The Growth of Human Teeth: A Simple Description of its Kinetics

Rast ljudskih zuba: jednostavan opis njihove kinetike

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Summary

The growth of human teeth is described by formalism devised for the analysis of a simple relaxational process. The basis for such a description are existing data on the masses of the teeth and on the duration of four characteristic phases in their formation. These data suggest that the slowing down (asymptotic) increase of the mass of a single tooth may be represented by an exponential function of time. Although the equations which follow from the model are derived from some parameters whose values have not yet exactly been determined, they describe the growth of teeth, and in particular the time dependence of its rate, in a plausible quantitative manner. The results are in accordance with common experience.

Key words: *human teeth, growth*

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Introduction

The formation, development and growth of human teeth is known to be a very intricate biological process. For a number of quite evident reasons, however, it has drawn the attention of various researchers and stimulated and enormous number of studies. Therefore, a respectable quantum of knowledge is available (see e.g.) (1). Though exceptionally complicated in detail and marked by innumerable individual differences the process has an important characteristic common to all people: it is remarkably regular. We shall illustrate this in just three features, out of several possible. Firstly, by the human teeth grow in two subsequent series called milk and permanent teeth, respectively. Secondly, the teeth in both series appear on after the other in a well-defined order. Thirdly by (most evident and of particular interest in this paper), the growth of each tooth is not uniform in time, being relatively fast as its beginning, and slowing down in any subsequent time interval.

The illustration of the speed of the rate of the process is given by the data on the time intervals needed for some phases in tooth development. For example, the intervals between the moments when the first minute quantity of a growing tooth is formed, when the formation of its crown is completed, when it emerges from the jaw surface, when its root or roots are formed are well known. This is sufficient for an impression, but not enough to give an insight into the kinetics of growth as a continuous process. Description of this kind would be given only by correlation showing how some quantum variable of a tooth or teeth (e.g. mass or volume) increases in time. Obviously such a correlation does not exist, since there are no direct measurement of any quantum variable during the long-lasting development of a tooth in the jaw. However, some important observations do exist. Thanks to many histological and x-ray investigations one can from another impression, this time on the shape and size of teeth in each characteristic phase of their development. The last quantity, the size of a growing tooth, could be used an appropriate quantum variable, although the impression, if possible, should somehow be "translated" into numbers. In this paper we propose a possible "translation" together with a simple mechanical model which enables correlation of the masses of teeth with time. As the final result we have derived a set of equations which, in spite of some degree of uncertainty, describe growth kinetics in agreement with common experience.

On the data

Prior to the explanation of the idea it is necessary to repeat some known facts on teething and to comment on the data used in our calculations. The comments concern (Figure 1) which shows sixteen permanent teeth (***dentes permanentes***) situated in one half of the upper and lower jaws of adult subjects. It is supposed that the left and right half of the jaws are symmetrical in any respect. The teeth denoted by 8 and 16 are so-called wisdom teeth (***dentes sapienthia***). Their development is very slow and they do not appear in the jaws of every man and woman. The number of milk teeth (***dentes decidui***) is not sixteen but ten (20 in total). More precisely, the teeth sketched in Figure 1 by dashed lines do not appear in the first series.

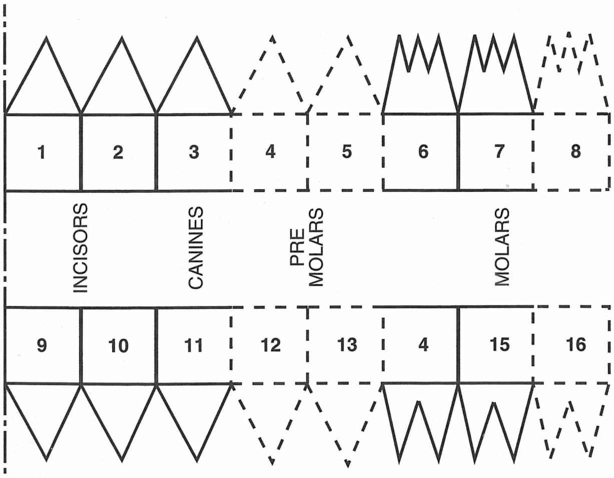

Figure 1. *Sketch of the teeth in one half of the upper and lower jaws, together with their names. The teeth drawn by dashed lines appear only as permanent*

