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COMPUTER REALIZATION OF THE CONTINUOUS PREISACH HYSTERESIS MODEL

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Preliminary notes
The Preisach hysteresis model is widely used in different engineering disciplines, for example: electrical engineering, mechanical engineering and so on. The Preisach model is a unique tool to describe in a mathematical consistent way the hysteresis phenomenon with non-linearities and memories. The Preisach hysteresis model is discrete, however this paper presents a continuous version and its computer realization. The paper also shows how the discrete model approximates the continuous solution and how the error can be reduced.

Keywords: *Preisach hysteresis model, geometric interpretation, Preisach triangle*

Računalno ostvarenje Preisachova modela kontinuirane histereze

Prethodno priopćenje
Preisachove model histereze je u širokoj uporabi u raznim tehničkim disciplinama kao što su elektrotehnika, strojarstvo itd. Preisachov model je jedinstven alat kojim se na matematički dosljedan način može opisati pojam histereze s nelinearnostima i memorijama. Preisachov model histereze je diskretan no u ovom se radu predstavlja kontinuirana verzija i njezin računalni prikaz. Rad također pokazuje kako diskretni model aproksimira kontinuirano rješenje i kako se greška može umanjiti.

Ključne riječi: *Preisachov model histereze, geometrijska interpretacija, Preisachov trokut*

1 Introduction

The first version of the Preisach hysteresis model was developed for magnetic materials by F. Preisach in 1935 [1]. That time it was a great idea to represent the magnetic nonlinearity through the properties of unit volumes (domains, clusters), however the mathematical formulation was a bit complicated for engineering applications. Mathematical formulation of the Preisach model was developed by two mathematicians M. A. Krasnozel'skii and A. V. Pokrovskii [2]. The wide spread of the method has started after the publication of the I. D. Mayergoyz book entitled 'Mathematical model of hysteresis' [3]. By this time not only the Preisach operator, but for example the Prandtl-Ishilinskii operator has been introduced as it is discussed in the books of A. Visintin [4], M. Brokate, J. Sprekels [5] and P. Krejci [6] and others [7]. By this time, the beginning of the 1970s a wide research work started on the development and simulation of different hysteresis models [8, 9, 10], as the Duhem model, the Stonar-Wohlfarth model [11], the Jiles-Atheron model [12], the Chua model [13] in magnetic materials. At the same time a wide range research started on modelling and simulation of steel and other mechanical and structural materials as well, from the empirical models [14], the hyperelasto-visco hysteresis [15] through the Ramberg-Osgood model [16] to the Richard-Abbott model [17] and the Bouc-Wen model [18]. Among all of the developed models the Preisach model is the most popular and widely used for modelling both magnetic materials and the nonlinear dynamical behaviour of mechanical structures.

2 The scalar Preisach model

In the definition of the scalar Preisach model this paper follows the work of Mayergoyz [3]. For the mathematical definition of the Preisach model a simple hysteresis operator is considered $\gamma_{\alpha\beta}$. This operator is represented by a rectangular loop, as it is shown in Fig. 1. This diagram shows the relation between an input and an output variable. The values of the input variables can be α and β, where it is also assumed that $\alpha \geq \beta$. The 'system' may switch between these two input values. The values of the output variable are +1 or –1, like a switch of relay. The input function $u(t)$ may monotonically increase along the path of A-B-C-D-E or decrease along the path of E-D-F-B-A, as it is shown in Fig. 1. Beside the hysteresis operator a weight distribution function is also usually defined: $\mu(\alpha, \beta)$. After these definitions if an infinite set of the hysteresis operators are considered then the Preisach model can be defined as the expected value of:

$$f(t) = \iint_{\alpha \geq \beta} \mu(\alpha, \beta) \gamma_{\alpha\beta} u(t) \mathrm{d}\alpha \mathrm{d}\beta. \tag{1}$$

In this way the Preisach model is a superposition of several very simple hysteresis operators.

