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Methodology for Optimal Mast and Standing Rigging Selection of a Racing Yacht Using *AHP* and *FEM*

Original scientific paper

The racing yacht design process, except the hull and appendages, involves the selection of sails, rigging and mast. Proper selection and scantling calculations of the mast and standing rigging is of crucial importance as it is the backbone and connection of sail driving load transferred to the hull. Optimal selection within a racing yacht is even more important because of the fact that such yachts are operated by professional crews that are capable to get out the maximum of the ship and its masts and rigging. Thus in this paper the authors present a novel methodology for the optimal selection of a mast and standing rigging based on the procedure in three main phases by using *AHP* method through the first two phases and FEM analysis in the last one. The first phase includes identification of possible design solutions, while the second one searches for the best design configuration and determines the stability of the chosen solution. The third phase is used for load calculations, scantlings determination, and final approval of the project variables. The proposed methodology is applied and tested on the selection of mast and standing rigging of a 40-foot racing yacht.

Keywords: *AHP optimization, design methodology, FEM analysis, mast and rigging, racing yacht*

Prijedlog metodologije za optimalni odabir jarbola i nepomične opute regatne jedrilice korištenjem AHP i FEM

Izvorni znanstveni rad

Proces projektiranja regatne jedrilice, osim trupa i privjesaka, podrazumijeva odabir i projekt snasti, tj. jedrilja, opute i jarbola. Pravilni odabir i dimenzioniranje jarbola i nepomične opute od presudne je važnosti budući da predstavljaju oslonac i vezu prijenosa opterećenja ‘pogona’ od jedrilja na trup. Regatnim jedrilicama upravlja uglavnom stručnija posada koja je sposobna od broda, pa tako i od jarbola i opute, izvući maksimum čime pravilan odabir dobiva još više na važnosti. U radu je prikazana metodologija za optimalan odabir jarbola i nepomične opute primjenom *AHP* metode kroz prve dvije faze i FEM analize u trećoj fazi. Prva faza uključuje identifikaciju mogućih projektnih rješenja, druga faza podrazumijeva odabir optimalne projektne konfiguracije i utvrđivanje stabilnosti odabranog rješenja, dok treća faza služi za proračun opterećenja, dimenzioniranje i konačno usvajanje projektnih varijabli. Predložena metodologija primijenjena je i provjerena na odabiru jarbola i nepomične opute 40-stopne regatne jedrilice.

Ključne riječi: *AHP optimizacija, FEM analiza, jarbol i oputa, projektna metodologija, regatna jedrilica*

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1 Introduction

Mast design for standard sailing yachts usually consists of a number of procedures based on empirical equations from relevant marine standards, rules and regulations, which are leading to the mast selection meeting aimed sail yacht characteristics [1]. However, mast selected in such way, will usually be of standard material and configuration that lead to a heavier mast and rigging. In such approach, inevitably, resulting mast structure will not be optimal and sail yacht performances will be decreased. Such procedure is often used for cruising sail yachts because it is fast and cheap [2]. However, for prototype racing yachts even before structural design, it is necessary to identify some other design variables such as: mast and rigging material, mast configuration, sail plan and deck configuration. If the design is constrained with the existing hull and appendages, optimisation procedure can develop towards three main directions regarding: material selection, specific competition regulations and structural design.

Therefore, in this paper, authors are proposing a novel methodology for determining optimal mast configuration, implementing the procedure consisting of three main phases, using specific methods, such as Analytical Hierarchy Process (*AHP*) and Finite Element Method (*FEM*).

Material selection in case of racing yacht is an important parameter within mast design and its equipment. A lighter mast leads to the smaller influence on the sail yacht angle of heel and moments of inertia and hydrodynamic characteristics will improve. The consequence is application of aluminium alloys for mast construction and stainless steel ropes and steel bars for standing riggings. The ultimate racing masts are made of carbon fibres and special light composite ropes for standing and running rigging [3]. Masts made from carbons fibres are lighter and have the same or even better mechanical characteristics compared to aluminium alloys masts, but the price is significantly higher as well as impact on specific competition regulations [4].

Furthermore, within sailing racing events significant diversity in hull form, racing yachts dimensions, sails, and equipment can be observed. Therefore, an obvious question arises: When different racing yachts compete between each other is it predictable that a bigger and faster racing yacht will reach the finish line first? Solution for such situation is founded in configuring special competition rules, respectively the handicap systems, which should in theory allow the equal chance for all yachts regardless racing yacht dimensions and characteristics. Within such handicap system the competition standings are calculated using “corrected time” generated by using calculated factors for each particular yacht. To determine how a single characteristic of the racing yacht is influencing its performances is a problem that has been tried to be solved since 1881, when such handicap system was introduced in Great Britain for the first time. Until today, a number of handicap systems have been introduced, however, only *ORC international* handicap system is based on a scientific approach. Therefore, *ORCi* handicap system best suits the real life situation introducing completely new approaches in racing yacht measuring and calculating the “corrected time”. This handicap system is not dealing with direct racing yacht comparison but is using mathematical model and large racing yachts database, which includes the hull form, stability, sail plan, etc. in order to calculate theoretical yacht speed for each wind speed and angle [5]. In such way the theoretical time for completing the race course is calculated for each yacht in the race. In theory a racing yacht will have as better corrected time as the crew will sail closest to the calculated theoretical time for their particular yacht. Practice has proven that certain racing yachts characteristics and equipment configuration are especially favourably influencing the racing yacht performances within used handicap system because of obvious limitations of existing mathematical model and used data base. Hence, the authors are especially emphasizing the importance of proposed methodology where such handicap limitations can be integrated in mast configuration selection.