Slika 1. *Zubi u jednoj polovici gornje i donje čeljusti i njihovi nazivi. Crtkano su označeni zubi koji se javljaju samo u trajnom obliku*

The data on the average mass of fully developed permanent and milk teeth are presented in Table 1. Although we found two groups of data according to nationality, those for Dutch people and Hungarians (2), the data in the first group seem to be more accurate since the masses were weighed on a huge number of (dry) specimens (11,260 upper and 10,535 lower teeth). Our calculations are thus based on this group of data. It should be mentioned, however, that the mass of a fully developed tooth in the jaw is not equal to the mass of the equivalent dry tooth refered to in Table 1. However, some important parameters are derived using fraction of masses where this difference becomes irrelevant.

Table1. *Masses m_{0i} permanent and milk teeth (grams)*
Tablica 1. *Mase m_{0i} trajnih i mliječnih zubi (grama)*

TOOTH (i)	PERMANENT	MILK
1	0.99	0.22
2	0.71	0.16
3	1.31	0.29
4	1.07	-
5	1.14	-
6	2.29	0.57
7	2.03	0.58
8	1.71	-
9	0.52	0.11
10	0.65	0.14
11	1.17	0.26
12	0.88	-
13	1.03	-
14	2.20	0.57
15	2.16	0.96
16	1.91	-

The second set data (3), on the duration of some characteristic phases in tooth formation, are shown in Table 2 for permanent and in Table 3 for milk teeth, respectively. Minimal and maximal observed values for four characteristic moments are referred to: T_{0i} - when the "mineralization" of the formation of the first minute quantity of the tooth mass is observed (a physicist would call this event - the "nucleation"). T_{1i} - when the tooth crown is formed,

Table 2. *Characteristic moments in the development of permanent teeht (years)*
Tablica 2. *Karakteristična vremena u razvoju trajnih zubi (godine)*

TOOTH (i)	T_{0i}	T_{1i}	T_{2i}	T_{3i}
1	0.25 - 0.33	3.3 - 4.1	6.7 - 8.1	8.6 - 9.8
2	0.83 - 1.00	4.4 - 4.9	7.0 - 8.8	9.6 - 10.8
3	0.33 - 0.50	4.5 - 5.8	10.0 - 12.2	11.2 - 13.3
4	1.50 - 2.00	6.3 - 7.0	9.6 - 10.9	11.2 - 13.6
5	2.00 - 3.00	6.6 - 7.2	10.2 - 11.4	11.6 - 14.0
6	(-0.16) - 0.17	2.1 - 3.5	6.1 - 6.7	9.3 - 10.8
7	2.50 - 4.00	6.9 - 7.4	11.9 - 12.8	12.9 - 16.2
8	7.00 - 10.00	12.8 - 13.2	17.0 - 19.0	19.5 - 19.6
9	0.25 - 0.33	3.4 - 5.4	6.0 - 6.9	7.7 - 8.6
10	0.25 - 0.33	3.1 - 5.9	6.8 - 8.1	8.5 - 9.6
11	0.33 - 0.50	4.0 - 4.7	9.2 - 11.4	10.8 - 13.0
12	1.50 - 2.00	5.0 - 6.0	9.6 - 11.5	11.0 - 13.4
13	2.00 - 3.00	6.1 - 7.1	10.1 - 12.1	11.7 - 14.3
14	(-0.16) - 0.17	2.1 - 3.6	5.9 - 6.9	7.8 - 9.8
15	2.50 - 4.00	6.2 - 7.4	11.2 - 12.2	11.0 - 15.7
16	8.00 - 10.00	12.0 - 13.7	17.0 - 19.0	20.0 - 20.8

