

Motion Control of a Redundant Flexure Based Mechanism Using Piezoelectric Actuators

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Original scientific paper

A 3-PRR flexure based mechanism which is used as a redundant mechanism providing only x - y micro positioning is designed and controlled in this paper. The aim of this work is to eliminate the unpredictable motions due to manufacturing and assembling errors by implementing sliding mode control (SMC) with disturbance observer (DOB) using piezoelectric actuator models. The system is designed to be redundant to enhance the position control. In order to see the effects of the redundant system firstly the closed loop control is implemented for 2 piezoelectric actuators and the remainder piezoelectric actuator is treated as a fixture. Then the position control is implemented for 3 piezoelectric actuators. As a result, our redundant mechanism tracks the desired trajectory accurately and its workspace is bigger. Finally we have compared the proposed position control with the conventional PID control. It is seen that SMC with DOB gives better results. We have achieved to make the position control of our mechanism, which has unpredictable position errors due to rough manufacturing, assembly, piezoelectric actuator hysteresis etc. The designed 3-PRR flexure mechanism can be used as a micro positioner with the available measurement in the laboratory.

Key words: Compliant Mechanism, Sliding Mode Control, Piezoelectric Actuator Control, Flexure Based Mechanism, Observer

Upravljanje gibanjem redundantnog mehanizma temeljenog na savijanju korištenjem piezoelektričnih aktuatora. U radu se projektira sustav mikro-pozicioniranja redundantnog mehanizma baziranog na savijanju TRR konfiguracije zglobova u x - y ravnini. Zadatak upravljanja je eliminirati nepredvidivo gibanje, koje je posljedica pogreške proizvodnje i montaže, korištenjem kliznog režima i obzervera poremećaja te matematičkih modela piezoelektričnih aktuatora. Sustav je projektiran redundantno kako bi se povećale upravljačke mogućnosti. Kako bi se ispitalo ponašanje sustava bez redundancije, najprije je proveden eksperiment upravljanja s uključena dva piezoelektrična aktuatora i fiksiran treći aktuator. Potom je implementirano upravljanje pozicijom uz uključena sva tri aktuatora. Pokazano je da redundancija u sustavu upravljanja rezultira boljim ponašanjem zadane putanje i većim radnim prostorom. Na posljetku smo predloženi sustav upravljanja pozicijom usporedili s PID regulatorom pozicije. Regulator baziran na kliznom režimu rada s uključenim poremećajnim obzerverom je postigao bolje ponašanje regulacijskog kruga. Ostvareno je upravljanje pozicijom eksperimentalnog postava koje ima nepredvidive pogreške pozicioniranja zbog odstupanja nastalih prilikom proizvodnje i montaže, utjecaja histereze piezoelektričnog aktuatora itd. Projektirani mehanizam TRR konfiguracije može se upotrijebiti za mikro-pozicioniranje korištenjem mjerenja dostupnih u laboratoriju.

Ključne riječi: klizni režim rada, upravljanje piezoelektričnim aktuatorima, mehanizam temeljen na savijanju, obzerver

1 INTRODUCTION

The developments in the field of micro and nano technologies initiated a new market for smaller and high precision positioning devices. As a result of these developments in micro and nano technologies high precision positioning devices with controlled motions at sub-micron and even at nano level is needed. Positioning of necessary parts became very important for micro/nano applications such as

cell manipulation, surgery, aerospace, micro fluidics, optical systems, micromachining and microassembly etc. [1-2].

The need of increased accuracy and precision requires the development of design and control methods simple enough that can be used in engineering practice. Traditional rigid body mechanisms start not to provide needed accuracy and precision. Then high precision mechanisms

with flexible joints are designed in which flexible joints transfer necessary motion or force in the mechanism. The desired motion is provided with the deflection of these flexible joints also called in the literature as “flexures” which provide high resolution, frictionless, smooth and continuous motion. Flexure based mechanisms are also called “compliant mechanisms” [3].

Mostly parallel kinematic structures are used for micro positioning stages because of their advantages but parallel kinematic structures have also important disadvantages such as having limited workspace and dexterity, nonlinear kinematics, difficult calculation of forward kinematics. However these drawbacks are not problems for flexure based (compliant) mechanisms because the motions are in micro range and due to the small flexure displacements the kinematics can be assumed as linear in the workspace range. The repeatability of these structures is eliminated with flexures because there is no backlash, friction problem in the joints as in rigid mechanisms. Various types of parallel kinematic structures have been used while designing compliant positioning stages in the literature. These structures are based on popular rigid body parallel mechanisms. A lot of 2 DOF planar parallel compliant mechanisms have been studied in [4-14]. The common problems of these stages are parasitic motions and the limited range of the motion of these stages. Amplification mechanisms have been designed to improve the range of motion of these stages. Three DOF planar parallel structures have also been developed for providing translation in x and y axes and rotation about z axis. These mechanisms are mostly based on triangular stages that have 3-RRR (three revolute joints) kinematic structure [15-21]. A triangular platform is actuated by three linkages which are at the corners of the stage. Each chain is composed of 3 revolute joints in a serial arrangement. The end-effector has translation motion along x - y direction and a rotation about the z axis. This type of parallel kinematic structure amplifies the motion of the actuators. The revolute joints were replaced with flexure hinges which were designed according to the desired parallel kinematic performance. Other types of x - y - θ planar compliant structures with amplification beams have also been designed in [22] and [23]. A very compact x - y - θ planar compliant mechanism which is actuated by one actuator is studied in [24]. 3-PRR kinematic structure is used in by J. K. Mills [25]. 3-PRR (one prismatic, two revolute joints) kinematic structure is also selected in this work to be used as a micromotion stage for our laser micromachining unit but instead of using flexible beams and linear motors as in [25] right circular flexures and flexure based 4-bar mechanisms are used as revolute and prismatic joints. The designed compliant 3-PRR mechanism is used as a redundant micro positioner in x - y axes although the mechanism has also rotational motion in z axis. We

have used the redundancy of the mechanism to improve the workspace and dexterity.

