

IMPROVING STRUCTURE DYNAMIC BEHAVIOUR USING A REANALYSIS PROCEDURES TECHNIQUE

Ezedine Allaboudi, Tasko Maneski, Natasa Trisovic, Todor Ergić

Original scientific paper

The present paper deals with the problem of improving dynamic characteristics for a sub-structure of bucket wheel excavator. The procedure is concerned with the analysis of the distribution of potential and kinetic energy in elements of the structure, which gives prediction for which elements reanalysis is needed. Reanalysis technique can be done for the structure using finite element methods. Information like material, geometry and boundary conditions should be prepared before making FE model. The main aim of dynamic modification is to increase natural frequencies and to increase the difference between them.

Keywords: *dynamics behaviour, FEM, kinetic energy, potential energy, reanalysis*

Poboljšanje dinamičkog ponašanja konstrukcija primjenom reanalize

Izvorni znanstveni članak

Ovaj rad se bavi problemom poboljšanja dinamičkih karakteristika podstrukture rotornih bagera. Postupak je zasnovan na analizi raspodjele potencijalne i kinetičke energije u elementima strukture, koja daje predviđanja za koje elemente je potrebna reanaliza. Reanaliza predstavlja skup metoda kod kojih je osnova metoda konačnih elemenata. Informacije kao što su vrsta materijala, geometrija i granični uvjeti trebaju biti pripremljeni prije formiranja modela konačnih elemenata. Glavni cilj dinamičke modifikacije je povećanje osnovnih frekvencija i povećanje razlike između njih.

Ključne riječi: *dinamičko ponašanje, kinetička energija, MKE, potencijalna energija, reanaliza*

1 Introduction

Nowadays structure design requirements have broad definitions because of high technology industry. For example, the development of materials with superior properties in exploitation conditions leads to extend the design requirements to involve structural integrity, reliability and life specification, in order to increase the life of structure. Structures which have a complicated design require massive efforts in analysing and diagnosing the defects. Thus, one should deal carefully with the factors affecting the structure. The external load is one of the important factors that have big influence on the structure and its response. Moreover, in the static analysis, strength and deformation of structure are governed by the value of the external load. Therefore, the strength and deformation should be always under control in the case of static load. Although the static analysis is very important, the complete significant solution requires a dynamic analysis to reach the best results especially when the structure is subjected to the dynamic load or under high revolution rates such as complex manufacturing systems in mines and power plants, aircrafts, ground vehicles, rail-road vehicles, etc.

Dynamic analysis is more complex than static analysis, and the design requirements must include dynamic properties such as vibration level, resonance range, response properties, eigenvalues, dynamic stability and modal forms [1]. To avoid dynamic problems, some modification will be done for structure in the process of reanalysis. Reanalysis is a technique through which the dynamic response of the structure is improved. Finite element method is a powerful method to perform these processes using simple procedures. Modelling of complex structures using finite elements method is a helpful approach in solving problems in short time with reliable results [2].

2 Reanalysis procedure

The procedure of reanalysis depends on the concept of energy distribution through the structure. Study of the energy distribution leads to finding out the right place and it will be conducted by some modifications to improve the eigenvalues of the structure. Therefore, determination of distribution of kinetic and potential energies on the elements of the whole structure is the main step in the reanalysis procedure. Complex structures need several steps during the analysis to reach the most accurate results, starting with initial rough analysis of a structure which is followed by the precise analysis based on the sensitivity of each element of the structure. The improvement of dynamic characteristics, during the reanalysis steps can be achieved by making some adjustment to the structure such as geometrical modifications, material properties and boundary conditions. The process of analysis is done using a computer program, based on using the finite element methods and the implementation of structure energy distributions. The distributions of potential and kinetic energies of elements of the whole structure give a clear view to the problem, which helps to make appropriate decision for structure modifications. The decision on the final modification can be made according to the structure dynamic behaviour during reanalysis steps and its obtained results. Several studies have been addressed to the subject of modal reanalysis and structure dynamic modifications. Paper [3] presented the basic theory to find out a solution existence for the structure optimization with frequency constraints. Based on this theory, natural frequencies do not change with uniform frame modification and key limitation for determination of optimal dynamic solution of frame structure modification is mostly that of eigenfrequencies. The optimization criterion for space frame structure with multiple