Table 3. *Characteristic moments in the development of milk teeth (years)*
Tablica 3. *Karakteristična vremena u razvoju mliječnih zubi (godine)*

TOOTH (i)	T_{0i}	T_{1i}	T_{2i}	T_{3i}
1	(-0.48) - (-0.42)	0.13	0.67 - 1.00	2.75
2	(-0.44) - (-0.41)	0.21	0.75 - 1.08	2.75
3	(-0.44) - (-0.38)	0.75	1.33 - 1.83	3.58
6	(-0.45) - (-0.40)	0.50	1.08 - 1.58	3.08
7	(-0.42) - (-0.26)	0.92	2.08 - 2.75	3.92
9	(-0.48) - (-0.42)	0.21	0.5 - 0.83	2.75
10	(-0.44) - (-0.41)	0.25	0.83 - 1.33	2.50
11	(-0.42) - (-0.38)	0.67 - 0.75	1.42 - 1.92	3.58
14	(-0.45) - (-0.40)	0.42 - 0.50	1.17 - 1.50	2.83
15	(-0.40) - (-0.34)	0.67 - 0.92	1.92 - 2.58	3.50

T_{2i} - when the tooth emerges out from the jaw surface, and T_{3i} - when the final development of the root or roots is observed. Time zero is placed at the moment of birth. Note that the number of decimal digits associated with the figures in Table 2 and Table 3 does not reflect the accuracy of the data. It is just a result the transformation of months or weeks (cited originally) into years, which are more convenient for the purpose of this paper. The meaning of Ts will henceforth be as follows: at the moments $t = T_{0i}$, $t = T_{1i}$, $t = T_{2i}$ and $t = T_{3i}$ ($i = 1,2,3\ldots16$) the mass of tooth i will be $m_i = 0$, $m_i = x_i\, m_{0i}$, $m_i = y_i\, m_{0i}$, $m_i = z_i\, m_{0i}$, respectively. Here m_{0i} is the final mass of tooth i given in Table 1 of Table 2, while x_i, y_i and z_i are the fractions of final masses whose values are somewhere between 0 and 1. The histological and x-ray investigations, resulting in many photographs and fine drawings of the interior of jaws during various stages of teething, suggest that x_i are around 0.5, at least for permanent teeth. It is, however, obvious that $y_i > x_i$ and $z_i > y_i$.

The model

A suggestion for a model capable of mimicking tooth growth may be found in the set of graphs showing the masses of teeth versus time which we constructed by the as yet arbitrary fractions x_i, y_i and z_i using the data for the minimal and maximal moments T_{0i}, T_{1i}, T_{2i} and T_{3i}. The graphs for three selected teeth are shown in Figure 2. In all three cases the fraction $x_i = x = 0.5$, $y_i = y = 0.8$ and $z_i = z = 1$ are chosen in rough correspondence with observation. Note that after T_{3i} the mass of each tooth is taken as constant up to the 25[th] year. However, the real process cannot be represented by such broken lines, so we further suppose that the mass of a tooth increases in time following a smooth, most probably exponential, curve. This means, in fact, that tooth growth is identified by a relaxational process that can formally be treated as similar to many other chemical or physical processes, e.g. as a chemical reaction of the first order, as the charging of a capacitor, or (best suited to our case) as the filling of an reservoir with a liquid whose flux diminishes in time. In short, ignoring the fine structure of the teeth, we suppose that the masses m_i increase in time according to the set of differential equations:

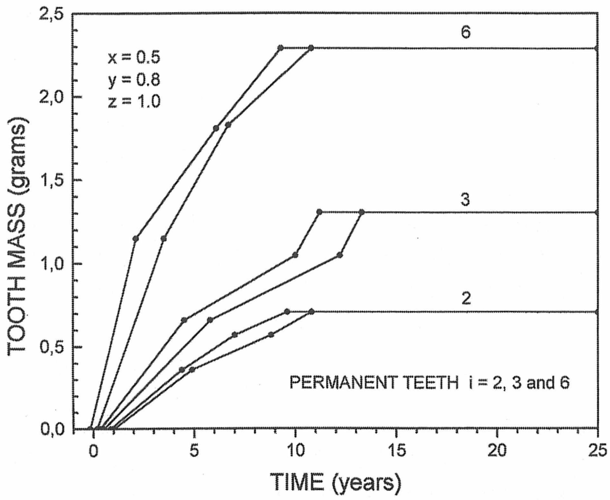

Figure 2. *The curves which show how the masses of three permanent teeth increase in time. The curves are constructed with the help of arbitrarily chosen fractions x, y and z*

Slika 2. *Vremenski porast mase trajnih zuba. Krivulje su konstruirane uz pomoć proizvoljno odabranih omjera x, y i z*

$$\frac{dm_i}{dt} = \frac{m_{0i} - m_i}{\tau} \qquad (1)$$

in which (i is defined as the relaxation time of the respective tooth i. The solutions of (1) read as

$$m_i = m_{0i}\left[1 - \exp\left(-\frac{t - T_{oi}}{\tau_i}\right)\right] \qquad (2)$$

As in any other realxational process, τ_i is a measure of its pace and has the following meaning: at the moment $t = \tau_i + T_{0i}$ the mass of tooth i is increased to the value $m_i = m_{0i}$ (1-1/e) or 63.2% of the final mass m_{01}. In our case, unfortunately, the set of these important characteristics of tooth development cannot be determined exactly since neither the masses m_i nor the fractions m_i/m_{0i} (equal to x_i, y_i or z_i) versus time are not known from any measurements. Thus, the only thing we can do is determine the ***reasonable*** values of relaxation times.

This may bye done ba the quite evident supposition that at the moment T_{3i} each tooth is not developed to its ***final*** state, but to a ***nearly final*** state. In other words, this may be done by taking m_i at the moment T_{3i} not as equal to the respective final mass m_{0i} (z = 1), but as equal to a slightly smaller fraction (e.g.z = 0.9). This should lead to fractions x_i and y_i which must also be in accordance with the previously mentioned histological and x-ray observations. Let us first prove the calculations for the permanent teeth. Using (2) and $z_i = z = 0.9$ we obtain the values for τ_i given in Table 4. In all cases the first number is obtained by using minimal values of T_{0i} and T_{3i} from Table 2. Similarly the second number is the result of using the maximal values of these quantities. Overall implications of the model are shown in Figure 3 where equation (2) is graphically represented for the same teeth as in Figure 2.

Table 4. *Relaxational times τ_i of permanent and milk teeth (years)*

Tablica 4. *Relaksacijska vremena τ_i trajnih i mliječnih zubi (godine)*

TOOTH (i)	PERMANENT	MILK
1	3.6 - 4.1	1.4
2	3.8 - 4.3	1.4
3	4.7 - 5.6	1.7
4	4.2 - 5.0	-
5	4.2 - 4.8	-
6	4.1 - 4.6	1.5
7	4.5 - 5.3	1.9 - 1.8
8	5.4 - 4.2	-
9	3.2 - 3.6	1.4
10	3.6 - 4.0	1.3
11	4.5 - 5.4	1.7
12	4.1 - 5.0	-
13	4.2 - 4.9	-
14	3.5 - 4.2	1.4
15	3.7 - 5.1	1.7
16	5.2 - 4.7	-

Three smooth curves are evidently not only more "beautiful" but also more probable than the broken lines in Figure 2. To test further whether the results are reasonable or not, one should calculate fractions x_i and y_i which - if the model is basically correct - must be more or less equal for all teeth. The calculations give the average values $x = 0.59 \pm 0.07$ and $y = 0.83 \pm 0.07$. They are compatible with observations and their standard deviations are extremely small for this kind of phenomena. Figure 3, where

