technique has been developed to solve the element scalar damage parameters α_j as well as the mode participation factors C_{ik} [1].

Once the mode participation factor C_{ik} is found, using equations (7) and (10) the eigenvectors for the damaged structure can be calculated as

$$\boldsymbol{\phi}_i^* = \boldsymbol{\phi}_i + \sum_{k=1,k\neq i}^{NM} C_{ik}\boldsymbol{\phi}_k, \tag{19}$$

where the pairing of the eigenvalues for the original structure and the damaged structure can be checked using the MAC factors (Modal Assurance Criterion), defined as

$$MAC(k,i) = \frac{\left|\boldsymbol{\phi}_k^{\mathrm{T}}\boldsymbol{\phi}_i^*\right|^2}{\left|\boldsymbol{\phi}_k^{\mathrm{T}}\boldsymbol{\phi}_k\right|\left|\boldsymbol{\phi}_i^{*\mathrm{T}}\boldsymbol{\phi}_i^*\right|}. \tag{20}$$

The highest $MAC(k, i)$ factors indicate the most possible pairings of the original mode k and the damaged mode i.

3 Direct Iteration Technique

Rewriting equation (16), yields

$$\sum_{j=1}^{NXE} S_{ij}\alpha_j = z_i, \tag{21}$$

where S_{ij} and z_i are the eigenmode-stiffness sensitivity matrix and vector, respectively, which are defined as

$$S_{ij} = a_{iji} + \sum_{l=1,l\neq i}^{NM} C_{il}a_{ijl}, \tag{22}$$

$$z_i = \Delta\lambda_i. \tag{23}$$

Similarly, equation (17) is rewritten as

$$C_{ik} = \frac{b_{ki} + \sum_{l=1,l\neq i,k}^{NM} C_{il}b_{kl}}{\lambda_i^* - \lambda_k - b_{kk}}, \tag{24}$$

where b_{kk}, b_{ki} and b_{kl} can be defined in a general form as

$$b_{kl} = \sum_{j=1}^{NXE} a_{kjl}\alpha_j. \tag{25}$$

The iterative solution procedure based on the above formulation is elaborated in Fig. 1. Depending on the number of available natural frequencies NF (number of equations) and the number of structural damage parameters NXE (number of unknows), the eigenmode-stiffness sensitivity matrix S_{ij} may not be square. Theoretically, if the number of available natural frequencies NF, is equal to NXE, the solution may be determined exactly. However, only a smaller number of natural frequencies can usually be measured. Hence, the number of the measured natural frequencies for the damaged structure NF is less than the number of structural damage parameters (finite-elements), ($NF<NXE$), which renders the equations underdetermined i.e. ill-conditioned. In order to find a solution for what is in general an ill-conditioned system, the Singular Value Decomposition (SVD) technique is applied.

A FORTRAN computer program for structural damage identification has been developed based on the knowledge of the computational procedure presented in [6] and in Fig. 1.

Step 1:

Assume the initial mode participation factors C_{ik}^0 to be zero, i.e., no changes in eigenvectors. Establish the initial values for α_j^1 and C_{ik}^1 from

$$\sum_{j=1}^{NXE} S_{ij}^1\alpha_j^1 = z_i, \text{ where } S_{ij}^1 = a_{iji} = \phi_i^{\mathrm{T}}K_j\phi_i;\ z_i = \Delta\lambda_i$$

$$C_{ik}^1 = \frac{b_{ki}^1}{\lambda_i^* - \lambda_k - b_{kk}^1}, \text{ where } b_{ki}^1 = \sum_{j=1}^{NXE} a_{kji}\alpha_j^1$$

Step 2:

Evaluate current estimate for α_j^n from

$$\sum_{j=1}^{NXE} S_{ij}^n\alpha_j^n = z_i, \text{ where } S_{ij}^n = a_{iji} + \sum_{l=1,l\neq i}^{NM} C_{il}^{n-1}a_{ijl}$$

Step 3:

Evaluate new modal participation factors C_{ik}^n from

$$C_{ik}^n = \frac{b_{ki}^n + \sum_{l=1,l\neq i,k}^{NM} C_{il}^{n-1}b_{kl}^n}{\lambda_i^* - \lambda_k - b_{kk}^n}, \text{ where } b_{ki}^n = \sum_{j=1}^{NXE} a_{kji}\alpha_j^n$$

and return to step 2 if solution has not converged.

