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Semi-Analytical Solution for Elastic Impact of Two Beams

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This paper presents semi-analytical solution for the problem of elastic impact of two beams. The solution is based on the finite element discretization of the structure and equation of motion solution using diagonalization method for solving a system of differential equations. This procedure avoids temporal discretization typical for numerical methods, and in this way eliminates influence of the time-step size on the solution and numerical stability problems that can occur if time step isn't defined properly. Computer code is developed for calculation, and application of the procedure is illustrated with the case of collision of communication bridge between two floating objects and safety structure called arrestor. The results are compared to those obtained by numerical procedure using the same discretization of the structure and applying energy conservation principle. Very good agreement of the results obtained by three methods is achieved.

Poluanalitičko rješenje elastičnog sudara dviju greda

Izvornoznanstveni članak

U članku je prikazano poluanalitičko rješenje problema elastičnog sudara dviju greda. Rješenje se temelji na fizičkoj diskretizaciji konstrukcije metodom konačnih elemenata i rješavanju jednadžbe gibanja metodom dijagonalizacije za rješavanje sustava diferencijalnih jednadžbi. Ovaj postupak omogućuje izbjegavanje vremenske diskretizacije koja je tipična za numeričke metode, i na taj način se eliminira utjecaj veličine vremenskog koraka na rješenje kao i probleme s numeričkom stabilnošću, koji se mogu pojaviti ako vremenski korak nije pogodno odabran. Razvijen je kod za provedbu proračuna, a primjena postupka je ilustrirana slučajem sudara komunikacijskog mosta između dvaju plovnih objekata i zaštitne konstrukcije zvane arestor. Rezultati su uspoređeni s rezultatima dobivenim pomoću numeričkog postupka koristeći identičnu diskretizaciju konstrukcije i primjenom principa očuvanja energije, te je uočeno njihovo jako dobro podudaranje.

Keywords
Diagonalization method
Elastic impact
FEM
Semi-analytical solution

Ključne riječi
Elastični sudar
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MKE
Poluanalitičko rješenje

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1. Introduction

Development of the procedure for solving elastic impact problem of two beams is inspired by practical engineering problem of analyzing impact response of communication bridge between two floating objects and a safety structure called arrestor. The problem is described in details in [1], where numerical procedure based on the FEM technique and modified linear acceleration method is presented. Impact problem of this type can be also solved by applying energy conservation principle [1, 2]. The former method is more suitable for detailed structural analysis, and the latter one is more convenient for standard engineering practice. It should be noticed that energy approach, which is very fast and especially appropriate in early design stage, gives lower stress level than numerical procedure since it can not capture local deformations and stresses, and it doesn't distinguish natural vibration modes and their influence on stress shape. Numerical procedure captures these effects, but it requires discretization of the structure and temporal discretization, which both influence the results. Moreover, the time step duration influences the stability of the solution. Concerning discretization of the structure into number of finite elements, it should be emphasized that longer finite elements close to contact location give lower values of contact force, so the topology of the finite elements should be properly adjusted to describe physical background of the problem in reliable way. Temporal discretization can be avoided by analytical solving differential equation of motion, and in this way one can get more reliable results. The semi-analytical procedure presented in this paper combines finite element technique for continuum discretization, and

Symbols/Oznake

A_c	- cross-section area of contact element - površina presjeka kontaktnog elementa
c	- integration constant - konstanta integracije
d	- horizontal distance between lower beam support and end - horizontalna udaljenost između oslonca i kraja donje grede
E	- Young's modulus - Youngov modul elastičnosti
F_c	- contact element load - opterećenje kontaktnog elementa
F_{din}	- dynamic contact force - dinamička kontaktna sila
g	- acceleration of gravity - gravitacijsko ubrzanje
h	- vertical distance between the upper beam centre of gravity at initial and at impact time instant - vertikalna udaljenost između težišta gornje grede u početnom trenutku i trenutku udara
h_{din}	- vertical distance between the upper beam centre of gravity at impact time instant and at maximum deflection instant - vertikalna udaljenost između težišta gornje grede u trenutku udara i maksimalnog progiba
i	- time step - vremenski korak
k_f	- flexion coefficient - koeficijent fleksije
l_c	- contact element length - duljina kontaktnog elementa
l_l	- lower beam length - duljina donje grede
l_u	- upper beam length - duljina gornje grede
m_i	- modal mass for certain vibration mode - modalna masa za određeni oblik vibriranja
m_l	- lower beam mass - masa donje grede
m_u	- upper beam mass - masa gornje grede
q_l	- lower beam distributed load - distribuirano opterećenje donje grede
q_u	- upper beam distributed load - distribuirano opterećenje gornje grede
t	- time variable - vrijeme
w_{din}	- dynamic deflection - dinamički progib
$y_{k,l}$	- degree of freedom of lower beam where contact is realized - stupanj slobode donje grede na mjestu kontakta
$y_{k,u}$	- degree of freedom of upper beam where contact is realized - stupanj slobode gornje grede na mjestu kontakta
$\mathbf{A}$	- auxiliary matrix - pomoćna matrica
$\mathbf{C}$	- damping matrix - matrica prigušenja
$\mathbf{C}_l$	- lower beam global damping matrix - globalna matrica prigušenja donje grede
$\mathbf{C}_u$	- upper beam global damping matrix - globalna matrica prigušenja gornje grede
$\mathbf{D}$	- auxiliary matrix - pomoćna matrica
$\mathbf{K}$	- stiffness matrix - matrica krutosti
$\mathbf{K}_c$	- contact element stiffness matrix - matrica krutosti kontaktnog elementa
$\mathbf{K}_l$	- lower beam global stiffness matrix - globalna matrica krutosti donje grede
$\mathbf{K}_u$	- upper beam global stiffness matrix - globalna matrica krutosti gornje grede
$\mathbf{M}$	- mass matrix - matrica mase
$\mathbf{M}_l$	- lower beam global mass matrix - globalna matrica masa donje grede
$\mathbf{M}_u$	- upper beam global mass matrix - globalna matrica masa gornje grede
$\mathbf{P}, \mathbf{Q}, \mathbf{R}$	- auxiliary matrices - pomoćne matrice
$\mathbf{S}$	- integration matrix - matrica integracije
$\mathbf{\Phi}$	- natural vibration modes matrix - matrica prirodnih oblika vibriranja
$\{F(t)\}$	- force vector - vektor sile
$\{F_l\}$	- lower beam nodal forces vector - vektor čvornih sila donje grede
$\{F_u\}$	- upper beam nodal forces vector - vektor čvornih sila gornje grede
$\{f\}_{i+1}$ $\{g_p\}$ $\{h_p\}$ $\{p\}$	- auxiliary vectors - pomoćni vektori
$\{\delta\}$	- displacement vector - vektor pomaka
$\{\delta_l\}$	- lower beam displacement vector - vektor pomaka donje grede
$\{\delta_u\}$	- upper beam displacement vector - vektor pomaka gornje grede
$\{\dot{\delta}\}$	- velocity vector - vektor brzine
$\{\dot{\delta}_l\}$	- lower beam velocity vector - vektor brzine donje grede