limitations in its natural frequencies is considered in [4]. Knott coordinates and cross sections of elements, although of different nature, have been treated simultaneously in unified design specification for the minimum weight of the structure. Optimal first criterion, developed for one limitation based on differentiation of Lagrange function, indicates that at the optimum all the variables are of the same efficiency. In order to solve multiple limitations of frequencies global numbers are introduced, avoiding in this way the calculation of Lagrange's multipliers. In final stage, the most efficient variables are identified and modified as priority. Using the minimal weight increment, optimal solution can be obtained from initial design solution. The procedure is also effective for repeated values of frequency. Based on reduced appreciative concept of improved method for approximation of eigenvalues and eigenvectors of first order, the model for modified dynamic structural system is presented in [5]. The expressions for local approximation based on Taylor's series are used as base vectors for eigenparameters perturbation approximation. Reduced system of eigenvalues is generated for each eigenvector using eigenvectors as a base and Ritz's vector approximation of first order. The equations for reanalysis are algebraic [6]. Paper [7, 14] introduced a new function to limit eigenvalues approximation in the procedure of structural optimization. The applied Rayleigh's ratio increases approximation quality for frequency limitations since it approximates eigenforms energy and kinetic energy instead of eigenvalues, producing faster and stable convergent solutions. The discussion of application of iterative method for design sensitivity analysis, including reanalysis structure procedure due to small perturbances of design variables is applied in numerical procedure, is presented in [8]. Simple algorithms with fast convergence to calculate displacements and stresses are given, as well as for eigenvalues and forms. The extension of the method to the sensitivity of eigenfrequencies with repeated values is convenient to avoid the conditions of matrix coefficients close to bifurcation points, which occurs when non-linear response of a structure is considered. An optimization procedure was presented in [9, 13, 15, 16] for the minimum weight optimization with discrete design variables for truss structures subjected to constraints on stresses, natural frequencies and frequency responses. Paper [10] presented the problem of determining the optimal joint positions and cross-sectional parameters of linearly elastic space frames with imposed stress and free frequency constraints. The sensitivity analysis of distinct as well as multiple frequencies was performed through analytical differentiation with respect to design parameters. The efficiency and accuracy of the optimization method was demonstrated through numerical study of space frames. The paper pointed out that the structure response is much more sensitive with respect to joint positions variation, and more effective designs can be generated by optimizing both shape and size parameters.

The main point of improving dynamic behaviour of a structure is increasing its natural frequencies and maximizing the interval between adjacent natural frequencies. This request, as previously mentioned, can be achieved by changing the design parameters of the

structure. The procedure used in this paper is concerned with distribution of potential and kinetic energy in all elements of structure. Calculations of main modes of oscillation were performed using Abacus software [17, 16], while those of the energy distributions using KOMIPS software [19]. Structure has a good dynamic behaviour when its first eigenvalue is high and the interval between adjacent eigenvalues is large.

3 Theoretical consideration

3.1 Potential and kinetic energy distribution over the principal modes of oscillation

For the system with no damping and no external force, the equation of motion in the matrix form is:

$$[M] \cdot \{\ddot{Q}(t)\} + [K] \cdot \{Q(t)\} = \{0\}. \tag{1}$$

Then, the eigenvalues of the previous differential equation for the r -th mode can be expressed as:

$$[K] \cdot \{Q_r\} - \lambda_r [M] \cdot \{Q_r\} = \{0\}. \tag{2}$$

Where λ_r - is the r -th eigenvalue, and Q_r - is the r -th eigenvector for the structure.

Now, by multiplying the left side of equation (2) by transposed value of r -th eigenvector and divided by 2 one can get:

$$\frac{1}{2} \{Q_r\}^T [K] \{Q_r\} = \frac{1}{2} \lambda_r \{Q_r\}^T [M] \{Q_r\}. \tag{3}$$

Eq. (3) is the balance equation of potential and kinetic energy for a structure in main modes of oscillation. Furthermore, the potential energy of a structure on the r -th main oscillation mode, having in mind the previous equation, can be rewritten as:

$$E_{p,r} = \frac{1}{2} \{Q_r\}^T [K] \{Q_r\}. \tag{4}$$

In the same way, the kinetic energy is:

$$E_{k,r} = \frac{1}{2} \lambda_r \{Q_r\}^T [M] \{Q_r\}. \tag{5}$$

Theoretically, the total energy conservation on main oscillation modes is:

$$E_{p,r} = E_{k,r} = E_r. \tag{6}$$

The kinetic and potential energy of the structure on the r -th main oscillation mode is the sum energy of all elements structure modelling and can be represented as:

$$E_{k,r} = \sum_{i=1}^N (e_{k,r})_i = \frac{1}{2} \sum_{i=1}^N \omega_r^2 \{q_r^s\}_i^T [m]_i \{q_r^s\}_i, \tag{7}$$

$$E_{p,r} = \sum_{i=1}^N (e_{p,r})_i = \frac{1}{2} \sum_{i=1}^N \{q_r^s\}_i^T [k]_i \{q_r^s\}_i.$$

Where are:

$(e_{p,r})_i = \frac{1}{2} \{q_r^s\}_i^T [k]_i \{q_r^s\}_i$ - potential energy of the i -th element on its r -th main oscillation mode,

$(e_{k,r})_i = \frac{1}{2} \omega_r^2 \{q_r^s\}_i^T [m]_i \{q_r^s\}_i$ - kinetic energy of the i -th element on the r -th main oscillation mode,

$\{q_r^s\}_i$ - is the corresponding r -th eigenvector, of the i -th element with s degrees of freedom.