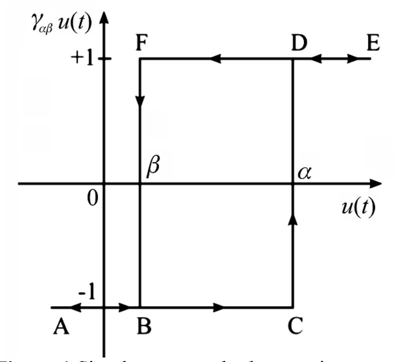

Figure 1 Simple, rectangular hysteresis operator

2 Geometric interpretation of the Preisach model

The computer realization of the Preisach model relies on the geometric interpretation of the model as it was discussed by Mayergoyz [3]. It should be mentioned, that in his work Mayergoyz considered an identity distribution function. In this case the difference between the continuous and the discrete Preisach model is zero. If some other distribution function is selected then a difference between the discrete and continuous realization of the Preisach model can be found. The method described in this paper makes it possible to create a continuous Preisach model. Basically there is a one-to-one correspondence between the elementary hysteresis operators $\gamma_{\alpha\beta}$ and the points determined by the (α, β) coordinates in a half space. The half space is visualized as a Preisach triangle, as it is shown in Fig. 2. This triangle is capable to represent the memory of the system. The figure also shows the initial state, when half of the hysteresis operators are in the 'down' state and half of the hysteresis operators are in the 'up' state. The operators in the 'up' state are denoted by the positive area and the light grey area in the figure, while the operators in the 'down' state are denoted by the negative area and the dark grey area. In this state the expected value of the distribution of the weights is zero and the output of the hysteresis operator is zero.

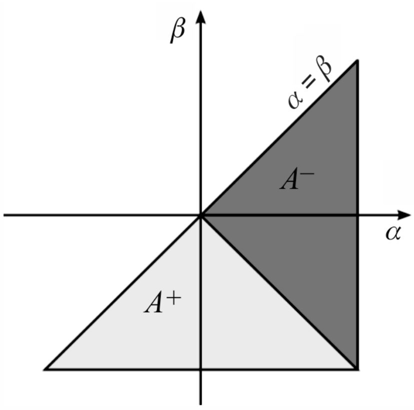

Figure 2 Geometric interpretation of the Preisach model

When the input parameter increases and some of the hysteresis operators are switched to the 'up' state, then the positive area is increased in such a way that the point on the $\alpha = \beta$ line is shifted to the right and therefore the interface between the negative and positive areas is shifted by a vertical line, α is increased. This state is shown in Fig. 3a. When further operators are switched to the 'up' state by increasing the input parameter then the vertical line shifts further to the right, as it is shown in Fig. 3b. In this example the local maximum is reached in Fig. 3b. Next, Fig. 3c and 3d show the states when the input parameter starts to decrease, β is decreased. In these cases the interface between the positive and negative areas is modified by shifting the horizontal line. When the input parameter is further decreased a state can be reached when the local maximum is deleted from the 'memory', since the vertical line disappears in the triangle, as it is shown in Fig. 3e. This is the wiping out property of the material memory. When all elementary operators are turned into the 'down' state, the sum of the area will be negative, as it is shown in Fig. 3f. This is the minimum value that can be reached by the system. It can be shown, that in a general state the interface between the positive and negative areas is a 'staircase' line. Fig. 4 shows a staircase interface and the corresponding hysteresis curve.