$\{\dot{\delta}_u\}$	- upper beam velocity vector - vektor brzine gornje grede	γ_i	- non-dimensional damping coefficient for certain vibration mode - bezdimenzijski koeficijent prigušenja za određeni oblik vibriranja
$\{\ddot{\delta}\}$	- acceleration vector - vektor ubrzanja	Δt	- interval duration - vremenski interval
$\{\ddot{\delta}_l\}$	- lower beam acceleration vector - vektor ubrzanja donje grede	ε_u	- upper beam angular acceleration - kutno ubrzanje gornje grede
$\{\ddot{\delta}_u\}$	- upper beam acceleration vector - vektor ubrzanja gornje grede	ω_i	- natural frequency of certain vibration mode - prirodna frekvencija određenog oblika vibriranja

diagonalization method for solving differential equation of motion. The commercial software package [3] is used to develop computer code for calculation, and numerical example of collision of communication bridge between two floating objects and arrestor is solved. Verification of the developed semi-analytical procedure is checked by correlating stress level obtained by numerical procedure and by applying energy conservation principle.

2. Formulation of the problem

Two elastic beams, i.e. upper and lower beam are considered, Figure 1. One end of the upper beam is sliding supported, and the other one is hinge jointed. If sliding support is moved off, upper beam is pitching and strikes the free end of the second beam. Another end of the lower beam is fixed. It is assumed that pitching of the upper beam is influenced only by gravitational force, but other excitation can also be taken into account. Dynamic interaction of the beams starts when upper beams strikes the lower one. The interaction can be treated as alteration of contact and non-contact stages, and basic equation of dynamic equilibrium (equation of motion) yields [4]:

$$\mathbf{K}\{\delta\}+\mathbf{C}\{\dot{\delta}\}+\mathbf{M}\{\ddot{\delta}\}=\{F(t)\}. \qquad (1)$$

Upper and lower beam are being discretized into a number of finite elements. Global stiffness and mass matrices for beams are defined according to [4], and global damping matrices for upper and lower beam are defined according to [5]. Constitution of damping matrix is rather complex, and study of damping influence on system dynamic response, conducted and presented in [1], showed that damping matrix can be derived by taking into account critical damping for all vibration modes. So, the global damping matrix can be calculated according to the following formula [5]:

$$\mathbf{C}=\mathbf{\Phi}^{-T}\left\lceil 2\gamma_i\,\omega_i m_i \right\rfloor\mathbf{\Phi}^{-1}. \qquad (2)$$

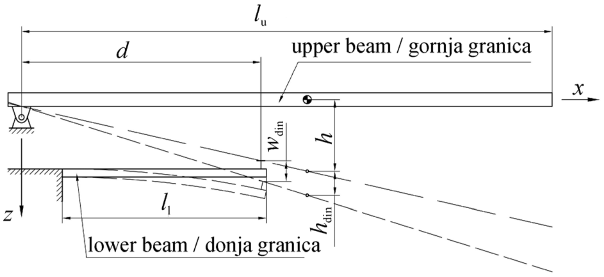


Figure 1. Disposition of upper and lower beam

Slika 1. Dispozicija gornje i donje grede

For standard vibration analyses, recommended value of non-dimensional damping coefficient is between 0,02 and 0,05.

Beam 4DOF finite element is used for beam modeling, and bar 2DOF finite element is used for contact element modeling. The role of the contact element is description of local behavior of both beams close to the impact place. Since the procedure intends to give global response, the influence of considered contact element on global deformations of the rest of the construction should be minimized. This can be achieved by choosing large stiffness of the contact element so that its deformation is within elastic domain.