Consequently, the dynamic analysis can be done according to the difference between potential and kinetic energy distribution ($e_p - e_k$) through all structure's elements.

3.2 Modification of dynamic parameters

For free vibration case the modified system can be described by a modified equation (perturbation equation) as:

$$[K] \{Q_r\}' = \lambda_i' [M] \{Q_r\}' \quad (8)$$

By introducing $[\Delta K]$ and $[\Delta M]$ are obtained the corresponding changes in stiffness and mass matrices respectively. Then,

$$[K]' = [K] + [\Delta K], \quad [M]' = [M] + [\Delta M] \\ \{Q_r\}' = \{Q_r\} + \{\Delta Q_r\}, \quad \lambda_i' = \lambda_i + \Delta \lambda_i. \quad (9)$$

Where: $\Delta \lambda$ and $\{\Delta Q_r\}$ are changes of eigenvalues and eigenvectors, respectively.

For a modified system equation (2) can be rewritten as:

$$([K] + [\Delta K])(\{Q_r\} + \{\Delta Q_r\}) = (\lambda_r + \Delta \lambda_r)([M] + [\Delta M])(\{Q_r\} + \{\Delta Q_r\}). \quad (10)$$

In the same manner, the balanced equation of potential and kinetic energy (3) can be rewritten in its perturbed form as:

$$(\{Q_r\} + \{\Delta Q_r\})^T ([K] + [\Delta K])(\{Q_r\} + \{\Delta Q_r\}) = (\lambda_r + \Delta \lambda_r)(\{Q_r\} + \{\Delta Q_r\})^T ([M] + [\Delta M])(\{Q_r\} + \{\Delta Q_r\}). \quad (11)$$

After some manipulations and neglecting the higher order terms [12], the change of the i -th eigenvalue under system modification can be expressed as:

$$\frac{\Delta \lambda_r}{\lambda_r} = \frac{\frac{1}{2} \{Q_r\}'^T [\Delta K] \{Q_r\}' - \frac{1}{2} \lambda_r' \{Q_r\}'^T [\Delta M] \{Q_r\}'}{\frac{1}{2} \lambda_r' \{Q_r\}'^T [M] \{Q_r\}'}. \quad (12)$$

Eq. (12) can be considered as a basic formula for reanalysis procedure to improve structure dynamic characteristics.

Furthermore, the next formula can be used for the unmodified system:

$$\frac{\Delta \lambda_r}{\lambda_r} = \frac{\frac{1}{2} \{Q_r\}'^T [\Delta K] \{Q_r\}' - \frac{1}{2} \lambda_r \{Q_r\}'^T [\Delta M] \{Q_r\}}{\frac{1}{2} \lambda_r \{Q_r\}'^T [M] \{Q_r\}}. \quad (13)$$

The denominator in equation (13) represents the kinetic energy of a certain oscillation mode and having in mind equation (3), it also represents the potential energy, for reasons of energy balance in the main oscillation modes.

The stiffness and mass matrices after the modification is done in the i -th finite element can be expressed as:

$$[k]_i' = [k]_i + [\Delta k]_i = [k]_i + \alpha_i [k]_i, \\ [m]_i' = [m]_i + [\Delta m]_i = [m]_i + \beta_i [m]_i. \quad (14)$$

Where α_i and β_i are values that define the modification of the i -th element: $[\Delta k]_i = \alpha_i [k]_i$, $[\Delta m]_i = \beta_i [m]_i$ and are called *modification coefficients* which represent changing of design parameters (example: thickness of the plate element). In this case, the members of stiffness matrices and mass matrices within the matrices of construction parameters are all equal to zero except for those corresponding to the i -th finite element, so that the nominator in equation (13) for the r -th oscillation mode becomes

$$\frac{1}{2} \{Q_r\}'^T [\Delta K] \{Q_r\}' - \frac{1}{2} \lambda_r \{Q_r\}'^T [\Delta M] \{Q_r\} = \\ = \frac{1}{2} \alpha_i \{q_r^s\}_i^T [k]_i \{q_r^s\}_i - \frac{1}{2} \beta_i \lambda_r \{q_r^s\}_i^T [m]_i \{q_r^s\}_i = \\ = \frac{1}{2} (\alpha_i e_{p,r} - \beta_i e_{k,r}). \quad (15)$$

Where: $\{q_r^s\}_i$ - is the corresponding r -th eigenvector of the i -th element with s degrees of freedom,

$e_{p,r} = \frac{1}{2} \{q_r^s\}_i^T [k]_i \{q_r^s\}_i$ is the potential energy of the i -th element in the r -th main oscillation mode without constructional modification, and

