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Performance Comparison of Different Control Algorithms for Robot Manipulators

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In this work are compared performances of five different robot control algorithms. The following controllers under consideration are: PD controller, PID controller, analytical fuzzy controller, classical adaptive controller and adaptive controller based on neural network. The mentioned controllers are used to control two different robot configurations with two rotational degrees of freedom (in horizontal and vertical plane). The basic performances for control algorithms comparisons are: tracking error, rate of convergence, robustness on structural changes of control object, complexity of stability criterion and complexity of implementation.

Usporedba performansi različitih algoritama upravljanja robotskim manipulatorom

Original Scientific Pape

U ovom radu uspoređuju se performanse pet različitih algoritama upravljanja robotom. Razmatraju se sljedeći regulatori: PD regulator, PID regulator, analitički neizraziti regulator, klasični adaptivni regulator i adaptivni regulator temeljen na neuronskoj mreži. Navedeni regulatori primijenjeni su na upravljanje dvjema različitim konfiguracijama robota s dva rotacijska stupnja slobode gibanja (u horizontalnoj i vertikalnoj ravnini). Osnovne performanse prema kojima se upravljački algoritmi uspoređuju su: pogreška vođenja, brzina konvergencije, robusnost na promjene strukture objekta upravljanja, složenost kriterija stabilnosti, te složenost implementacije.

1. Introduction

A lot of sophisticated control algorithms for robot manipulators such as adaptive control, fuzzy control and neural network control have been developed last twenty years. Despite of that fact, most industrial robots today still use conventional controllers like PD and PID controllers. Basic reason for PD/PID wide applications lies in their simplicity and relatively satisfactory performances in control actions.

From the point of implementation and stability analysis PD controller is the simplest one [1]. It enables asymptotic stabilization for robots with moving in horizontal plane (SCARA robots), but for robots with rotational degrees of freedom moving in vertical plane has permanent tracking error. To remove permanent tracking error is necessary to add integral action to PD controller. PID controller is also simple for implementation, but stability analysis is much complicated than for PD controller due to integral action [2].

Better performances of linear PID controller can be achieved using fuzzy PID controller. Adequate choice of controller parameters results much better characteristics of transient process, faster response and smaller tracking error. However, implementation and stability analysis of fuzzy controller is much more complicated then for linear PID controller [3].

Linear and fuzzy PID controllers enable removing permanent tracking error just in case of tracking constant reference trajectory. In the case of tracking time variable reference trajectory, with the assumption of unknown robot model parameters, it is necessary to apply some form of adaptive control. Classical adaptive robot control [4] enables asymptotic tracking of arbitrary continuous time variable trajectory, but with high cost of knowing regressive matrix of robot dynamical model. Using regressive matrix in control algorithm means knowledge of dynamical model, at indefiniteness of their parameters. Otherwise that means regressive matrix depends on specific robot configuration and it can not

Symbols/Oznake

$\boldsymbol{q}$	- vector of generalized coordinates - vektor unutrašnjih koordinata robota
$\mathbf{M}$	- matrix of inertia - inercijska matrica
$\mathbf{C}$	- matrix of Coriolis and centrifugal forces - matrica Coriolisovih i centrifugalnih sila
$\boldsymbol{g}$	- vector of gravity forces - vektor gravitacijskih sila
$\boldsymbol{u}$	- vector of control forces - vektor upravljačkih sila
$\boldsymbol{Y}$	- regresion matrix of dynamical model - regresijska matrica dinamičkog modela
λ_{a}	- minimal matrix eigenvalue - minimalna svojstvena vrijednost matrice
λ_{M}	- maximal matrix eigenvalue - maksimalna svojstvena vrijednost matrice
$\tilde{\boldsymbol{q}}$	- tracking error - pogreška pozicije
$\boldsymbol{q}_{\mathrm{d}}$	- desired reference position - željeno referentno stanje
$\mathbf{K}_{\mathrm{P}}$	- matrix of proportional gains - matrica pojačanja proporcionalnog člana
$\mathbf{K}_{\mathrm{D}}$	- matrix of derivative gains - matrica pojačanja derivacijskog člana
$\mathbf{K}_{\mathrm{I}}$	- matrix of integral gains - matrica pojačanje integracijskog člana
$\boldsymbol{\theta}$	- vector of unknown parameters for dynamic robot model - vektor nepoznatih parametara dinamičkog modela robota
$\hat{\boldsymbol{\theta}}$	- estimation of the vector of model parameters - procjena vektora parametara modela
$\mathbf{W}$	- weight matrix of neural network (one layer) - težinska matrica neuronske mreže (s jednim slojem)
$\hat{\mathbf{W}}$	- estimation of weight matrix of neural network - procjena težinske matrice neuronske mreže
$\boldsymbol{\phi}$	- vector of activation function - vektor aktivacijskih funkcija
s_{ji}	- membership functions of input fuzzy sets - funkcije pripadnosti ulaznih neizrazitih skupova
K_{Cj}	- gain of output fuzzy sets centers - pojačanje centara izlaznih neizrazitih skupova
N_j	- number of fuzzy sets that belong to j -th input variable - broj neizrazitih skupova koji pripadaju j-toj ulaznoj varijabli
I_j	- area of the j-th output fuzzy set - površina j-tog izlaznog neizrazitog skupa
y_{Cj}	- position of output fuzzy sets centers - pozicija centara izlaznih neizrazitih skupova
l	- length of robot link - duljine članaka robota
m	- mass of robot link - masa članka robota

be used for adaptive control of robots with some other configuration.