During the pitching of the upper one, the considered beams are two independent bodies. Only during the contact these beams can be treated as one structure. It is also obvious that the beams are not in contact from the impact time instant till the resting time instant. Moreover, the interaction of considered beams consists of a number of contact and non-contact periods. In other words, we can say that the contact of beams can sustain only compressive but not the tensile force. This fact should be taken into account when defining the system of equations of motion. Therefore, by means of Eq. (1) two systems with two possible stages of beams interaction should be defined, i.e. the first one without the contact and the second one with the contact. Governing equation for the first stage, where beams act as two independent bodies, yields:

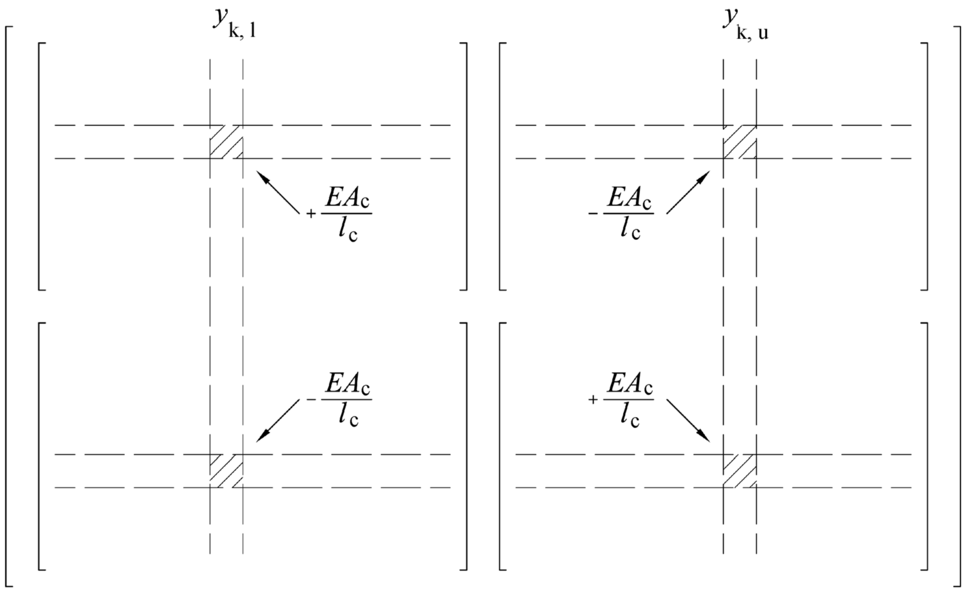
$$\begin{bmatrix} \mathbf{K_l} & \mathbf{0} \\ \mathbf{0} & \mathbf{K_u} \end{bmatrix} \begin{Bmatrix} \{\delta_l\} \\ \{\delta_u\} \end{Bmatrix} + \begin{bmatrix} \mathbf{C_l} & \mathbf{0} \\ \mathbf{0} & \mathbf{C_u} \end{bmatrix} \begin{Bmatrix} \{\dot{\delta}_l\} \\ \{\dot{\delta}_u\} \end{Bmatrix} + \\ + \begin{bmatrix} \mathbf{M_l} & \mathbf{0} \\ \mathbf{0} & \mathbf{M_u} \end{bmatrix} \begin{Bmatrix} \{\ddot{\delta}_l\} \\ \{\ddot{\delta}_u\} \end{Bmatrix} = \begin{Bmatrix} \{F_l\} \\ \{F_u\} \end{Bmatrix}. \tag{3}$$

When considered beams are in contact, inertial and damping forces take effects on both beams independently, but restoring forces are coupled. So, the coupling should be introduced in Eq. (3) through new stiffness matrix, and it is achieved by imposing the 2DOF bar finite element stiffness matrix into the old one. The stiffness matrix of 2DOF bar finite element yields [4]:

$$\mathrm{K_c} = \frac{EA_c}{l_c} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \tag{4}$$

and it is added up together with matching terms in the old stiffness matrix. Addition of matching terms into old stiffness matrix can be described in the following form:

K=



(5)

For the purposes of the calculation execution and because of code development requirements it is necessary to describe alteration of contact and non-contact stages by appropriate condition which expresses appearance and disappearance of compressive load in contact element, and it is given by the following formula:

$$F_c = \frac{EA_c}{l_c}\left(y_{k,u} - y_{k,l}\right) < 0. \tag{6}$$

It is more suitable to replace previous equation by using the following one:

$$y_{k,u} - y_{k,l} < 0. \tag{7}$$

Thus, if the condition in Eq. (7) is satisfied, there is contact between the beams and vice-versa.

