CODEN STJSAO
ZX470/1579
ISSN 0562-1887
UDK 621.313.52-136:534.1.013

Transient Torsional Oscillations of Turbine-Generator Electromechanical System due to Network Faults

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Keywords *Electrical System Linear Model Mechanical System Three-phase Fault Turbine-generator Transient Torsional Oscillations Two-axis Theory*

Ključne riječi
Dvoosna teorija
Električni sustav
Linearni model
Mehanički sustav
Prijelazne torzijske oscilacije
Trofazni kratki spoj
Turbogenerator

Received (primljeno): 2007-11-15
Accepted (prihvaćeno): 2008-02-05

Original scientific paper

In the paper electromechanical torsional oscillations of the turbine-generator's shaft-line due to network faults are addressed. Basics of the problem are outlined with special emphasis on the practical part of the issue. Problems of calculation of torsional oscillations are discussed. A mathematical model of the electromechanical system, suitable for the analysis of torsional oscillations due to the power system's symmetrical faults, is established. An implementation of the model by the "Matrixx" software package is briefly described. Capabilities and restrictions of the presented model are discussed. Results of an example of computer simulation of transient torsional torques in the shaft-line due to a three-phase fault and the subsequent fault clearing, as obtained by the model, are presented. The effects of chosen fault clearing time are analyzed and discussed.

Prijelazne torzijske oscilacije elektromagnetskog sustava turbogenaratora zbog kvarova u mreži

Izvornoznanstveni članak

Prikazane su prijelazne torzijske oscilacije osovinskog voda turbogeneratora nastale zbog kvarova u mreži. Osnova problema je ocrtana s posebnom značajkom na realnoj primjeni stvari. Obrađen je problem proračuna torzijskih oscilacija. Postavljen je matematički model elektromehaničkog sustava podesnog za analizu torzijskih oscilacija pri simetričnim kvarovima. Ukratko je opisana primjena modela pomoću Matrix programskog paketa. Razmatrane su mogućnosti i ograničenja prezentiranog modela. Prikazani su rezultati primjera kompjutorske simulacije prijelaznog torzijskog momenta u osovinskom vodu pri trofaznom kratkom spoju i sekvencijalnom isključenju kvara. Analizirane su i raspravljane posljedice izabranih vremena isključenja kvarova.

Symbols/Oznake

FCT - fault clearing time
- vrijeme isključenja kvara

$J_{m,i}$ - mechanical time constant of *i*-lumped elements of the turbine-generator rotor, p.u.
- mehanička vremenska konstanta *i*-te rotacijske mase osovinskog voda, p.u.

GEN - generator
- generator

k - viscous damping coefficient, N·s/m
- koeficijent viskoznog prigušenja

HP - high-pressure turbine
- visokotlačni dio turbine

K_{dq0} - transformation matrix
- matrica transformacija

IP - intermediate-pressure turbine
- srednjotlačni dio turbine

L_0 - inductance of zero component, H
- induktivitet nulte komponente

LP1 - low-pressure turbine no.1
- niskotlačni dio turbine br. 1

L_D, L_Q - inductance of damping windings in *d* and *q*-axis respectively, H
- induktivitet prigušnog namota u *d* i *q*-osi

LP2 - low-pressure turbine no.2
- niskotlačni dio turbine br. 2

L_d, L_q - inductance of armature windings in *d* and *q*-axis respectively, H
- induktivitet armaturnog namota u *d* i *q*-osi

c_i - torsional stiffness, N·m/rad
- torzijska konstanta krutosti

L_f - inductance of field winding, H
- induktivitet uzbudnog namota

D - damage rate
- stupanj oštećenja

M_{dD} - mutual inductance between armature and damping windings in *d*-axis, H
- međuinduktivitet između armaturnog i prigušnog namota u *d*-osi

D_{dop} - allowed damage
- dopušteno oštećenje

M_{fd} - mutual inductance between field and armature windings in *d*-axis, H
- međuinduktivitet između uzbudnog i armaturnog namota u *d*-osi

E_q - notional e.m.f. proportional to excitation voltage, V
- fiktivni inducirani napon proporcionalan uzbudnom naponu

M_{fD} - mutual inductance between field and damping windings in *d*-axis, H
- međuinduktivitet između uzbudnog i prigušnog namota u *d*-osi

e_q - notional e.m.f. proportional to excitation current, V
- fiktivni induciran napon proporcionalan uzbudnoj struji

M_{qQ} - mutual inductance between armature and damping windings in *q*-axis
- međuinduktivitet između armaturnog i prigušnog namota u *q*-osi

n_i - number of stress cycles of block *i*
- broj ciklusa naprezanja bloka *i*

i_0 - current of zero component, A
- struja nulte komponente

N_{izr} - axis durability
- izdržljivost osovine

i_a, i_b, i_c - armature current of phase *a, b i c* respectively, A
- struja armaturnog namota faze *a, b i c*

r - resistance of armature winding per phase, Ω
- otpor armaturnog namota po fazi,

i_D, i_Q - damping current in *d* and *q*-axis respectively, A
- struje prigušnog namota u *d* i *q*-osi

r_D, r_Q - resistance of damping windings in *d* and *q*-axis respectively, Ω
- otpor prigušnog namota u *d* i *q*-osi

i_d, i_q - armature current in *d* and *q*-axis respectively, A
- struje armaturnog namota u *d* i *q*-osi

r_f - resistance of field winding, Ω
- otpor uzbudnog namota

i_f - field current, A
- uzbudna struja

r_0 - resistance of zero component, Ω
- otpor nulte komponente

J_i - moment of inertia of *i*-lumped elements of the turbine-generator rotor, kg·m^2
- moment tromosti *i*-te rotacijske mase osovinskog voda