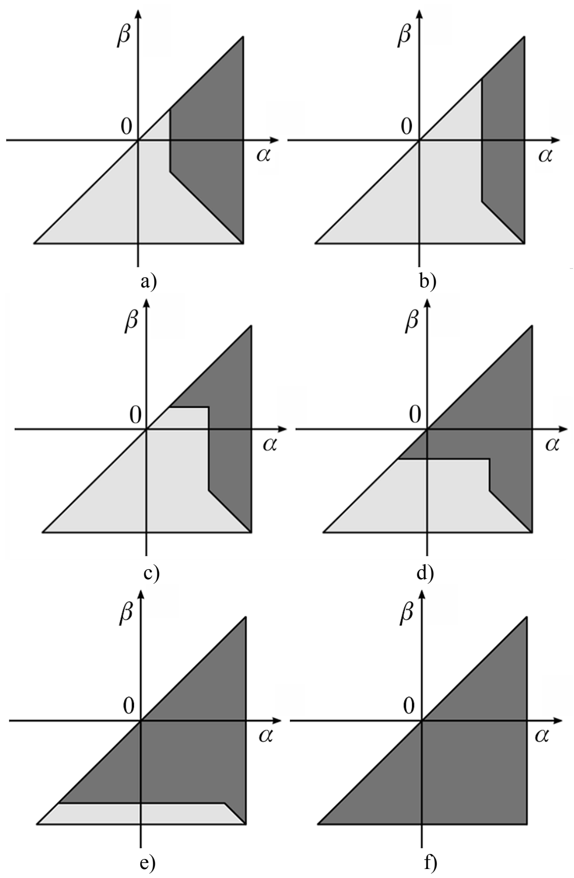

Figure 3 Different states of the Preisach triangle when the input variable is increased and decreased afterwards to the absolute minimum

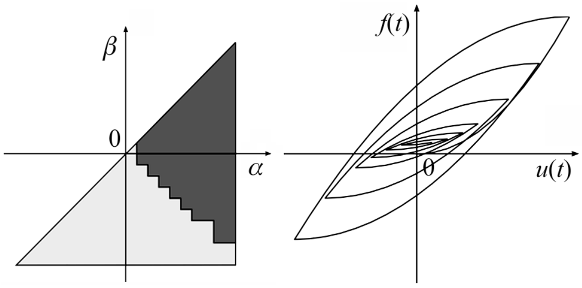

Figure 4 Staircase interface in the half space and the corresponding curve

4 Implementation of the Preisach triangle

For the implementation, the triangular shape is divided into pieces of square areas. Fig. 5 shows the division of the space. The sides of the triangle are divided into $2 \cdot n$ pieces. By the division a grid is created, for which data can be stored in a matrix form. An element of the matrix can be accessed directly by using two indices, for example *i* and *j*. Fig. 5 also shows the variation of index *i* and *j*. Since only a triangle has to be implemented, therefore only the elements of the matrix with indices $i \le j$ will be used. The elements of the matrix are classified as:

- full cells, denoted by white squares in Fig. 5;

- boundary cells that are marking diagonal edge of the Preisach triangle, denoted by dark grey squares in Fig. 5; and
- starting cells, used at the beginning of the simulation, denoted by light-grey square in Fig. 5.

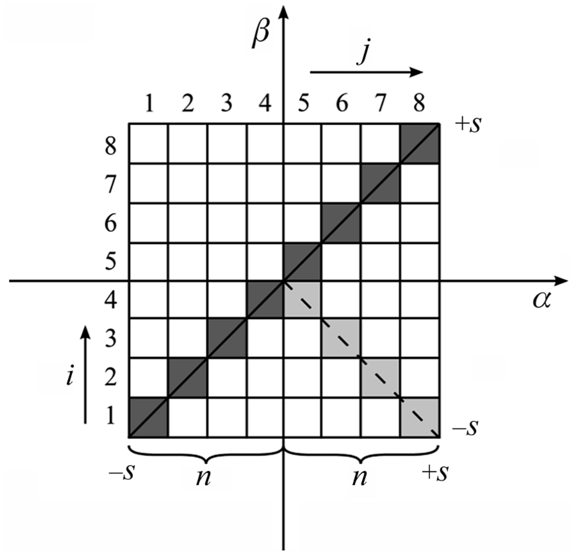

Figure 5 Discretised Preisach triangle

Although the half space is divided into square areas, in this implementation a square area can also be divided further by the 'staircase' line between the positive and negative area. To account for this behaviour, an element of the matrix, or a square area, stores six data:

1) type of the cell;
2) coordinates of the start point (top-left corner point);
3) coordinates of the end point (bottom-right corner point);
4) vector of coordinates, representing other points between the start and the end point;
5) the number of points between the start and the end point; and
6) weight value at the centre of the cell.