As mentioned before, external load is caused only by gravitational force, and taking into account that cross-section of beams is assumed to be uniform, the weight is uniformly distributed per length. In that case the load vector can easily be defined, and for distributed load of the upper and the lower beam one can write:

$$q_u(x) = \frac{m_u g}{l_u}; \quad q_l(x) = \frac{m_l g}{l_l}. \tag{8}$$

3. Solution of differential equation of motion

Equation (1) is a nonhomogenous second-order differential equation (system of differential equations) which can be reduced to a first-order system. If Eq. (1) is multiplied by $\mathbf{M}^{-1}$ from the left we can write:

$$\mathbf{M}^{-1}\mathbf{K}\{\delta\} + \mathbf{M}^{-1}\mathbf{C}\{\dot{\delta}\} + \{\ddot{\delta}\} = \mathbf{M}^{-1}\{F(t)\}. \tag{9}$$

To reduce Eq. (9) to a first-order differential equation we introduce the following substitution:

$$\{\dot{\delta}\}_1 = \{\delta\}_2. \tag{10}$$

Thus, one can write:

$$\{\dot{\delta}\}_2 = -\mathbf{M}^{-1}\mathbf{C}\{\delta\}_2 - \mathbf{M}^{-1}\mathbf{K}\{\delta\}_1 + \mathbf{M}^{-1}\{F(t)\}. \tag{11}$$

Having in mind Eqs. (10) and (11) one can obtain:

$$\begin{Bmatrix} \{\dot{\delta}\}_1 \\ \{\dot{\delta}\}_2 \end{Bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix} \begin{Bmatrix} \{\delta\}_1 \\ \{\delta\}_2 \end{Bmatrix} + \begin{Bmatrix} \{0\} \\ -\mathbf{M}^{-1}\{F(t)\} \end{Bmatrix}. \quad (12)$$

Because of simplicity we introduce the following substitutions:

$$\{p\} = \begin{Bmatrix} \{\delta\}_1 \\ \{\delta\}_2 \end{Bmatrix}, \quad (13)$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad (14)$$

$$\{g_p\} = \begin{Bmatrix} \{0\} \\ -\mathbf{M}^{-1}\{F(t)\} \end{Bmatrix}. \quad (15)$$

Now, after the reduction of equation of motion to a first-order nonhomogenous system we write:

$$\{\dot{p}\} = \mathbf{A}\{p\} + \{g_p(t)\}. \quad (16)$$

There are several methods on disposal for solving Eq. (16), as for example method of undetermined coefficients, method of the variation of parameters or method of diagonalization [6]. Here, the method of diagonalization is applied, since this kind of solving of equations requires eigenvectors and eigenvalues calculation, which is very similar to ordinary engineering problem of analyzing free vibrations, i.e. natural modes and natural frequencies calculation. The idea of this method is to "decouple" the n equations of a linear system, so that each equation contains only one of the unknown functions, i.e. deflections p_1, …, p_n and thus can be solved independently of the other equations. Matrix $[A]$ has a basis of eigenvectors $\{x\}^{(1)}$, …, $\{x\}^{(n)}$. Then, one can write:

$$\mathbf{D} = \mathbf{X}^{-1}\mathbf{A}\mathbf{X}. \quad (17)$$

where $[D]$ is diagonal matrix with eigenvalues λ_1, …, λ_n of $[A]$ on the main diagonal, and $[X]$ is the n x n matrix with columns $\{x\}^{(1)}$, …, $\{x\}^{(n)}$. It should be noted that $[X]^{-1}$ exists since these columns are linearly independent.

To apply diagonalization to (16) we define the new unknown function:

$$\{z\} = \mathbf{X}^{-1}\{p\}. \quad (18)$$

Then we can write:

$$\{p\} = \mathbf{X}\{z\}. \quad (19)$$

Substituting (19) into (16) we obtain:

$$\mathbf{X}\{\dot{z}\} = \mathbf{A}\mathbf{X}\{z\} + \{g_\mathrm{p}\}. \quad (20)$$

Now, we have to multiply (20) by $[X]^{-1}$ from left, obtaining:

$$\{\dot{z}\} = \mathbf{X}^{-1}\mathbf{A}\mathbf{X}\{z\} + \{h_\mathrm{p}\}. \quad (21)$$

where

$$\{h_p\} = \mathbf{X}^{-1}\{g_\mathrm{p}\}. \quad (22)$$

Because of (17) we can write this:

$$\{\dot{z}\} = \mathbf{D}\{z\} + \{h_\mathrm{p}\}, \quad (23)$$

and in components:

$$\dot{z}_\mathrm{j} = \lambda_\mathrm{j} z_\mathrm{j} + h_{\mathrm{p,j}}, \quad (24)$$

where $j = 1, …, n$. We can solve each of these n linear equations and then we have:

$$z_\mathrm{j}(t) = c_\mathrm{j} e^{\lambda_j t} + e^{\lambda_j t} \int e^{-\lambda_j t} h_{p,j}(t)\,\mathrm{d}t. \quad (25)$$

These are the components of $\{z(\mathrm{t})\}$, and from them we obtain the solution of (19).

It is already mentioned that execution of the calculation consists of solving differential equation of motion in coupled and uncoupled form, since contact and non-contact stages are altering. This fact enables us determination of integration constants c_j, which are for each stage determined from the antecedent one. When starting the calculation, the displacement vector is known, and bearing in mind Eqs. (19) and (25), we can write:

$$\{p^0\} = \mathbf{X}\left(\{c\}e^{\{\lambda\}t} + e^{\{\lambda\}t}\int e^{-\{\lambda\}t}\{h_p\}\mathrm{d}t\right), \quad (26)$$

$$\{c\}e^{\{\lambda\}t} + e^{\{\lambda\}t}\int e^{-\{\lambda\}t}\{h_p\}\mathrm{d}t = [X]^{-1}\{p^0\}, \quad (27)$$

and finally:

$$\{c\} = \mathbf{X}^{-1}\{p^0\}e^{-\{\lambda\}} - \int e^{-\{\lambda\}t}\{h_p\}\mathrm{d}t. \quad (28)$$

4. Outline of numerical procedure and energy conservation principle approach

Numerical procedure is detailed described in [1], and description of energy conservation principle is given in [2]. Here, only basic remarks and formulae are given.