Relatively direct way to overcome mentioned constraints of classical adaptive controller is to apply neural network [5]. Adaptive control algorithm using one-layer neural network has vector of activation functions instead regressive matrix, which is independent on specific robot configuration. In this way it is possible to apply adaptive control based on neural network to general class of robots.

In spite of a lot of works that deal with control algorithms mentioned above there is relatively low number of works that deal with their performance analysis. In this work are considered performances of these controllers using example of control the robot with two rotational degrees of freedom. Under consideration are two configurations: a) in horizontal plane (SCARA robot) and b) in vertical plane (PELICAN robot) [6]. The reason for comparison control algorithms at two different robot manipulators is establishment of algorithm robustness at structural changes of control object.

2. Robots and controllers mathematical models

2.1. Robot dynamical model

Dynamics of robot with n degrees of freedom is described by the next expression

$$\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u}. \tag{1}$$

Where $\boldsymbol{q} \in \mathbb{R}^{n}$ is vector of inner robot coordinates, $\mathbf{M}(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is inertia matrix, $\mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \mathbb{R}^{n\times n}$ is matrix of Coriolis and centrifugal forces, $\boldsymbol{g}(\boldsymbol{q}) \in \mathbb{R}^{n}$ is gravitational forces vector and $\boldsymbol{u} \in \mathbb{R}^{n}$ is control forces vector. Matrix $C(q,\dot{q})$ is defined by Christoffel's symbols.

Complexity of nonlinear dynamical robot model extremely grows with increasing number of degrees of freedom n. Despite of that, dynamical robot model possesses some properties that are general, independently of specific robot configuration [1, 6]:

S1) Matrix $\mathbf{M}(\boldsymbol{q})$ is symmetric.

S2) Matrix $\dot{\mathbf{M}}(\boldsymbol{q})-2\mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is antisymmetric.

S3) $\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{g}(\boldsymbol{q})=\mathbf{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\theta}$, where $\boldsymbol{\theta}\in\mathbb{R}^{\mathrm{p}}$ is constant parameter vector and $\mathbf{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\in\mathbb{R}^{\mathrm{n}\times\mathrm{p}}$ is regressive matrix of dynamical model. Given property is the key property for classical control and is named as *linear parameterization* of dynamical robot model. Furthermore, robots with rotational degrees of freedom worth next criterions for particular elements of dynamical model:

S4) $\lambda_{\mathrm{m}}\{\mathbf{M}\}\|\boldsymbol{z}\|^2\le\boldsymbol{z}^{\mathrm{T}}\mathbf{M}(\boldsymbol{q})\boldsymbol{z}\le\lambda_{\mathrm{M}}\{\mathbf{M}\}\|\boldsymbol{z}\|^2$, for each $\boldsymbol{q},\boldsymbol{z}\in\mathbb{R}^{\mathrm{n}}$, where is $\|\cdot\|$ vector's Euclid norm and , $\lambda_{\mathrm{m}}\{\cdot\},\lambda_{\mathrm{M}}\{\cdot\}$ are symbols for minimal and maximal matrix eigenvalue, respectively.

S5) $\|\mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}\|\le k_{\mathrm{C}}\|\dot{\boldsymbol{q}}\|^2$, where k_{C} is some positive constant.

S6) $\|\boldsymbol{g}(\boldsymbol{x})-\boldsymbol{g}(\boldsymbol{y})\|\le k_{\mathrm{g}}\|\boldsymbol{x}-\boldsymbol{y}\|$, for each $\boldsymbol{x},\boldsymbol{y}\in\mathbb{R}^{\mathrm{n}}$ where k_{g} is some positive constant.

Properties of dynamical model mentioned above are crucial for stability analysis and enable to make stability criterion that is valid for general class of robots with rotational degrees of freedom.

2.2. PD controller

PD controller is the simplest type of controller that can achieve relatively satisfactory performances of robot control

$$\boldsymbol{u}=-\mathbf{K}_{\mathrm{P}}\tilde{\boldsymbol{q}}-\mathbf{K}_{\mathrm{D}}\dot{\tilde{\boldsymbol{q}}}, \tag{2}$$

where $\tilde{\boldsymbol{q}}=\boldsymbol{q}-\boldsymbol{q}_{\mathrm{d}}$ is position tracking error, $\boldsymbol{q}_{\mathrm{d}}$ is desired reference trajectory of the robot position, $\mathbf{K}_{\mathrm{P}}\in\mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ is positive diagonal matrix of proportional gain and $\mathbf{K}_{\mathrm{D}}\in\mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ is positive diagonal matrix of derivative gain. Stability criterion for robot manipulator (with rotational degrees of freedom) in closed loop with PD controller [6] is

$$\lambda_{\mathrm{m}}\{\mathbf{K}_{\mathrm{p}}\}>k_{\mathrm{g}}, \tag{3}$$

where $\lambda_{\mathrm{m}}\{\mathbf{K}_{\mathrm{p}}\}$ is minimal eigenvalue for matrix of proportional gains (which is the same as minimal value of gains for diagonal matrix). In the case of constant position reference trajectory $\dot{\tilde{\boldsymbol{q}}}=\dot{\boldsymbol{q}}$, derivative element can be interpreted as artificial (virtual) friction which can adjust performances like rise time and overshoot. Using PD controller it is possible asymptotic control just for robots with moving in horizontal plane (SCARA). For general class of robots with rotational degrees of freedom moving in vertical plane produces permanent tracking error. Gravitation is cause for permanent tracking error in vertical plane in contrast to asymptotic tracking error in horizontal plane.