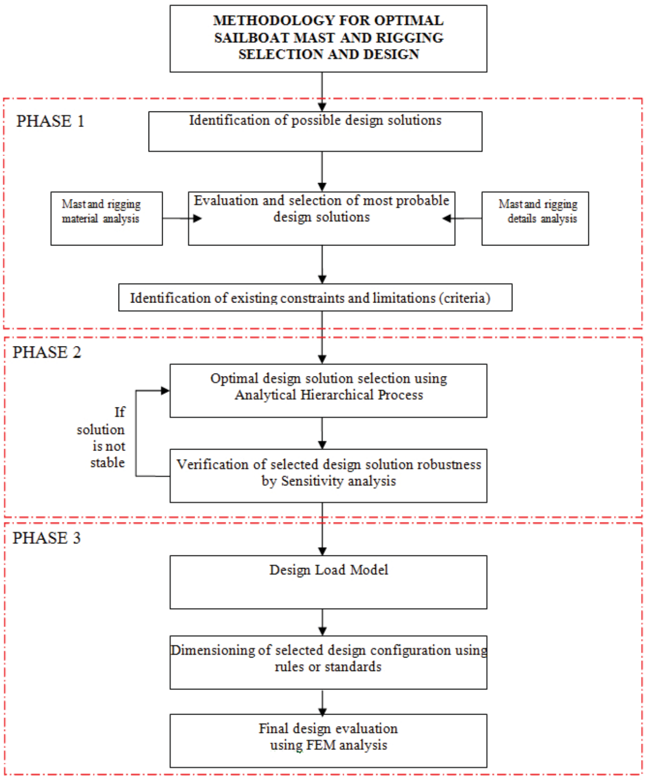
Final design aspect is a structural strength assessment. The mast and rigging in design have a great influence on racing yachts performances, firstly because of the height of the centre of gravity that decreases yacht stability, then because its deflection under loads influences the flying shape of the sail and hence the sail efficiency. The mast designer has thus to deal with two opposite objectives: to obtain a lightweight structure, with a low centre of gravity that can be efficiently controlled by the crew, and to obtain enough robustness to handle all the conditions that the yacht may encounter. To face that challenge, mast designers have traditionally two main kinds of tools: Finite Element Method (*FEM*) or Euler’s Formulas modified and adopted for easy use through classification rules and regulations [6], [7] and [8]. The first method represents the state of the art of structural engineering and provides very good results as long as loads are accurate. Practically, these methods are only used to check the validity of a candidate design or to, carry out a refined analysis of a design before construction. The second method, the use of Euler's Formulas, which deals with mast stability, in combination with designer’s experience, appears to be a "rule of thumb" and leads to over built rigs and overestimated safety coefficients as to avoid any risk of failure due to the various approximations made along the design process. In this context the proper combination of these two tools may lead to satisfactory results, because it eliminates the need for complex and expensive aerodynamic and hydrodynamic load models. In such a way it is important to prepare correct structural model for a *FEM* analysis using all the capabilities of a modeller and the *FEM* software.

2 Methodology for optimal mast and standing rigging selection of a racing yacht

A novel methodology for racing yacht mast and rigging selection is based on conducting three phases to reach an optimal design solution. Such design solution is the basis for further technical documentation development.

The procedure, methods and techniques of the developed methodology are explained in this section. Furthermore, the proposed methodology pattern of the procedure is shown in Figure 1.

Figure 1 **Proposed methodology Pattern of Procedures**
Slika 1 **Shematski prikaz procedura predložene metodologije**



2.1 Possible design solutions identification - phase I

Within the first phase of the developed methodology the design solutions are identified based on possible mast and rigging design variables. Such variables are: mast material, mast profile length and cross section design, number of spreaders, position of the mast foot step, rigging material, and sail plan configuration. All possible solutions include combinations of the mentioned variables. The number of possible different design solutions can be more than 300, but there are practical constraints that reduce this number to most feasible ones.

The criteria in most cases will be analysed through the influence on sailing performance, handicap system, building costs, useful exploitation life time, and maintenance costs, Table 1. Each criterion will have particular weight factor K_i calculated by expert approach within *AHP* method [9].

Table 1 **Proposed criteria for further calculations**
Tablica 1 **Prijedlog kriterija za daljnji proračun**

Criterion number	*Criterion name*	*Criterion weight factor*
1	Influence on sailing performance	K_1
2	Influence on handicap system	K_2
3	Influence on building and maintenance costs	K_3
4	Influence on useful exploitation life time	K_4

Constraints in considerations will normally suggest no more than n=50 most feasible design solutions i.e. design alternatives, Table 2.

Table 2 **Possible design solutions, alternatives**
Tablica 2 **Moguća projektna rješenja, alternative**

Alternative	*Mast characteristics*			
	Mast material	*Mast length,* m	*Rigging material*	*No. of spreaders*
1	Mast 1	L_1	Rig 1	1
2	Mast 2	L_2	Rig 2	2
3	.	.	.	.
.	Mast *m*	L_s	Rig *r*	*p*
.	.	.	.	.
.	Mast 1	L_1	Rig 1	1
.	Mast 2	L_2	Rig 2	2
n-1	.	.	.	.
n	Mast *m*	L_s	Rig *r*	*p*

2.2 Optimal configuration selection – phase II

2.2.1 Optimal configuration selection applying AHP

In the second phase of the proposed methodology, for optimal design solution selection, the authors suggest using Analytical Hierarchy Process (*AHP*) [9]. The *AHP* method as one of multi-attribute decision making approaches is a structured technique for dealing with complex decisions. Rather than prescribing a “correct” decision, the *AHP* helps the decision makers find the one that best suits given constraints (criteria). In order to select the optimal design among previously selected probable solutions it is necessary to involve relevant constraints which this design has to satisfy optimally. These criteria are included in *AHP* model development and based on them an optimal solution, as the method goal, will be found among the chosen design solutions.

Hierarchical model structurally consists of the following levels: goal, criteria, sub-criteria and alternatives, Figure 2. The goal is placed on the highest hierarchical level and it is not compared to any other element of the hierarchical structure. On the first level there are k criteria which are compared to each other in pairs regarding the directly superior element – goal. The $k \cdot (k - 1)/2$ of comparison is required. The same procedure is repeated for the next hierarchical level, all the way down to the last r level, until all comparisons of all solutions with respect to the superior criteria, down to r-1 level, is completed.

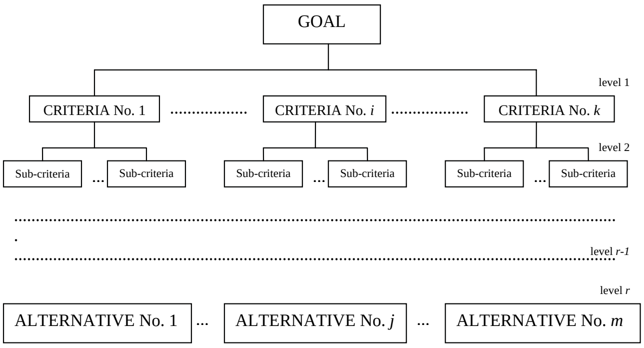


Figure 2 ***AHP* hierarchical model**
Slika 2 ***AHP* hijerarhijski model**

Each comparison of two elements of the hierarchical model is done by Saaty’s scale of relative importance as shown in [9]. The results of elements comparison on the observed hierarchical level are organised in matrix form as follows:

If n elements are compared to each other with respect to the superior corresponding element on a higher hierarchical level, then, when comparing i element to j element using Saaty’s scale of relative importance, numerical coefficient a_{ij} is determined and placed in its adequate position in matrix A:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdot & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot \\ a_{n1} & a_{n2} & \cdot & \cdot & a_{nn} \end{bmatrix} \tag{1}$$

Inverse result value is placed on position a_{ji} as to maintain consistency of decision making. Detailed description of *AHP* method can be found in [9] and [10].