4.1. Numerical procedure

Solution of the Eq. (1) can be obtained by assuming linear acceleration in specified time interval [3]. Velocity and displacement vector at time instant t_{i+1} are being calculated by using the following formula [1, 3]:

$$\left\{\dot{\delta}\right\}_{i+1}=\left\{\dot{\delta}\right\}_i+\frac{1}{2}\Delta t\left(\left\{\ddot{\delta}\right\}_i+\left\{\ddot{\delta}\right\}_{i+1}\right),$$
$$\left\{\delta\right\}_{i+1}=\left\{\delta\right\}_i+\Delta t\left\{\dot{\delta}\right\}_i+\frac{\Delta t^2}{3}\left\{\ddot{\delta}\right\}_i+\frac{\Delta t^2}{6}\left\{\ddot{\delta}\right\}_{i+1}. \tag{29}$$

Acceleration vector in (29) is obtained from:

$$\left\{\ddot{\delta}\right\}_{i+1}=\mathbf{S}^{-1}\left\{f\right\}_{i+1}, \tag{30}$$

where:

$$\left\{f\right\}_{i+1}=\left\{F(t)\right\}_{i+1}-\left(\mathbf{P}\left\{\delta\right\}_i+\mathbf{Q}\left\{\dot{\delta}\right\}_i+\mathbf{R}\left\{\ddot{\delta}\right\}_i\right). \tag{31}$$

Matrix [S] in (30) is usually called integration matrix, and it is defined by the expression:

$$\mathbf{S}=\mathbf{M}+\frac{\Delta t}{2}\mathbf{C}+\frac{\Delta t^2}{6}\mathbf{K}. \tag{32}$$

The expressions for auxiliary matrices yield:

$$\mathbf{P}=\mathbf{K},$$
$$\mathbf{Q}=\mathbf{C}+\Delta t\,\mathbf{K},$$
$$\mathbf{R}=\frac{\Delta t}{2}\mathbf{C}+\frac{\Delta t^2}{3},\mathbf{K}. \tag{33}$$

The load is defined by (8), and definition of initial conditions is described in [1] in details.

4.2. Energy conservation principle

For the stress determination of the upper and lower beam, contact dynamic force has to be calculated. Potential energy of the upper beam transforms due to gravity force into the kinetic energies and strain energies of the upper and lower beam. If one assumes that upper beam has several times higher stiffness than lower beam and that the lower beam has zero velocity at highest deflection (the proof is given in [1]), we can write:

$$m_{\mathrm{u}}g\left(h+h_{\mathrm{din}}\right)=\frac{1}{2}F_{\mathrm{din}}w_{\mathrm{din}}. \tag{34}$$

The contact dynamic force is largest at maximum deflection of the lower beam, and it is calculated according to simple expression:

$$F_{\mathrm{din}}=k_{\mathrm{f}}w_{\mathrm{din}}. \tag{35}$$

Flexural stiffness in previous equation is equal to:

$$k_{\mathrm{f}}=\frac{3EI_1}{l_1^3}. \tag{36}$$

Bearing in mind equation (34) and geometrical relations shown in Figure 1, deflection is obtained from:

$$w_{\mathrm{din}}=\frac{m_{\mathrm{u}}g\,l_{\mathrm{u}}\pm\sqrt{m_{\mathrm{u}}g(8d^2hk_{\mathrm{f}}+m_{\mathrm{u}}g\,l_{\mathrm{u}}^2)}}{2\,d\,k_{\mathrm{f}}}. \tag{37}$$

5. Numerical example

The application of semi-analytical solution is illustrated with the case of collision of communication bridge between two offshore units and safety structure called arrestor, Figure 2. When offshore units are being installed on the exploitation location, seakeaping and anchoring calculations are required. Since external forces, which are assessed by stochastic type calculations, acting on the offshore objects and cause their movement, it is obvious that the case when distance between objects exceeds the bridge length can occur. In that case, the bridge is pitching and strikes the end of arrestor. The bridge is braced structure and it is made of high-tensile steel [1, 7], Figure 3. Arrestor consists of two longitudinal beams and one transverse beam [1], Figure 4. Both, the bridge and the arrestor are modelled as uniform beams and their properties are given in Table 1. Maximum bending stress of material of the bridge and arrestor is σ_{al}=533 MPa, maximum shear stress is τ_{al}=355 MPa, and Young's modulus equals E =2,1·10^5 MPa. Vertical distance between the bridge centre of gravity at initial and at impact time instant is equal to 0,57 m. The discretization of the bridge structure is done with 14 finite elements, and arrestor is discretized into 7 finite elements, Figure 5. One finite element is used for contact element simulation. The length of contact element is chosen to be 0,3 m, and it's cross-section area is equal to 0,04 m^2. Non-dimensional damping coefficient is taken to be 0,3.

