I. TANACKOV, F. SINANI, V. BOGDANOVIĆ, Đ. ĆOSIĆ, B. LALIĆ

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METASTABILITY – MARKOVIAN APPROACH

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A complete system of events of nonlinear processes in complex dynamical systems describes the evolution of the distribution. Different stages of evolution of the distribution declared stable, metastable and unstable systems. Variance of probabilistic distribution plays a crucial role in determining the state of the system. It was found that the system is metastable, when it carried Markovian processes in continuous time. Metastability is the original metallurgical phenomenon and actively exists in the structure of the materials. In a metastable state, the structural distances in material are exponentially distributed.

Key words: system stability, distributive evolution, material structure

INTRODUCTION

Standard rating system stability is consistent with the principles of Aristotelian logic: the principle of identity, the principle of contradiction and the principle of non-existence of the third. Analogously, in theory declare the system steady state and unsteady state. The usual absence of the third principle is absent.

Systems are declared as stable when the resources and activities of the system achieve goals of system. Complementary, if the system does not implement the goals, the situation is unstable. Control of systems always focus on maintaining stability and preventing the destabilization of the system.

In theory of the system, phenomenon of metastability is a temporally declared. In a qualitative sense, the metastable state is not the most stable state of the system. In the quantitative sense, the time that the system spends in the metastable state is always longer than the time that the system spends in a stable condition.

According to available data, the phenomenon of metastability was first used in the literature almost 100 years ago, just in metallurgy - in 1915 [1, 2].

In accordance with the early findings of the metallurgical industry, the phenomenon of metastability have a basic source in the system structure. This structure is variable, especially in chemical or thermal processing of materials. In analyzes of phase transitions in polymer [3], martensitic transformation [4], exponential decay as a function of surface separation in materials [5], corrosion process [6], crystal growth kinetics [7], etc. metastability phenomenon takes the form of an exponential expression. In continuous time, exponential processes are the basis of Markovian processes. The paper explained the Markovian basis of metastable state in the system structure.

I. Tanackov, V. Bogdanović, Đ. Ćosić, B. Lalić, Faculty of Technical Sciences, University of Novi Sad, Novi Sad, Serbia
F. Sinani, Ministry of Transport and Communications, Skopje, Republic of Macedonia

DISTRIBUTION EVOLUTION

The system structure or flows structure in the system, have different forms in the time domain (Figure 1). Part of the complex system structure have a flow dynamics analogy. This dynamics start from the equal distance between successive identical parts or events (Figure 1a), small and medium differences between successive distances (Figure 1b), a large number of small distances and small number of large distances (Figure 1c), initial and expression clusters (Figure 1d) and singularities in one cluster (Figure 1e).

At random, timed stochastic domain [0, T] derived bijection between intervals of homogeneous events *n* and axiomatic probability space [0, 1] (Figure 2). The basic random variable is equal to the quotient of your real length subintervals and the total time $t_i = T_i / T$. Applied bijective function is not bicontinuous function.

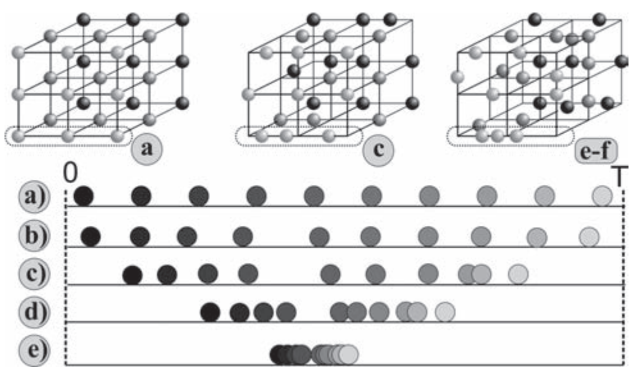

Figure 1 System structure or flows structure

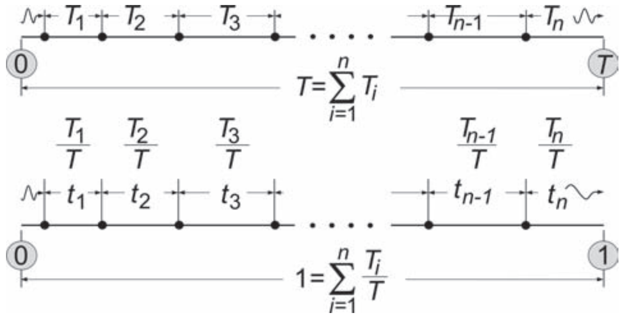

Figure 2 Biection of real stochastic flow

These system structure or flows structure in the system can be described by probability distributions and densities. Flow with identical distances between successive events are described with Dirac delta function (Figure 3a). Small and medium differences of successive distances are described by a Normal or Erlang distribution (Figure 3b). A large number of small distances and a small number of large distances are described the exponential distribution (Figure 3c). For a description of the initial and distinct clustering (Figure 3d) and singularity (Figure 3e) is necessary to choose probability density function which in defined domain [0, 1] has minimum.