The authors used specialised software, *ExpertChoice11*, [11] for solving the considered case study. Based on determined priorities the solution with the highest value is selected and such solution is considered to be the optimal one.

$$P_i = A_{1-i} \cdot K_1 + A_{2-i} \cdot K_2 + A_{3-i} \cdot K_3 + A_{4-i} \cdot K_4 \tag{2}$$

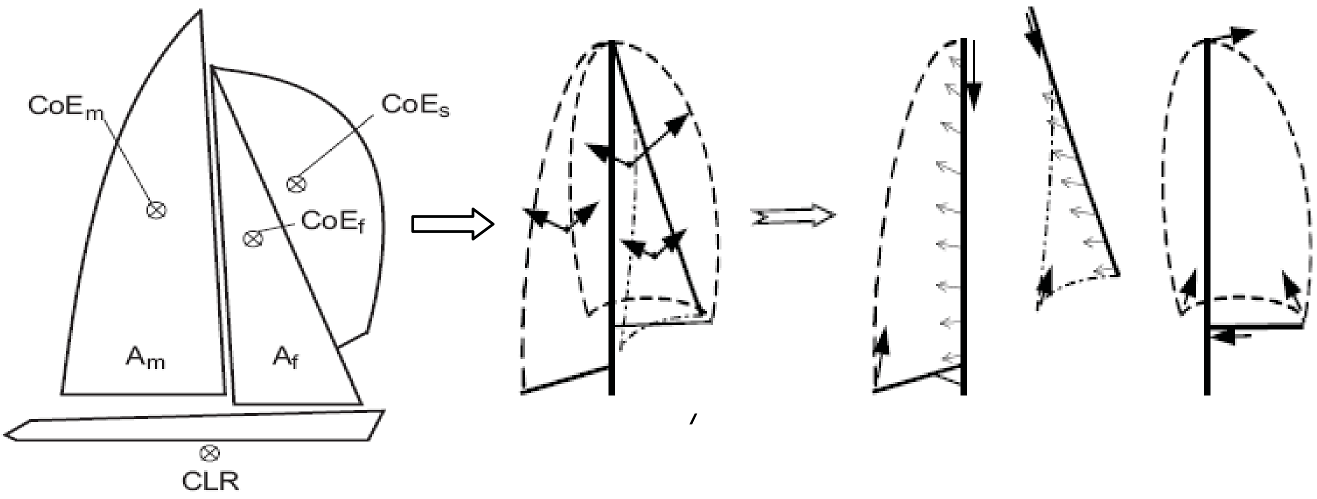


Figure 3 **Centre of efforts, sail areas, and load transfer from sails to mast and rigging**
Slika 3 **Položaji težišta i površine jedara i prijenos sila na jarbol i oputu**

2.2.2 *Stability determination of chosen solution by sensitivity analysis*

To conclude if the suggested rank list of the design solution is stable, the Sensitivity Analysis (*SA*) is conducted. Sensitivity Analysis belongs to Operation Research methods within linear programming and is used for analysing how changes of model parameters are influencing the optimal solution [12].

There are two types of *SA* as follows [13]:

- Analytical *SA*, used for well defined systems and solving problem using partial derivation (3),

$$S_x^F = \frac{\partial F}{\partial x}, \tag{3}$$

where *S* defines sensitivity function (change intensity) of goal function *F* related to changes of parameter *x*.

- Empirical *SA*, used when the influence of parameter values change on optimal solution is analysed by experiments, such type of *SA* is more applicable to complex systems [13].

Within the proposed methodology the empirical *SA* is suggested because the mast and rigging configuration present a complex system that cannot be analytically well defined [13]. For conducting *SA* the *Expert Choice* software can be used with the following empirical *SA* types: Dynamic, Performance, Gradient and Head to Head analysis [11]. Using *SA* within phase 2, the selected optimal design solution can be confirmed as stable and therefore as a final solution being an input for the final phase. In case that the solution is not stable, i.e. there are two or more solutions with the same priority value, the iteration loop returns towards the previous phase for further analysis of criteria and the weight factors, Figure 1.

2.3 Strength assessment – phase III

Strength assessment - phase III, within the proposed methodology represents a part of a standard theory for marine structural design procedure [14]. Mast and rigging, as marine structural elements are subjected to ultimate strength and fatigue assessment, as well as to structural reliability. These particular tasks are included by applying safety coefficients (panel factor, staying factor, etc.) through available standards and rules. Therefore, they are within safety margins, and could be improved. Although the ultimate strength and fatigue are recognised as design drivers in the design process, the present methodology can be extended to the future work. A standard *FEM* analysis, together with ultimate strength and fatigue would make the main loop in the optimisation process. But this could be done for the case of completely new racing yacht design that is not constrained with existing hull parameters.

2.3.1 *Design load model*

One of the major issues when dealing with mast design is the ability of the designer to define design or maximal loads applied on the structure. To determine the sail loads on the rigging, a load model is to be developed based on the yacht performance. Therefore, various velocity prediction programs [5], [15], [16] are used for the prediction of forces generated by each sail, Figure 3, [15]. Later on these forces are translated to forces acting on the mast and rigging. For a sailing yacht rigging the following conditions, which a robust mast has to be able to face, belong to the critical ones: closed hauled, broad reach, and running under spinnaker.

The first and the most important step in all these conditions, with small exceptions for downwind, is to evaluate the transversal force T_f on each sail and its centre of effort $CoE_{m,s,f}$, Figure 4. The sum of each sail moment has to balance the righting moment *RM* of the yacht depending on the sailing condition:

$$T_f(m,f,s) = \frac{A_{m,f,s} RM}{\sum\limits_{i=m,f,s} A_i \left(CoE_i - H \right)} \tag{4}$$

Once the transversal force on a sail is known, the distribution of the loads at the sail corners with respect to the sail balance has to be performed. As the sail has a quite high aspect ratio, one can assume that the clew and tack points are located at the same heights. That load is then applied to the node carrying the headsails stay, or it is distributed on small portion of the mast at headsail location in the case of mainsail. Loads on the mast from the mainsail stem, not only from transverse force but also from leech tension, induce longitudinal load that cannot be neglected. This load highly depends on mainsail twist, adjusted when sailing through the mainsheet tension and traveller, and also on the sail roach and on sail cloth.