$e_{k,r} = \frac{1}{2} \omega_r^2 \{q_r^s\}_i^T [m]_i \{q_r^s\}_i$ is the kinetic energy of the i -th element in the r -th main oscillation mode without constructional modification. Consequently, equation (13) can be written as:

$$\frac{\Delta \lambda_r}{\lambda_r} = \frac{\frac{1}{2} \{Q_r\}'^T [\Delta K] \{Q_r\}' - \frac{1}{2} \lambda_r \{Q_r\}'^T [\Delta M] \{Q_r\}}{\frac{1}{2} \lambda_r \{Q_r\}'^T [M] \{Q_r\}} = \\ = \frac{\alpha_i e_{p,r} - \beta_i e_{k,r}}{E_{k,r}}. \quad (16)$$

The previous equation has an important definition to understand the procedures of reanalysis and to define the position of elements that require modifications to improve the dynamic behaviour of the structure. Because the

denominator has the same value, the numerator is the main interest of analysis. Therefore, the natural frequency of the structure increases or decreases according to the values of α_i and β_i . When α_i has a positive value, hence increased rigidity, the natural frequency is increased. When α_i has negative values, hence decreased rigidity, the natural frequency is decreased. On the other hand, when β_i has a positive value, hence increased mass, the natural frequency is decreased. When β_i has negative values, hence decreased mass, the natural frequency is increased. Consequently, the modification (increase/decrease structure rigidity or mass) which will be done for the structure depends on the sign value of numerator in equation (16). The main point of improving dynamic behaviour of the structure is increasing its natural frequencies and maximizing the interval between adjacent natural frequencies. Hence, study of energy distribution will be done for each element in the structure to determine places of modification.

4 Developed procedure of dynamic modification

Structural Dynamics Modification is a very effective technique to improve a structure's dynamic characteristics such as natural frequency, mode shape and frequency response functions. Although this topic has been widely studied in the previous decades, the methodology of modification (reanalysis) of constructions is still under intense development. The dynamic behaviour of the structure can be improved by predicting the modified behaviour making some modification parts like rigid links, beams, lumped masses, dampers etc. The present paper shows Structural Dynamics Modification procedures that can be successfully applied for all types of constructions. These procedures have been applied on a complex real problem to improve dynamic response of the structure. The obtained results, by applying reanalysis procedure for the structure under study, achieve the purpose of this analysis.

4.1 Dynamic analysis and diagnostics of model and its groups

The procedure which is used in this paper is concerned with distribution of potential and kinetic energy in all elements of the structure which gives predictions for reanalysis.

The procedure used in this paper is developed in PHD thesis [11] and it is shown shortly in the rest of this paper [11].

The following cases should be considered for reanalysis of similar constructions:

- a) Elements in which the kinetic and potential energies (and the difference in their increase) are negligible with respect to other elements.
- b) Elements in which the kinetic energy is dominant compared to potential energy.
- c) Elements in which the potential energy is dominant compared to kinetic energy.
- d) Elements in which the potential and kinetic energy exist and are not negligible in comparison with other elements.

4.1.1 Reanalysis algorithm

The following algorithm is established based on the previous analysis as illustrated in the following steps:

Step 1: The observed structure is divided into appropriate number of finite elements for which kinetic

$$e_{k,r} = \frac{1}{2} \omega_r^2 \{q_r^s\}^T [m] \{q_r^s\} \text{ and potential}$$

$$e_{p,r} = \frac{1}{2} \{q_r^s\}^T [k] \{q_r^s\} \text{ energies are calculated}$$

separately, on those main modes which are of interest in the analysis.

Step 2: Comparing the values of potential and kinetic energy over zones or elements, as well as corresponding energy differences, based on which the following courses of analysis are formed:

Step 3: In elements for which is valid:

$e_{p,r} \rightarrow 0, e_{k,r} \rightarrow 0$, there are no possibilities for successful modifications with respect to increasing eigenfrequencies. These elements do not have significant effect on dynamic behaviour of structure, but they might be suitable for other types of optimizations. In general, reducing the mass of those elements lightens the weight of the whole structure without endangering its dynamical behaviour.

Step 4: For those elements where $e_{k,r} \gg e_{p,r}$, eigenvalues can be increased by increasing the stiffness of structure. The modifications to increase these values are not arbitrary, but they are done according to the principle of energy distributions through the elements of structure.

Step 5: For those elements where $e_{k,r} \gg e_{p,r}$, eigenvalues can be increased by decreasing the mass of structure. Also, this operation can be done based on distribution of energy through the elements of structure. According to many criteria, decreasing of mass is a generally desired type of modification.