2.3. PID controller

In the case of constant reference tracking it is possible to achieve asymptotic control by adding integral action to PD controller. Then, control law for PID controller is given by the next expressions

$$\boldsymbol{u}=-\mathbf{K}_{\mathrm{P}}\tilde{\boldsymbol{q}}-\mathbf{K}_{\mathrm{D}}\dot{\tilde{\boldsymbol{q}}}-\mathbf{K}_{\mathrm{I}}\boldsymbol{v}, \tag{4}$$

$$\dot{\boldsymbol{v}}=\tilde{\boldsymbol{q}}, \tag{5}$$

where $\mathbf{K}_{\mathrm{I}}\in\mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ is positive diagonal matrix of integral gain. Local stability criterion for robot manipulator (with rotational degrees of freedom) in closed loop with PID controller [8] is

$$\left(\lambda_{\mathrm{m}}\{\mathbf{K}_{\mathrm{P}}\}-k_{\mathrm{g}}\right)\lambda_{\mathrm{m}}\{\mathbf{K}_{\mathrm{D}}\}>\lambda_{\mathrm{M}}\{\mathbf{M}\}\lambda_{\mathrm{M}}\{\mathbf{K}_{\mathrm{I}}\}. \tag{6}$$

It is possible to achieve global asymptotic stability using corresponding nonlinear modifications of PID controller [7-8]. Using PID controller is not possible asymptotic tracking of time variable position reference trajectory.

2.4. Classical adaptive controller

With assumption of unknown robot model parameters, classical adaptive control offers asymptotic tracking of time variable position reference trajectory. Adaptive controller control law is given by the next expression

$$\boldsymbol{u}=\mathbf{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_{\mathrm{r}},\ddot{\boldsymbol{q}}_{\mathrm{r}})\hat{\boldsymbol{\theta}}-\mathbf{K}_{\mathrm{D}}\boldsymbol{r}, \tag{7}$$

Where are

$$\boldsymbol{r}=\dot{\tilde{\boldsymbol{q}}}+\boldsymbol{\Lambda}\tilde{\boldsymbol{q}}, \tag{8}$$

$$\dot{\boldsymbol{q}}_{\mathrm{r}}=\dot{\boldsymbol{q}}_{\mathrm{d}}-\boldsymbol{\Lambda}\tilde{\boldsymbol{q}}, \tag{9}$$

$\boldsymbol{\Lambda}\in\mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ is constant positive definite diagonal matrix, and $\hat{\boldsymbol{\theta}}\in\mathbb{R}^{\mathrm{p}}$ is estimation of model parameter vector, till $\mathbf{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_{\mathrm{r}},\ddot{\boldsymbol{q}}_{\mathrm{r}})\in\mathbb{R}^{\mathrm{n}\times\mathrm{p}}$ is regressive matrix defined by the next expression

$$\mathbf{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_{\mathrm{r}},\ddot{\boldsymbol{q}}_{\mathrm{r}})\boldsymbol{\theta}=\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}_{\mathrm{r}}+\mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_{\mathrm{r}}+\boldsymbol{g}(\boldsymbol{q}), \tag{10}$$

where $\boldsymbol{\theta}\in\mathbb{R}^{\mathrm{p}}$ is unknown parameter vector of dynamical robot model.

Law of adaptive parameter adjusting is given by the next expression

$$\dot{\hat{\boldsymbol{\theta}}}=-\boldsymbol{\Gamma}\mathbf{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_{\mathrm{r}},\ddot{\boldsymbol{q}}_{\mathrm{r}})^{\mathrm{T}}\boldsymbol{r}, \tag{11}$$

where $\mathbf{\Gamma} \in \mathbb{R}^{p\times p}$ is some constant positive definite matrix. Adaptive control law enables global asymptotic tracking of arbitrary continuous trajectory with any choice of positive definite matrices $\mathbf{K}_D$, $\mathbf{\Lambda}$, $\mathbf{\Gamma}$.

At first sight the fact that adaptive controller stability criterion is simpler then PID controller stability criterion is a bit surprising. However dealing with the fact that adaptive control law is, in contrast to PID controller, derived from Lyapunov stability analysis where linear parameterization of dynamical robot model has main role (10). This includes knowing regressive matrix that means knowing dynamical model with indefiniteness of its parameters. According recursive Newton-Euler algorithm, symbolic deriving of dynamical robot model with n>2 rotational degrees of freedom needs 92n–127 multiplying operations and 81n–117 adding operations which include trigonometric functions of inner coordinates and robot parameters. That means that for robot with n=6 degrees of freedom is needed about 425 multiplying operations and 369 adding operations. In the other words, classical adaptive controller is without any doubt the most demanded controller for implementation in real time.

2.5. Adaptive controller based on neural network

One specific way to overcome problems of classical adaptive control is applying neural network which is used instead regressive matrix. Basic assumption for adaptive control with neural network is existence of neural network weight matrix (with one layer), $\mathbf{W} \in \mathbb{R}^{k\times n}$ and activation function vector $\boldsymbol{\phi}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}_r) \in \mathbb{R}^k$ such that

$$\mathbf{Y}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}_r)\boldsymbol{\theta} = \mathbf{W}^T\boldsymbol{\phi}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}_r). \tag{12}$$

Besides that assumption, adaptive control law based on neural network with one layer has next form

$$\boldsymbol{u} = \hat{\mathbf{W}}^T\boldsymbol{\phi}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}_r) - \mathbf{K}_D\boldsymbol{r}\,, \tag{13}$$

where $\hat{\mathbf{W}} \in \mathbb{R}^{k\times n}$ is estimation of neural network weight matrix.