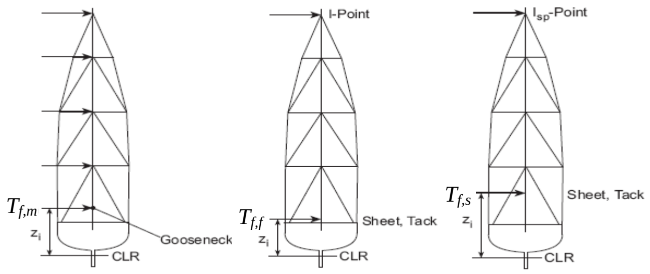


Figure 4 **Mainsail, foresail, and spinnaker point loads**
Slika 4 **Poprečno opterećenje od glavnog jedra, prednjeg jedra i spinakera**

From this point two paths exists. The first one is to use the classification societies rules and/or available standards regarding these design loads and it is the path integrated in the proposed methodology presented within the paper and shown through the presented case study. The second one is more demanding and requires a more complex dynamic load model and fluid-structure interaction calculations, or results from a tank model or some real structures.

2.3.2 Scantlings through available rules

Compression from the halyard is one of the main sources of compression encountered in the mast tube, so it has a great influence on mast stability and buckling coefficients. The maximum compression force in the lower mast panel and the tensile force in the windward stay are used to determine the required stay dimensions and the panel bending stiffness (*EI*), in both the transverse and longitudinal direction. The general method for the bending stiffness is to use the Euler buckling formula:

$$P_{cr} = P_E = \frac{\pi^2 E \cdot I}{k \cdot l^2} \tag{5}$$

A distinction must be made between the transverse and longitudinal direction due to different support lengths [16].

$$\begin{aligned} I_x &= k_1 \cdot m \cdot PT \cdot l(n)^2, \ \ \mathrm{mm}^4 \\ I_y &= k_2 \cdot k_3 \cdot m \cdot PT \cdot h^2, \ \ \mathrm{mm}^4 \end{aligned} \tag{6}$$

An important remark is that the Euler buckling method is a linear representation of a non linear phenomenon. The formula is a theoretical approach of the buckling or instability load of a compression column. It is only valid for ideal undisturbed structures under a pure compression force. In practice, like in the case of sailing yacht rigging, this ideal situation never occurs. A distributed force of the mainsail or point loads from boom or stays make that a mast is never in a pure compression state. Right from the beginning there is a certain bending and an axial displacement. For the dimensioning of the rest of the rigging, mast, and windward rigging, it is considered as a static determined structure. The heeling moment at deck level is the result of heeling forces acting at the hinges between panels and spreaders. Distributed forces from the mainsail and forces acting between panels need to be translated to forces acting at the hinges as shown in Figure 6. With equilibrium equations the transverse stay and panel forces can now be determined and so the required dimensions as well. The dimensioning of the longitudinal stays is also based on the transverse stability.

2.3.3 Final design evaluation

Finally, the mast is a structure subjected to combined load, compression and bending. Therefore, the analytical solution of the differential equation that describes the nature of the problem becomes more complex. The ultimate evaluation of the preliminary design configuration includes application of appropriate finite element analysis regarding stress state, which means the proper choice of finite elements for structure modelling, meshing and definition of boundary condition. An additional option is to use nonlinear analysis instead of linear analysis. Nonlinearity can be included through geometry, meaning application of large deformations, which is an ordinary practice, or by material nonlinearity that requires an extra knowledge of the user. It is a common practice to model the mast, spreaders, boom and tack pool with a 3D beam element, as well as shrouds with rod or tension only elements. Cables are nonlinear for they fall slack when compressed. The exception is the forestay that is modelled with a beam element in order to control the sagging and get closer to the real sailing condition of the deformed rigging. Plate thin shell elements may be used instead of the beam element in the case of more precise information of stresses in some particular structural details. This type of element is necessary when modelling and analyzing the composite structure. The special attention has to be put on mast and rigging composite materials although not all solvers are capable of solving orthotropic or anisotropic relations.

The primary role of the shrouds is to support the mast in cooperation with the spreaders. Spreaders are small compression struts whose only purpose is to correct or improve the shrouds attack angle to the mast. Therefore, the shrouds are designed as tension only elements and have an advantage in comparison with the element that has to withstand the compression load, buckling or some other nonlinear loads. Shrouds are made of wire or thin solid rods having very low flexural stiffness. Modelling of such a structure is difficult while using FEM analysis because the finite elements stiffness, according to Hooke's law cannot take the zero values that are real situations when shrouds fall slack i.e. the load cannot become negative. Final dimensions evaluation comprises deformation control and stresses calculation over the model based on classification rules preliminary dimensions.

3 Application of proposed methodology – case study

The proposed methodology was applied to a real problem regarding the selection of the mast and rigging for a racing yacht, type ILC40, with a goal to optimize its design for best performances in racing within specific *ORCi* handicap system. Considering the methodology determined criteria in Table 1, using the proposed expert approach, numbers of mast design alternatives were generated. In this particular case, two types of materials were considered for the mast construction, carbon fibre and aluminium alloy material. Furthermore, two different lengths of the mast tube were analysed due to their direct influence on the sail size and therefore on *ORCi* corrected time calculations. Then, three type of materials for standing riggings were analyzed, i.e. composite materials (*PBO, Kevlar*, etc.), special stainless steel ropes *Dyform*, and bars from stainless steel. Finally, the total number of mast spreaders was analysed, because of the significant influence on handicap system calculation, and later on it will be important in the mast strength assessment. Mutual parameter combination conducted for 24 possible design alternatives are shown in Table 3.