The position tracking control of the compliant micro motion stages is very important because of the high performance requirements in high precision applications. The complexity of modeling of these mechanisms leads to be hard to control its position because of being lack of computing the accurate model. Therefore, a usable method should be defined for controlling the mechanism or the control should eliminate the nonlinearities of the mechanism. There are researches going on simplifying the models that can be computed while real time control is running. The most popular one is pseudo rigid body model which computes the stiffness value of the flexures that are equivalent to joints with torsional springs and rest of the mechanism is treated as a rigid body mechanism [1]. Howell and Mathilda have developed loop closure theory which uses the complex number method to model the mechanism [26]. Handley et al. have used this model to make the position control of the mechanism [27]. A linear scheme method is presented by Her and Chang for the displacement analysis of micropositioning stages which linearize the geometric constraint equations of the stages [28]. Zhang et al. developed the work and came up with the idea of constant Jacobian method for computing the kinematics of the mechanism [29]. This method has been used for the PID control of a 3-RRR flexure based mechanism [30]. Goldfrab has made the position control simulation of a compliant mechanism by using a sliding control [31]. A four bar mechanism is designed for micro/nano manipulation and a robust adaptive control methodology is applied by Liaw et al [32]. Another adaptive control has been used by Shieh and Huang to emulate the unwanted behaviors of the mechanism [33]. Chang et al. have designed a x - y - θ piezo micropositioner and used a feedback control to eliminate the hysteresis, nonlinearity and drift of piezoelectric effects [34].

In this work, instead of deriving and using a nonlinear model that mimics the behaviour of the flexure based mechanism, a control method with disturbance observer is implemented to get rid of the unwanted motions. The implemented position control of designed flexure based parallel mechanism is sliding mode control (SMC) by using end-effector measurement. The reason of why SMC is implemented is that it is a suitable high precision control method for complex high order dynamic plants having uncertainty conditions [35]. Q. Xu and Y. Li in [36] also implemented SMC control with velocity observer for their decoupled $x - y$ parallel micro positioning stage but we have used also SMC for the observer to eliminate the parasitic motions of the stage, misalignments of the actuators, errors of manufacturing and hysteresis of the system. Sliding mode observer provides the estimation of the states

which is driven by the control input and the difference between the output of the assumed plant and the output of the plant.

The mechanism is actuated using 3 piezoelectric actuators to achieve 2 DOF motion which cause redundancy. The redundancy here is used in the sense of having more actuators than the controlled end-effector motion not in the conventional meaning. We have experimentally determined the kinematics of the system which indicates the relationship between the displacement of the actuators and the end-effector displacements in x - y axes.

Firstly, open loop control for the linear model of the piezoelectric actuators is implemented to compare the results with closed loop control results. SMC with DOB is also implemented for using 2 piezoelectric actuators and the remaining piezoelectric actuator is used as a fixture support then the control method is implemented for 3 piezoelectric actuator. Finally we have compared our position control which is SMC using DOB with PID control. It is seen that our control gives better results. We have achieved to make the position control of our mechanism, which has unpredictable position errors, in the accuracy of our measurement system that is available in our laboratory.

In Section 2 the flexure based micromotion stage is introduced, the experimental setup is presented in Sec. 3. The experimentally determined workspace and the transformation matrix which connects the end-effector x - y motion with the displacement vectors coming from piezoelectric actuators are shown in Sec. 4. The modeling of the piezoelectric actuators and the control design is explained in Sec. 5. The results for open loop control, the closed loop controls for using 2 and 3 piezoelectric actuators and finally the comparison of proposed SMC with DOB and PID control are presented in Sec. 6. Finally, in Sec. 7 a conclusion have been made based on the results.

2 FLEXURE BASED MICRO MOTION STAGE DESIGN

We have used a 3 DOF parallel mechanism called 3-PRR in Fig. 1a (one prismatic – two revolute joints for each link) concept for our micromotion stage design. Right circular flexures are used as revolute joints and simple linear spring structures based on 4 bar mechanism using circular flexure hinges are used as prismatic joints as shown in Fig. 1b.

The end-effector of the mechanism is the triangular stage which connects the three links and has translational motion in x - y directions and a rotation about z -axis. The stage is actuated by piezoelectric actuators which drive the prismatic joints by creating forces F_1 , F_2 and F_3 and create the center displacements in the direction of u_1 , u_2 and u_3 vectors respectively as shown in Fig. 1b. The desired

x - y motion of the triangular stage can be generated by the combination of the “ u ” displacements.

A hexagonal case is also designed outside the mechanisms range so that it can be fixed to the experimental setup properly. The mechanism is manufactured by using wire electrical discharge machining (Wire EDM) technique by using Aluminum 7075 for the material and the shortest thickness of the flexure is 0.8 mm. The overall thickness of the mechanism in z axis is 10 mm.