Figure 1 Computational procedure for the direct iteration technique

Table 1 Experimental natural frequencies for four-span bridges of different ages [7]

Mode	Frequency / Hz					
	Bridge age / years					
	0	5	12	18	25	30
1	2,153	2,091	2,011	1,957	1,903	1,850
2	7,203	7,051	6,434	6,031	5,630	5,442
3	18,635	17,930	17,310	16,220	15,920	14,790
4	32,483	31,570	30,220	29,400	28,440	26,380
5	48,321	46,760	44,510	43,050	41,810	39,420

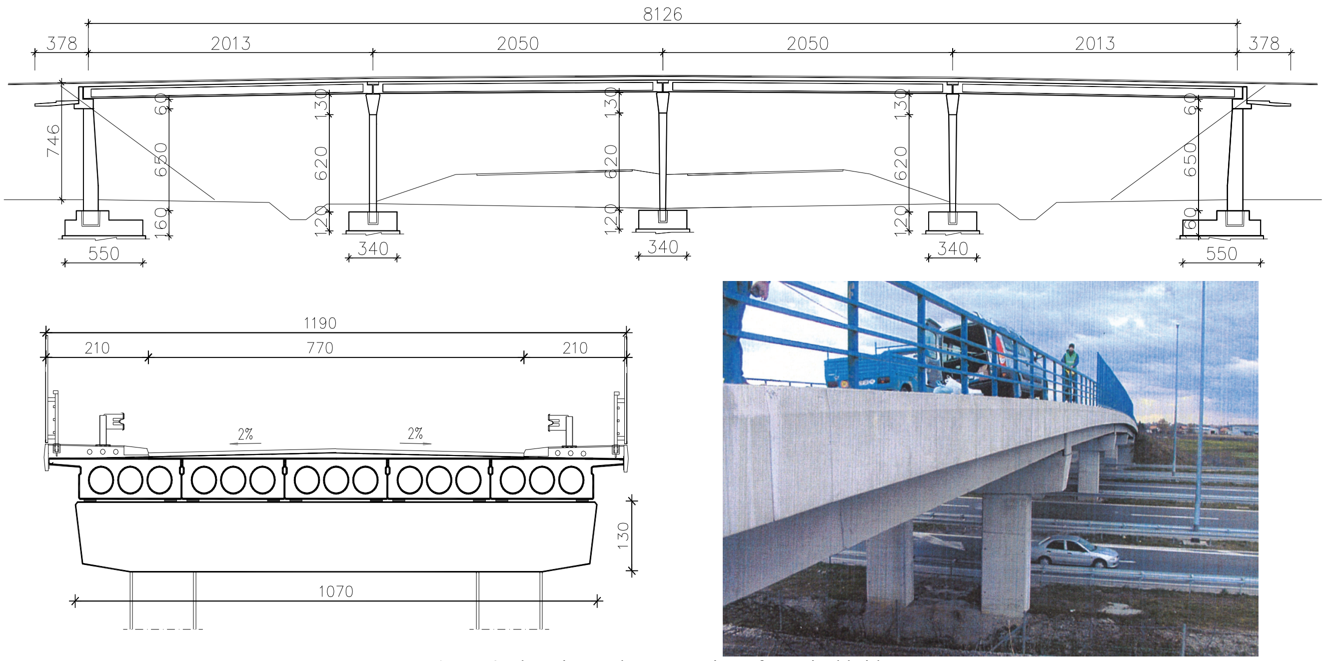


Figure 2 Elevation and cross section of a typical bridge

4 Bridges on which experiments were conducted
4.1 Description of bridges and dynamic tests

The dynamic tests were conducted by [7] on the four-span simply supported prestressed concrete plate girder bridges of different ages, crossing the highway in Croatia. A total of six concrete girder bridges aged 0 years, 5 years, 12 years, 18 years, 25 years and 30 years were tested. A typical bridge has an overall length of 88,82 m, a width of 11,90 m, and two traffic lanes. The two main spans are 20,50 m each, and the two side spans are 20,13 m each. The length of each simple span is 20 m. The superstructure consists of 5 plate girders supported on neoprene bearings. Sketches of the elevation and the cross section of a typical bridge are shown in Fig. 2.