S - apparent power, V·A
- prividna snaga

t - time, s
- vrijeme

φ_i - angle between *i*-lumped inertial elements and synchronous axis
- kut između osi i-te rotacijske mase *i* sinkrone osi

T - torque, N·m
- zakretni moment

ψ_0 - flux linkages of zero component, Wb
- ulančani tok nulte komponente

$T_{T,i}$ - torsional torque between *i*- and (*i*-1)-lumped inertial elements, N·m
- torzijski moment između *i*-te i (*i*-1)-te rotacijske mase

ψ_d, ψ_q	- flux linkages of armature windings in *d* and *q*-axis respectively, Wb - ulančani tok armaturnog namota u *d* i *q*-osi
u_0	- voltage of zero component, p.u. - napon nultog sustava
ψ_D, ψ_Q	- flux linkages of damping windings in *d* and *q*-axis respectively, Wb - ulančani tok prigušnog namota u *d* i *q*-osi
u_a, u_b, u_c	- armature voltages of phase *a, b* and *c* respectively, V - napon armaturnog namota faze *a, b i c,*
ψ_f	- flux linkages of field winding, Wb - ulančani tok uzbudnog namota
u_d, u_q	- armature voltages in *d* and *q*-axis respectively, V - naponi armaturnog namota u *d* i *q*-osi
ω	- electrical rotational speed, rad/s - električna kutna brzina
u_f	- excitation voltage, V - uzbudni napon
ω_m	- mechanical rotational speed, rad/s - mehanička kutna brzina
Z	- impedance, Ω - impedancija

Indices / Indeksi

x_{fd}	- mutual inductance between field and armature windings in *d*-axis, Ω - međuinduktivitet između uzbudnog i armaturnog namota u *d*-osi
B	- basic value - bazna vrijednost
γ	- electrical angle between phase a and rotating *d*-axis - električni kut između osi faze a i rotirajuće *d*-osi
a, b, c	- phase a, b and c - faze a, b i c
σ_{ai}	- stress amplitude of block *i*, MPa - amplituda naprezanja bloka *i*
d	- direct-axis - uzdužna os
σ_D	- stess at 10^7 cycle on Woehler's curve, MPa - naprezanje kod 10^7 ciklusa na Woehlerovoj krivulji
q	- quadrature-axis - poprečna os

1. Introduction

An electric power-generating drive consists of several mechanical and electrical machines. As a utility's power plant commonly comprises large generating units, any outage of the unit is associated with high costs for the utility.

Costs of unexpected outages of the generator set can be prevented by the following measures:

- Checking operating security of the generating system by calculations. The calculations have to consider the system as a whole, as well as all possible cases of operation.
- Planning continuous monitoring of the system's condition in order to evaluate the condition of a single component. On the basis of measured values one can **obsere** the inception and developing of degradation at some element of the system. This can be used for making plans for protection from further more severe damages or drive outage.

In the paper practical calculations of the shaft torsional oscillations, as one of the potential risks for the power generating unit, are discussed. The phenomenon of torsional oscillations of a turbine-generator set which occurs during a regular operation as well as at unexpected network faults has been intensively investigated for the last 20 years. Initiated by operating experiences and damages which were the result of subsynchronous resonance (SSR), the comprehensive theoretical investigations have revealed the complex relationship between the electrical network/generator system on one side and the torsional oscillating system of the rotor shaft on the other side [1].

Transient phenomena in a turbine-generator set are a function of phenomena in the complex system of turbine-generator and electrical network. Disturbances in the electrical network cause torsional oscillations in the shafts of the turbine-generator set. These oscillations, if excessive, can cause fatigue in the life reduction of the shaft. The shafts of the turbine-generator set are sensitive to low cyclic excitation torques because of a small low cyclic oscillation damping. This problem does not affect hydrogenerator sets due to their larger shaft stiffness and relatively small inertia of the hydro-turbine compared to the generator.

A shaft-line of a turbine-generator consists of a rotational electrical machine (generator) and a mechanical part, i.e. single parts of a steam- or a gas-turbine. Checking torsional security of any of the drives, one has to consider all the predictable operating conditions: starting up to nominal speed, turning off and retardation process, all the characteristics of nominal operation, i.e. change of the load or the speed, and, finally, all kinds of either electrical or mechanical disturbances. All the predictable disturbances should be considered - ones that occur frequently as well as the occasional ones. By these calculations it has to be confirmed that the system will not collapse or become irreparably damaged in any of its parts. Even the operating conditions which can occur with a very small probability must not be omitted. It is obvious that for these calculations one has to comprehend thoroughly the technological process, its control logic, the behaviour of its mechanical and electrical machines as well as the behaviour of the electrical network. It should be pointed out here that insufficient data flow among manufacturers of single parts of the system and power plant staff impose practical problem, as large number of details involving manufacturing process, functioning and operating conditions of each machine are necessary for the analysis.