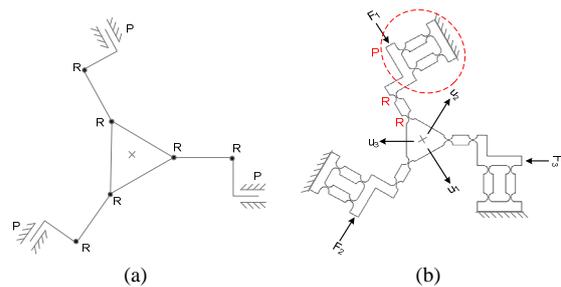


Fig. 1. Rigid 3 PRR mechanism (a), Flexure based 3 PRR mechanism (b)

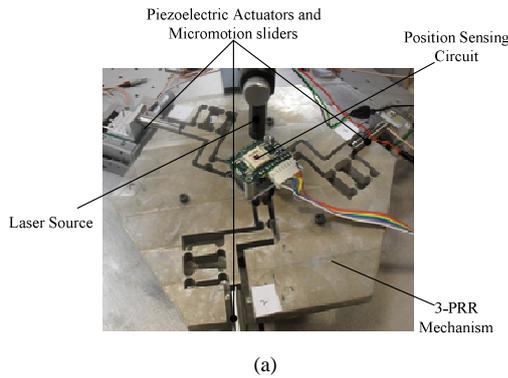
3 THE EXPERIMENTAL SETUP

The setup shown in Fig. 2 is composed of the mechanism, three piezoelectric actuators, a base table, three sliding stages with micrometers, a laser position sensor and a middle base. The used piezoelectric motor used is piezomechanik's PST 150/5/40 VS10 type which has max stroke $55 \mu\text{m}$ for semibipolar $-30 \text{ V}/+150 \text{ V}$ activation and $40 \mu\text{m}$ stroke for unipolar $0 \text{ V}/+150 \text{ V}$ activation. Piezomechanik's analog amplifier SVR 150/3 is also used for actuating the piezoelectric actuators. PI's P-853 piezoelectric micrometer drives with sliding stages are put in x and y axes according to the each PRR chain link connected the triangular stage so that we can manually preload the mechanism and drive the prismatic joints correctly. A $4\text{mm} \times 4\text{mm}$ dual axis position sensing diode on a PCB (DL 16-7PCBA3) is placed on the triangular stage's center for the displacement measurement of the center of the triangular stage using a laser source which is assembled on the top of the position sensing diode.

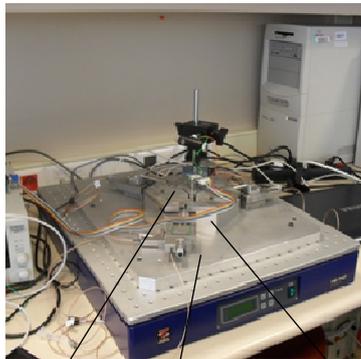
The piezo amplifiers inputs and the laser dual axis position outputs are connected to dSPACE 1103 controller board through DACs and ADCs. Control Desktop is used for CPU calculations for the controller. The schematic diagram of the connections is shown in Fig. 3.

4 THE WORKSPACE AND THE KINEMATICS OF THE MECHANISM

The workspace of 3-PRR compliant mechanism is determined by giving the maximum strokes to the piezoelec-



(a)



(b)

Fig. 2. Manufactured 3-PRR mechanism and the experimental Setup

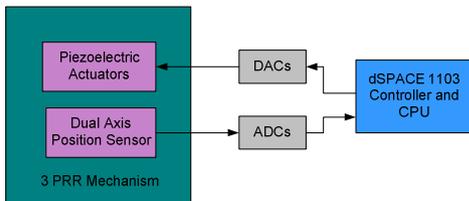


Fig. 3. Schematic diagram of the connections with CPU

tric actuators individually and to the combinations of them when all piezoelectric actuators are assembled to the mechanism. The measurements are taken by using the dual axis position sensing unit assembled to the center of the triangular stage. The output of the measurements in x and y axes are filtered by using a 2nd order filter in order to get a usable measurement data. The shape of the workspace is drawn in Fig. 4, which is a distorted hexagonal because of the misalignments of the actuators and the manufacturing errors which we should take into account for the position control of the mechanism.

T. S Smith says in his work that no matter how crude

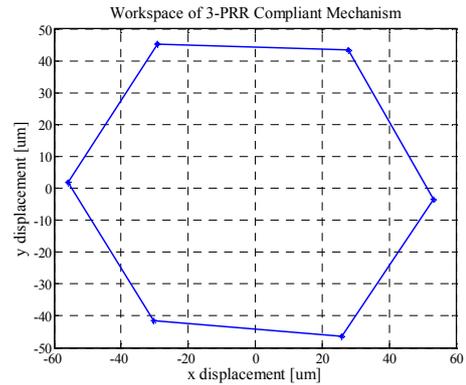


Fig. 4. The workspace of 3-PRR compliant mechanism

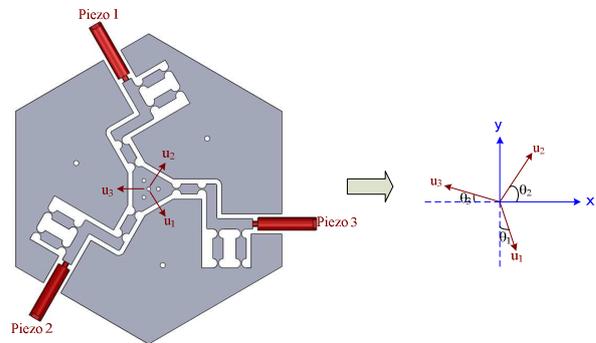


Fig. 5. The displacement vectors of the links connected to the triangular stage

the machining, the displacement characteristics of compliant mechanisms will remain linear, the axis of the motion will change [37]. We have experimentally determined the direction of the displacement vectors u_1 , u_2 and u_3 to have the kinematics of the mechanism shown in Fig. 5. After calibration of laser position sensor, we have applied respectively 30, 60, 90, 120 and 150 Volts to the piezoelectric actuators when all the piezoelectric actuators are assembled to the mechanism and preloaded before starting actuation. The x - y displacement results of the center of the stage depending on the piezoelectric actuation are presented in Fig. 6. The transformation matrix \mathbf{A} which relates the motions u_1 , u_2 and u_3 to x - y motion of the end-effector can be written as in Eqn. 1:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} \sin \theta_1 & \cos \theta_2 & -\cos \theta_3 \\ -\cos \theta_1 & \sin \theta_2 & \sin \theta_3 \end{bmatrix}}_{\mathbf{A}} \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (1)$$