Table 3 **Possible design solutions, case study alternatives**
Tablica 3 **Moguća projektna rješenja, alternative realnog problema**

Alternative	*Mast characteristic*			
	Mast tube material	*Length (sail height), m*	*Standing riggings material*	*Spreaders number*
1	Carbon	16.7	Composite material rope	3
2	Carbon	15.7	Composite material rope	3
3	Carbon	16.7	Dyform steel rope	3
4	Carbon	15.7	Dyform steel rope	3
5	Carbon	16.7	Steel bar	3
6	Carbon	15.7	Steel bar	3
7	Carbon	16.7	Composite material rope	4
8	Carbon	15.7	Composite material rope	4
9	Carbon	16.7	Dyform steel rope	4
10	Carbon	15.7	Dyform steel rope	4
11	Carbon	16.7	Steel bar	4
12	Carbon	15.7	Steel bar	4
13	Aluminium	16.7	Composite material rope	3
14	Aluminium	15.7	Composite material rope	3
15	Aluminium	16.7	Dyform steel rope	3
16	Aluminium	15.7	Dyform steel rope	3
17	Aluminium	16.7	Steel bar	3
18	Aluminium	15.7	Steel bar	3
19	Aluminium	16.7	Composite material rope	4
20	Aluminium	15.7	Composite material rope	4
21	Aluminium	16.7	Dyform steel rope	4
22	Aluminium	15.7	Dyform steel rope	4
23	Aluminium	16.7	Steel bar	4
24	Aluminium	15.7	Steel bar	4

Results obtained using *AHP* and *SA* method, based on the chosen criteria and determined 24 design alternatives, are shown in Figure 5. On the left side of Figure 5 the criteria and relevant weight factors are shown as green horizontal bars. The most important criterion for selecting the optimal design alternative is the influence on *ORCi* handicap system (K_2=33.1%). The next important criterion is the influence of the design solution on overall costs (K_3=30.6%). The third important criterion is related to the influence of the design solution on yacht performances (K_1=20.7%) and the least significant one is related to the exploitation long term reliability (K_4=15.7%). On the right side of Figure 5, all design alternatives are shown as coloured horizontal bars with corresponding percentage. The largest percentage corresponds to the *Design Alternative 18,* which represents the solution that optimally meets defined criteria and the project goals.

Design Alternative 18, as shown in Table 4, is the one where the mast material is of aluminium alloy (*Al.6083*), with three mast spreaders and the height of the main sail of 15.7 m, and with fixed standing rigging from steel bars (*Nitronix50*). Illustration of the selected mast design is shown on right side of Figure 6. The considered yacht with such mast design configuration obtained through the proposed phase is recognized as slower for approximately 4 seconds per nautical mile by *ORCi* theoretical background in comparison with the old design, Figure 6 left. In practical race situations this represents a distance of 8 to 14 yacht lengths depending of the wind or yacht speed, which is obviously an important bonus regarding final ranking. However, due to the crew and designer experience and knowledge it is expected that this yacht will actually be faster than before. It can be concluded that the goal of selecting the optimal mast configuration regarding the particular *ORCi* handicap system is achieved.

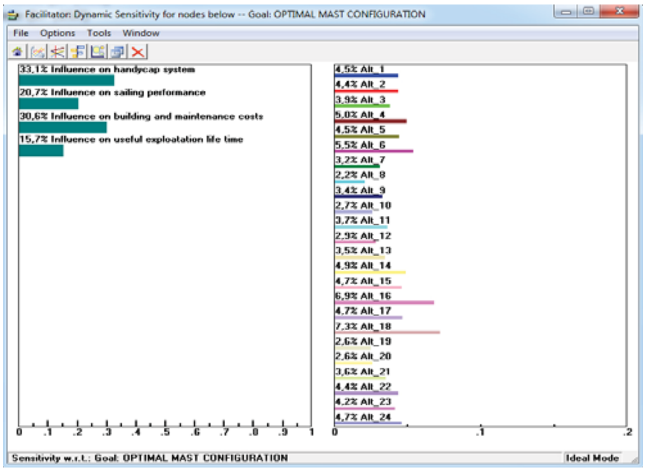


Figure 5 **Results of *AHP/SA* method**
Slika 5 **Rezultati *AHP/SA* metode**

For the selected mast design configuration, structural analysis is conducted in order to obtain and verify the scantlings of structural elements, which are: mast, forestay, backstay, runner and shrouds, Figure 7. Modified *Skene* method is used through the available *Nordic Boat Standards* [6], [17] and partially by use of ISO standard 12215-9 [7]. Allowable values of stresses are set to 90% of yield strength or 70% of ultimate strength, whatever is the lesser. Mechanical characteristics of the selected material, as well as allowable stresses are shown in Table 4.

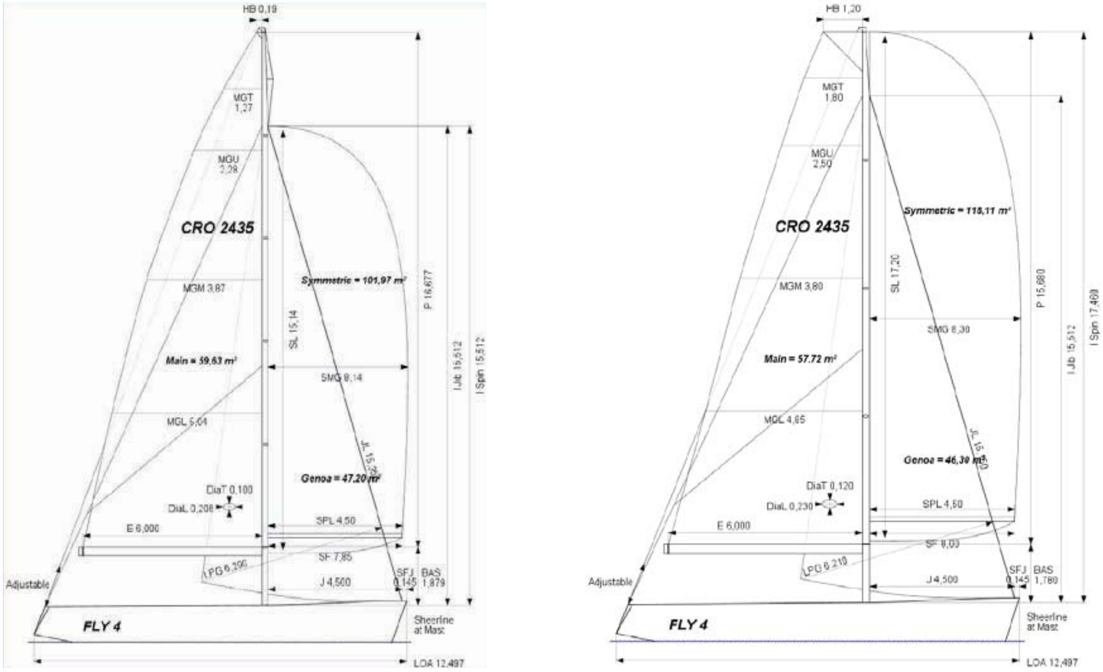
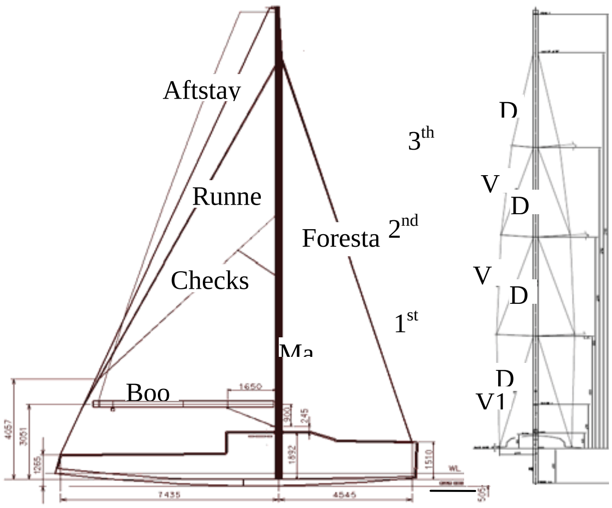