Step 6: Most often, elements appear in structure for which the values of $e_{k,r}, e_{p,r}$ are not negligible. Therefore, the situation is more complex and those elements are suitable for reanalysis. In this case, the reanalysis of structure is done based on the differences in increases of potential and kinetic energy $\Delta e_{p,r} - \Delta e_{k,r}$ between modified and original system. The modification parameters α and β are independently calculated for each element. It has been shown that modification parameters depend on type of cross sectional area, type of material used, and boundary conditions. Reanalysis formula can be applied to achieve the purpose of increase eigenvalues.

$$\Delta \lambda_1 = \frac{\sum_{j=1}^N (\alpha_j e_{p,j}^{(1)} - \beta_j e_{k,j}^{(1)})}{\{Q_1\}^T [M] \{Q_1\}} \tag{17}$$

Step 7: When the desired value of increase is achieved, it is possible to conduct the check of modified structure by running the software based on the finite element analysis, with modified parameters. Then, the evaluation of modified structure can be obtained based on new energy distribution schemes. If the difference of energy increase on the redesigned places is less than the previous that

means that the procedure converges, and vice versa. Convergence is the goal of every optimization procedure.

5 Case study

Bucket wheel excavators are complex systems, with numerous functionally important components. This wheel excavator is working in cement factory BFC Lafarge Beocin. In this paper the diagnostic of dynamic behaviour of the bogie rotary excavator has been done in order to achieve the appropriate reconstruction. Fig. 1 shows the first exact model which is the model of the original

structure. Calculations of main modes of oscillation were performed using Abaqus [18] while the energy distributions using KOMIPS [19]. In this analysis plate finite elements are used.

This study consists of seven models for structure reanalysis. Model 1 is referred to the original structure. Fig. 2 shows the obtained results for the first mode of oscillation of this model (bending). Potential and kinetic energies have been calculated using Equations (4) and (5) and the differences in increment were determined, as presented in Fig. 2.

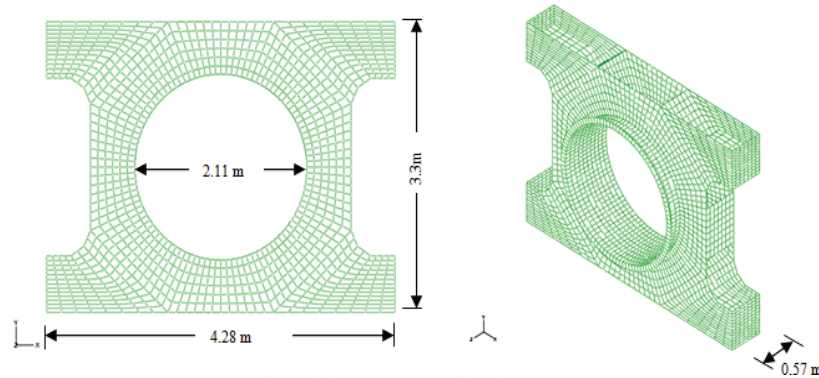


Figure 1 The existing structure of bogie rotary excavator

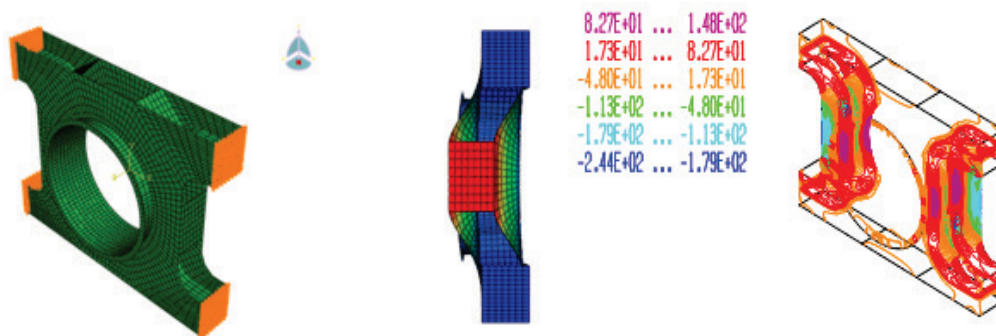


Figure 2 FEM of model 1. The first frequency is $f_{01} = 63,132$ Hz. Difference between potential and kinetic energy (N·m)

Model 2 represents the first proposed modifications for the structure. The additional materials were added around the hole in the centre. Fig. 3 shows the obtained results of this model. Based on the distribution of energy through the structure, it can be noticed that the zones which have positive values in the difference between potential and kinetic energy (red and violet colours) require increasing in the stiffness. Therefore, the stiffness

of the structure was increased in model 3 (Fig. 4) by increasing the distance between the upper and lower plates. According to the obtained results of model 3, it is clear that the dynamic behaviour of the structure has been improved, where the value of the first frequency for this model is 92,993 Hz while the first frequency for model 1 was 63,132 Hz.