A numerical proof of safety regarding torsional stresses due to oscillations can be induced from the equations of motion of the overall electromechanical system, and it essentially depends on the boundary conditions. An electrical machine generates an electromagnetic field which, at rotation, generates a torsional torque in the air-gap between the stator and the rotor. If this torque, due to any cause, alternates or pulsates, it will induce torsional oscillations in the shaft line. Due to the electromagnetic field in the air-gap, the rotor may, as a result of torsional oscillations, influence the shape of the electromagnetic field in the machine. The result is a link of the mechanical and the electrical part of the system. This is similar to the hydraulic machines where there is a link between the motion of both "hydraulic" and mechanical system of the shaft-line. The medium produces an excitation (the change of the pressure and the change of the flow speed) on the one hand, and on the other hand it contributes to the system's inertia and increases the damping of the system [2]. It should be kept in mind that at hydraulic machines, as long as they do not operate optimally, there is a substantial flow separation which, among other things, produces a pulsating torsional torque [3]. Besides, the rotor's blades cause an excitation torque as they pass near the stator's blades [4 - 6]. Characteristics of this excitation have still not been numerically described well enough. Besides, measured results are very few.

Torsional oscillations of the shaft-line of an electrical drive can be calculated by means of three different mechanical models:

- model with discrete inertial elements (lumped mass model),
- model with finite elements,
- continuous parameters model.

It is not always an easy task to choose the number and places of lumped inertial elements. The application of finite elements is recommended for systems with high speed electrical motors whose stators are supplied at a variable frequency, as their high natural frequencies could be excited. Owing to its complexity the model with continuous parameters is seldom applied.

2. Mechanical Model

A lumped mass mechanical model for the analysis of torsional oscillations of the turbine-generator's shaft-line is described by the following parameters:

- moment of inertia with respect to the axis of rotation; distribution of the lumped masses is questionable,
- torsional stiffness of the shaft's parts; difficulties can occur with noncylindrical cross-sections as well as at a large change of the cross-section surface, damping (internal or external); the hypothesis, distribution and the value are questionable.

The calculation of the moment of inertia for cylindrical and finally processed elements is elementary. Insignificant errors can occur due to tolerances and nonhomogeneity of the material. A great number of elements cannot be manufactured with a high precision, e.g. casted wheels for pumps or hydro-turbines where, depending on type, size and type of processing, the moment of inertia can vary from ± 2 % to ± 20 %. Erosion of the material can also have significant influence on the rotor's moment of inertia e.g. at the pumps with substantial cavitation.

The divergence of values in manufacturing of electrical machines is, e.g. for parts of the laminated core, windings, insulation materials, from ±1 % to ±2 %.

The calculation of the torsional stiffness, which depends on the torsional moment of the cross-section's surface, equal only in special cases to the polar moment of inertia, is also elementary. Difficulties occur at the cross-sections which cannot be described by the simple elements in torsional train, and specially at the overlapped or wedged conjunctions. In such cases only boundary values can be quantified and it is incorrect to take average values.

It is especially difficult to include damping into the mechanical model. Both its physical phenomenology and its mathematical description are questionable. However, in technical applications the (linear) viscous damping is often used. This damping can be:

- internal (proportional to the relative oscillation speed), e.g. in the material, i.e. steel parts of the shaft-line or at the contact surfaces,
- external (proportional to the absolute oscillation speed) e.g. damping of the surrounding medium: water, air, the electromagnetic field.

While the value of internal damping is sufficiently covered by the so-called Rayleig's assumption, the external damping is questionable, especially at hydraulic machines for which there is still no adequate description of the torsional damping mechanism.

It should be pointed out that in this case any link between the rotor and the housing and fundaments is neglected, which means that only the link between the rotor and the network is taken into account. The link between the rotor's torsional and lateral oscillation, existing in all cases where there is eccentricity of the rotor parts in regard to his axis, is also neglected.

Excitation of the torsional oscillation due to a short circuit at the generator's mains e.g., depends primarily on the moment in the alternate voltage diagram at which a fault has occurred. The values of the excitation torque can disperse significantly, depending on the operating condition of the generator (loaded or no-loaded) as well as on the fact if the electrical part is mathematically modelled. To avoid this dispersion of the excitation torque a comprehensive mathematical model involving the electrical generator and the related part of the power system should be used. In modelling the generator, it has become common to use the two-axis theory of electrical machines.

3. Computation using damage accumulation hypothesis

The basis of such computation is a linear damage accumulation hypothesis, since vibration load block causes axis damage which is accumulated from all collectors until critical values are reached, when fatigue axis fracture occurs. Damage rate D is defined by contribution of each block $D_i = \frac{n_i}{N_i}$ for n_i block cycle with amplitudes σ_{ai} and average stresses σ_{mi}, where N_i=$N_i(\sigma_{ai}, \sigma_{mi})$ is number of stress cycle of block i on Woehler's curve, i.e. where axis fracture occurs caused only by block i. Axis fracture caused by all block stresses is determined as follows:

$$D = \sum_{i(\sigma_{ai} > \sigma_D)} \frac{n_i}{N_D} \left(\frac{\sigma_D}{\sigma_{ai}} \right)^{-k} + \sum_{i(\sigma_{ai} < \sigma_D)} \frac{n_i}{N_D} \left(\frac{\sigma_D}{\sigma_{ai}} \right)^{-(2k-1)}.$$

Where σ_D is strain at 10^7 cycle on Woehler's curve.