The adaptive adjustment of neural network weight matrix is given by the next expression

$$\dot{\hat{\mathbf{W}}} = -\mathbf{\Gamma}\boldsymbol{\phi}(\boldsymbol{q},\dot{\boldsymbol{q}},\dot{\boldsymbol{q}}_r,\ddot{\boldsymbol{q}}_r)\boldsymbol{r}^T, \tag{14}$$

where $\mathbf{\Gamma} \in \mathbb{R}^{k\times k}$ is some constant positive definite matrix. Using assumption (12), adaptive control law with one-layer neural network enables global asymptotic tracking of arbitrary continuous trajectory for any choice of positive definite matrices $\mathbf{K}_D$, $\mathbf{\Lambda}$, $\mathbf{\Gamma}$. In the case of multi-layer neural networks [9], or one-layer neural network with activation functions adaptive adjustment, stability criterions are more complicated. Because vector of activation functions does not depend on robot dynamical model, in contrast to regressive matrix, adaptive control algorithm based on neural network can be applied to any robot configuration.

2.6. Analytical fuzzy controller

Major problem using conventional fuzzy controller is the problem of exponential increase of behavior rules by increasing number of input-output system variables. As consequence, classical fuzzy controller application on multivariable systems like robot becomes very demanded from the standpoint of computing complexity as complexity of controller synthesis. Analytical fuzzy controller [10-11] overcomes that problem using analytical functions for determining centers of output fuzzy sets instead defining bases of behavior rules. Analytic function enables direct procedure of nonlinear mapping from input variables to centers of output fuzzy sets, which can be simply implemented in the control algorithm. In that way there is no behavior rule base, so the number of input and output variables as the number of fuzzy sets is not constrained with exponential growth of the number of behavior rules. Control variable of analytical fuzzy regulator is defined by the next expression

$$u(x_1,...,x_n) = \frac{\sum_{j=1}^{n} y_{Cj}(x_j)\omega_j(x_j)I_j}{\sum_{j=1}^{n}\omega_j(x_j)I_j}, \tag{15}$$

where x_1,...., x_n are input variables,

$$\omega_j(x_j) = \sum_{i=1}^{N_j} S_{ji}(x_j)\,, \tag{16}$$

represents activation function for output fuzzy set in the form of *sum-prod* operator over membership functions of input fuzzy sets $s_{ji}(x_j)$, and

$$y_{Cj}(x_j) = K_{Cj}\left(1-\frac{\omega_j(x_j)}{N_j}\right)\mathrm{sign}(x_j)\,, \tag{17}$$

represents position of centers for output fuzzy sets, where K_{Cj} is gain of centers of output fuzzy sets, N_j is number of fuzzy sets that belong to j-th input variable, and I_j is area of j-th output fuzzy set. In the case of analitical fuzzy PID controller, input variables are $\tilde{\boldsymbol{q}}, \dot{\tilde{\boldsymbol{q}}}$ i $\int_0^t \tilde{\boldsymbol{q}}(\tau)\mathrm{d}\tau$.

From the standpoint of implementation, analytical fuzzy controller is more complex then linear PID controller, but simpler then adaptive algorithms. On the other hand, stability criterions of robot manipulator controlled by analytical fuzzy controller are much more complicated than in case of other controllers mentioned above [12-13]. Difference from other algorithms mentioned above, control law of fuzzy controller has saturation property of control variable.

3. Simulation results

Dynamical robot model with two rotational degrees of freedom moving in vertical plane (Figure 1) is given by expression (1), where are:

$$\mathbf{M}(\boldsymbol{q}) = \begin{bmatrix} m_1 l_1^2 + m_2 l_1^2 + m_2 l_2^2 + 2 m_2 l_1 l_2 \cos q_2 & m_2 l_2^2 + m_2 l_1 l_2 \cos q_2 \\ m_2 l_2^2 + m_2 l_1 l_2 \cos q_2 & m_2 l_2^2 \end{bmatrix},$$

$$\mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \begin{bmatrix} -m_2 l_1 l_2 \sin(q_2)\dot{q}_2 & -m_2 l_1 l_2 \sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ m_2 l_1 l_2 \sin(q_2)\dot{q}_1 & 0 \end{bmatrix},$$

$$\boldsymbol{g}(\boldsymbol{q}) = \begin{bmatrix} m_1 g l_1 \sin q_1 + m_2 g l_1 \sin q_1 + m_2 g l_2 \sin(q_1 + q_2) \\ m_2 g l_2 \sin(q_1 + q_2) \end{bmatrix}.$$

Vector $\boldsymbol{q}=[q_1 \quad q_2]^{\mathrm{T}}$ represents inner rotational coordinates of robot, l_1 and l_2 are lengths of robot links, and m_1 and m_2 are masses of the first and second robot link, respectively.

Numerical parameter values are: l_1=0.3 m, l_2=0.2 m, m_1=9.5 kg and m_2=5 kg.

Regressive matrix for robot model in horizontal plane ($g(q)$ =0) je

$$\mathbf{Y}(q_2,\dot{q}_1,\dot{q}_2,\dot{q}_{1\mathrm{r}},\dot{q}_{2\mathrm{r}},\ddot{q}_{1\mathrm{r}},\ddot{q}_{2\mathrm{r}}) = \\ = \begin{bmatrix} \ddot{q}_{1\mathrm{r}} & (2\ddot{q}_{1\mathrm{r}} + \ddot{q}_{2\mathrm{r}})\cos(q_2) - [\dot{q}_{1\mathrm{r}}\dot{q}_2 + \dot{q}_{2\mathrm{r}}(\dot{q}_1 + \dot{q}_2)]\sin(q_2) & \ddot{q}_{2\mathrm{r}} \\ 0 & \ddot{q}_{1\mathrm{r}}\cos(q_2) + \dot{q}_{1\mathrm{r}}\dot{q}_1 \sin(q_2) & \ddot{q}_{1\mathrm{r}} + \ddot{q}_{2\mathrm{r}} \end{bmatrix}$$