Using these data it is possible to model precisely the staircase line. For example Fig. 6 shows the start, the end points and all other points in between. In the figure the three states of a square area are: fully negative state, general state and fully positive state. In the figure the grey colour denotes the area that is added to the positive area. In this implementation only the positive area of the elements is summed, since the negative area can be calculated at the end from the full triangle.

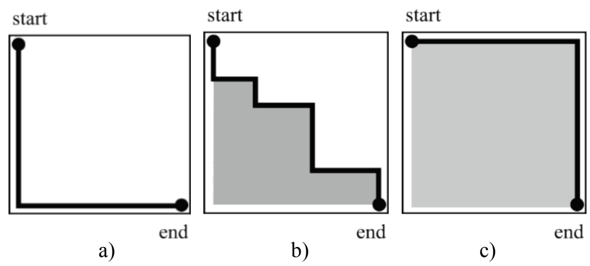

Figure 6 Three states of a full cell

Fig. 7 shows a general cell with a staircase interface between the positive and the negative areas. For the area calculation let's consider point i and $i+1$, which determine a rectangular area as it is shown by a dark-grey area in Fig. 7a. To calculate the area under the interface the following general formula is used:

$$A^{+} = \left(x_{i+1} - x_i\right) \cdot \left[\frac{\left(y_{i+1} - y_{\mathrm{b}}\right) + \left(y_i - y_{\mathrm{b}}\right)}{2} \right], \tag{2}$$

where y_{b} is the base height for the cell. This formula is also valid for the situations when there is a triangle in the interface, for example as it is shown in Fig. 7b.

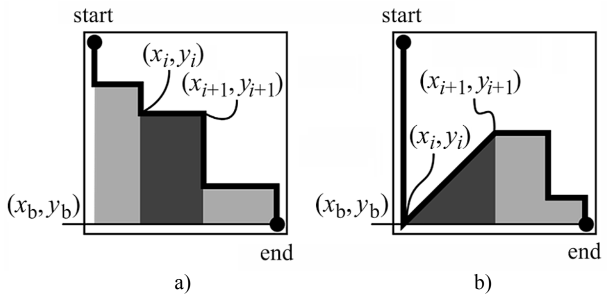

Figure 7 Calculation of the positive area under the interface line: a) staircase interface, b) boundary interface

4.1 Full cells

A full cell is the most general form of a square area from the triangular half space. The word: 'full' means, that the area of the cell is fully inside the triangular half space. On the scale of different states of a full cell, at one end there is the state, when all of its area contributes only to the negative area, as it is shown in Fig. 6a. The other end of the scale is the state, when all of its area contributes only to the positive area. This state is also shown in Fig. 6c. In all other cases there is a staircase line going through the cell. This state is represented in Fig. 6b.

4.2 Boundary cells

The boundary cells are special cells and they cannot change their type. In the case of the boundary cells it is always valid - according to the numbering scheme in Fig. 5 - that the two indices are equal, *i=j*. Fig. 8 shows three different states of a boundary cell. In Fig. 8a the cell contributes only to the negative area, while in Fig. 8c the cell contributes only to the positive area. Fig. 8b shows a general state, when only a part of the cell contributes to the positive area.