The angles of the each direction of u_i ($i=1,2,3$) vectors are found as $\theta_1 = 25^\circ$, $\theta_2 = 26^\circ$ and $\theta_3 = 1.5^\circ$. The

inverse kinematics of the mechanism will be derived by using the pseudo inverse of the matrix **A**.

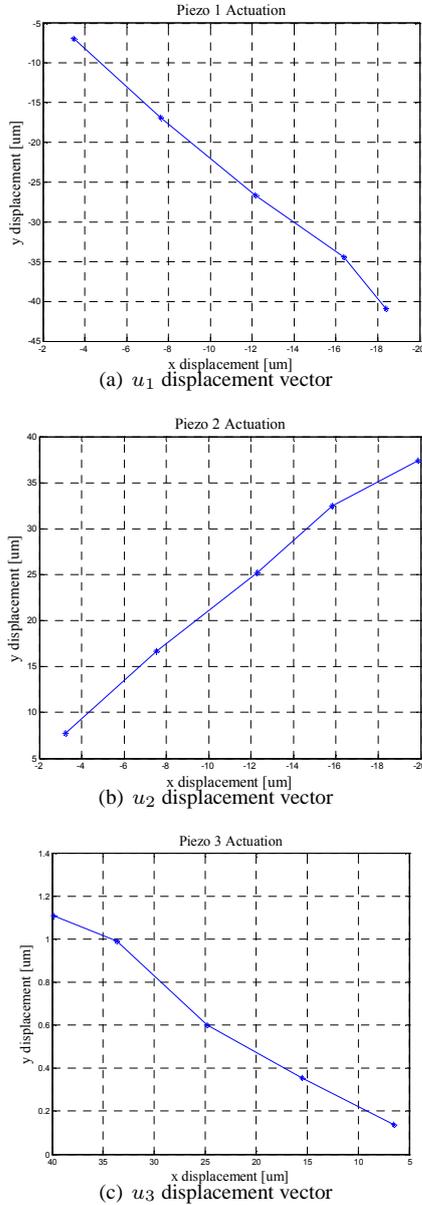


Fig. 6. x-y center displacement results for each actuation of piezoelectric actuator

5 PIEZOELECTRIC ACTUATOR MODELING AND POSITION CONTROL

The 3-PRR kinematic structure decouples the stiffness between actuators which gives the advantage of controlling the actuators separately. Therefore, we can use 3 independent single input single output (SISO) controllers for each piezoelectric actuator. First the piezoelectric actuator

model is presented then simple PID control is introduced. Finally, the proposed method based on SMC using DOB is explained in this section.

5.1 Piezoelectric Actuator Model

Piezoelectric actuators electromechanical lumped model can be defined by the equations (2)-(7) [38]. The model is shown in Fig. 7 where v is the total voltage across the actuator, v_p is the piezoelectric voltage and v_h is the hysteresis voltage. T is the electromechanical transformation ratio that connects electrical part and mechanical part of the model. q is the total charge in the actuator, q_p is the charge transduced due to mechanical motion, H is the hysteresis function that depends on q , F_p is the force of the piezoelectric effect and F_{ext} is the external force on the actuator. According to equation (7) u is the displacement, m_p , c_p and k_p are respectively the equivalent mass, damping and stiffness of the piezoelectric actuator. Finally, F_c is the control force and F_{dis} is the disturbance force.

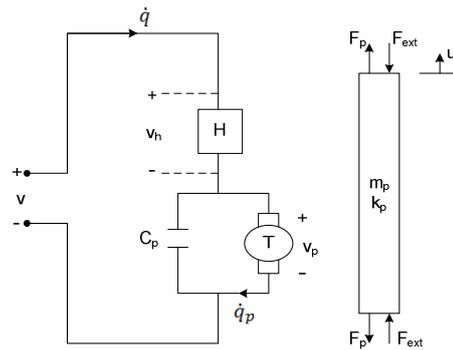


Fig. 7. Piezoelectric actuator model [38]

$$v_p = v - v_h \tag{2}$$

$$v_h = H(q) \tag{3}$$

$$q = C v_p + q_p \tag{4}$$

$$q_p = T u \tag{5}$$

$$F_p = T v_p \tag{6}$$

$$m_p \ddot{u} + c_p \dot{u} + k_p u = \underbrace{T v}_{F_c} - \underbrace{T v_h - F_{ext}}_{F_{dis}} \tag{7}$$

5.2 PID Position Control

Simple PID control is implemented for the position control of the center of the triangular stage to compare the results with the proposed control methodology based on SMC and DOB. The block diagram of the PID control is shown in Fig. 8. We have made the position control of

each piezoelectric actuator by using discretized PID controller as follows:

$$v_{ini}(t) = K_p \cdot e_i(t) + K_i \int_0^t e_i(t)dt + K_d \frac{de_i(t)}{dt} \quad (8)$$

$$e_i(t) = u_{refi}(t) - u_i(t) \quad (9)$$

where v_{ini} is the input voltage for piezoelectric actuator, e_i is the position error, K_p is the proportional control constant, K_i is the integral control constant and K_d is the derivative control constant. The subscript i determines the number of the piezoelectric actuator. The position reference is given in x - y coordinates as x_{ref} , y_{ref} and the corresponding reference positions for the actuation directions, u_{refi} ($i=1,2,3$), are calculated with the pseudo inverse of the transformation matrix which is A^\dagger . Similarly, the actuation positions u_i ($i=1,2,3$) is calculated by taking the center position results (x, y) and multiplying with A^\dagger .