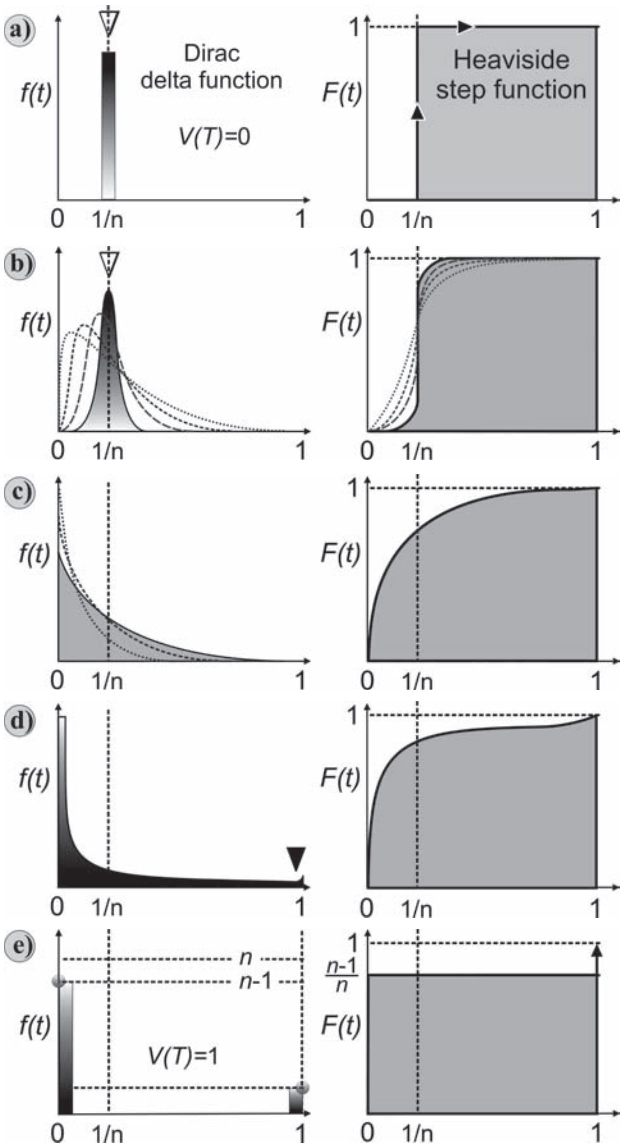

Figure 3 Distribution evolution

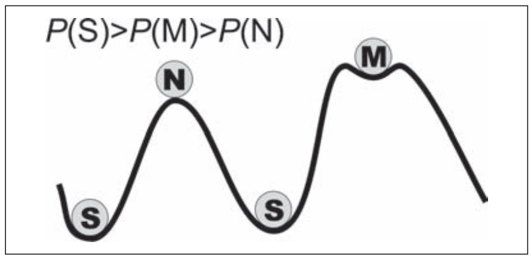

Figure 4 Graphical representation of the system stability, S-stable, M-metastable, N-unstable

The maximum probability density is most likely the state of the system. If the state of the system can be described by probability density function which has at least one maximum value, the system is stable. Forecasting the state has the highest certainty.

Minimum probability density is the least likely state of the system. If the state of the system can be described by probability density function which has at least one minimum, the system is unstable. Predicted state of the system has the largest uncertainty.

If there is no probability density function of the extreme value (or minimum or maximum), his condition is forecasting a meta-case, and the system is metastable.

Analog graphic interpretation of stability (Figure 4), the following declaration proposed for the assessment of stability, metastability and instability of the system.

In line with the evolution of the continuum distribution (Figure 3), a stable state of "a" to "b", metastable states are "c", and the unstable situation of the "d" and "e".