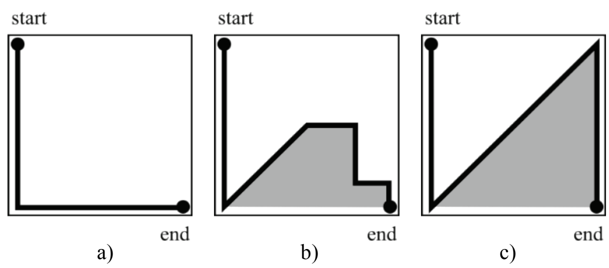

Figure 8 Three states of a boundary cell

4.3 Starting cells

Starting cells are only used until the absolute maximum or minimum of the input is not reached once. When the absolute maximum or minimum of the input is reached, the starting cells change their type and they become a full cell. Fig. 9a shows the starting state of a starting cell, and Fig. 9b shows a general state of a starting cell, while there are only small changes in the input value of the system. When the input value of the system increases continuously at the end a starting cell will reach the state that is shown in Fig. 6c. If the input value of the system is decreased a starting cell can also reach another state that is shown in Fig. 6a. In either case the starting cell becomes a full cell, since the original diagonal line is not reconstructed again.

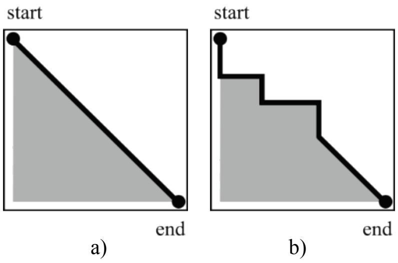

Figure 9 Two states of a starting cell

4.4 Calculation steps during input value changes

In the implementation of the Preisach triangle a matrix that is shown in Fig. 5 is created with cells in their initial state. Cells above the $i=j$ line are full cells in the state of Fig. 6a. Cells, denoted by the dark grey colour are boundary cells in the state of Fig. 8c. Cells below the $i=j$ line are full cells in the state of Fig. 8c, except the cells denoted by the light grey colour, since these cells are starting cells, initially, as it is shown in Fig. 9a.

After the initialization the input parameter of the system is increased or decreased. When the input parameter is increased the steps in Tab. 1 are performed. When the input parameter is decreased the steps in Tab. 2 are performed. These algorithms ensure that a valid and correct staircase line is maintained throughout the simulation. In the algorithms whenever points are added, they are added to the front of the list.

Table 1 Algorithm for the case when the input parameter of the system is increased

if $x_{\text{new}} > x_{\text{end}}$ **then**
delete every intermediate point
if the cell is a starting cell **then**
change the cell type to full
end if
if the cell is a full or starting **then**
create the state of Fig. 6c by
adding the top right corner as an intermediate point
else if the cell is a boundary cell **then**
create the state of Fig. 8c by
adding two intermediate points
end if
else if $x_{\text{new}} < x_{\text{start}}$ **then**
there is nothing to do
else
delete every intermediate point that $x_i < x_{\text{new}}$
if the cell is a full cell **then**
add two points that create a vertical line
$(x_{\text{new}}, y_{\text{start}})$ and $(x_{\text{new}}, y_{\text{end}})$
else if the cell is a boundary cell **then**
add three points that create a vertical line
$(x_{\text{new}}, y_{\text{start}})$ and
$\left(x_{\text{new}}, \frac{x_{\text{new}} - x_{\text{start}}}{x_{\text{end}} - x_{\text{start}}} (y_{\text{start}} - y_{\text{end}}) + y_{\text{end}} \right)$ and
$(x_{\text{new}}, y_{\text{end}})$
else if the cell is a starting cell **then**
add two points that create a vertical line
$(x_{\text{new}}, y_{\text{start}})$ and
$\left(x_{\text{new}}, \frac{x_{\text{end}} - x_{\text{new}}}{x_{\text{end}} - x_{\text{start}}} (y_{\text{start}} - y_{\text{end}}) + y_{\text{end}} \right)$
end if
if the cell is a boundary cell **then**
if there are more than 3 intermediate points **then**
$y_3 = y_4$
end if
else
if there are more than 2 intermediate points **then**
$y_2 = y_3$
end if
end if
end if