Axis durability N_{izr} is computed from allowed damage for which value D_{dop}= 0,2 is recommended $N_{izr} = \frac{\sum_i n_i}{D} D_{dop}$. It represents the total number of cycles of all load blocks which the axis may endure.

4. Mathematical model of the system

Transient torsional oscillations of the turbine-generator set due to a three-phase fault in the network can be analysed by a simplified electromechanical model. In a model commonly used, as well as in the one here presented, the electromagnetic conditions in the air-gap are simplified i.e. the magneto motive force is taken to be sinusoidal along the air-gap edge, the air-gap is constant, the saturation may or may not be taken into account, and the electric loading is equally and continuously distributed along the air-gap's circumference. The mechanical model of the rotor is discretized by linearly elastic shafts whose mass is neglected and by the stiff discs whose elasticity is neglected.

Electromagnetic processes in the generator's windings are described by a system of voltage nonlinear differential equations, which involve three equations for the armature windings, one equation for the excitation windings and two equations for the damping windings. Written as matrixes they are:

$$[u] = [R][i] + \frac{\mathrm{d}[\psi]}{\mathrm{d}t}. \tag{1}$$

If the voltage of transformation is separated from the voltage of rotation, the following form of the equation (1) is obtained :

$$[u] = [R][i] + [L]\frac{\mathrm{d}[i]}{\mathrm{d}t} + \omega \frac{\mathrm{d}[L]}{\mathrm{d}\gamma}[i], \tag{2}$$

where: $[u] = [-u_a \ -u_b \ -u_c \ u_f \ 0 \ 0]^T$ is the voltage vector, $[i] = [i_a \ i_b \ i_c \ i_f \ i_D \ i_Q]^T$ is the current vector, $[R]$ = diag $[r, r, r, r_f, r_D, r_Q]$ is the diagonal matrix of the active resistances, $[L]$ is the quadrate matrix (6×6) of the inductances, with the stator and the rotor self-inductances on the diagonal, and the mutual-inductances of corresponding windings on the remaining places, $\mathrm{d}\omega = \mathrm{d}\gamma / \mathrm{d}t$ is the electrical angular speed, γ is the electrical angle between the effective axis of the first phase and the rotor's d-axis.

Elements of the matrix of the inductances are mostly the functions of the rotor's position thus making the system of the voltage equations more complex. By an efficient transformation of the stator parameters (indexes a, b, c) this matrix obtains a more convenient structure in the $dq0$ rotating coordinate system. This coordinate system rotates, in relation to the stator, by the speed of the rotor which is not constant in transient states. Figure 1 shows the windings of a synchronous machine and the referential d-q rotational axes.

The matrix of transformation for the stator's values from one coordinate system to the another is (similar to [9]):

$$\left[K_{\mathrm{dq0}}\right] = \begin{bmatrix} \cos\gamma & \sin\gamma & 1 \\ \cos(\gamma - \frac{2\pi}{3}) & \sin(\gamma - \frac{2\pi}{3}) & 1 \\ \cos(\gamma + \frac{2\pi}{3}) & \sin(\gamma + \frac{2\pi}{3}) & 1 \end{bmatrix}. \tag{3}$$

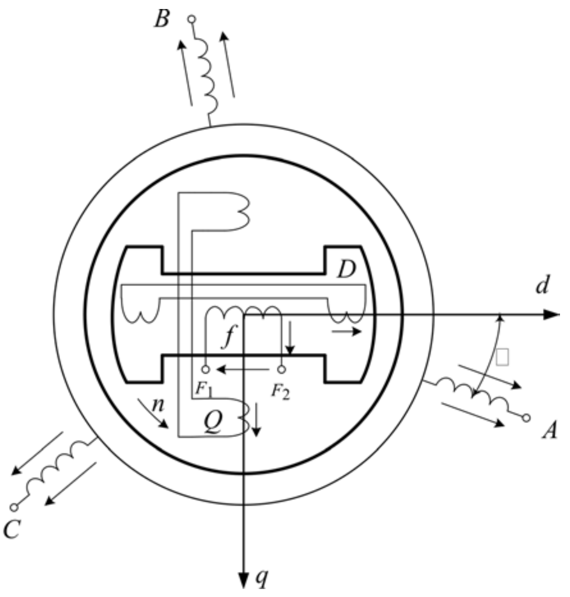

Figure 1. Windings in a synchronous generator and the *d-q* rotational axes

Slika 1. Namoti sinkronog generatora i *d-q* rotacijske osi

The correlations between the voltages and the currents in these coordinate systems are:

$$[u_{\mathrm{abc}}] = [K_{dq0}]\,[u_{dq0}], \tag{4}$$

$$[i_{\mathrm{abc}}] = [K_{dq0}]\,[i_{dq0}], \tag{5}$$

or

$$[u_{dq0}] = [K_{dq0}]^{-1}\,[u_{\mathrm{abc}}], \tag{6}$$

$$[i_{dq0}] = [K_{dq0}]^{-1}\,[i_{\mathrm{abc}}], \tag{7}$$

respectively.