FUNCTION OF EVOLUTION

One of the functions defined on the interval [0, 1] that can get normal forms, to move to the eccentricity and forms an Erlang distribution, than to exponential, and then take on a "U" shape is – β function dual substituent parameter with defined functions of the probability density and parameters (1):

$$\beta(a,b,t)=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}t^{a-1}(1-t)^{b-1},\ 0\le t\le 1,$$
$$E(t)=\frac{a}{a+b},\ V(t)=\frac{ab}{(a+b)^2\,(a+b+1)} \qquad (1)$$

β function is defined in the domain [0, 1], according to the bijection of any stochastic flow [0, T] in axiomatic probabilystic domain [0, 1]. For n events defined on the interval [0, 1], the mathematical expectation of successive differe-nces is known and always is $E(t) = 1 / n$. On the basis of these facts, it is to have a functional relationship between the parameters a and b proposed functions (1), the value basic parametars (2) and one-parameter form of β function (3):

$$E(t)=\frac{1}{n}\Leftrightarrow b=\frac{a}{n-1},\ V(t)=\frac{(n-1)}{n^2(an+1)} \quad (2)$$

$$f(t)=\frac{\Gamma(an)}{\Gamma(a)\Gamma(an-a)}t^{a-1}(1-t)^{a(n-1)-1} \quad (3)$$

Deterministic flow is described identical intervals $t_i = t_j$, $\forall i, j\in[1, n]$ identical successive differences between events (Figure 3a). In the deterministic flow, the mean length of the interval is $E(t) = n^{-1}$, $\forall i\in[1, n]$ variance of the deterministic flow is zero, $V(t) = 0$. Deterministic flow can be described by the Dirac delta probability function density form (4):

$$\delta_c(t)=\lim_{c\to 0}\frac{1}{c\sqrt{\pi}}e^{-\frac{t^2}{c^2}},\ \delta_c(t)=\begin{cases}+\infty, & t=0\\ 0, & t\neq 0\end{cases} \quad (4)$$

that by substitution $c = \sigma\sqrt{2}$ and translation for the period μ reduced down to a normal distribution $N(\mu,\sigma)$. Being known the mathematical expectation distance customers during the interval [0, 1] and is always equal to the reciprocal of clients $1/n$, the density distribution deterministic flow becomes (5):

$$\delta_{\sigma\sqrt{2}}(t-\frac{1}{n})\cong\lim_{a\to\infty}\frac{\Gamma(an)t^{a-1}(1-t)^{a(n-1)-1}}{\Gamma(a)\Gamma(an-a)} \quad (5)$$

Distribution function of a deterministic flow, Dirac delta function is the Heaviside step function is 0 for $t<1/n$, and 1 for $t\geq 1/n$.

This stage of evolution of the distribution is called Deterministic phase and is characterized by the value of the flow variance $V(t) = 0$.

Deterministic phase is a special form of the normal phase. Minimal differences in the lengths of intervals t_i under the influence of many other factors, leads to an increase in the standard deviation σ, infinitesimal but finite value that is greater than zero, $\sigma\geq 0$. The intervals between events are no longer identical. Normal course of development begins. Mathematical expectation is universal and is always equal to the reciprocal value of the number of events, $1/n$. (Figure 3b). The density distribution has a maximum and is not eccentric. For a large number of events and finite n, the condition, which gives to the normal flow symmetric function probability density, must be fulfilled koji normalnom toku daje simetričnu funkciju gustinu verovatnoće: $\mu-3\sigma\geq 0$. According to (2), parameter „a" of distribution β becomes $a\cong 9$. Interval Variance of flow in Normal phase of distribution evolution is from 0 to $1/9n^2$, $0 < V(t)\leq 1/9n^2$.

Violation of the conditions of Normal phase flow $\mu-3\sigma\leq 0$ expresses eccentircity density functions of the distribution, but the probability density function is still a maximum. Increasing eccentricity follows the rise of the probability density function of the ordinate, increasing eccentricity (Figure 3b) and the final position of the maximum of the asymptotic ordinate (Figure 3c). At this stage of evolution of the distribution, the key issue is the density function is lost when the maximum ordinate value in t=0 (6), which is for a finite number of events n possible only when $a\cong 1$ (7):

$$\left[\frac{\Gamma(an)}{\Gamma(a)\Gamma(an-a)}t^{a-1}(1-t)^{a(n-1)-1}\right]'=0 \quad (6)$$

$$\Leftrightarrow\left(t^{a-1}(1-t)^{a(n-1)-1}\right)'=0\Leftrightarrow a=\frac{n}{n-1}\cong 1 \quad (7)$$

One-parameter β function gets the parameters of mathematical expectation, variance and standard deviation have a unique relationship characterized by an exponential distribution (8) (note: $a\cong 1$):

$$E(t)=\frac{1}{n},V(t)=\frac{(n-1)}{n^2(an+1)}\approx\frac{1}{n^2} \quad (8)$$

This interval the evolution of the distribution is called Erlang flow. Boundary condition at the maximum loss is exponential. From probability theory it is known that convolution exponential distribution gives Erlang, and that under certain conditions Erlang distribution can be approximated by a normal distribution. Also, depending on the parameters, the Erlang distribution has adequate ordinanty rise and eccentricity.