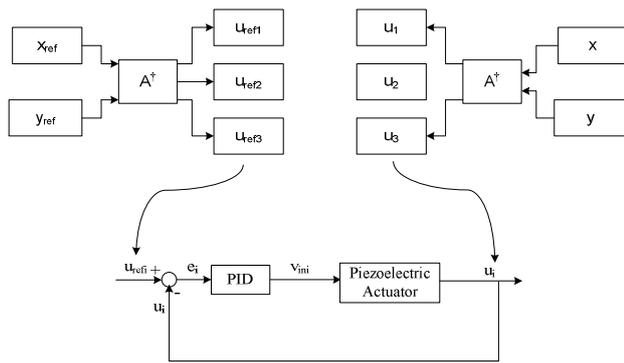


Fig. 8. Block diagram of the PID position control of compliant mechanism

5.3 Sliding Mode Control with Disturbance Observer

5.3.1 Disturbance Observer

We are able to eliminate disturbances by modeling an observer so a linear model is defined by using nominal parameters of actuator as in equation (10). The displacement u for every piezoelectric actuator can be measurable by using laser position sensor and inverse of the transformation matrix. The supply voltage is also measurable. The linear model of the piezoelectric actuator is:

$$m_n \ddot{u} + c_n \dot{u} + k_n u = T_n v - F_d \quad (10)$$

We can define F_d as hysteresis force, external force and the uncertainties of the plant parameters which are

Δm , Δc , Δk and ΔT . These parameters are assumed as bounded and continuous.

$$F_d = T_n v_h + F_{ext} + \Delta T (v_h - v) + \Delta m \ddot{u} + \Delta c \dot{u} + \Delta k u \quad (11)$$

The observer can be designed as a position tracking system in which F_d is replaced with an observer control $T_n v_{obs}$ because u and v_{in} can be measured and the observer transfer function is written as following equation.

$$m_n \ddot{\hat{u}} + c_n \dot{\hat{u}} + k_n \hat{u} = T_n v_{in} - T_n v_{obs} \quad (12)$$

\hat{u} is the estimated position, v_{in} is the plant control input, v_{obs} is the observer control input. When \hat{u} tracks u , F_d equals to $T_n v_{obs}$. A sliding manifold is selected for that purpose which is $\sigma = \dot{u} - \dot{\hat{u}} + C_{obs}(u - \hat{u})$. The Lyapunov function is taken as $v_L = \sigma^2/2$ which is positive definite and the derivative of Lyapunov function is taken as $-D_{obs}\sigma^2$ which is negative definite. We will get equation (13) by equating the above results and simplifying:

$$L = \sigma \dot{\sigma} = -D_{obs}\sigma^2 \Rightarrow \dot{\sigma} + D_{obs}\sigma = 0 \quad (13)$$

If we insert sliding mode manifold into the equation (13):

$$(\ddot{u} - \ddot{\hat{u}}) + (C_{obs} + D_{obs})(\dot{u} - \dot{\hat{u}}) + C_{obs}D_{obs}(u - \hat{u}) = 0 \quad (14)$$

When we subtract the equations (12) from (11) and insert the result into the above equation (14) we can find the equivalent control v_{eqc} which keeps system motion in manifold $\sigma + \dot{\sigma} = 0$.

$$v_{eqc} = \frac{1}{T_n} \left\{ F_d + [c_n - m_n(C_{obs} + D_{obs})](\dot{u} - \dot{\hat{u}}) + [k_n - m_n C_{obs} D_{obs}](u - \hat{u}) \right\} \quad (15)$$

Equation (15) tells us that when $\sigma \rightarrow 0$ then $u \rightarrow 0$ and $T_n v_{eqc} \rightarrow F_d$. For the implementations discrete form of sliding mode control is used as:

$$v_{(k)} = v_{(k-1)} + K_{uobs} \left(D_{obs}\sigma_{(k)} + \frac{\sigma_{(k)} - \sigma_{(k-1)}}{dT} \right) \quad (16)$$

K_{uobs} is a design parameter that optimizes the controller and dT is the sampling interval for discrete time control. The system and the observer can be summarized as in equations (17-19):

$$m_n \ddot{u} + c_n \dot{u} + k_n u = T_n v_{in} - F_d \quad (17)$$

$$m_n \ddot{\hat{u}} + c_n \dot{\hat{u}} + k_n \hat{u} = T_n v_{in} - T_n v_{obsc} \quad (18)$$

$$v_{in} = v_c + \frac{\alpha}{T_n} v_{obsc} \quad (19)$$

5.3.2 Sliding Mode Position Control

The sliding manifold is selected to be as in equation (20) and when the sliding manifold is reached the closed loop control showed in equation (21) and the system is described by equation (22).

$$\sigma_x = (\dot{u}_{ref} - \dot{u}) + C_x(u_{ref} - u) \quad (20)$$

$$v(k) = v(k-1) + K_{ux} \left(D_x \sigma_x(k) + \frac{\sigma_x(k) - \sigma_x(k-1)}{dT} \right) \quad (21)$$

$$(\ddot{u}_{ref} - \ddot{u}) + (C_x + D_x)(\dot{u}_{ref} - \dot{u}) + C_x D_x (u_{ref} - u) = 0 \quad (22)$$

Figure 9 presents our proposed control method which is the combination of the sliding mode position control with the disturbance observer based on sliding mode control for each piezoelectric actuator. The *i* subscript defines the number of piezoelectric actuator and its actuation direction.