In the theory of synchronous machines the relative, nondimensional values (per unit, p.u.) are used to simplify the comparison among synchronous machines of different ratings and speeds, i.e.: $A(\mathrm{p.u.}) = \dfrac{A(\text{original value})}{A_{\mathrm{B}}(\text{basic value})}$.

In the transient analysis, the most commonly basic values used are the following: $U_{\mathrm{aB}} = U_{\mathrm{B}} = \sqrt{2}\, U_{\mathrm{n\,phase}} = \sqrt{2}\, U / \sqrt{3}$, the armature voltage, $I_{\mathrm{aB}} = I_{\mathrm{B}} = \sqrt{2}\, I_{\mathrm{n\,phase}}$, the armature current, $\omega_{\mathrm{B}} = \omega_{\mathrm{s}} = 2\pi f$, the electrical angular speed, $\omega_{\mathrm{mB}} = \omega_{\mathrm{s}} / \mathrm{p}$, the mechanical angular speed, and the deduced basic values: $S_{\mathrm{B}} = (2/3)\, U_{\mathrm{B}} I_{\mathrm{B}} = S_{\mathrm{n}}$, $\psi_{\mathrm{B}} = U_{\mathrm{B}} / \omega_{\mathrm{B}}$, $Z_{\mathrm{B}} = U_{\mathrm{B}} / I_{\mathrm{B}}$, $L_{\mathrm{B}} = Z_{\mathrm{B}} / \omega_{\mathrm{B}}$, $T_{\mathrm{B}} = S_{\mathrm{B}} / \omega_{\mathrm{mB}}$, $c_{\mathrm{B}} = \mathrm{p}\, S_{\mathrm{B}} / \omega_{\mathrm{mB}}$, $k_{\mathrm{B}} = \mathrm{p}\, S_{\mathrm{B}} / \omega_{\mathrm{mB}}^2$, $t_{\mathrm{B}} = 1 / \omega_{\mathrm{B}}$.

To determine relative values of the rotor's windings, it is necessary to choose arbitrarily the six basic values for the voltages and currents of the excitation and damping windings U_{fB}, I_{fB}, U_{DB}, I_{DB}, U_{QB}, I_{QB}. A suitable choice of basic values is the one for which stator and rotor inductances can be connected into one substitute scheme which keeps the real ratios between the values.

If the standard basic values are chosen, the real inductances matrix is both symmetric and equal to the real reactances matrix:

$$\left[\underline{L}_{dq0fDQ}\right] = \left[\underline{x}_{dq0fDQ}\right] = \begin{bmatrix} \underline{x}_d & & & & & \\ 0 & \underline{x}_q & & & \text{sim.} & \\ 0 & 0 & \underline{x}_0 & & & \\ \underline{x}_{fd} & 0 & 0 & \underline{x}_f & & \\ \underline{x}_{dD} & 0 & 0 & \underline{x}_{fD} & \underline{x}_D & \\ 0 & \underline{x}_{qD} & 0 & 0 & 0 & \underline{x}_Q \end{bmatrix}. \tag{8}$$

The following values are also widely used: $\underline{E}_{\mathrm{q}} = \underline{u}_{\mathrm{f}}\, \underline{x}_{\mathrm{fd}} / r_{\mathrm{f}}$ the armature voltage at stationary state induced by the stationary excitation current ("rotor's voltage", "fictive induced voltage") and $\underline{e}_{\mathrm{q}} = \underline{i}_{\mathrm{f}}\, \underline{x}_{\mathrm{fd}}$ the armature voltage in stationary state induced by the momentary value of the excitation current.

Now the voltage differential equations written in *dq*0 rotating coordinates are:

$$-\underline{u}_d = \underline{r}\,\underline{i}_d + \frac{\mathrm{d}\underline{\psi}_d}{\mathrm{d}t} + \underline{\omega}\,\underline{\psi}_q, \tag{9}$$

$$-\underline{u}_q = \underline{r}\,\underline{i}_q + \frac{\mathrm{d}\underline{\psi}_q}{\mathrm{d}t} + \underline{\omega}\,\underline{\psi}_d, \tag{10}$$

$$-\underline{u}_0 = \underline{r}\,\underline{i}_0 + \frac{\mathrm{d}\underline{\psi}_0}{\mathrm{d}\underline{t}}, \tag{11}$$

$$\underline{E}_0 = \underline{e}_0 + \frac{\underline{x}_{fd}}{\underline{r}_r}\frac{\mathrm{d}\underline{\psi}_0}{\mathrm{d}\underline{t}}, \tag{12}$$

$$0 = \underline{r}_D\,\underline{i}_D + \frac{\mathrm{d}\underline{\psi}_D}{\mathrm{d}\underline{t}}, \tag{13}$$

$$0 = \underline{r}_Q\,\underline{i}_Q + \frac{\mathrm{d}\underline{\psi}_Q}{\mathrm{d}\underline{t}}, \tag{14}$$

where flux linkages are:

$$\underline{\psi}_{\mathrm{d}} = \underline{L}_{\mathrm{d}}\underline{i}_{\mathrm{d}} + \underline{M}_{\mathrm{fd}}\underline{i}_{\mathrm{f}} + \underline{M}_{\mathrm{dD}}\underline{i}_{\mathrm{D}}, \tag{15}$$

$$\underline{\psi}_{\mathrm{q}} = \underline{L}_{\mathrm{q}}\underline{i}_{\mathrm{q}} + \underline{M}_{\mathrm{qQ}}\underline{i}_{\mathrm{Q}}, \tag{16}$$