Both ambient (random) and controlled traffic were used to provide excitation to the bridge structures. When using controlled traffic, a 27,3 ton truck was provided. For most bridges, a total of 4 ÷ 6 vehicle runs were carried out at various speeds (20 ÷ 50 km/h) along the centreline of the bridge. Because of traffic considerations the ambient vibration test was mainly due to the traffic under the bridges (traffic on the highway induces vibration to the foundations and the abutments of the bridges).

The response of the bridges was measured with an accelerometer (Kistler) connected to a PC based oscilloscope/spectrum analyser (*PicoScope* software). All data were recorded in the notebook computer with the *Picolog* data acquisition software. The accelerometer was placed on the sidewalk in the middle of one of the two main spans.

The first five vertical bending modes are identified from the truck tests as summarized in Tab. 1 [7]. Columns 2 ÷ 7 in Tab. 1 list the experimental natural frequencies for one span of four-span bridges of different ages. The 0 (zero) years in second column means the new, undamaged bridge. The experimentally obtained natural frequencies were then used in the direct iteration technique (see Fig. 1) to calculate the extent of damage for each of finite-element i.e. structural damage parameters.

4.2 Finite Element Model

The three-dimensional grillage finite-element (3D FE) model of a typical bridge is developed using the Sofistik software. Since the bridge superstructure is simply supported (only the slab portion of the deck is being made continuous), one span of the bridge is modelled.

The elasticity modulus of concrete assumed for the un-tuned FE model was as the design one E = 3,4×10^{7} kN/m^{2}. The first five bending mode shapes of the model are shown in Fig. 3.

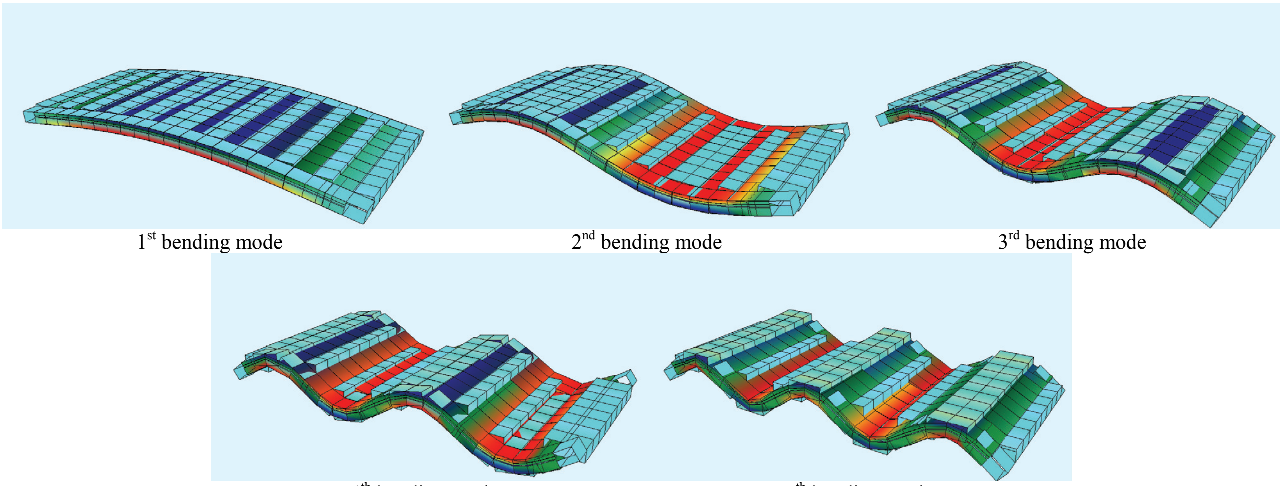
The natural frequencies of undamaged bridge determined from the finite-element analyses are summarized in Tab. 2 and compared to the measured results. Most of the computed natural frequencies correlated well with the test results as shown in Tab. 2.

4.3 Damage identification

Based on the measured natural frequencies for the undamaged and the damaged structure, a direct iteration technique was employed for structural damage identification for each of the damaged bridges aged 5, 12, 18, 25 and 30 years.

