DEAF CHILDREN AND MATHEMATICS

PAUL ARNOLD

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Deaf children are retarded in mathematics relative to their hearing peers. There are two possible and interrelated reasons for this: 1. their relatively poor linguistic skills and secondly, the social consequences of deafness (lower teacher expectations, the use of less than optimal and possibly inappropriate teaching methods). These two elements, the linguistic and the social, no doubt interact in complex ways. At the same time, some experimental work appears to suggest that there is no absolute obstacle to the deaf child learning mathematics. This inconsistency between the deaf child's potential and actual performance is discussed and ways of optimizing their mathematical skills are suggested.

INTRODUCTION

It is, perhaps, surprising that recent textbooks on the psychology of the deaf and partially-hearing child and education (for example, Conrad, 1979; Rodda & Grove, 1987; and Marchark, 1993) contain no reference to learning mathematics. In fact, mathematics has been a largely neglected area in the education of deaf and hearing-impaired children (Bunch, 1987; Fridriksson & Stewart, 1988). This is a serious omission, as industry and commerce now require a higher level of numeracy than was previously the case. The purpose of the present paper is to outline what is known about the psychology of learning mathematics by deaf pupils and what might be done to improve heir mathematical skills. Skemp (1971, page 134) argued that "If it is agreed that genuine mathematics is simply a specialised form of intelligent activity, then we need no longer wonder why it should be enjoyable for its own sake." We must ask to what extent deaf children are capable of this intelligent activity; and how they can be encouraged to develop further.

The mathematical abilities of deaf children have been overshadowed by the intense interest and effort in improving their communication skills. Fridriksson and Stewart (1988) have suggested reasons why mathematics has received so little attention. Teacher training programs tend to emphasize speech and language skills, and students on these courses are not provided with an adequate foundation in mathematics teaching. Secondly, mathematics in the classroom is often only pencil-and-paper learning with little hands on experimentation or computer use. Thirdly, mathematics is not seen as important, and progress in mathematical understanding is not stressed. It would appear that underlying these reasons is a fundamentally pessimistic view of the potential of deaf children. Outside the school classroom the skills required in industry and commerce are becoming increasingly more complex and technology based, and at the same time jobs requiring numeracy, and computing skills based on numeracy, increase as manual employment declines. A knowledge of mathematics is then important for deaf and hearing-impaired people.

Dr. Paul Arnold is a professor at the Department of Psychology, University of Manchester, England
Correspondence to: Dr. Paul Arnold, Department of Psychology, University of Manchester, Manchester M13 9PL, England.

EXPERIMENTAL WORK

Hitch, Arnold and Phillips (1983) investigated orally educated deaf children's simple addition and concluded that their reasoning processes are very similar to those of hearing children. Hitch et al . measured the time required for both deaf and hearing children to verify whether problems in the form x + y + z were correct or incorrect. The deaf and the hearing produced remarkably similar patterns of response, and both groups relied heavily on the MIN counting model, whereby the subject begins from the larger digit and increments it a number of times equal to the smaller:

(RT= a (min (x,y)) + b).

So in the case of 7 + 3 = ? the child would begin with 7 and count on to 8, to 9, and up to the correct answer 10. Mulhearn and Budge (1993) modified the method used by Hitch et al. (1983). The deaf children in their sample used signed English. No screening procedure was employed. Instead, the deaf and hearing subjects were matched according to chronological age. Finally, the subjects were required to produce answers to addition problems in the form x + y = ? Mulhearn and Budge concluded that deaf children did not appear to be at any major disadvantage relative to their hearing contemporaries. They appeared to use similar computational mechanisms to those used by the hearing, both groups relying on the **MIN** counting model.

The conclusion from these two studies is that the addition skills of both the deaf and the hearing children show no qualitative differences and that the deaf children are simply developing more slowly. This suggests that there may be no other qualitative differences in the basic processes between the deaf and hearing, although this, of course, requires further investigation.

I will now review some of what is known about the psychology of deafness and mathematics, and tentatively suggest ways of teaching maths to deaf children more optimally.