$$\underline{\psi}_0 = \underline{L}_0\underline{i}_0, \tag{17}$$

$$\underline{\psi}_{\mathrm{f}} = \underline{L}_{\mathrm{f}}\underline{i}_{\mathrm{f}} + \frac{3}{2}\underline{M}_{\mathrm{df}}\underline{i}_{\mathrm{d}} + \underline{M}_{\mathrm{fD}}\underline{i}_{\mathrm{D}}, \tag{18}$$

$$\underline{\psi}_{\mathrm{D}} = \underline{L}_{\mathrm{D}}\underline{i}_{\mathrm{D}} + \frac{3}{2}\underline{M}_{\mathrm{dD}}i_{\mathrm{D}} + \underline{M}_{\mathrm{fD}}\underline{i}_{\mathrm{f}}, \tag{19}$$

$$\underline{\psi}_{\mathrm{Q}} = \underline{L}_{\mathrm{Q}}\underline{i}_{\mathrm{Q}} + \frac{3}{2}\underline{M}_{\mathrm{qQ}}\underline{i}_{\mathrm{Q}}. \tag{20}$$

If (11) is omitted, since neither of the armature currents affects it (symmetrical phenomena), the five voltage equations are left, two for the armature, one for the excitation windings and two for the damping windings.

The equations of motion of the considered discrete torsional model of the mechanical part of the system are:

$$\underline{J}_{m,i}\frac{\mathrm{d}\underline{\omega}_{m,i}}{\mathrm{d}\underline{t}} = \underline{c}_{i+1}(\varphi_{i+1}-\varphi_i)-\underline{c}_i(\varphi_i-\varphi_{i-1})+ \underline{k}_{i+1}(\underline{\omega}_{m,i+1}-\underline{\omega}_{m,i})-\underline{k}_i(\underline{\omega}_{m,i}-\underline{\omega}_{m,i-1})+\underline{T}_i \tag{21}$$

i=1(1)n,

where: $\underline{J}_{m,i} = J_i / (S_B / \omega_{mB}^3)$, i =1(1)n - the mechanical time constants of the lumped inertial elements of the turbine-generator's rotor, $\underline{T}_\mathrm{i} = \underline{\psi}_\mathrm{q}\underline{i}_\mathrm{d} - \underline{\psi}_\mathrm{d}\underline{i}_\mathrm{q}$ - the relative air-gap torque of the generator, and $\underline{\omega}_{m,i} = \frac{\mathrm{d}\varphi_i}{\mathrm{d}\underline{t}}$, i =1(1)n - the mechanical speed of the lumped inertial elements.

A real mechanical angular speed ω_m of the generator's rotor is for a number of pole pairs less than angular speed ω and in the case of the turbine-generator they are usually equal since the number of the pole pairs usually equals one.

A torsional torque in the shaft or in the connection elements between two lumped inertial elements is calculated as: $\underline{T}_\mathrm{Ti} = \underline{c}_\mathrm{i}\,(\underline{\varphi}_\mathrm{i} - \underline{\varphi}_{i-1})$.

5. Computer Program

Since the model involves a system of differential equations, it could be best solved by a numerical integration on a digital computer. A number of commercial soflware packages with different features are nowadays available for this purpose. The results presented in the paper are obtained by implementation of the model onto a SUN Sparc Workstation by a Matrixx, a general purpose modelling software package widely used in the analysis of control circuits. The model is designed for analysing the symmetrical faults only and is based on the Second Benchmark model suggested by the Working Group 11-O 1 of the CIGRE Study Committee 11 [10]. Different models of the generator's excitation and the speed control can be optionally included. In the model design we aimed at obtaining a versatile model which can be modified to match requirements for different types of analyses by changing it minimally.

The model's structure, consisting of a number of different blocks, allows for changes and upgrades of the model (adding control circuits, changing the number of lumped masses, modelling the saturation) to be made in a simple way. It also enables the user to simply choose the number of outputs from the model to be presented during or after the simulation. In cases when a great number of simulations have to be done, the whole process can be automated by the simple user's program routines, and if the same model structure can be kept in all simulations, calculation speed can be improved by generating the model's "hyper code".

The implemented model enables optional inclusion of the saturation effect. If the effect of saturation is of no interest or there is no required data available, reactances are considered as constants provided by the generator input data. The effect of saturation is included by a function which describes magnetization curve of the generator involved. The parameters of the function have to be calculated in a presimulation procedure according to the generator's magnetization curve. From the obtained comprehensive function, the generator's reactances are calculated in each integration step depending on the working point. The effect of saturation is applied to the d-axis reactances only.

The power system presentation involves a step-up transformer and an optional number of parallel transmission lines connecting the generator to the infinite bus bars, and it is modelled by differential equations in dq-axis. Both the transmission lines and the transformer are characterized by their lumped resistance and reactance.

6. Example of numerical analysis

A numerical analysis of torsional transients in a turbine generator set is carried out by the computer program described in section 4. The analysis is carried out by the Second Benchmark model suggested by the Working Group 11-01 of the CIGRE Study Committee 11. The electrical part of the model is presented in Figure 2, and the mechanical part in Figure 3.