The conversion between the actuation positions *u_i* and the center positions (x,y) are carried out like explained in Section 4 as:

$$u_i = A^\dagger [x \ y]^T \quad (23)$$

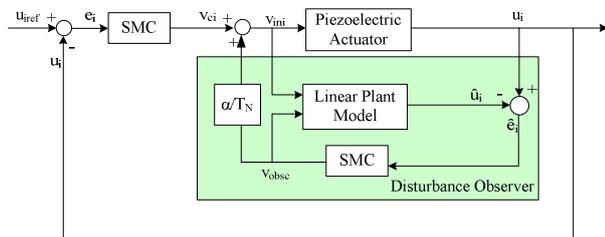


Fig. 9. Closed loop control block diagram

6 RESULTS

The experiments are done for open loop and closed loop control of the 3-PRR mechanism to compare and see clearly the effects of the observer and the sliding mode

Table 1. Parameters of piezoelectric actuator

Parameter	Quantity	Value
<i>m_p</i>	Mass	6.16×10^{-4} kg
<i>c_p</i>	Damping	1027.5 Ns/m
<i>k_p</i>	Stiffness	12×10^6 N/m
<i>T</i>	Transformation ratio	4.738 N/V

control. Figure 10 shows the experiments done for seeing the advantage of redundancy which is in the sense of having 3 actuators for the control of *x – y* motion. The red colored piezoelectric actuators are the active ones (working) and the blue one is passive (not working). First 2 piezoelectric actuators are controlled as shown in Fig. 10a then all piezoelectric actuators are controlled as shown in Fig. 10b to give an answer to the question “is it possible to achieve the same motion by using 2 and 3 actuations?”.

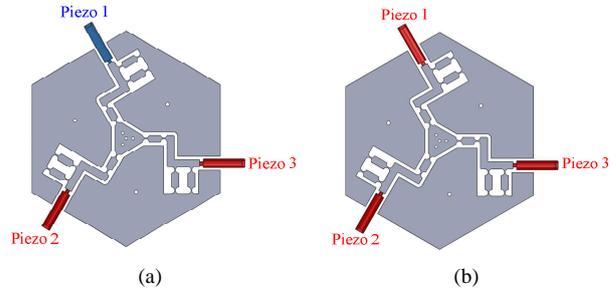


Fig. 10. 2 piezoelectric actuators are activated (a), 3 piezoelectric actuators are activated (b)

The parameters of the piezoelectric actuators are presented in Table 1. These parameters are used for nominal parameters in linear model of the piezoelectric actuators for open and closed loop experiments. A circular trajectory having 20 μm diameter of circle is given as a reference to the mechanism by setting references as $x_{ref} = 10 + 10\sin(0.2\pi t)$ and $y_{ref} = 10 + 10\cos(0.2\pi t)$ for all of the experiments. The pseudo inverse of transformation matrix **A** as in equation (1) is used for calculating the necessary references for the *u₁*, *u₂* and *u₃* displacement vectors. The control input voltages is saturated between 0 V to 150 V to use the bipolar actuation property of the piezoelectric actuators.

6.1 Open Loop Results

Inverse of the linear model of the plant is used for estimating the necessary voltage input to the piezoelectric actuators as shown in Fig. 11. The errors in *x* direction and *y* direction are shown in Fig. 12a and 12b. The *x – y* motion is presented in Fig. 12c.

It can be seen from the results that open loop control with the inverse of linear models of the piezoelectric ac-

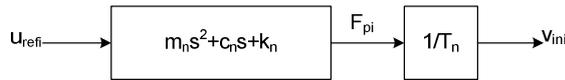


Fig. 11. Open loop control block diagram

tuators doesn't have enough accuracy for the end-effector motion of our flexure based mechanism. The error in x direction is between $3 \mu\text{m}$ and $-8 \mu\text{m}$ and the error in y direction is between $2 \mu\text{m}$ and $-8 \mu\text{m}$. Besides, when we look at the motion result we can examine that the reference trajectory is shifted and it's not in a circular shape.

6.2 Closed Loop Results

Closed loop control is applied to the mechanism for position control of the end-effector which is the center of the triangular stage. SMC with DOB shown in Fig. 9 and simple PID shown in Fig. 8 are implemented to the mechanism.

The linear model with the nominal parameters as presented in Table 1 is used for the observer with SMC to kill the hysteresis and unwanted disturbances of the system and another SMC is used for tracking the reference positions. As in open loop control the necessary reference positions of piezoelectric actuators for tracking reference $x - y$ motion is calculated by using pseudo inverse of transformation matrix \mathbf{A} . Firstly, the mechanism is actuated as a non-redundant mechanism using 2 piezoelectric actuators (the 2nd and 3rd ones according to the Fig. 4) while the other piezo (the 1st one) is attached to the mechanism as a rigid support. Then all of the actuators are controlled for the redundant case of the mechanism.

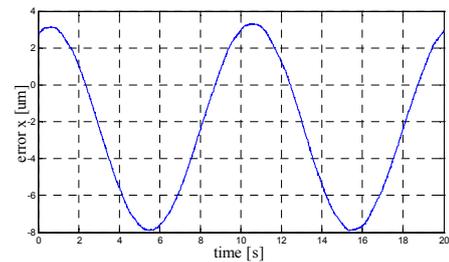
The control parameters are tuned in an heuristic way by taken into account the accuracy of our measurement which is $0.06 \mu\text{m}$ and the performance of the SMC with DOB is compared with PID control.