Table 2 Comparison of measured and finite-element natural frequencies for undamaged bridge

Mode	Test / Hz	3D FE model / Hz	Relative error / %	2D FE model / Hz	Relative error / %
1	2,15	2,17	0,9	2,15	0,0
2	7,20	8,67	20,4	8,62	19,6
3	18,64	19,49	4,6	19,42	4,2
4	32,48	34,56	6,4	34,66	6,7
5	48,32	53,65	11,0	54,55	12,9


1^{st} bending mode 2^{nd} bending mode 3^{rd} bending mode

4^{th} bending mode 5^{th} bending mode

Figure 3 First five vertical bending mode shapes of 3D FE model (Sofistik)

The computer program used here for dynamic analysis and damage identification was developed for the solution of two-dimensional (2D) framed structures which have used beam-column elements. The framed structures are made up of members arbitrarily inclined to one another. Loading of such structures results in displacements due to both axial and bending effects. These elements each have six degrees of freedom, incorporating two translations and a rotation at each node, although the axial displacement is ignored in this example.

It should be noted that, since the structure discussed here is almost symmetric, convergence difficulties may arise. From equation (18), it can be shown that if a structure is symmetric, the set of governing equations (16) becomes singular, and the identification process can not proceed. If a structure is 'almost' symmetric, the governing equations (16) are ill-conditioned, and in such a case the information of a larger number of natural frequencies for the damaged structure (greater than *NXE*) are required to correctly determine the location and amount of structural damage. Due to ill-conditioned governing equations (16), damage parameters α_j converge very slowly, leading to convergence difficulties in some cases. In order to avoid such difficult cases, some methods may be used to desymmetrise the structure, such as non-symmetric element mesh generated, suitable boundary conditions selected, and additional concentrated mass applied. Consequently, the proposed method is also practicable for symmetric or near symmetric structures and a smaller number of natural frequencies for the damaged structure is required to determine structural damage.

In order to avoid problems associated with structural symmetry, 9 non-symmetric finite-elements with 10 nodes and a total of 18 degrees of freedom, are generated (Fig. 4). All structural members have the same material properties with experimentally estimated elastic modulus E = 3,4 × 10^7 kN/m^2 and density ρ = 2,5 t/m^3 and the same cross sectional area A = 5,798 m^2 and second moment of area I = 128,142 × 10^{-3} m^4. The first five natural frequencies for the undamaged structure are listed in Tab. 2.

The information about five measured modified frequencies is used and all original eigenvectors are considered to obtain the structural damage parameters for damaged bridges aged 5 ÷ 30 years, as shown in Fig. 5. The correlation between eigenvectors for the original (undamaged) structure and the damaged structure is checked using the *MAC* factors (Modal Assurance Criterion). The *MAC* value always lies between 0 and 1. A *MAC* value of 1 indicates an excellent correlation whereas a *MAC* value of 0 indicates that the modes do not show any correlation. The diagonal of the matrix *MAC*(*k*, *i*) obtained from (20) should have a high value (>0,95) for good correlation. It is seen that the modes for the damaged structure obtained from the direct iteration technique match very well the corresponding modes for the original structure.

Parameters of the Problem	
Total degrees of freedom	18
Structural members	9
Damage parameters	9

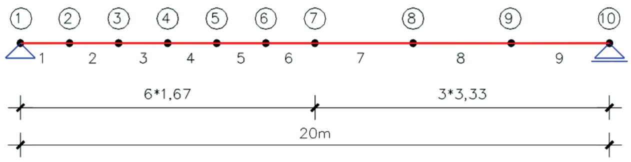


Figure 4 One-span (of four-spans) girder bridge, divided in 9 non-symmetric finite-elements with 10 nodes

The results of the damage prediction in Fig. 5 are obtained from the direct iteration technique. At the horizontal axes in Fig. 5 there are 9 parts of girder i.e. 9 finite-elements. Those 9 parts of girder are positioned in the bridge structure according to Fig. 4. Damage amounts (%) at vertical axes for every one of 9 parts of girder in Fig. 5 represent the change in structural stiffness expressed as the damage parameter α_j calculated by the computational procedure for the direct iteration technique presented in Fig. 1. Negative value in Fig. 5 represents the reduction in stiffness and positive value represents false change in the stiffness, which can never be produced by damage. From the results, it can be seen that the prediction of structural damage is significantly sensitive to the quality of the measured natural frequencies, which is caused by the ill-conditioned system of governing equations.