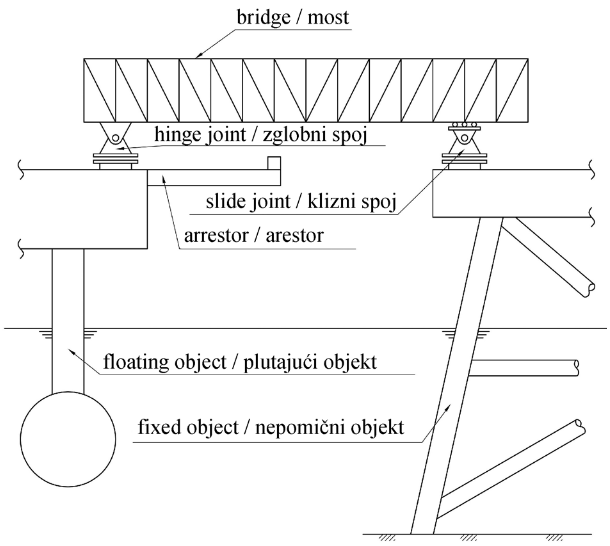


Figure 2. Bridge and arrestor – schematic view

Slika 2. Most i arestor – shematski prikaz

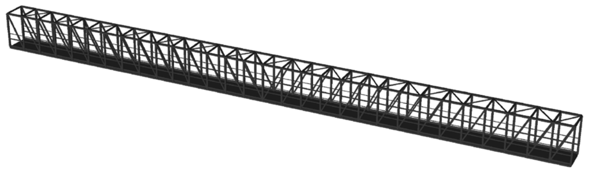


Figure 3. Bridge structure

Slika 3. Struktura mosta

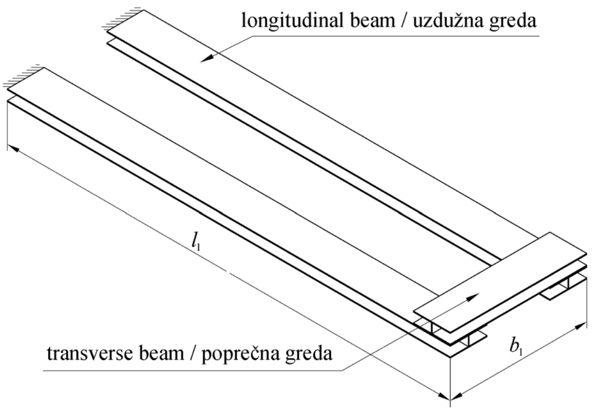


Figure 4. Arrestor structure

Slika 4. Arestor

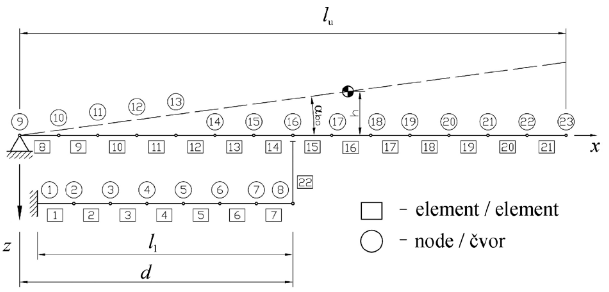


Figure 5. FEM model of the bridge and arrestor

Slika 5. Model konačnih elemenata mosta i arestora

Motion of the end of the arrestor is represented by motion of node 8, while node 16 represents impact location of the bridge, Figure 5. Deflections of the considered nodes, obtained semi-analytical approach are represented in Figure 6. Further on, normal stresses at critical cross-sections of the bridge and arrestor, shear stress at critical cross-section of the arrestor as well as contact element stresses are shown in Figures 7, 8 and 9, respectively. Since shear stresses of braced structures are usually neglected, only shear stresses in the critical cross-sections of the arrestor have been calculated. Very good agreement between the results obtained by both, semi-analytical and numerical procedure in arrestor and bridge deflections is achieved, Figures 10 and 11. The same can be also concluded for time histories of arrestor and bridge bending stresses, shear stresses in the arrestor critical cross-section as well as for time history of contact element stresses, Figures 12, 13, 14 and 15. The value of deflection, obtained by semi-analytical approach equals 0,658 m, which is higher than value obtained by applying energy conservation principle, but lower then the value obtained by the numerical procedure, Table 2. This is also valid for bending stress levels of the arrestor, while semi-analytical procedure gives slightly higher values of the bridge bending stresses and arrestor shear stresses comparing to numerical procedure and energy conservation principle approach.