Figure 6 **Old (left) and new optimal (right) sail plan with mast configuration [5]**
Slika 6 **Stari (lijevo) i novi optimalni (desno) plan jedrilja i konfiguracija jarbola [5]**

Table 4 **Mechanical characteristics of mast and rigging material**
Tablica 4 **Mehaničke osobine materijala jarbola i nepomične opute**

Material /element	Ultimate strength, $_B$, MPa	Yield strength, $_y$, MPa	Allowable stress, $_D$, MPa
Al.6083 / mast	300	255	210
Nitronic50 / standing rigging	690	380	340

Figure 7 **Mast and rigging elements**
Slika 7 **Prikaz elemenata snasti za dimenzioniranje**



Regarding the selected design configuration with three spreaders and mast topology, Figure 8, the resultant values of transversal force for the two worst loadcases (foresail and reefed mainsail) are shown in Table 5. The forces were calculated analitically and graphically (Cremona-Maxwell) in the nodes conecting spreaders to the mast. Later, these forces were checked directly within the model [18], Figure 8.

Table 5 **Transverse loads**
Tablica 5 **Poprečno opterećenje**

Force	Load case 1 (foresail)	Load case 2 (reefed mainsail)	Value, kN
F_1	0	T_{bu}	0.88
F_2	0	T_{hl}	0.60
F_3	0	T_{hu}	2.61
F_4	$T_{f,f}$	0	3.64

The selected fractional rigging with three straight spreaders, runner, and checkstay has a breaking strength of at least selected ultimate strength presented in Table 6, where dynamic factors are according to [17].

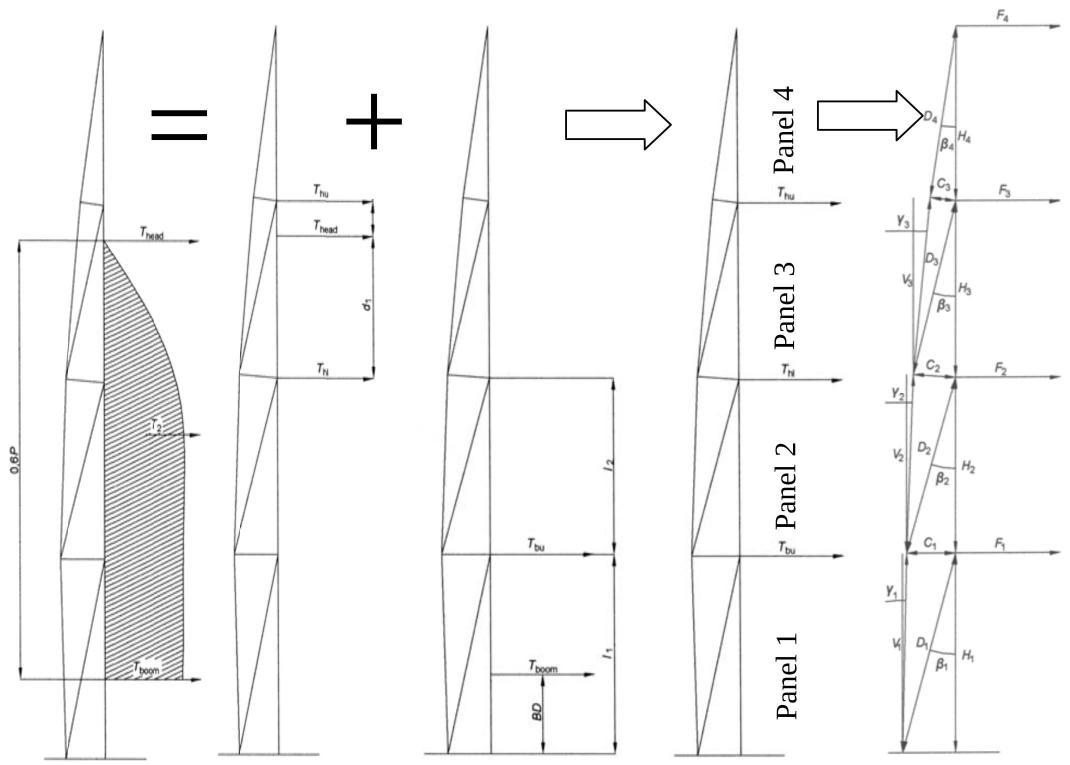


Figure 8 **Main sail force (T_2=T_m) transfer for mast and shrouds dimensioning**
Slika 8 **Prijenos sila od glavnog jedra za dimenzioniranje jarbola i pripona**

Table 6 **Standing rigging scantlings**
Tablica 6 **Dimenzije nepomične opute**

Standing rigging (shrouds)		Calculated (max. value 1 and 2)	Selected (dynamic factor included [17])	
		Ultimate strength, kN	Ultimate strength, kN	Diameter *D*, mm
Lower diagonal	*D1*	18.8	52.6	7.1
First intermediate diagonal	*D2*	10.1	23.3	5.7
Second intermediate diagonal	*D3*	9.2	21.1	5.0
Upper diagonal	*D4*	19.3	57.9	7.1
Lower vertical	*V1*	35.3	112.9	11.1
First intermediate vertical	*V2*	25.8	77.4	8.4
Second intermediate vertical	*V3*	19.2	57.6	7.1
Forestay	P_{fo}	50.7	50.7	7.1
Backstay	P_a	24.5	24.5	5.0
Runner	P_{a1}	38.6	38.6	6.4

Mast cross section dimensions are calculated based on the required longitudinal I_y and transversal I_x moment of inertia [17]. The fundamental parameter for the transversal moment of inertia is the compression force on each mast segment (panel), Figure 8, while the longitudinal one deals with the compression force over the total free buckling length that is the height of the mast above the deck or superstructure. The results are shown in Table 7.