Table 2 Algorithm for the case when the input parameter of the system is decreased

if $y_{\text{new}} < y_{\text{start}}$ **then**
there is nothing to do
else if $y_{\text{new}} < y_{\text{end}}$ **then**
delete every intermediate point
create state of Fig. 6a by
adding the bottom left corner as an intermediate point
if the cell is a starting cell **then**
change the cell type to full
end if
else
delete every intermediate point that $y_i > y_{\text{new}}$
if the cell is a full cell **then**
add two points that create a horizontal line
$(x_{\text{start}}, y_{\text{new}})$ and $(x_{\text{end}}, y_{\text{new}})$
else if the cell is a boundary cell **then**
add three points that create a horizontal line
$(x_{\text{start}}, y_{\text{end}})$ and
$\left(\frac{y_{\text{new}} - y_{\text{end}}}{y_{\text{start}} - y_{\text{end}}} (x_{\text{end}} - x_{\text{start}}) + x_{\text{start}}, y_{\text{new}} \right)$ and
$(x_{\text{end}}, y_{\text{new}})$
else if the cell is a starting cell **then**
add two points that create a horizontal line
$(x_{\text{start}}, y_{\text{new}})$ and
$\left(\frac{y_{\text{start}} - y_{\text{new}}}{y_{\text{start}} - y_{\text{end}}} (x_{\text{end}} - x_{\text{start}}) + x_{\text{start}}, y_{\text{new}} \right)$
end if
end if
if the cell is a boundary cell **then**
if there are more than three intermediate points **then**
$x_3 = x_4$
end if

```
else
   if there are more than two intermediate points then
      x2 = x3
   end if
end if
```

Once one of the algorithms in Tabs. 1 and 2 is executed, and the staircase line is updated, then the positive area under the staircase line is calculated using Eq. (2) in every cell. The negative area is the difference between the full triangle and the positive area.

5 Examples

The above described algorithm has been tested in structural dynamic problems, for example [19], [20]. When the input variable oscillates between the absolute minimum and maximum values (Fig. 10) then the resulting hysteresis curve can be seen in Fig. 11. In the case when the input variable oscillates between continuously decreasing maximum and minimum values (Fig. 12) the resulting curve is shown in Fig. 13 and the corresponding Preisach triangle can be seen in Fig. 14.

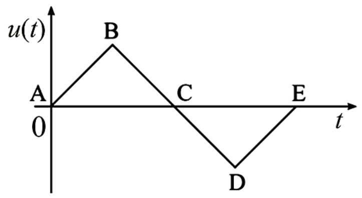

Figure 10 The oscillation of the input variable between the absolute minimum and maximum values

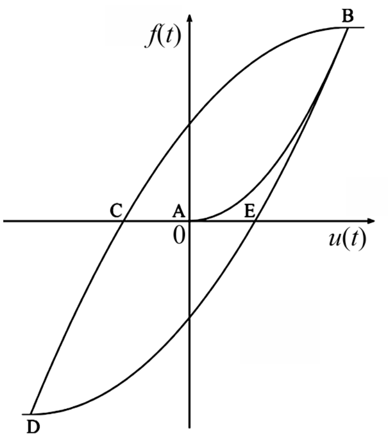

Figure 11 Hysteresis loop when the input variable oscillates between the absolute minimum and maximum values

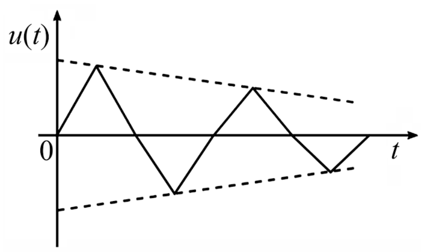

Figure 12 The oscillation of the input variable between continuously decreasing extreme values

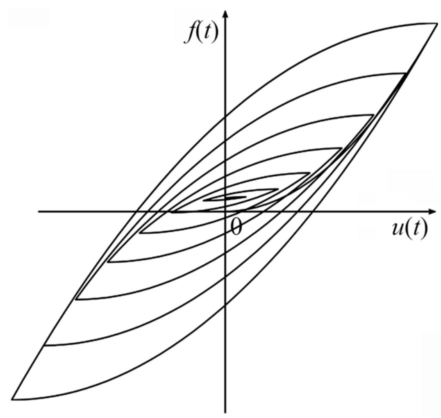

Figure 13 Hysteresis loop when the input variable oscillates between continuously decreasing extreme values