Interval variance Erlang phase flow in the evolution of the distribution is from $1/9n^2$ to n^{-2}.

Deterministic phase, normal phase, and Erlang phases are phases in which the system is stable. Loss of maximum and minimum non-existence of the next phase of the evolution of the distribution system is the metastability period.

With further reduction of the parameter „a" the exponential evolution in the distribution function se intensively approaches the ordinate (Figure 3c). Exists a large number of small intervals and a small number of medium and large intervals. Such a distribution of the initial interval corresponds to the phenomenon of clustering (Figure 3d). The next crucial task in the course of evolution the distribution is a description of the probability density of these random variables t_i with a function that receives a minimum. Differentiation is expressed in clusters and "disappearing" of middle distance. Number of small and large distances is greater than the number of "middle" range. If we consider the probability density (Figure 3e), in underexponential part we expect convergence of the distribution of density values to the values of a distance and the first appearance of a minimum close to this Singular flow (Figure 3d) have full concentration in n events to one arbitrary point on the interval [0, 1]. This means that the $(n-1)$ distance equal to those of the random variable t_i is zero, and the remaining distance is equal to 1. Minimum variance in which this is possible is obtained for the parameters $a = (n-1)^{-1}$, and according to the conditions (2), za b=1. Minimum variance at which a sufficient number of events and the final n for the first time in the evolution of the distribution occurs a minimum (9):

$$V(t)\approx\frac{ab}{(a+b)^2(a+b+1)}=\frac{(n-1)^2}{2n^3-n^2}\approx\frac{1}{2n} \quad (9)$$

Interval Variance flow in the exponential phase of the evolution of the distribution is from $1/n^2$ to $1/2n$. It should be noted that for large enough and final n, the value of the parameter $a \approx 1/n$.

The end of the exponential phase is the beginning stages of system instability. After the onset of minimum performance sub-exponential phase of evolution of the distribution. The phenomenon of concentration of all the events at a single point on the interval [0,1] is borderline phenomenon of singularity flow, with maximum value of the variance $V(t)=1$.

Interval Variance flow in sub-exponential phase of evolution of the distribution is from $1/2n$ to 1.

The approximate capacity distribution at this stage is not adequate and viable only up to the value of a parameter $a \geq \delta$, $\delta \rightarrow 0$. For condition (2) the parameter $b \rightarrow 0$. For parameters for the mathematical expectation $E(t) = 1/n$ and variance $V(t)=1$, parametar value is $a = -n^{-1} < 0$, which is not in accordance to basic definition of β distribution. Singular flow distribution in the phase of evolution may also be described by two Dirac delta function. We know that $(n-1)$ distance is equal to „0” and that one distance is equal to „1”. β distribution does not have this capability, and therefore its overall role is catalytic. At the same time, completely ambiguous position in which the interval [0, 1] forms singularity. Therefore, the state of the system is completely unstable.

CONCLUSION

Variance plays a key role in assessing the stability of the system. General criteria for the evaluation of the system are given in Table 1:

Table 1 **System stability and variance**

System states	Variance, V(t)
Stable	$0 \leq V(t) \leq 1/n^2$
Metastable	$1/n^2 \leq V(t) \leq 1/2n$
Unstable	$1/2n \leq V(t) \leq 1$

Quantitatively, the variation interval in which the system is stable is less than the interval in which the system is metastable or unstable. From the above criteria indicate that the stability of the system decreases with the square of the elements. Exponential phase of the evolution of the distribution is system metastability phase. In the base of Markov continuous process is exponential distribution. Metastable phase is Markov system [8, 9]. The finding in this study is not the first and a unique relationship and metastability of Markovian processes [10-13].

Generalization in a flow structure for one dimensional form, can be extended to the material structure for two-dimensional form (surface) or three-dimensional form (volume). According to the findings, metastable states in the structures of materials are analogous to Markovian processes. In a metastable state, the structural distance are exponentially distributed. Explicit metric system for metastability – based on the probabilistic concept, is established.

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