The mechanical part consists of five lumped inertial elements: a high-pressure turbine (HP), an intermediate-pressure turbine (IP), two low-pressure turbines (LP1, LP2) and a generator.

The following characteristic sequence of disturbance is analysed: a turbine-generator set operates at nominal load and supplies the network via two parallel transmission lines; a three-phase short circuit at one of the parallel lines occurs and is followed by a fault clearing of the short-circuited transmission line. The influence of the fault clearing time (FCT) after the short-circuit is also analysed. The FCT is varied from 3T to 10T, where T = 1 / 50 s or a period of the network frequency, i.e. from 60 ms to 200 ms. The excitation control response and the governor have not been taken into account at the calculation. The results of the analysis are presented in Figures 5, 6 and 7.

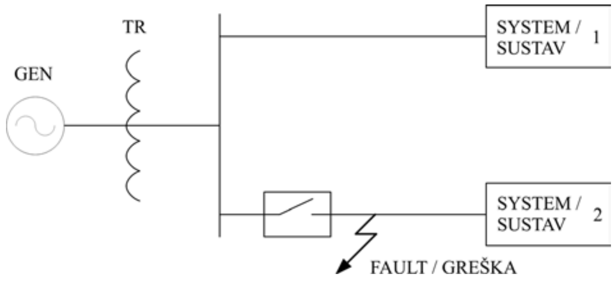

Figure 2. Electrical network representation

Slika 2. Prikaz električne mreže

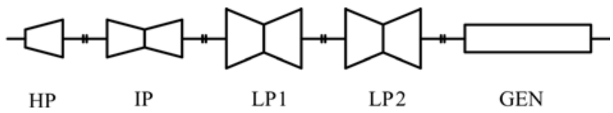

Figure 3. Model of the shaft-line

Slika 3. Model osovinskog voda

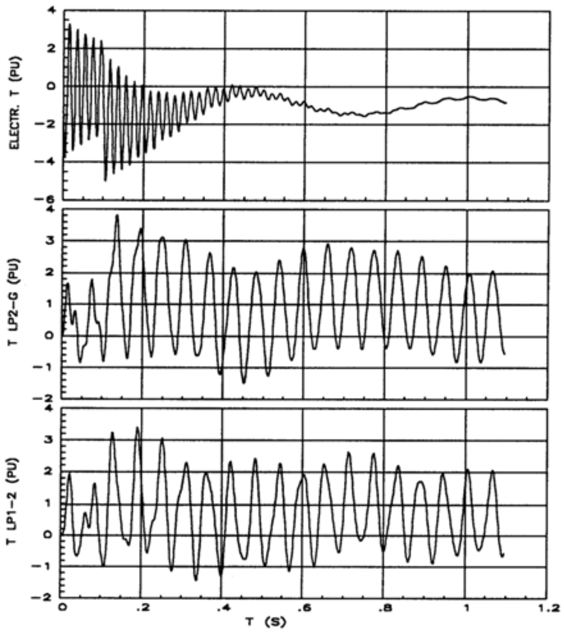

Figure 4. Transients in electrical air-gap torque and torsional torques in the shafts LP2-GEN and LPl-LP2 after a three-phase short circuit at ***t*** = 0 and FCT at ***t*** = 0,095 s (FCT = 4,75 *T*)

Slika 4. Prijelazna pojava elektromagnetskog momenta i torzijskih momenata u osovinskom vodu LP2-GEN i LPl-LP2 poslije trofaznog kratkog spoja u $t = 0$ i FCT u $t = 0{,}095$ s (FCT = 4,75 *T*)

The basic values for the analysed turbine-generator set, calculated according to section 4, are: $S_B = 850$ MV·A, $T_B = 2{,}707 \cdot 10^6$ N·m, $c_B = 2{,}706 \cdot 10^6$ N·m/rad, $k_B = 8{,}612 \cdot 10^3$ N·m·s/rad, $\omega_B = 314{,}16$ rad/s.

The values of basic parameters of the analysed turbine-generator set are: $\underline{J}_{m1} = 1{,}312$ s, $\underline{J}_{m2} = 1{,}602$ s, $\underline{J}_{m3} = 1{,}602$ s, $\underline{J}_{m4} = 0{,}255$ s, $\underline{J}_{m5} = 0{,}102$ s, $\underline{c}_2 = 42{,}5$, $\underline{c}_3 = 88{,}7$, $\underline{c}_4 = 94{,}25$, $\underline{c}_5 = 59{,}14$, $\underline{k}_2 = \underline{k}_3 = \underline{k}_4 = \underline{k}_5 = 1$, $\underline{T}_2 = 0{,}2075$, $\underline{T}_3 = 0{,}2075$, $\underline{T}_4 = 0{,}2296$, $\underline{T}_5 = 0{,}203$.

Figure 4 presents the air-gap torque and the torsional torques in the shafts between the generator (GEN) and the low-pressure turbine 2 (LPT2), and between the two low-pressure turbines (LPT1, LPT2) at the analysed disturbance. The wave-forms are obtained for the case of fault clearing at 95 ms (4,75*T*) after the fault occurrence. It can be seen that at the short circuit both the peak air-gap torque and the torsional torques reach high values, and after the fault clearing the values are additionally increased. The peak values of torsional torques exceed four-times nominal value (0,8 p.u.) for several cycles for the chosen value of internal damping coefficient ($k = 1$ p.u.). The contribution of analysed disturbance sequence to the shaft fatigue can be calculated from the waveforms of the torsional torque.