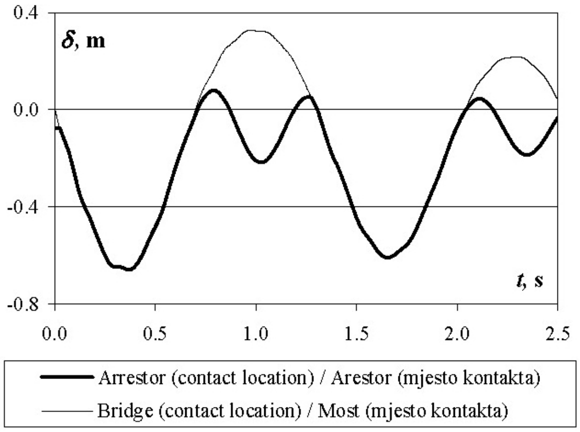


Figure 6. Deflections of arrestor and bridge at contact location

Slika 6. Progibi arestora i mosta na mjestu kontakta

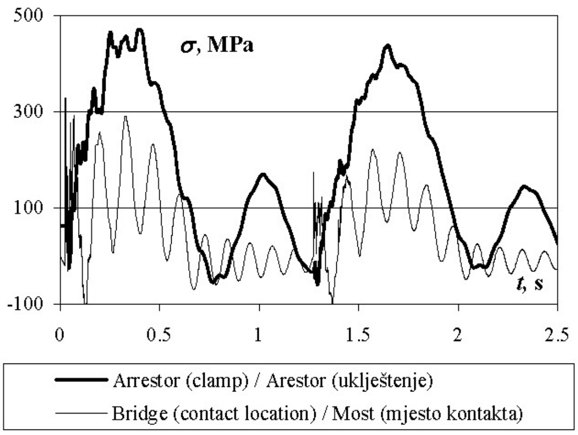


Figure 7. Bending stresses at critical cross-sections of arrestor and bridge

Slika 7. Savojna naprezanja u kritičnim presjecima arestora i mosta

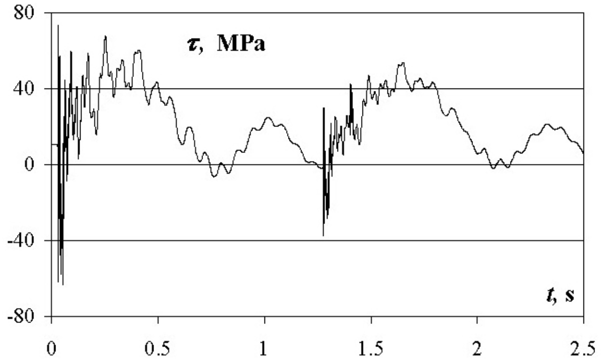


Figure 8. Shear stress of arrestor at critical cross-section

Slika 8. Smično naprezanje arestora u kritičnom presjeku

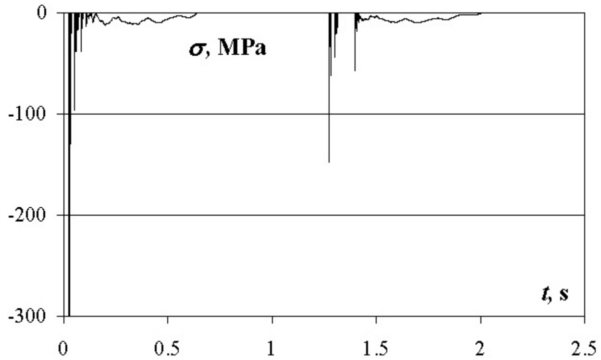


Figure 9. Contact element stress

Slika 9. Naprezanja u kontaktnom elementu

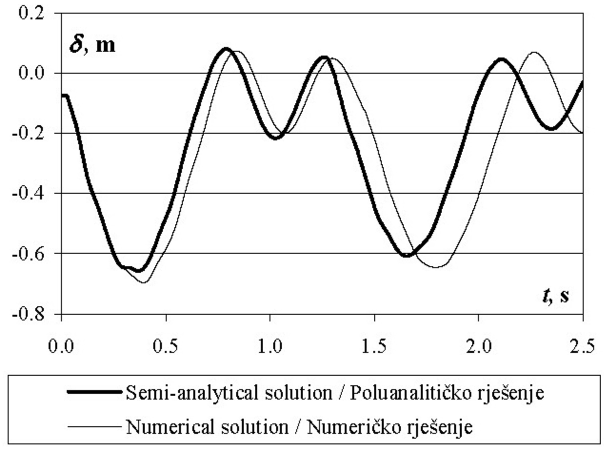


Figure 10. Comparison of arrestor deflections obtained by different methods

Slika 10. Usporedba progiba arestora izračunatih različitim metodama

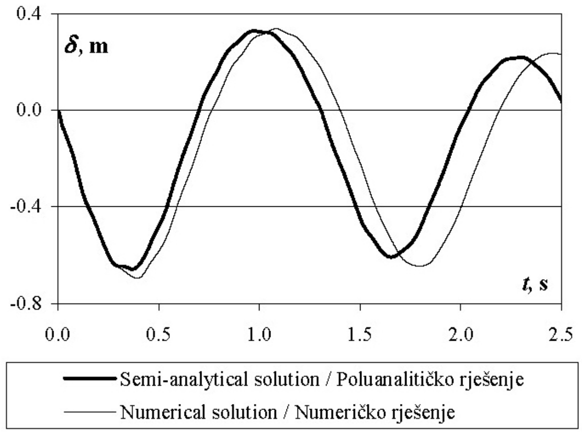


Figure 11. Comparison of bridge deflections obtained by different methods

Slika 11. Usporedba progiba mosta izračunatih različitim metodama

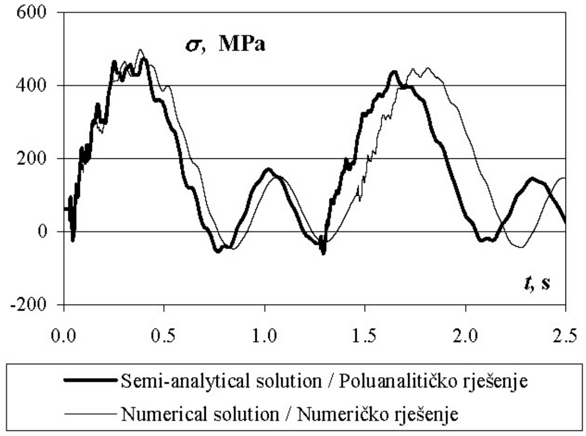


Figure 12. Comparison of arrestor bending stresses obtained by different methods

Slika 12. Usporedba savojnih naprezanja arestora izračunatih različitim metodama

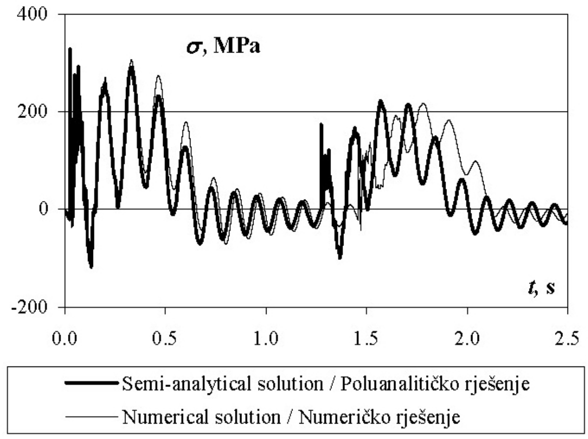


Figure 13. Comparison of bridge bending stresses obtained by different methods

Slika 13. Usporedba savojnih naprezanja mosta izračunatih različitim metodama

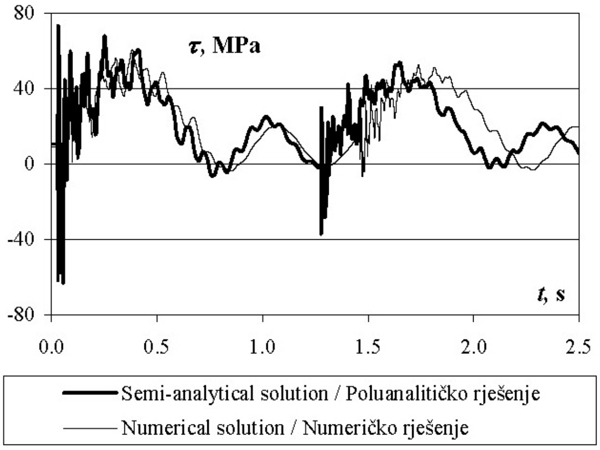


Figure 14. Comparison of arrestor shear stresses obtained by different methods

Slika 14. Usporedba smičnih naprezanja arestora izračunatih različitim metodama

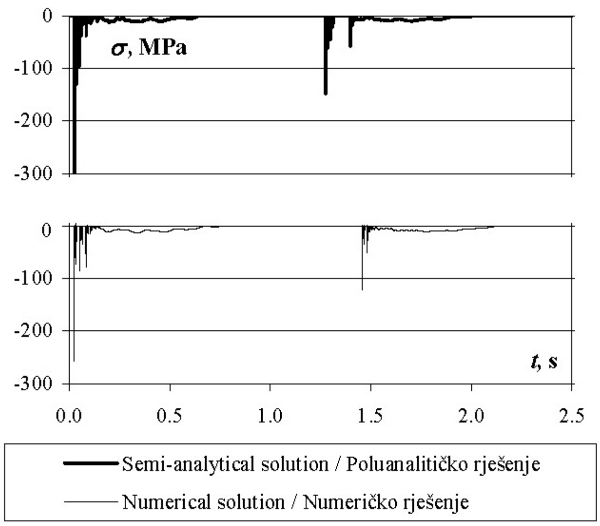


Figure 15. Comparison of contact element stresses obtained by different methods

Slika 15. Usporedba naprezanja u kontaktnom elementu izračunatih različitim metodama

Table 1. Properties of bridge and arrestor

Tablica 1. Značajke mosta i arestora

	Bridge / Most	Arrestor / Arestor
Mass / Masa	12 t	11,7 t
Length / Duljina	29 m	12 m
Cross-section moment of inertia / Moment tromosti poprečnog presjeka	0,01831 m^4	0,002075 m^4