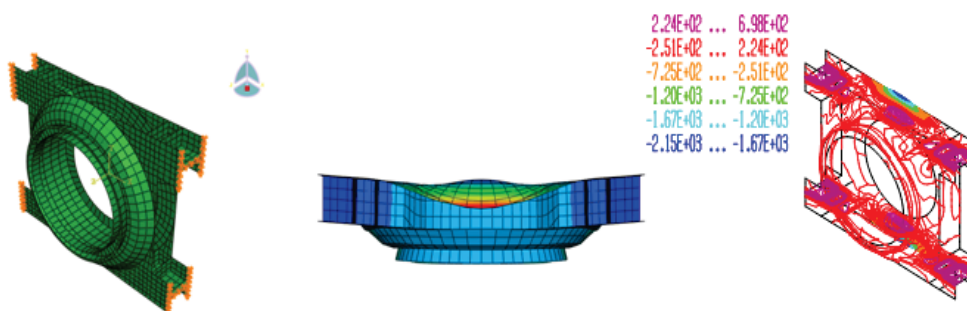


Figure 3 FEM of model 2. The first frequency is $f_{01} = 88,975$ Hz. Difference between potential and kinetic energy (N·m)

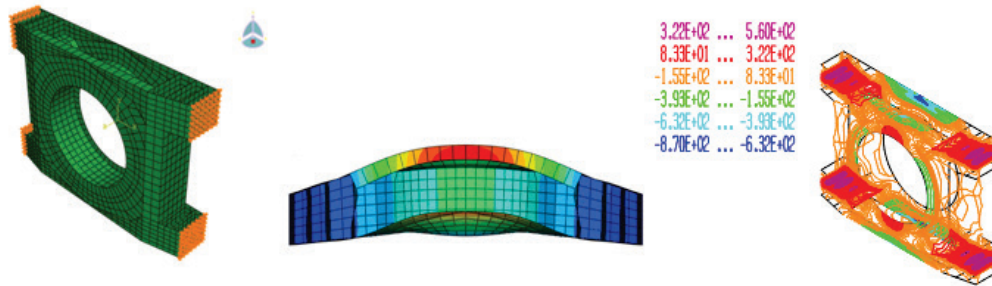


Figure 4 FEM of model 3. The first frequency is $f_{01}= 92,993$ Hz. Difference between potential and kinetic energy (N·m).

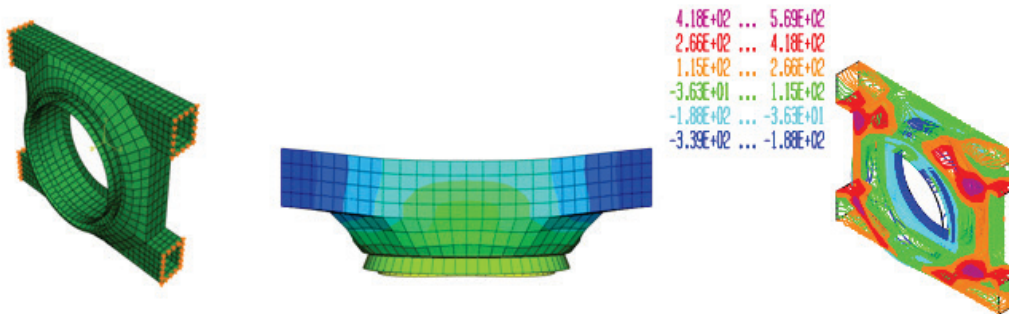


Figure 5 FEM of model 4. The first frequency is $f_{01}= 101,88$ Hz. Difference between potential and kinetic energy (N·m).

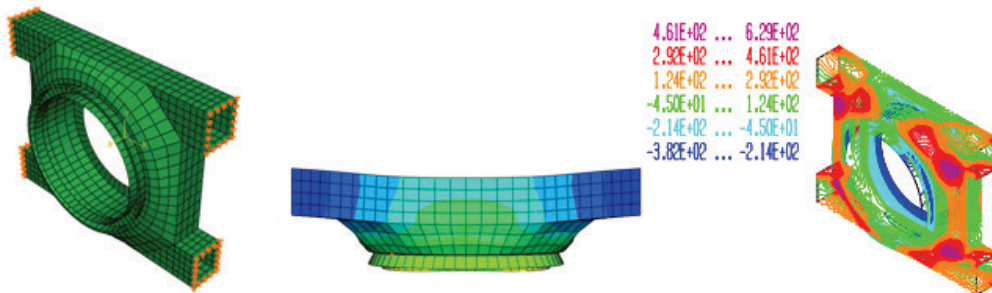


Figure 6 FEM of model 5. The first frequency is $f_{01}= 107,4$ Hz. Difference between potential and kinetic energy (N·m).

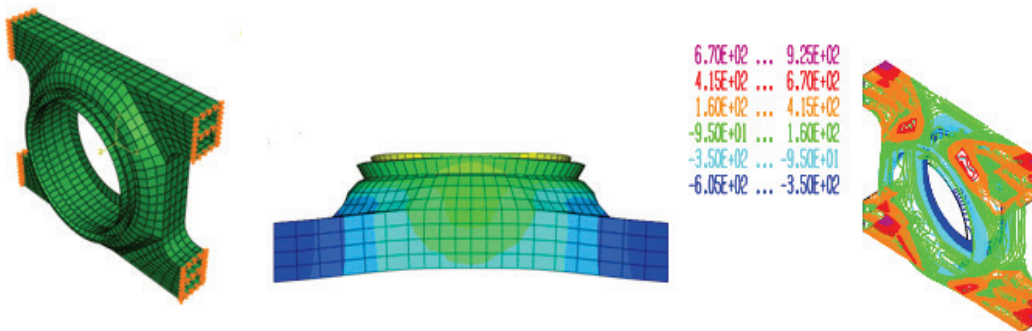


Figure 7 FEM of model 6. The first frequency is $f_{01}= 129,42$ Hz. Difference between potential and kinetic energy (N·m).