THE NOTTINGHAM GROUP'S WORK

Wood, Wood, Kingsmill, French, and Howarth (1984) reported their work on the mathematical achievements of deaf children in different kinds of schools. This work is also described in the book by Wood, Wood, Griffiths, and Haworth (1986). They used the Vernon-Miller Graded Arithmetic-Mathematics Test (1976), which uses little written language and covers the age range from 5 to 17 years. They tested 1,005 English children in their final school year, of whom 540 were hearing-impaired. Those who attended special (deaf) schools had a mean hearing loss of 92 dB, and a maths age of 12.1. Those who attended partially hearing units (attached to regular schools) had an average loss of 68 dB and a maths age of 12.8. The mainstreamed hearing-impaired had a loss of 48 dB and a maths age of 14 years. The hearing children had a mean maths age of 15.5 years. In summary, the hearing children did the best, and the unit and special school children, taken together, were three years behind them. The mainstreamed children were about 18 months behind the hearing. The authors asked why the deaf children perform less well. They found that in fact the degree of deafness was not very important. The maths scores of the children from units and special schools (12.1 and 12.8) were not significantly different. The authors compared children with losses of between 50 and 70 dB from each of the three kinds of school and found no difference in maths score between them. Despite the fact that their hearing losses ranged from 30 to 120 dB, the amount of variance in the maths scores accounted for by their degree of deafness was only 7%. The severity of hearing loss was a poor predictor of any individual's maths age, and the type of school attended made little difference to their math scores.

These findings suggest that deafness is not a barrier to the development of mathematical competence. This is potentially an optimistic finding, which should renew and energize efforts to optimize the teaching of mathematics to deaf and hearing-impaired children. The authors looked at the highest scores achieved by children with different levels of hearing - loss. They divided the whole sample into nine 10 dB hearing loss bands and found that within each 'band' there were children who reached the 'ceiling' (the highest maths age on the test). There was no correlation between hearing loss and the 'top scores' within each band. There was, however, a significant correlation between hearing loss and 'lowest scores' in each band, suggesting that at the lower end of the ability range hearing loss exerts its most retarding effect. It may then asked: If hearing loss and the type of school attended do not appear to have much effect on the maths scores, why do the deaf children lag so far behind the hearing children? Wood and his colleagues argue that the relatively small influence of hearing loss suggests, but does not conclusively prove, that fluency in the English language is not an important factor in deaf maths achievement. But if this is so, why are there differences between them and the hearing? Wood et al. reply that it cannot be explained by just one factor, but by several factors interacting together. Wood (1988) concluded that the deaf find learning more difficult because the process of communication with their teachers is more difficult and so their acquisition of knowledge is impeded.

IS THE VERNON-MILLER TEST MEASURING THE SAME SKILLS IN THE DEAF AND THE HEARING?

The authors examined all of the test items and recorded whether the deaf and hearing tended to get the same items right, and others wrong. They then examined whether the errors made by the two groups were different in kind, and again there was no evidence that they are. Thirdly, they asked if the deaf children tended to "persevere senselessly" on the test. Did they tend to proceed with the test even when they were getting only a few answers right? In general, there were no overall differences between the deaf and hearing. It cannot be concluded that, on the Vernon-Miller test at least, that deaf children's reasoning was different in kind, nor that they were more "impulsive" in test taking than the hearing.

TEACHER PREDICTIONS AND PUPIL PERFORMANCE

All of the teachers of the deaf were asked to show which questions they thought their students would find difficult, and to specify whether the hypothesized difficulty would be the result of mathematical or linguistic problems. Although the maths test had less written language than other similar tests, it still contained problems with up to 33 words in them. The teachers' predictions were compared with the children's performance on each item. The authors found very high correlations between teacher predictions of mathematical difficulties and children's success rates, and these predictions applied just as well to the hearing. When, however, they looked at the predictions of difficulties due to linguistic problems, they found the relationship was far weaker on the Senior test and non-existent on the Junior version. Although teachers tended to agree about where such linguistic problems would occur, their judgements did not agree with what the children actually did. The children, in fact, found the linguistic problems much easier than the teacher predictions suggested.

THE ANALYSIS OF CHILDREN'S ERRORS

The children made common errors on most of the questions. This suggests that the errors were not random, but rather the result of systematic, but invalid or incomplete, mathematical procedures.

When the errors of the deaf and the hearing were compared, two main facts emerged. First, on the great majority of the questions, the same errors appeared in both deaf and hearing groups. This suggests that the ways in which children think about or work through maths problems and produce errors is similar, whether they are deaf or hearing. The second finding was that, although a similar pattern of errors occurred in both groups, the actual proportion of errors of each type tended to vary. The authors suggest the following explanations for this. First, the deaf might be less mathematically "sophisticated". This can be explored in two ways. One is to compare the deaf group with a sample of the hearing who achieved the same average maths score. The second, which involves moreguess work, is to try to explain how errors are produced. This is what they found.

The "Logic" of Error.

The authors ask us to consider the following question:

3 x 4 = 6 x ......

Common errors on this problem were as follows (Wood et al.' s explanations are in brackets).

72 (3 x 4 x 6)

12 (3 x 4)

18 (3 x 4 + 6)

6 (3 x 4 - 6)

If their explanations are correct, then this enables them to make some predictions. So the incorrect answer of 72 (see above) is the result of the child ignoring the equation (=) sign. A general feature of the results was that many of their errors were systematic and related, in understandable ways, to the answers given on other problems. There is then a certain "logic" underlying many errors. The results lead the authors to suggest that deaf children go through the same stages as hearing children on the way to mastery of mathematical processes, but that their progress is slower. Why are they slower? The reason lies, say the authors, in the teaching-learning process, which is itself impaired. Deaf children are only "impulsive" where their knowledge of the task at hand, for example reading, is limited. In the maths tests where they have relatively normal (if delayed) levels of ability, they are no more selective nor more impulsive than hearing children.