Where are:

$$\dot{q}_{1\mathrm{r}} = \dot{q}_{1\mathrm{d}} - \lambda_1 \tilde{q}_1,\ \dot{q}_{2\mathrm{r}} = \dot{q}_{2\mathrm{d}} - \lambda_2 \tilde{q}_2,\ \ddot{q}_{1\mathrm{r}} = \\ = \ddot{q}_{1\mathrm{d}} - \lambda_1 \dot{\tilde{q}}_1,\ \ddot{q}_{2\mathrm{r}} = \ddot{q}_{2\mathrm{d}} - \lambda_2 \dot{\tilde{q}}_2,$$

and λ_1, λ_2 $\in$ diag ($\boldsymbol{\Lambda}$) while vector of unknown parameters is $\boldsymbol{\theta}$ = [θ_1 θ_2 θ_3], with components.

$$\theta_1 = m_1 l_1^2 + m_2 l_1^2 + m_2 l_2^2,\quad \theta_2 = m_2 l_1 l_2,\quad \theta_3 = m_2 l_2^2$$

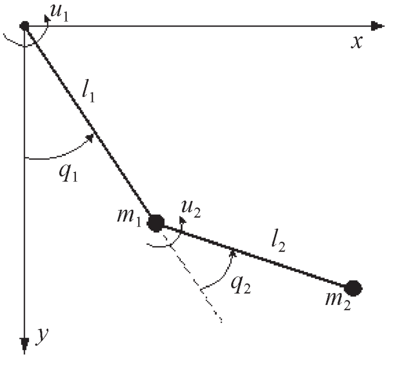


Figure 1. Two revolute joints robot

Slika 1. Robot s dva rotacijska stupnja slobode gibanja

Control algorithms are tested on two different robot configurations with two rotational degrees of freedom (in horizontal and vertical plane). Dynamical model and robot parameters are taken form [2]. Regressive matrix of adaptive controller has been derived from dynamical model of the robot in horizontal plane. The only difference between dynamical model in horizontal and vertical plane is value of gravitational vector, which is equal zero for configuration in horizontal plane.

In regard to that, such control performances depend on characteristics of referent signal, so the control algorithms are tested first on the problem of tracking constant position reference trajectory (Figure 2), and than tested on time variable position reference trajectory (Figure 3).

Figures show logarithm of absolute value tracking error, $\log|\tilde{q}|$, in time t , as the application with different control algorithms (PD controller, classical adaptive controller, PID controller, adaptive controller based on neural network). Reason using logarithmic scale lies in fact that in linear scale can not be clearly seen asymptotic (exponential) convergence from convergence with permanent tracking error. Furthermore, logarithmic scale gives clear insight in speed convergence of control algorithms.

3.1. Simulation results in the case of tracking constant reference position trajectory

Asymptotic robot stabilization to the desired constant position reference state is possible for all controllers mentioned here but just if robot moves in horizontal plane (Figure 2, left). PID controller has the slowest convergence, in contrast to others.

If the robot moves in vertical plane, asymptotic robot stabilization to the desired constant position reference state is possible for PID controller and adaptive controller based on neural network (Figure 2, right). Also, convergence of all mentioned algorithms depends a little on structural changes of control object. As expected, PD controller and classical adaptive controller have permanent tracking error.

If robustness of control algorithm is defined as insensibility of control performances to control object structural changes, than PID controller and adaptive controller based on neural network are robust at tracking constant position reference trajectory (Table 1).

One layer neural network has 13 radial base activation functions. Initial weight matrix is equal zero. Change of parameters does not influence essentially on tracking error.