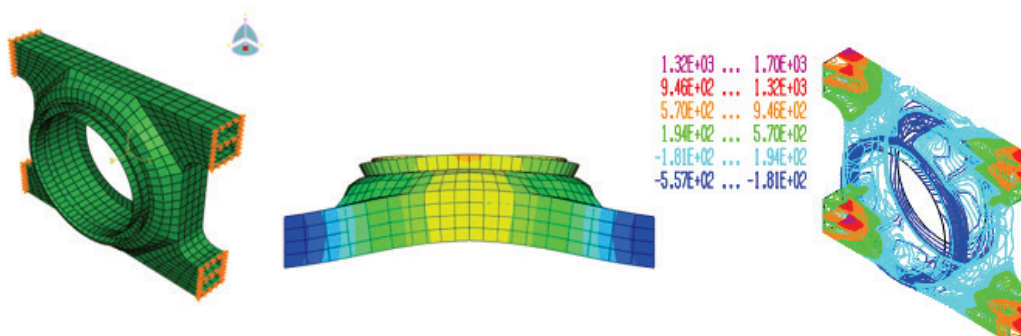


Figure 8 FEM of model 7. The first frequency is $f_{01}= 143,72$ Hz. Difference between potential and kinetic energy (N·m).

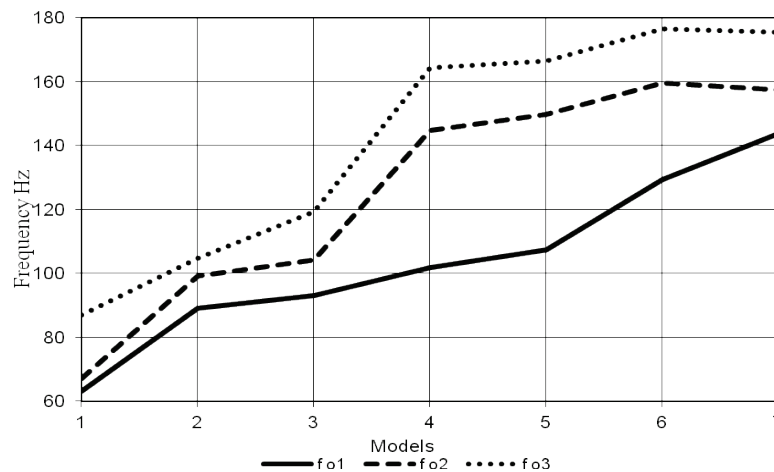


Figure 9 Comparison between models considering the differences between adjacent frequencies

To get better results some modifications have been done to the structure, where both sides of structure were covered by additional plates. Figs. 5, 6 and 7 show the effect of these modifications on models 4, 5 and 6.

Model 7 is the final proposed modification model for the structure. The additional stiffeners have been added to both sides of the Bucket wheel excavator as shown in Fig. 8. This model has the best results compared with other previous models. Fig. 8 shows the obtained results of this model. The first frequency of this model is $f_{01} = 143,72$ Hz which is considered a higher value in all models.

Although the height of first frequency is a good criterion for improving the structure's behaviour, the difference between frequencies is also a very important factor as mentioned before. Therefore, in order to observe the difference between adjacent frequencies, the first three frequencies have been determined for all models. The comparison between all models is shown in Fig. 9.

6 Conclusion

Distribution of potential and kinetic energy in main oscillation modes is the base methodology for improving dynamic behaviour of structure using reanalysis procedures technique. Study of distribution of potential and kinetic energy of structure gives obvious prediction which elements need some modifications to achieve the best dynamic characteristics. The main point of improving dynamic behaviour of a structure is increasing its natural frequencies and maximizing the interval between adjacent natural frequencies.

The algorithm of reanalysis has the following aspects:

- Elements in which the kinetic and potential energies (and the difference in their increase) are negligible with respect to other elements.
- Elements in which the kinetic energy is dominant compared to potential energy.
- Elements in which the potential energy is dominant compared to kinetic energy.
- Elements in which the potential and kinetic energy exist and are not negligible in comparison with other elements.

According to the results obtained from the dynamic behaviour of the bogie rotary excavator after the

modifications have been done on the base structure, it can be clearly concluded that the study of distribution of potential and kinetic energy gives a clear definition for interest zones and elements for modifications.

The new solution of structure increases the first main mode about 2,2 times of the original structure. As a result, the improving of the structure's dynamic behaviour was achieved.