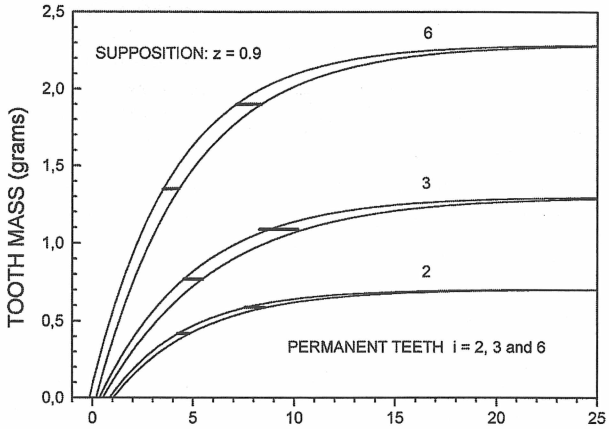

Figure 3. *Three curves form Figure 2 smoothed by the formalism of our relaxational model. Horizontal bars show how well average x,y and z are incorporated in to the curves*

Slika 3. *Tri krivulje sa Slike 2. izglađene formalizmom relaksacijskog modela.. Deblje horizontalne crte pokazuju da se usrednjeni x, y i z dobro uklapaju u krivulje*

the average fractions are denoted as horizontal bars, shows that they are very well incorporated into the model. The same procedure for milk teeth gives the ralaxation times whose values are also presented in Table 4. The average values of associated fractions x and y are as follows: x = 0.44 ± 0.06 and y = 0.68 ± 0.08. Again, the figures are reasonable and their errors acceptable for the phenomenon in question.

What remains is a short comment on the obtained results. Since the growth of a tooth in our model is identified with a relaxational process, the associated relaxation time is a quantitative measures of growth kinetics. Thus, the set of data in Table 4 suggests, for example, that the formation of incisors (i = 1,2 and i = 9,10) is faster than the formation of all other teeth. Somewhat slower is the growth of premolars (i = 4,5 and i = 12,13), first molars (i = 6 and i = 14) and canines (i = 3 and i = 11). Third molars (i = 8 and i = 16) are characterized by the slowest development. At the same time, their relaxation times are dispersed in a larger interval than those of other teeth, which suggest very pronounced individual differences. It seems, generally, that the development of teeth in the lower jaw is somewhat faster than in the upper jaw. Finally, relaxation times of milk teeth are, as they should be, drastically shorter than those of permanent teeth. Although these findings reflect some basically known facts, our model gives them a quantitative background.

The growth of a group of teeth

The word "growth", however, may require a look not only at the formation of any single tooth but also the development of a group of teeth, best of the teeth in the upper and lower jaws separately. In other words, to get an insight into the respective kinetics one should know how the total mass of the in the jaw increases in time. This correlation may easily be obtained by the summation of masses given by (2), i.e. by the formula

$$M = \sum_i m_{0i} \left[1 - \exp\left(- \frac{t - T_{0i}}{\tau_i} \right) \right] \qquad (3)$$

in which M is the total mass of the chosen group of teeth. To the obtain the correct time dependence of M, this summation must be done by taking into account two facts: first by the starting moments T_{0i} are not the same for all teeth, and second, by two sets of values are available (see Table 2). For illustration, the results of summation are shown in Figure 4 and Figure 5 for permanent teeth in the upper

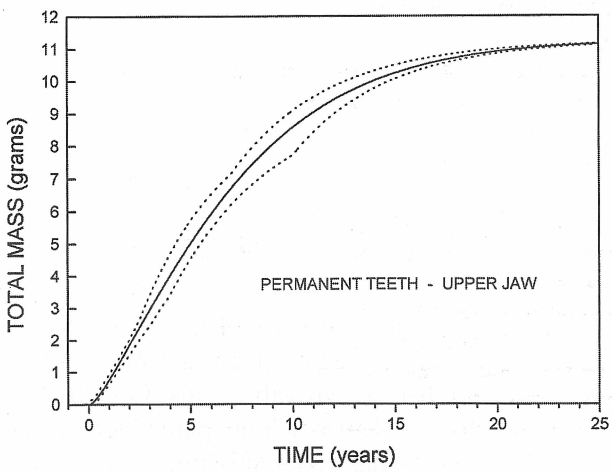

Figure 4. *Total mass of the permanent teeth in the upper jaw versus time. Broken lines show the results of summation according to formula (3). Full line is the fitting curve (4).*