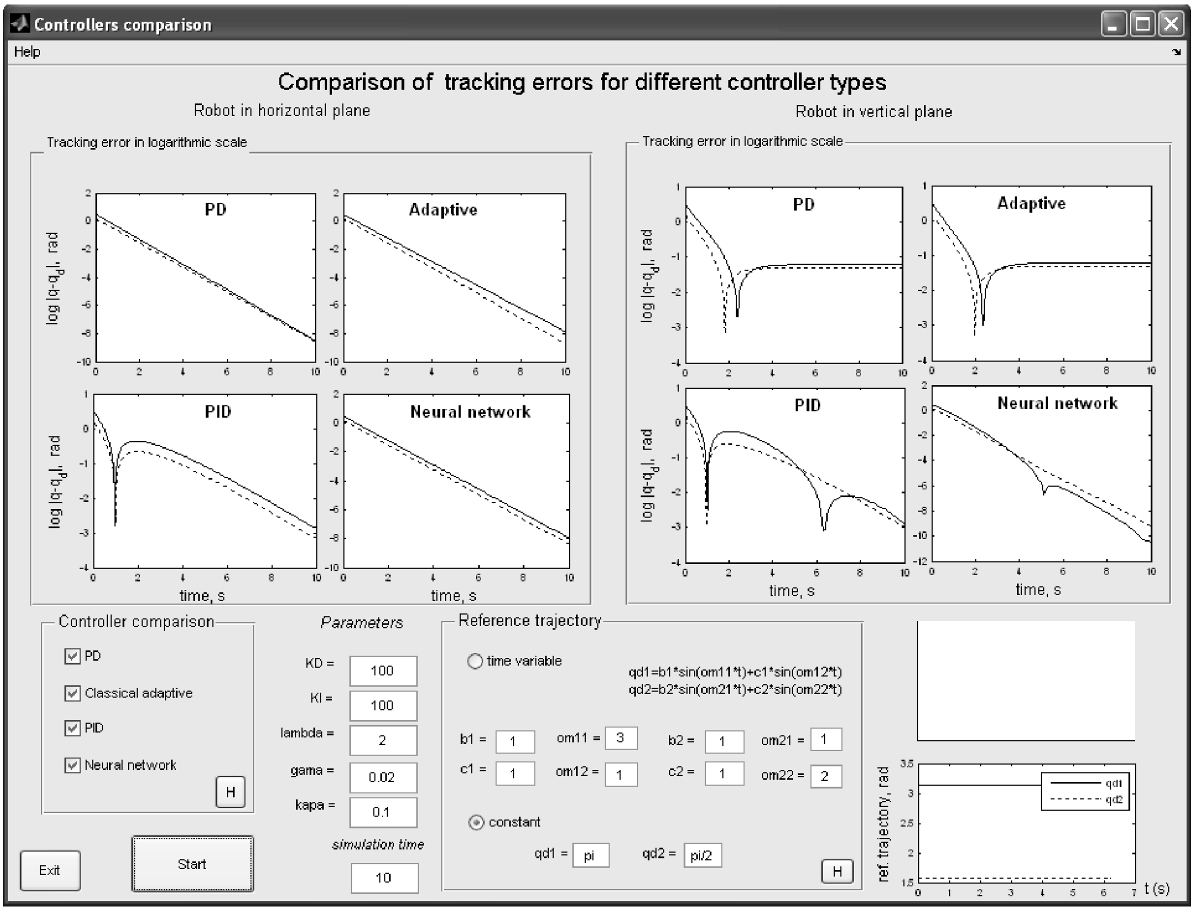


Figure 2. A comparison of tracking error for the PD controller, adaptive controller, PID controller and adaptive controller based on neural network in the case of time constant referent signal.

Slika 2. Usporedbe pogreške vođenja kod primjene PD regulatora, klasičnog adaptivnog regulatora, PID regulatora i adaptivnog regulatora zasnovanog na neuronskoj mreži u slučaju vremenski konstantnog referentnog signala

3.2. Simulation results in the case of tracking variable reference position trajectory

Time variable reference trajectory is represented by superposition of two sinus signals with different amplitudes and frequencies. For such reference trajectory adaptive controller based on neural network with radial base function does not have better performances than PD or PID controller, so in this work is used neural network with activation functions in form of combinations of functions $\sin(\tilde{\boldsymbol{q}}), \cos(\tilde{\boldsymbol{q}}), \dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}_{\mathrm{r}}, \ddot{\boldsymbol{q}}_{\mathrm{r}}$, because such elements are also elements of regressive matrix, [15]. Modificated law for adjusting weight matrix is applied due to need for additional improvement of algorithm convergence [5, 15],

$$\dot{\hat{\mathbf{W}}} = -\boldsymbol{\Gamma}\boldsymbol{\phi}(\boldsymbol{x})\boldsymbol{r}^{\mathrm{T}} - \kappa \cdot \boldsymbol{\Gamma}\|\boldsymbol{r}\|\hat{\mathbf{W}}, \tag{18}$$

where κ is positive constant.

Classical adaptive controller for robot control in horizontal plane can achieve relatively fast asymptotic convergence of tracking error. Adaptive controller based on neural network has also asymptotic convergence, but much slower than classical adaptive controller. PD and PID controllers have permanent tracking error (Figure 3)

All controllers have permanent tracking error in the case of robot control in vertical plane.

3.3. Simulation results for analytical fuzzy PID controller

Linear PID controller and adaptive controller based on neural network are compared with analytical fuzzy PID controller (Figures 4 and 5).

Asymptotic robot stabilization at desired position of reference trajectory can be achieved with faster convergence using PID fuzzy controller instead linear PID controller (Figure 4). Also, after specific time interval, convergence of analytical fuzzy PID controller becomes slower than for adaptive controller based on neural network.