Table 7 **Minimal required moment of inertia for the mast**
Tablica 7 **Proračun minimalnog momenta inercije jarbola**

	Compression force, kN	Mast cross section area moment of inertia	
		transversal, I_x, cm^4	longitudinal, I_y, cm^4
Panel 1	77.0	303	-
Panel 2	60.3	327	-
Panel 3	48.5	262	-
Panel 4	38.6	208	-
Mast	77.0	-	1465

Structural model is created using *Nastran* [18]. Geometry of the model, boundary conditions and loads combination in transversal and longitudinal vertical plane (load case 2) are shown in Figure 9. *FEA* required 18 computer models that include 6 load models built up from the longitudinal and transversal vertical plane and included pretension with the combination of two different mast construction materials (aluminium and composite). The

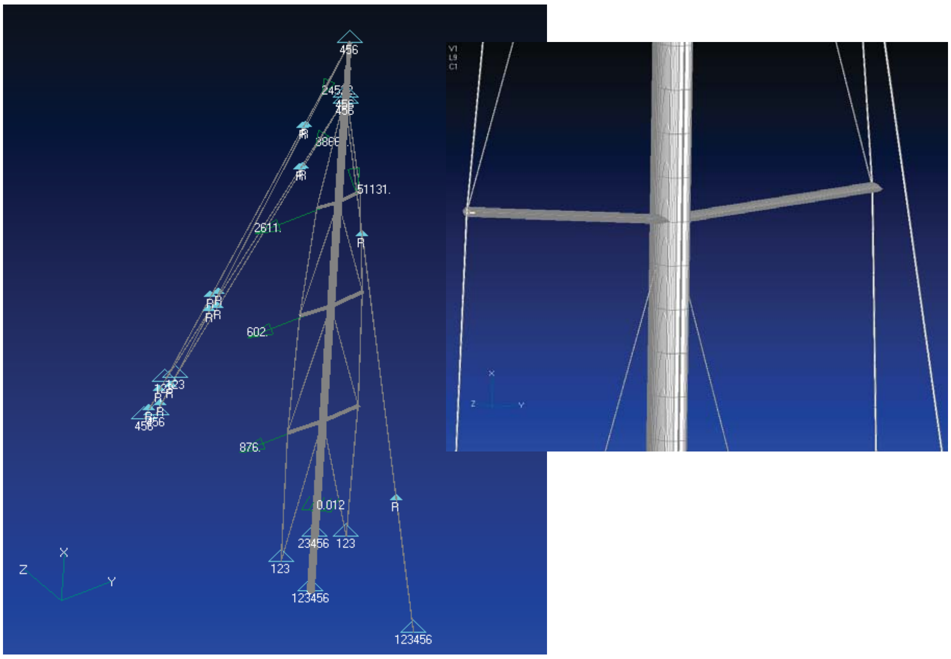
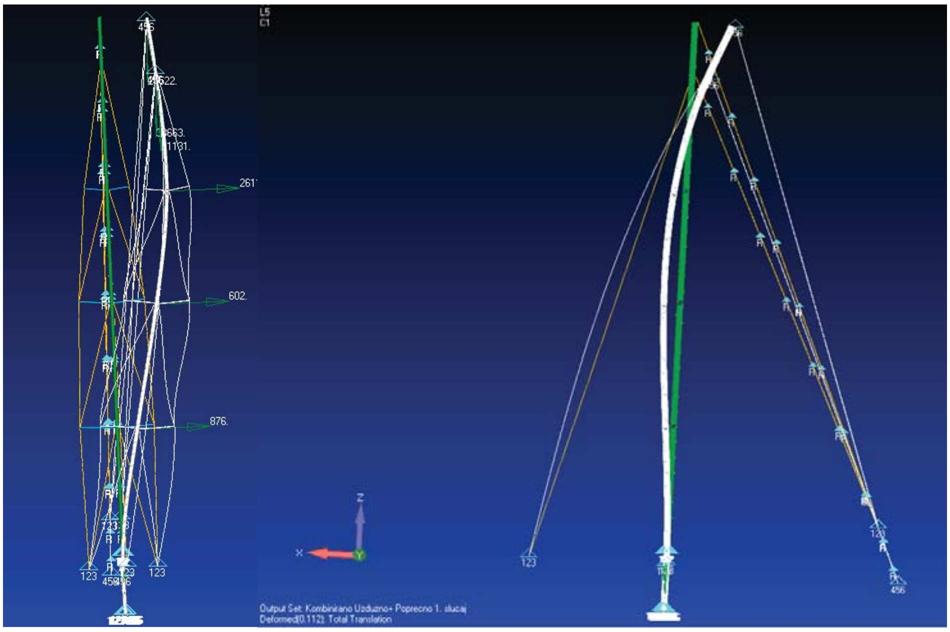


Figure 9 **Model geometry, longitudinal and transversal load (case 2) and boundary conditions**
Slika 9 **Geometrija, uzdužno i poprečno opterećenje (stanje 2) modela i rubni uvjeti**

results in the form of deformations, Figure 10, and stresses, Figure 11, are presented for the selected design configuration only (aluminium mast and spreaders, *Nitronic 50* rod for standing rigging). The non-linear *FEA* was applied and only maximal values of deformation for the load case 1 (longitudinal + transversal 1) and stresses for the load case 2 (longitudinal + transversal 2) are shown.

Maximal total deformation for the loadcase 1 (longitudinal load and transversal load from the foresail only) is 0.112 m (aft and leeward). The length of forestay is 16.5 m and the total sagging is 0.7% of the forestay length, which is relatively small. The purpose of the calculation is to control the forestay sagging line in comparison to the real sailing condition. Second part of the results is related to the stresses in the mast and standing rigging structural elements for the both loadcases. The maximal stresses occurred in load case 2 within intermediate diagonal shrouds *D2* (186 Nmm^{-2}), Figure 11.