EDUCATIONAL IMPLICATIONS

The authors wondered why the teachers were so much better at predicting probable success based on mathematical ability and, in contrast, so poor when linguistic factors were involved. They suspected that teachers could predict success based on mathematical ability because they knew that the children have never been taught particular mathematical skills. The suspicion is that "Rather, teaching tends to concentrate on the basic operations (addition, subtraction, etc.) and on topics such as buying and change with some work on fractions, and perhaps, a bit on decimals and a little on simple graphical representations. More complex mathematics (e.g. algebra, trigonometry), more demanding uses and analyses of graphs, etc., seldom seems to be taught." They refer to the work of Suppes (1974) who found that using computer-assisted teaching of maths, deaf and hearing children of similar maths ages showed different rates of progress. The deaf made more rapid gains when the new mode of teaching was adopted.

The studies of the Nottingham Group and that of Suppes taken together indicate that deaf children have a greater potential for mathematical learning than is generally believed. Teachers often underestimate the deaf child's ability to handle or to bypass linguistic difficulties in maths. These pioneering studies have opened up the area, but there are many new questions produced by their work. The practical problem remains of overcoming the teaching-learning difficulties.

THE CAMBRIDGE GROUP

Bishop and Barham (Barham, 1987; Bishop and Barham, 1987; Barham, 1988; Barham, 1990) have described their research on mathematics and much younger deaf children. On an early visit to a unit for small deaf children, one of the teachers gave them a surprising answer to their enquiry about the problems the children had in learning mathematics - "The trouble is, so many of them are spoiled at home." They continued, "But most teachers of deaf children will agree that deaf youngsters tend to be impulsive, unreflective, often finding it difficult to think a situation through in a reasoned way. Yet the whole wealth of primary school mathematical activities demand that children do just that." The members of the Cambridge Group are convinced that mathematics in the classroom must be taken beyond the "What they need to know" stage to encourage the children to enter the exciting realm of mathematical discovery. To enable them to face confidently the world of uncertainty, surprise, and excitement. So maths is not then just an intellectual exercise. Emotion and motivation are intertwined with intellectual progress or failure.

They began the project by asking school and unit teachers what they felt their greatest problem was. "Invariably we receive the answer 'Language'". They found that very young deaf children have difficulty in putting numbers in order - they may recognize a 3, but cannot relate it to 2 and 4. On the basis of a series of visits the authors wrote a set of computer programs. Each program was designed to encourage a particular intellectual activity.

Their program "Odd Man Out", shown on a television monitor, a set of three shapes, one of which is different from the other two by a single feature - colour, shape or size. The child is asked "Which is different?", and responds by pressing key 1, or 2, or 3. The problems gets progressively more difficult and the number of difference factors is increased. Their programs on sequencing first of all show children three pictures which tell a simple story. They are invited to tell the story in either words or signs. The computer then mixes up the order of the picture and the child has to rearrange the order correctly using a light-pen. Other programs help develop the idea of causality of "If... then", and also "Why?.. because." The authors claim that the appeal of the programs lay in their visual nature. The programmer made "clear, dynamic screen images with the minimum of written language and using colour wherever appropriate." "The programs were made on the basis of the hypothesis that visual skills would be among the children's relative strengths, and it was certainly felt by the team that this hypothesis has been substantiated."

SOME WAYS FORWARD

The contrast between the young deaf children in the Cambridge sample, spoiled and impulsive, and the achievements of the young people in the Nottingham sample must be a source of satisfaction to the teachers who contributed to this change. It still remains a problem, however to improve maths education further and to reduce the lag of18 months for the mainstreamed, and that of up to 3 years for the unit and deaf school children. It is clear that mainstreaming does not in itself remove the mathematical retardation. The Cambridge group stress the strength of the deaf child's visual system. Could this be used to aid mathematical learning?