The sampling time is taken as $100 \mu\text{s}$ due to calculations caused by the 2nd order filter that is used for the laser position measurement.

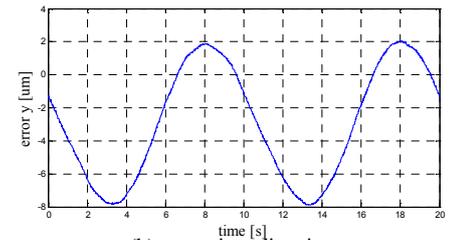
6.2.1 Non-Redundant Control Results

The control method based on SMC with DOB is implemented for the 2-piezoelectric actuators and the third one is treated like a rigid support. The control parameters are presented in Table 2.

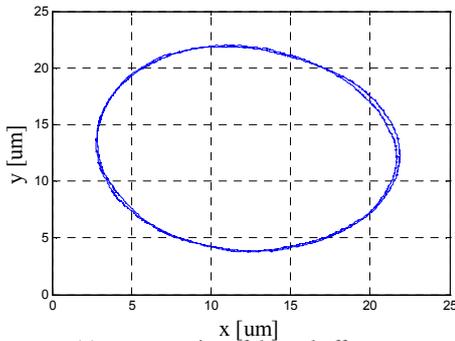
The errors in x direction as showed in Fig. 13a are between $0.05 \mu\text{m}$ and $0.55 \mu\text{m}$ and the errors in y direction shown in Fig. 13b are between $0.05 \mu\text{m}$ and $0.5 \mu\text{m}$. The measured $x - y$ motion of the end-effector is presented in



(a) errors in x direction



(b) errors in y direction



(c) $x - y$ motion of the end-effector

Fig. 12. Open loop control results

Fig. 13c. The errors are smaller than open loop control of 3 piezoelectric actuators but there is still a shift from the reference trajectory. When looking at the control outputs (piezoelectric actuator inputs) the piezoelectric actuator that creates u_2 displacement vector reaches to 150V which is the maximum stroke voltage. This means that the workspace is almost limited to a circle having $20 \mu\text{m}$.

6.2.2 Redundant Control Results

The same control method and the same parameters presented in Table 2 is implemented for 3 piezoelectric actua-

Table 2. SMC and SM observer parameters

Sliding Mode DOB Parameters		SMC for Position Parameters	
Kobs	0.000008	Kx	0.005
Cobs	10	Cx	80000
Dobs	200	Dx	0.0001

Table 3. Tuned PID, SM Observer and SMC parameters

PID Control Parameters		Sliding Mode DOB Parameters		SMC for Position Parameters	
Kp	0.005	Kobs	0.000002	Kx	0.02
Ki	0.0001	Cobs	1	Cx	40
Kd	0.0001	Dobs	50	Dx	3000

tors.

The errors in x direction is between $-0.15 \mu\text{m}$ and $0.25 \mu\text{m}$ as shown in Fig. 14a and the errors in y direction is between $0.06 \mu\text{m}$ and $0.25 \mu\text{m}$ as shown in Fig. 14b. When looking at the errors and measured $x - y$ motion results compared with the reference motion shown in Fig. 14c. The maximum control input voltages for piezoelectric actuators are respectively 90 V, 50 V and 50 V. This shows us that the voltages that are coming to the piezoelectric actuators are not close to 150V so the workspace is bigger than a circle with $20 \mu\text{m}$ diameter. So this means that the redundancy allows us to extend the workspace when compared to the results of 2 actuators results.

6.2.3 Comparison with PID Control

Finally we have made a comparison of our proposed control method shown in Fig. 9 with simple PID control shown in Fig. 8. The results are tuned so that the errors can be as much as close to the accuracy of our measurement which is $0.06 \mu\text{m}$. The tuned PID and SMC with DOB parameters are presented in Table 3.

The errors in x and y direction are shown in Figure 15a and 15b. We can extract from the figures that the tuned PID control has position errors between $0.25 \mu\text{m}$ and $-0.38 \mu\text{m}$ in x direction and between 0 and $-0.25 \mu\text{m}$ in y direction. Whereas, we have lowered those position errors using SMC with DOB control method. It can be seen in Fig. 15 that the errors in x direction is decreased to $\pm 0.12 \mu\text{m}$ while the errors in y direction are decreased between $0.17 \mu\text{m}$ and $-0.13 \mu\text{m}$.

7 CONCLUSION

A micro motion stage is designed based on 3-PRR kinematic structure to be used as a planar micromotion stage. The position control of the mechanism is analyzed experimentally without modeling the mechanism.

Firstly, the workspace and the kinematics of the mechanism are determined experimentally and it's seen that the motion of the end-effector which is the center of the stage has distorted motions due to manufacturing and assembling errors. It's clear that we need to kill those unpredictable motions so that it can be used as a micro positioning stage using our dual position sensor.