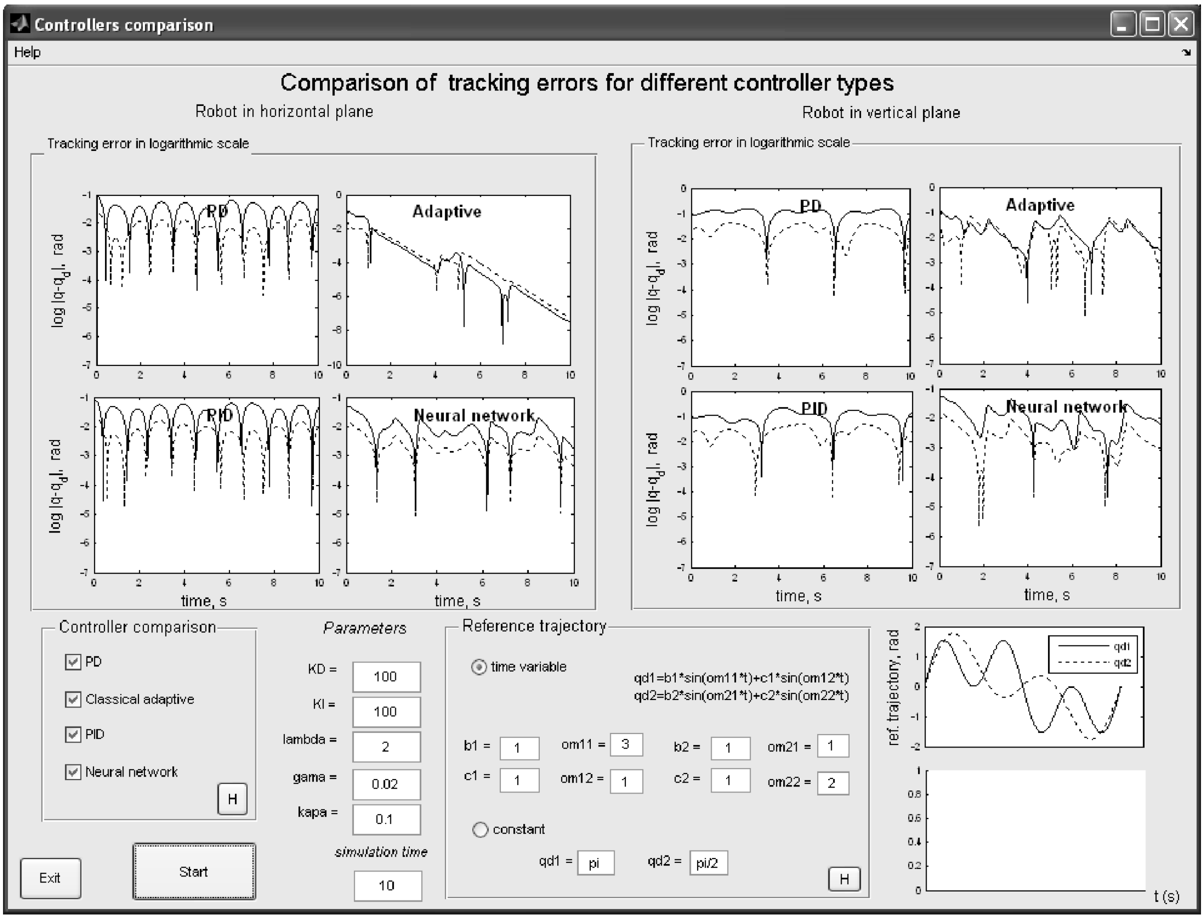


Figure 3. A comparison of tracking error for the PD controller, adaptive controller, PID controller and adaptive controller based on neural network in the case of time variable referent signal.

Slika 3. Usporedbe pogreške vođenja kod primjene PD regulatora, klasičnog adaptivnog regulatora, PID regulatora i adaptivnog regulatora zasnovanog na neuronskoj mreži u slučaju vremenski promjenjivog referentnog signala

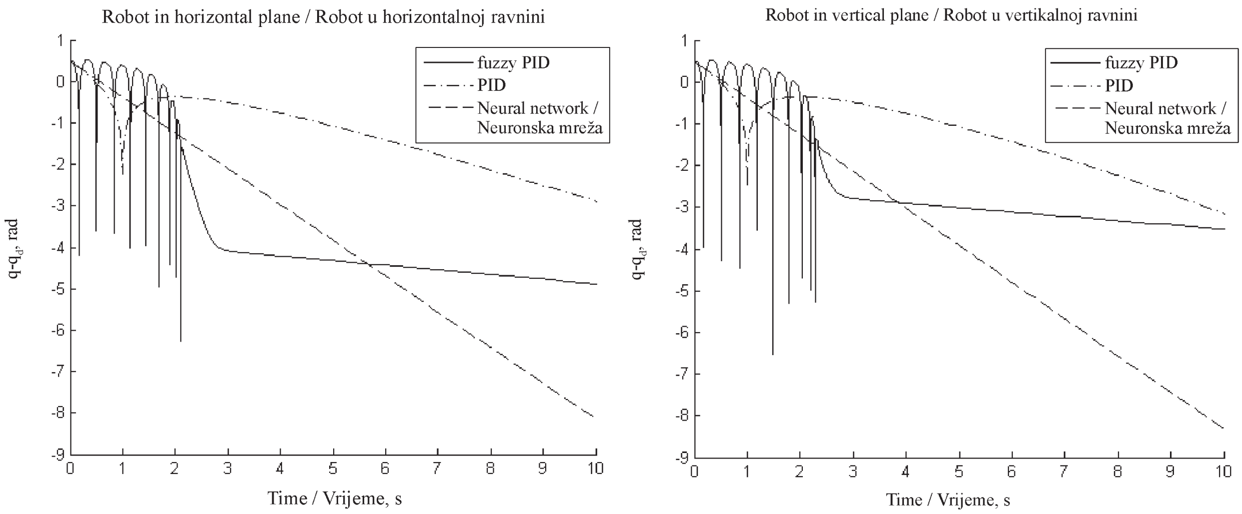


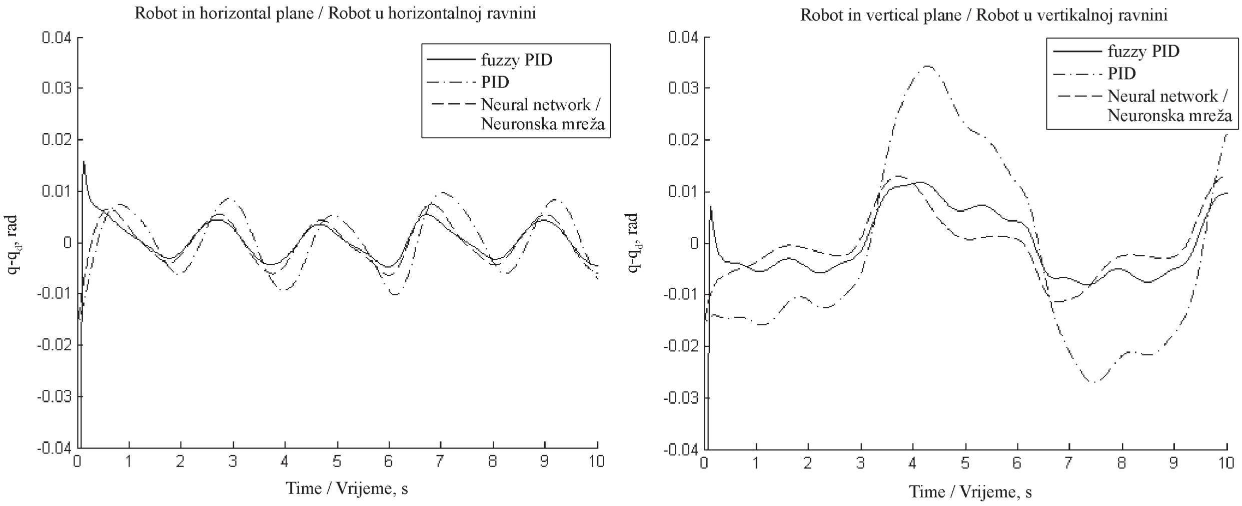
Figure 4. A comparison of tracking error for the fuzzy PID controller, linear PID controller and adaptive controller based on neural network in the case of time constant referent signal.