LANGUAGE OR VISUAL IMAGERY

Maths school textbooks are full of language, of English (or other) printed sentences, even the most elementary of them. Some of the questions in the test used by the Nottingham Group contained 33 words. The child's ability to read and to do maths may easily be confused by an observer. Does maths have to be so entwined with, and dominated by, the written and printed language of the hearing community? Many less able hearing children, and possibly their hearing-impaired peers, would be very pleased to learn that this does not have to be so. Since the work of Francis Galton in the 1880s, we have known that (hearing) people differ greatly in their mental imagery. Our mistaken (hearing) tendency is to assume that deaf people must think in just the way we do. We tend to forget that deaf children often have poor speech, but may have fluency in Sign Language, a viaully based language. Rather we might hypothesise that deaf children use visual imagery to an equal extent to hearing people, and quite possibly to a greater extent. The general view to emerge from the Nottingham Group's work is the fact that the deaf students were much better at maths than they were at English. This may suggest that they were using some alternative code or symbol system to perform the maths tests. Makshark (1993, pages 172-175) has dicussed imagery and deafness, but does not consider the realtionship between mathematics and deafness. Skemp (1971), a psychologist and mathematician, discussed the role of imagery in hearing people's maths. He distinguished visual symbols from verbal symbols. As he observes "as soon as words are written down, they become things to be seen, not heard. Nevertheless words are primarily auditory symbols, and their preferred mode of communication is by word-of-mouth, not word-on-paper. A reader usually turns them into sub-vocal speech...." He continues, "visual symbols are clearly exemplified by diagrams of all kinds, particularly geometrical figures. But into which category should we put algebraic symbols like these?

sin x dx
{x:x >0}

"Both visual and verbal symbols are used in mathematics, together and apart. Thus we find diagrams with verbal explanations and, say, trigonometric calculations; we find curves together with their equations: but we also find page after page of algebra with no kind of figure or diagram. Indeed, a recent and highly - thought - of - book on geometry also contains not a single figure! It looks as if verbal (including algebraic) symbols are indispensable, but visual symbols are not." (pages 95-97). Skemp continues: "Even if they are not indispensable, however, there is no doubt that visual symbols are often useful, and may be a great deal more understandable than a verbal-algebraic representation of the same ideas. One sometimes also has the impression that the avoidance of diagrams is a demonstration, perhaps unconscious, that the writer needs no such props to his thinking: an academic "Look boys - no hands!" Skemp argues that it isa reasonable working hypothesis to assume that the functions of visual and verbal symbols are different, and perhaps complementary. We should find out what these functions are, with a view to using them and combining them to the best advantage. At this point I wish to make it clear that I am not advocating an exclusive visual method of teaching of maths to deaf children, but only that the balance between language and visual imagery should be changed for the deaf at certain stages of the mathematical learning process. According to Skemp, "Visual symbols would appear to be more basic, at least in their primitive form of representation of actual objects."

Hayes (1973) has discussed the function of visual imagery in elementary mathematics. Hayes develops the idea of Skemp's (1971) that visual imagery is important not just in geometrical problems, but also in algebraic ones. Spatial symbolism, Skemp wrote, "finds its way into every detail of the verbal-algebraic system." Skemp cites the example of position in a division number, and the position of a multiplier digit, say 271, indicates how each digit is to be interpreted: the 2 is worth 200 but the 7 only 70, and the 1 just 1. If this fundamental notion of place value can be demonstrated, in part visually, then the function of visual imagery is worth examining in more depth. The importance of verbal factors and visual imagery is confirmed by Hadamard (1945), for professional mathematicians, and by Syer (1953) for hearing students. Hayes considers in some detail the extent to which visual imagery "penetrates into the structure of the problem solving process. "

VISUAL IMAGES MAY BE INVOLVED IN ELEMENTARY MATHEMATICAL PROCESSES

Hayes goes on to give evidence that visual imagery may in some cases play an important role in elementary maths processes. Tentatively, Hayes suggests that "all our results are consistent in indicating an important function for notation-related imagery in the solution of elementary mathematical problems." If hearing, verbally fluent, fully literate adults use visual imagery, then its use may help deaf young people as an alternative to, and as an addition to, the English language. Perhaps it provides another route to mathematical understanding, which does not rest entirely on linguistic skills.

DEVELOPING SPATIAL ABILITY

Wheatley and Wheatley (1979) studied whether low achieving hearing 14 - year - old's spatial ability could be improved. In their maths classes they had no number work for a month, but instead engaged in activities intended to improve their spatial processing. These activities included tangrams, tessellations, hexaflexagons, polyominoes, and making three-dimensional cardboard shapes. Trial-and-error hands-on methods were used. The authors claim, anecdotally, that the effect of these experiences were dramatic. The authors also justify the comparison and manipulation of shapes as providing practice in gestalt thinking. These methods may be suitable for deaf children.

CONCLUSIONS

The first conclusion is that there appears to be no absolute reason why deaf children cannot become good mathematicians. This fact and the requirements of modern work demand that improved methods of teaching mathematics to deaf children must be developed. The message is that every maths lesson should be a maths lesson, and not primarily a language lesson, although maths may assist the child to acquire the language of the hearing society. It is possible that that the child's visual imagery should be enlisted in the struggle to learn mathematical concepts and skills. It is also possible that Sign Languages could play a valuable part in the acquisition of mathematical concepts.

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