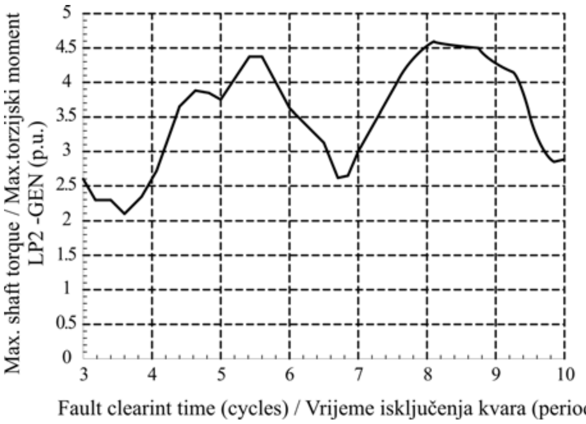

Figure 5. Maximum torsional torque in the shaft LP2-GEN as a function of chosen FCT after a three-phase short circuit at one of the two parallel transmission lines

Slika 5. Maksimalni torzijski moment u osovinskom vodu između niskotlačnog dijela turbine i generatora (LP2-GEN) u ovisnosti o izabranom vremenu isključenja kvara (FCT) poslije trofaznog kratkog spoja na jednom od dva paralelna prijenosna voda

By a series of simulations obtained by varying the value of the fault clearing time between 3*T* and 10*T* the maximum torsional torques presented in Figures 5 and 6 are obtained.

Maximum torsional torques in the shafts between the generator and the low-pressure turbine 2, and between the two low-pressure turbines are shown in Figure 5 and 6 respectively. It can be clearly seen that the maximum shaft torque depends on the moment of fault clearing. Thus, the maximum shaft torque in the shafts of the analysed turbine-generator set will be the lowest if the fault clearing time is set to be between 6.2 and 7 cycles (124-140 ms). The worst case is when FCT is between cca 7.5 and 9 cycles.

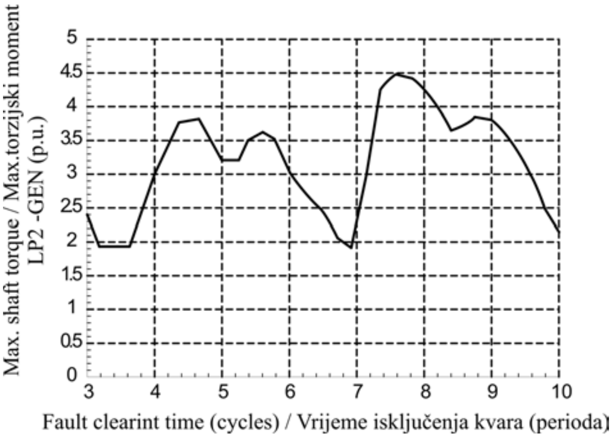

Figure 6. Maximum torsional torque in the shaft LPl-LP2 as a function of chosen FCT after a three-phase short circuit at one of the two parallel transmission lines

Slika 6. Maksimalni torzijski moment u osovinskom vodu između dva dijela niskotlačne turbine (LP1-LP2) u ovisnosti o izabranom vremenu isključenja kvara (FCT) poslije trofaznog kratkog spoja na jednom od dva paralelna prijenosna voda

From [11], it can be clearly seen that an individual calculation should be carried out for each turbogenerator as the results (max. shaft torque vs. FCT) vary for different sets of turbogenerator and network data.

7. Conclusion

Torsional oscillations in the shafts of a power-generating unit induced by various types of operational conditions can, due to the fatigue mechanism, jeopardize availability of the unit and lead to unplanned costs. Therefore, the operating security of the generating system should be checked by calculations in order to evaluate the condition of each component. From the values obtained possible inception or development of degradation of the system's elements can be assessed. This can be used for making plans for protection either from further, more severe, damages or drive outage.

It is possible to analyse numerically the air-gap torque and the torsional torques in the shafts of a turbine-generator set at different transient states by a mathematical model. The model should involve both the electrical and the mechanical characteristics of a turbine-generator set since their influences on the torsional behaviour of the set are linked.

In an analysis of the torsional oscillations some uncertainties of the model and its parameters must be taken into account. Further research on nature and modelling of damping, particularly one related to the external damping, is requied.

Since the model for an analysis of torsional oscillations involves a system of differential equations, the analysis should be carried out by a numerical integration. For different purposes of analysis, some additional details can be built into the model.

The presented example of analysis of the torsional oscillation of the turbine-generator set shows that some typical disturbances like a three-phase short circuit followed by a fault clearing can cause serious torsional torques, which can add to the fatigue life reduction of a shaft line. The example shows that maximum torsional torques in the shafts of a turbine-generator set, at a fault clearing following a three-phase short circuit at a transmission line, substantially depend on chosen fault clearing time. Therefore, the results of such an analysis can be used for an optimal setting of network protection devices with a risk of excessive torsional stresses in the shafts of a generating unit taken into account. Finally, it should be noted that the minimum of the maximum torsional torques at a turbine-generator set and a power system with different parameters occurs at different fault clearing times. Thus the analysis of the air-gap and torsional torques should be carried out individually for each combination of the turbine-generator and the network parameters.

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