Slika 4. *Ovisnost ukupne mase trajnih zubi u gornjoj čeljusti o ovisnosti o vremenu. Crtkane linije predočuju krivulju sumacije prema formuli (3), a puna linija krivulju prema formuli (4).*

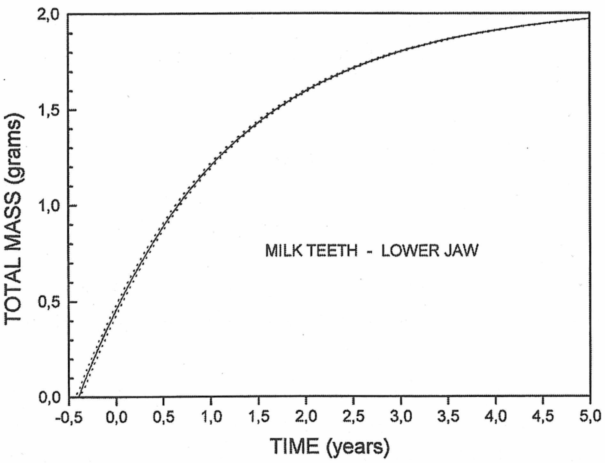

Figure 5. *Total mass of milk teeth in the lower jaw versus time. Broken lines show the results of summation according to formula (3). Full line is the fitting curve (4)*

Slika 5. *Ovisnost ukupne mase mliječnih zubi u donjoj čeljusti u ovisnosti o vremenu. Crtkane linije predočuju krivulju sumacije prema formuli (3), a puna linija krivulju prema formuli (4).*

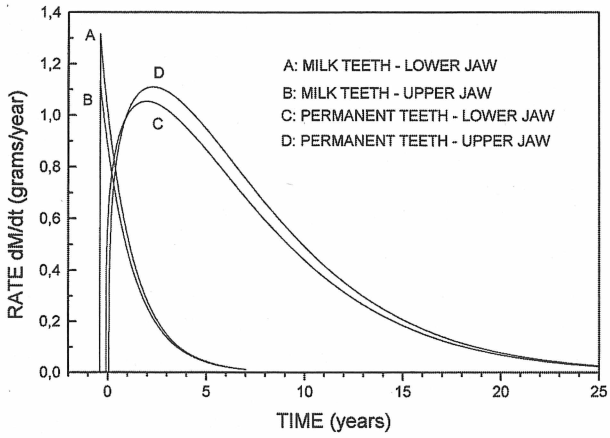

Figure 6. *The rate of teething versus time*

Slika 6. *Tempo ozubljivanja u ovisnosti o vremenu*

jaw, and for milk teeth in the lower jaw, respectively.

Due to the unusual type of summation it is hard to identify any physical meaning behind the formula (3). This, however, need not prevent us from loking for a mathematical function to which the results may by fitted and in which a possible meaning can easier be recognized. Thus in order to preserve the concept of relaxation time and the starting moment of growth we decided to fit the results to the equation

$$M = M_0 \left\{ 1\text{-exp} \left[- \left(\frac{t - T_0}{\Theta} \right)^K \right] \right\} \quad (4)$$

in which M_0 is the total mass of the chosen group of teeth (sum of the values given in Table 1). T_0 is the moment when the growth begins, Θ is the "relaxation time" and K is a helping parameter (see later). The equation has apparently been chosen well since the procedure of nonlinear fitting (10) gives the results loaded with acceptable errors:

PERMANENT - UPPER:	T_0=0.06±0.01	Θ=7.49±0.01	K=1.271±0.002
PERMANENT - LOWER:	T_0=-0.10±0.01	Θ=7.39±0.01	K=1.252±0.002
MILK - UPPER:	T_0=-0.407±0.001	Θ=1.625±0.001	K=1.004±0.001
MILK - LOWER:	T_0=-0.401±0.001	Θ=1.571±0.001	K=1.005±0.001

Overall fitting quality, measured by chi-square is also good. This quantity ranges from about 6.10^{-3} for permanent to as small as 5.10^{-6} for milk teeth, respectively. Such small errors encourage us to say that the figures just mentioned are in a sense the "natural" constants of the "average" human being. It must not be forgotten, however , that all the calculations are based on the fraction z = 0.9. This value is chosen quite reasonably, but its accuracy needs and independent control.

The rate

Having thus obtained the correlation of M versus time we can easily obtain its derivative dM/dt, or the rate of growth:

$$\frac{dM}{dt} = \frac{kM_0}{\Theta} \left(\frac{t - T_0}{\Theta} \right)^{K-1} \text{exp} \left[- \left(\frac{t-T_0}{\Theta} \right)^K \right] \quad (5)$$

It is the quantity best suited to describe the kinetics of the process. The rate is represented in Figure 5 which is, in our opinion, very instructive. It shows some known and evident, but also some not-so-evident, facts. First, of all, one can see that the formation of permanent teeth lasts much longer than the development of milk teeth. This is a trivial fact but the syntagm "much longer" has now gained a quantitative measure. While the cessation of teething activities of milk and permanent teeth is quite distant in time (approximately the seventh year for milk, and twenty-fifth year for permanent teeth), the position t_m of their respective maxima are not so distant (around year zero in the former, and around the third year in the latter case). Since teething activity is extremely slow after the twenty-fifth year, the full development of wisdom teeth, on average, is improbable. In fact, individual differences are the key factor determining how will be developed. Last but not least, although the masses of milk teth are drastically smaller than those of permanent teeth, the rates of growth are in both cases almost the same: 1.1 - 1.3 grams per year. However, looking at the fine details, one can see that the milk teeth grow at a somewhat greater rate in the lower jaw than in the upper. Conversely, the growth rate for permanent teeth has a greater maximum in the upper jaw, although the maximum for the lower jaw appears slightly earlier. Finally the value $K \approx 1$ obtained for milk teeth suggests that their growth is almost a "pure" relaxational process.

Conclusions

To the best of out knowledge there have been no attempts in literature to describe any aspect of the growth of human teeth in a quantitative manner. Insight into the growth rate, for example, doubtlessly an important characteristic of the process, has been possible only through the widely scattered and often controversial data on the duration of some phases in tooth development. Our mechanical model conversely, although meant not to explain but to describe the process, gives a much clearer insight. Despite the fact that the calculations are performed using a quantity ($z = 0.9$) the accuracy of which, has still to be tested, the model offers some parameters (starting moments of the growth and "relaxation times") which may, with reasonable confidence, be accepted as characteristic constants of the "average" human being. Finally, we believe, that the model may used for educational purposes.

RAST LJUDSKIH ZUBA: JEDNOSTAVAN OPIS NJIHOVE KINETIKE

Sažetak

Rast ljudskih zuba opisan je uz pomoć formalizma koji se inače upotrebljava pri raščlambi jednostavnih relaksacijskih procesa. Osnovu za takav pristup čine postojeći podaci o masama pojedinih zuba i o trajanju četiriju karakterističnih faza u njihovu razvoju. Iz tih se podataka može zaključiti da asimptotski porast mase pojedinog zuba slijedi eksponencijalnu funkciju vremena. Premda su relacije koje proistječu iz modela izvedene uz pomoć parametara, čije vrijednosti zasad nisu točno određene (no moguće ih je dobro procijeniti), one opisuju rast zuba, posebice njegovu kinetiku, na uvjerljiv kvantitativan način. Rezultati se dobro slažu s iskustvom.

Ključne riječi: *ljudski zubi, rast*

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