Slika 4. Usporedba pogreške vođenja neizrazitog PID regulatora, linearnog PID i adaptivnog regulatora primjenom neuronske mreže kod praćenja konstantnog referentnog signala

Tracking errors of adaptive controller based on neural network are for time variable reference trajectory (Figure 5) almost the same as for fuzzy PID, when linear PID tracking errors are some larger than other two compared. Because performances do not essentially depend on robot configuration, it can be told that they are robust on control object structural changes (Table 1).

3.4. Discussion of simulation results

Although performances of the most mentioned algorithms are as expected, it is a little bit surprising slow convergence and relatively bigger tracking error for adaptive controller based on neural network. Additional simulation tests made by classical adaptive controller give explanations of that problem. It is shown that very



Slika 5. Usporedba pogreške vođenja neizrazitog PID regulatora, linearnog PID i adaptivnog regulatora primjenom neuronske mreže kod praćenja vremenski varijabilnog referentnog signala

Figure 5. A comparison of tracking error for the fuzzy PID controller, linear PID controller and adaptive controller based on neural network in the case of time variable referent signal

Table 1. Qualitative comparison of controller performance (ratings in brackets are used for case of time variable referent signal; VP means vertical plane)

Tablica 1. Kvalitativne usporedbe performansi regulatora (ocjene u zagradama vrijede u slučaju vremenski promjenjivog referentnog signala; VR znači vertikalna ravnina).

	Tracking error / Regulacijsko odstupanje	**Algorithm convergence / Konvergencija algoritma**	**Algorithm robustness / Robusnost algoritma**	**Algorithm complexity / Složenost algoritma**	**Stability criterions / Kriteriji stabilnosti**
Linear PD controller / Linearni PD regulator	exists in VP (exists) / postoji u VR (postoji)	fast (fast) / brza (brza)	weak (very good) / slaba (vrlo dobra)	complex simple / vrlo jednostavan	simple / jednostavni
Linear PID controller / Linearni PID regulator	removed (exists) / otklonjeno (postoji)	slow (slow) / spora (spora)	very good (very good) / vrlo dobra (vrlo dobra)	simple / jednostavan	complicated / složeni
Classical adaptive controller / Adaptivni regulator-– klasični	exists in VP (exists in VP) / postoji u VR (postoji u VR)	fast (fast) / brza (brza)	very weak (very weak) / vrlo slaba (vrlo slaba)	very complex / vrlo složen	very simple / vrlo jednostavni
Adaptive controller – neural netw. / Adaptivni regulator – neuronska m.	removed (exists) / otklonjeno (postoji)	very fast (slow) / vrlo brza (spora)	good (weak) / dobra (slaba)	complex / složen	very simple / vrlo jednostavni
Analytical fuzzy PID controller / Analitički neizraziti PID regulator	removed (exists) / otklonjeno (postoji)	relatively fast (relatively fast) / relativno brza (relativno brza)	very good (very good) / vrlo dobra (vrlo dobra)	less complex / manje složen	very complicated / vrlo složeni

small deviations of regressive matrix elements from their exact value make worse the performances of classical adaptive controller. In regard to that neural network with finite dimension represents approximation of regressive matrix, this can be sufficient reason for essentially worse performances. Besides, slow convergence is partly consequence of neural network weight matrix $\hat{\boldsymbol{W}}$ that has much more elements than parameter vector $\hat{\boldsymbol{\theta}}$.

It is interesting to mention that application of controller based on neural network does not provide essentially better performances using additional adjusting of width and position of radial basis functions [14].

Table 1 shows performance indexes of controllers, qualitatively. Different performances depend on tracking constant or time variable reference trajectory.

4. Conclusion

This work analyzes performances of different algorithms for robot manipulator control. Linear PID enables asymptotic robot stabilization at constant reference state, but with relatively slow convergence. Convergence properties can be improved using analytical fuzzy PID controller, but with cost of implementation complexity and stability criterions complexity. In the case of time variable reference trajectory only classical adaptive controller enables asymptotic tracking, but for specific robot configuration and cost of knowing very complex regressive matrix. Adaptive controller based on neural network does not depend on robot configuration, but has relatively slow convergence and relatively large tracking error. In the other words, every analyzed algorithm has specific advantages in regard to other controllers, but the same goes for disadvantages. Choice of appropriate controller in practice will prior depend on demands of positioning accuracy and robustness, and on the implementation cost of control algorithm.

Future work will be oriented as the comparison of mentioned controllers to performances of actuation variables, and sensitivity to measurement noise as output disturbances.

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