Section modulus / Moment otpora	0,009978 m^3	0,01384 m^3

Table 2. Results comparison

Tablica 2. Usporedba rezultata

	Semi-analytical solution / polu-analitičko rješenje	Numerical procedure / Numerički postupak	Energy approach / Energetski pristup
$w_{din,a}$	0,658 m	0,691 m	0,628 m
$\sigma_{din,a}$	471 MPa	492 MPa	412 MPa
$\tau_{din,a}$	73 MPa	59 MPa	40 MPa
$\sigma_{din,b}$	328 MPa	295 MPa	219 MPa

6. Conclusion

The semi-analytical solution for elastic impact of two beams, combining finite element technique for continuum modelling and diagonalization method for solving differential equation of motion, is presented. Developed solution is illustrated by solving numerical example of collision of communication bridge interconnecting two offshore units and safety structure called arrestor. In this illustrative example it is assumed that pitching of the bridge is influenced only by gravitational force, but the presented solution, as well as numerical procedure, enables taking into account other kind of external forces.

Semi-analytical solution gives very reliable results. It enables avoidance of temporal discretization and one can say that it is unconditionally stable comparing to numerical procedure, but it requires higher computing capabilities. This solution also has some advantages comparing to energy conservation principle approach, as it can take into account local deformations and stresses close to contact location as well as distinguishing of natural modes and their influences on deformations and stresses. The energy conservation principle based approach can not take these effects into account, but it is very simple and fast and in this way useful in preliminary design stage.

Semi-analytical solution, presented here, and numerical procedure are more suitable for detailed structural analysis of impulsive loaded structures, while application of energy conservation law based procedures seems to be more convenient for standard engineering practice.

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