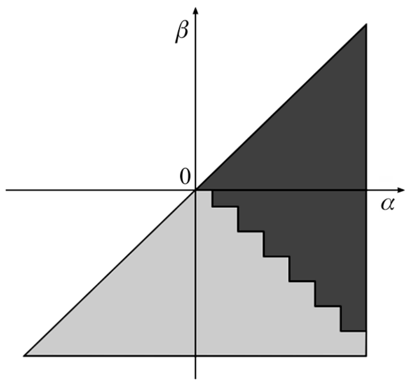

Figure 14 Preisach triangle for the final state of the curve in Fig. (13)

It can be also demonstrated how the error is reduced in the case of a discrete Preisach model. Fig. 15 shows a Preisach triangle divided into three cells. For this Preisach triangle the following distribution function will be considered:

$$P(\alpha,\beta)=\left[1-\left(\frac{\alpha}{1}\right)^2\right]\left[1-\left(\frac{\beta}{1}\right)^2\right]. \quad (3)$$

For the cells in Fig. 15 the values have been determined numerically (discrete way) and analytically (continuous realization). The same calculation has been performed with twice as many cells as is shown in Fig. 16. The error between the discrete and the continuous Preisach model with the distribution function (3) is shown in Fig. 17. The figure shows the error in cell 1 (h1), cell 2 (h2) cell 3 (h3) and the total error.

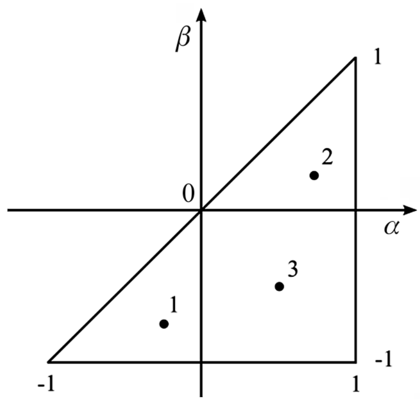

Figure 15 Discrete model of a Preisach triangle

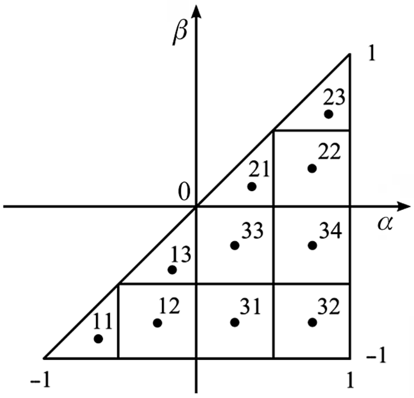

Figure 16 Discrete model of a Preisach triangle with more elements

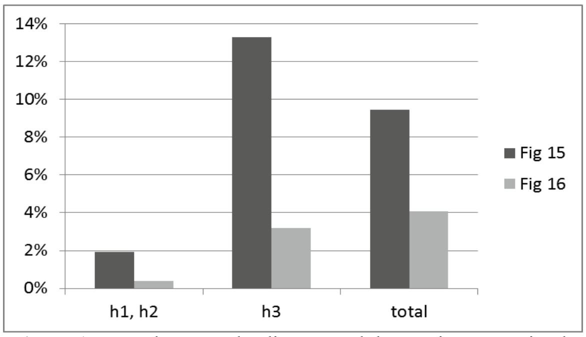

Figure 17 Error between the discrete and the continuous Preisach model shown in Fig. 15 and 16.

6 Conclusion

The paper has presented the scalar Preisach model and its geometric interpretation by a Preisach triangle. Although the Preisach triangle is usually discretized, in this paper the continuous implementation is discussed in details. Finally some examples are presented to demonstrate the use of the algorithm. This algorithm will be used in the analysis of a hysteresis phenomenon of a tolling bell tower.

7 References

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