Figure 10 **Longitudinal and transverse deformations (loadcase 1)**
Slika 10 **Uzdužne i poprečne deformacije (stanje 1)**



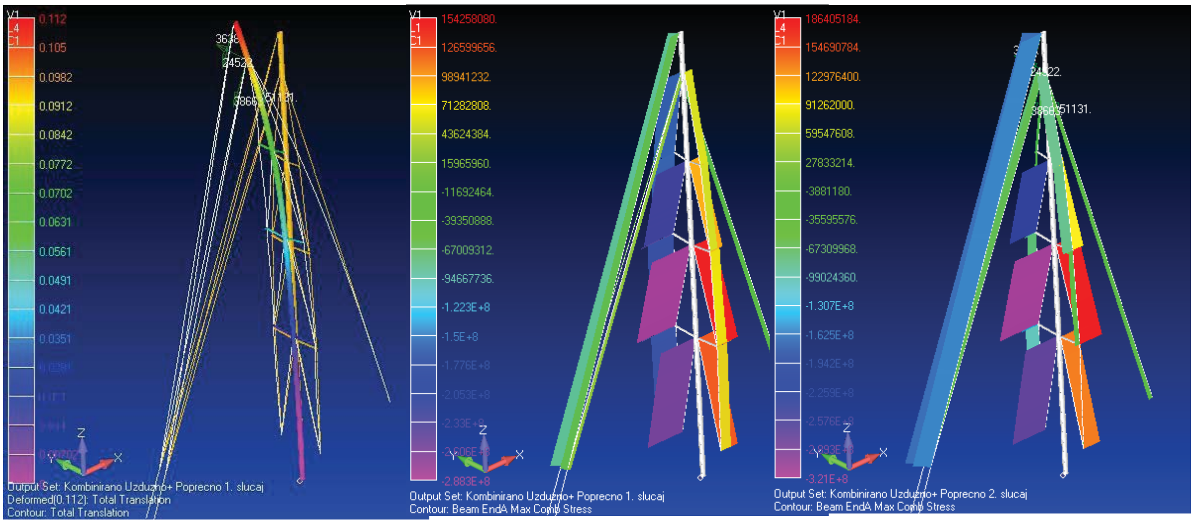
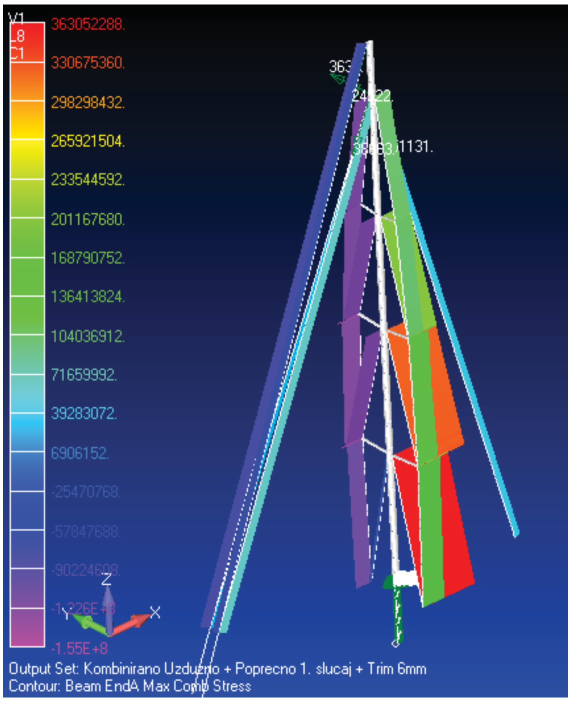


Figure 11 **Maximum stresses, loadcase 2 (left) and loadcase 1 (right)**
Slika 11 **Maksimalna naprezanja, stanje 2 (lijevo) i stanje 1 (desno)**

If the result is compared against maximal permissible stresses, Table 4, it can also be concluded that some strength reserve exists. For the final strength evaluation of the structural elements, stresses due to pretension (dock tune) have to be added. Pretension is used to restrict standing rigging to fall slack and it is achieved by use of hydraulic mast jack to lift up the mast foot. Depending on sailing condition and shrouds position, forces in shrouds are to be from 15% to 30% of the breaking strength. The presented case study is based on 6 mm pretension which brings maximal stress in lower diagonal *D1* to 363 Nmm^{-2}, Figure 12.

Figure 12 **Maximum stresses, loadcase 2 + 6 mm pretension**
Slika 12 **Maksimalna naprezanja, stanje 2 + početno trimanje od 6 mm**



According to the performed analysis, the final shape and geometrical characteristics of the mast are adopted and the scantlings of the standing rigging are slightly modified (Table 6), therefore the diameter of the first intermediate diagonal (*D2*) is increased to 6.4 mm instead of 5.7 mm, the forestay is set up to 8.4 mm instead of 7.1 mm, lower vertical (*V1*) is decreased from 11.1 mm to 9.5 mm, and finally the second intermediate vertical (*V3*) and upper diagonal (*D4*) are both decreased form 7.1 mm to 6.4 mm.

4 Conclusions

The proposed methodology for the sailing yacht mast and rigging selection consists of three phases. In the first two phases multi criteria analysis was performed using expert approach, *AHP* method, and sensitivity analysis in order to select the optimal mast configuration regarding defined criteria. The proposed approach for such configuration selection is influenced by the existing yacht hull and related structural parameters that have to remain unchanged. The set of remaining design parameters that were considered here, such as materials of mast and rigging construction, height of the mast, and number of the spreaders in combination with defined criteria, were analysed through the described phases, and resulted in the selection of the optimal design alternative (*Design Alternative 18*). Further, this optimal design alternative was used as the input for strength assessment and final design evaluation. Last phase represents the combination of the conventional approach to scantlings determination using classification rules and/or available standards and nonlinear finite element analysis. According to the adopted design parameters configuration, the final evaluation of the mast and rigging cross section characteristics is performed. Also, *FEA* is used for the evaluation of the effects that conventional approach is not able to predict such as pretension, local buckling and general behaviour of the mast and standing rigging. The proposed methodology together with the obtained results represents a verified platform for successful mast and standing rigging design, as shown on the real structure [19]. The presented case study was confirmed in practice. The yacht with such an optimized mast achieved the 4th place in the *ORCi* Mediterranean Championship [20].

The presented methodology can be improved within the third phase where ultimate strength and fatigue would be included through direct calculation instead of using safety factors, and where the *FEM* would be a part of optimisation loop instead of final evaluation only.

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Nomenclature

AHP - Analytic Hierarchy Process
A_{1i} - local priority of the *i*-class alternative regarding criterion 1,
A_{2i} - local priority of the *i*-class alternative regarding criterion 2,
A_{3i} - local priority of the *i*-class alternative regarding criterion 3,
A_{4i} - local priority of the *i*-class alternative regarding criterion 4,
A_{5i} - local priority of the *i*-class alternative regarding criterion 5,
a_{ij} - Saaty’s intensity of relative importance
∂F - goal function
$K_{1\text{-}4}$ - criteria weight factor
P_i - overall priority of *i*-class,
S_x^F - sensitivity function
k_1 - panel factor
k_2 - staying factor
k_3 - mast end (foot) factor
m - factor of material
PT - compression force, N
l - actual panel length, m
h - mast height, m.