Acknowledgment

This work has been performed within the projects TR35040, TR35011 and ON74001 which are supported by the Ministry of Education and Science of the Republic of Serbia, whose financial help is gratefully acknowledged.

7 References

- Raczynski, S. Uncertainty, Dualism and Inverse Reachable Sets. // *International Journal of Simulation Modelling*, 10, 1(2011), pp. 38-45.
- Contuzzi, N.; Campanelli, S. L.; Ludovico, A. D. 3D Finite Element Analysis in the Selective Laser Melting Process. // *International Journal of Simulation Modelling*, 10, 3(2011), pp. 113-121.
- Tong, W. H.; Jiang, J. S.; Liu, G. R. Solution Existence of the Optimization Problem of Truss Structure with Frequency Constraints. // *International Journal of Solids and Structures*, 37, 30(2000), pp. 4043-4060.
- Wang, D.; Zhang, W. H.; Jiang, J. S. Truss Optimization on Shape and Sizing with Frequency Constraints. // *AIAA Journal*, 42, 3(2004), pp. 622-630.
- Nair, B. P.; Keane, A. J.; Langley, R. S. Improved First-Order Approximation of Eigenvalues and Eigenvectors. // *AIAA Journal*, 36, 9(1998), pp. 1722-1727.
- Wang, B. P.; Pilkey, W. D. Eigenvalue Reanalysis of Locally Modified Structures Using a Generalized Rayleigh's Method. // *AIAA Journal*, 24, 6(1986), pp. 983-990.
- Canfield, R. A. High-Quality Approximation of Eigenvalues in Structural Optimization. // *AIAA Journal*, 28, 6(1990), pp. 1116-1122.
- Yoon, B. G.; Belegundu, A. D. Iterative Methods for Design Sensitivity Analysis. // *AIAA Journal*, 26, 11(1988), pp. 1413-1417.
- Tong, W. H.; Liu, G. R. An Optimization Procedure for Truss Structure with Discrete Design Variables and

- Dynamic Constraints. // Computers and Structures, 79, 2(2001), pp. 155-162.
- [10] Sergejev, O.; Mroz, Z. Sensitivity Analysis and Optimal Design of 3D Frame Structures for Stress and Frequency Constraints. // Computers and Structures, 75, 2(2000), pp. 167-185.
- [11] Trisovic, R. N. Modification of the Dynamics Characteristics in the Structural Dynamic Reanalysis, PhD, Thesis, University of Belgrade, Serbia, 2007.
- [12] Ki, I. K. Nonlinear Inverse Perturbation Method in Dynamic Redesign, PhD, Thesis, Michigan University, USA, 1983.
- [13] Sedaghati, R.; Suleman, A.; Tabarrok, B.; Structural Optimization with Frequency Constraints Using the Finite Element Force Method. // AIAA Journal, 40, 2(2002), pp. 382-388.
- [14] Mateus, H. C.; Mota Soares, C. M.; Mota Soares, C. A. Sensitivity Analysis and Optimal Design of Thin Laminated Composite Structures. // Computers and Structures, 41, 3(1991), pp. 501-508.
- [15] Gaćeša, B.; Milošević-Mitić, V.; Maneski, T.; Kozak, D.; Sertić, J. Numerical and Experimental Strength Analysis of Fire-Tube Boiler Construction. // Tehnicki vjesnik - Technical Gazette 18, 2(2011), pp. 237-242.
- [16] Vladić, J.; Đokić, R.; Kljajin, M.; Karakašić, M. Modelling and Simulations of Elevator Dynamic Behaviour. // Tehnicki vjesnik - Technical Gazette 18, 3(2011), pp. 423-434.
- [17] Chen, G. FE Model Validation for Structural Dynamics, PhD, Thesis, University of London, UK, 2001.
- [18] Software Abaqus /standard version 6.7-1
- [19] Maneski, T. KOMIPS Computer modeling and structures calculation, Monograph, Faculty of Mechanical Engineering, University of Belgrade, 1998, ISBN 86-7083-319-0.

Authors' addresses

Ezedine Allaboudi, PhD student

University of Belgrade
Faculty of Mechanical Engineering
Kraljice Marije 16
11120 Belgrade 35, Serbia
E-mail: zdn1971@yahoo.com

Dr Taško Maneski

University of Belgrade
Faculty of Mechanical Engineering
Kraljice Marije 16
11120 Belgrade 35, Serbia
E-mail: tmaneski@mas.bg.ac.rs

Dr Nataša Trišović

University of Belgrade,
Faculty of Mechanical Engineering,
Kraljice Marije 16,
11120 Belgrade 35, Serbia
E-mail: ntrisovic@mas.bg.ac.rs

Dr Todor Ergić

J. J. Strossmayer University of Osijek
Mechanical Engineering Faculty in Slavonski Brod
Trg Ivane Brlic Mazuranic 2
35000 Slavonski Brod, Croatia
E-mail: tergic@sfsb.hr