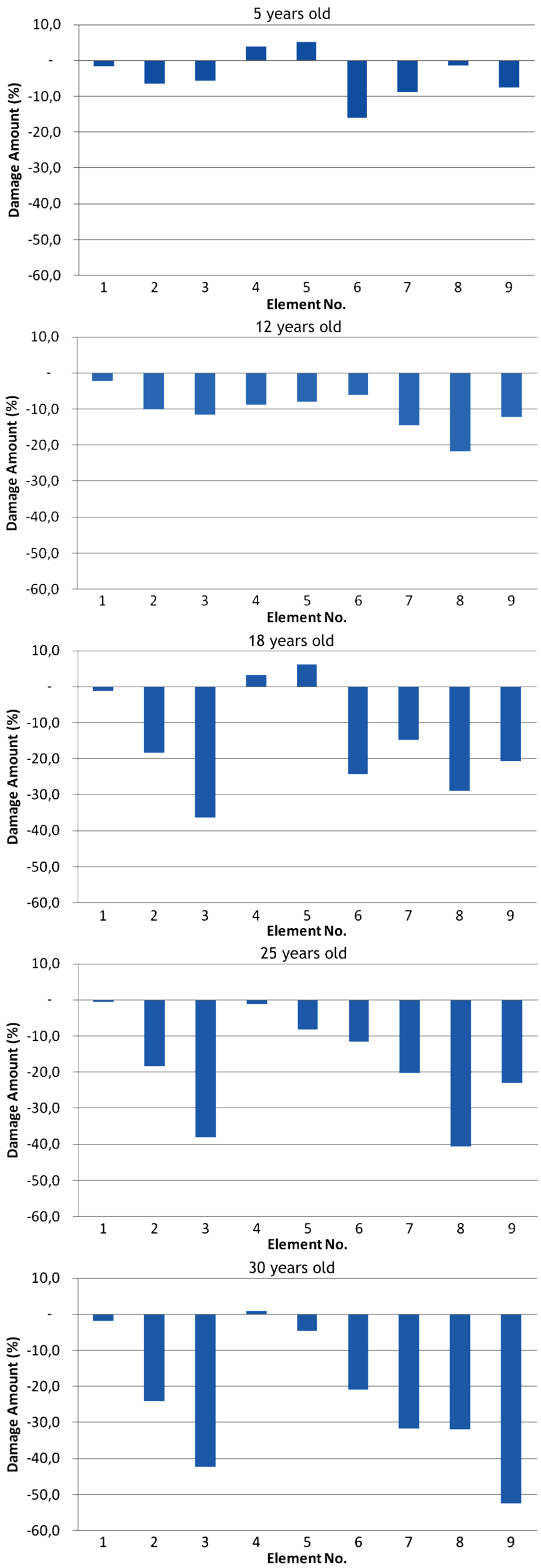


Figure 5 Inverse damage predictions from direct iteration technique for typical bridges of different ages, 5 experimental natural frequencies used

It is found that only a limited number of natural frequencies for the damaged structure are required, even 5 measured natural frequencies are sufficient to predict correctly structural damage.

With an assumption that the stiffness degrades proportionally to the damage, i.e. the change in the stiffness matrix can be expressed in the form of equation (12), the results show that structural stiffness decreases with age. It is obvious that the 30 years old bridge has sustained a very serious damage (damage parameter α ranging from 30 to 50 % stiffness reduction).

For the analysed bridges, cracks, corrosion losses, concrete spalls, and changes in boundary conditions are typical damage events [7]. All of these reduce structural stiffness.

Results of the visual inspection for accessible areas of 25 years and 30 years old bridges also confirm the need for further in-situ tests and detailed laboratory analysis.

5 Conclusion

The presented computational technique for structural damage identification using only measured natural frequencies can be successful in determining both the location and the size of damage, even when only a limited number of natural frequencies for the damaged structure are adopted. Moreover, the proposed approach is suitable for symmetric structures, if a non-symmetric element mesh is generated.

The practicability of the proposed method has been demonstrated by employing experimental measured natural frequencies and applying them to real engineering structures, such as bridges.

Finally, the results of this research indicate that the concept of a monitoring technique using the measured vibration frequencies for the evaluation of structural systems is feasible and practicable.

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