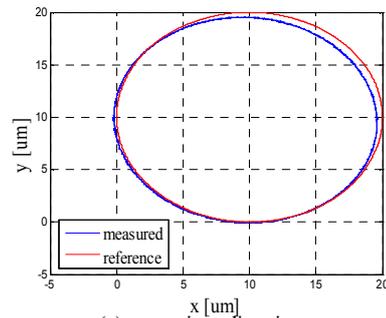
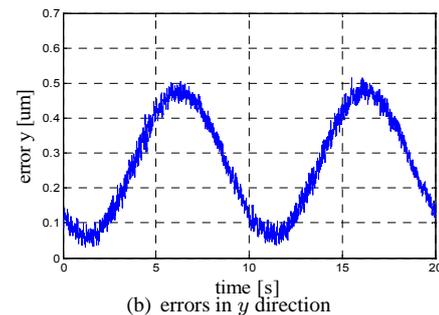
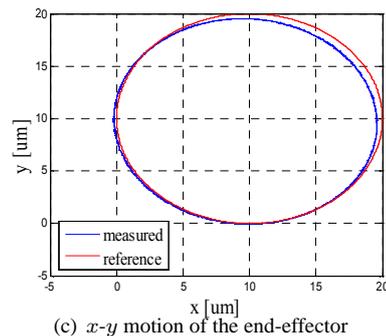
(a) errors in x direction(b) errors in y direction(c) x - y motion of the end-effector

Fig. 13. Non-redundant control results

The mechanism is designed as to be redundant to eliminate the manufacturing errors, hysteresis, assembling errors etc. and improve the workspace. SMC using DOB and PID controls are implemented for the closed loop control. In order to see the effects of closed loop control and having 3 actuators for x - y position, firstly we have implemented the open loop control and then respectively 2 piezoelectric actuators and 3 piezoelectric actuators are controlled using SMC with DOB for closed loop control. Finally we have compared the proposed control results with the simple PID control.

The proposed closed loop control eliminates the nonlinearities of the system. When we have 2 piezoelectric actuators active; the maximum position errors are in 550 nm whereas for the same control parameters when 3 piezoelectric actuators are active the maximum error is decreased to 250 nm. The control output voltages are compared and

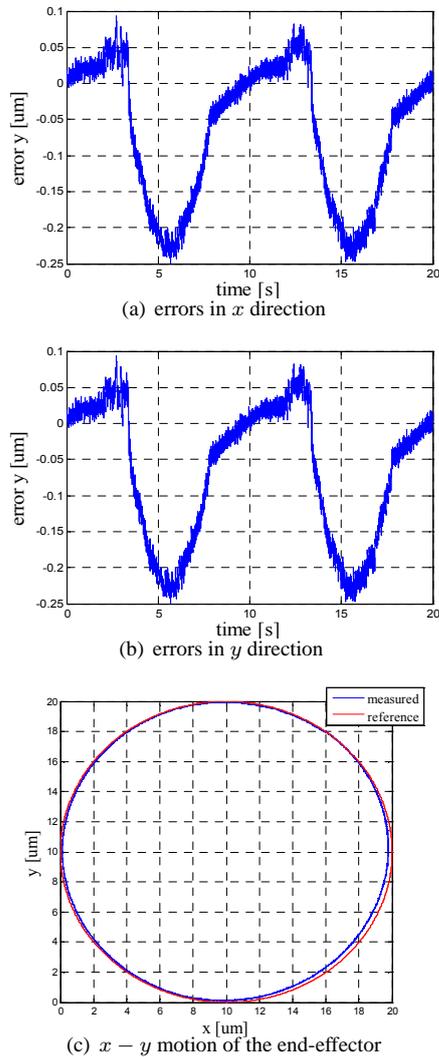


Fig. 14. Redundant control results

it's seen that our redundant mechanism provides a larger workspace because the voltages are not close to 150 V which is the limitation of piezoelectric actuators. Finally, The SMC with disturbance observer parameters and the PID control methods are implemented for 3-PRR compliant mechanism with tuned parameters and it's clearly seen that our proposed control gives better results.

Shortly, the experimental results tells us that making our mechanism redundant gives us better results for tracking the reference motion and the work space of the redundant mechanism is bigger. We have also clearly seen that our SMC with a sliding mode DOB is also a good choice for position control of micro motion stages although a mathematical model of the full mechanism is not extracted.

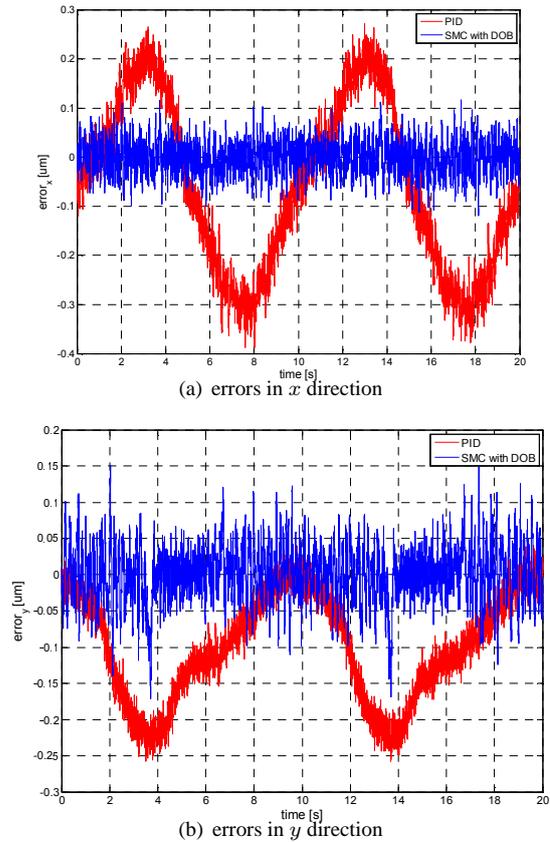


Fig. 15. Comparison of PID and SMC with DOB results

For the future work, the mechanism will be modeled by using finite element analysis and the results will be compared with the experimental results to see the erroneous motion. It's thought that this will lead to develop our position control strategy for the compliant micro positioners.

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