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# Modified Net Present Value (MNPV): A New Technique for Capital Budgeting

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***Abstract:*** In this paper we show that, when the firm's opportunity rate of reinvestment is different from its financing rate. The risk-adjusted discount rate method (RADR) of computing Net Present Value (NPV) leads to the same type of incorrect results that required new methodology for modifying Internal Rate of Return (IRR) calculations (i.e. Modified Internal Rate of Return (MIRR)). Specifically, the current method of calculating NPV is biased in favour of (against) high-risk projects and against (in favour of) low-risk projects when the risk-adjusted discount rate is greater (less) than the firm's average marginal cost of capital (WACC) for two reasons. First, it incorrectly assumes that the reinvestment rate for project cash inflows is the risk-adjusted discount rate of the project instead of the firm's opportunity rate of reinvestment. Second, it incorrectly assumes that projected cash outflows, for the time periods after the initial outlay, are also discounted at the risk-adjusted discount rate. We propose a new methodology to establish a MNPV to eliminate the RADR methodology bias and derive a generalised rate of return (k*) under all combinations of reinvestment and financing rates for a firm. In addition we derive and demonstrate the linkages and consistency between the new MNPV, k*, Profitability Index (PI) and MIRR methodologies.

***JEL Classification:*** M21, M41

***Key words:*** capital budgeting, cost inflows, reinvestment rate, total initial outlay

## Introduction

The Net Present Value (NPV) and the Internal Rate of Return (IRR) are two of the most widely used techniques in capital budgeting decision making. The problems of IRR have been widely investigated and various modified internal rate of return

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(MIRR) models were devised as an alternative measure of rate of return and addresses many of the shortcomings of IRR (see Lin 1976; McDaniel, McCarty and Jessell 1988, and Beaves 1988 and 1993). On the other hand, the major criticism of NPV to date has been its failure to take into account the managerial option to abandon or extend a project and hence the NPV method underestimating the true NPV of the projects cash flow. Many of today's corporate finance textbooks already addressed these issues and many have integrated the application of options theory to capital budgeting problem (e.g. Pinches 1994 and Van Horne 1995). The interaction of financing and investment decisions has also been addressed by numerous researchers and has lead to Adjusted NPV method, where the Adjusted NPV is the sum of the NPV to equity and the present value of financing effects (Myers 1974). Solomon (1956) and Renshaw (1957) have shown that one of the reasons that IRR and NPV give a conflicting recommendation is due to the implicit reinvestment assumption embedded in these two approaches. In contrast, Dudley (1972) and Biedleman (1984) argue that there is no implicit reinvestment rate assumption in NPV or IRR methodology. However, they show that it is necessary to make an explicit reinvestment rate assumption when selecting from competing projects.

More recently, Beaves (1988, 1993) has developed a generalised net present value formula that explicitly accounts for the reinvestment of project cash flows and an overall rate of return. However, he assumes that net cash outflows that occur after time zero are financed by positive net cash inflows that occurred subsequent to time zero cash outflows but before the next cash outflows. Based on this definition the project's initial total outlay (initial wealth) depends not only on the sign of cash flows but the order of the cash flows. The determination of total initial outlay needed to finance the project should depend only on expected net cash outlays regardless of the source of financing. His definition of project total outlay underestimates the total financing cost for project and underestimates the terminal value (end of project wealth). Although Beaves (1993) incorporates uncertainty, he was more concerned with uncertainty about the term structure and not the question of reinvestment rate when the risk-adjusted discount rate method (RADR) is used to determine project NPV. To our knowledge no one has fully addressed the question of the correct reinvestment rate for determining the NPV of a project's cash flow when the discount rate is different from the firm's Weighted Average Cost of Capital (WACC). One of the reasons that NPV is considered superior to IRR is that, unlike IRR, the NPV uses the correct reinvestment rate, which is the firm's WACC. That is in fact true when the project under consideration is an average risk project and the discount rate is the WACC.

When the project is perceived to be above (below) average risk there are currently two approaches to determine the NPV of the project's net cash flows. The first approach is the risk-adjusted discount rate method (RADR), which adjusts the discount rate for risk by subjectively adding (subtracting) a given percentage to (from) the firm's WACC (which is already risk-adjusted) and uses this rate to determine the project's NPV. The second approach is the certainty equivalent method (CE), and this method adjusts the project's cash flows for risk and uses the risk-free rate to determine the project's NPV. The risk-adjusted discount rate (RADR) methodology of computing NPV assumes that the risk-adjusted discount rate, the financing rate and the firm's reinvestment rate are the same, and hence tends to overestimate (underestimate) the NPV of above (below) average risk projects. The main reason for the overestimation (underestimation) is the assumption that cash flows from high (low) risk projects will be reinvested at the higher (lower) risk-adjusted discount rate, hence leading to overestimation (underestimation) of above (below) average risk project's NPV. This may lead to accepting more high-risk projects and rejection low-risk projects.

Our thesis is that the risk-adjusted discount rate method (RADR) of computing NPV leads to incorrect results that are biased in favour of (against) high-risk projects and against (in favour of) low-risk projects, when the risk-adjusted discount rate is greater (less) than the firm's average marginal cost of capital (WACC), for two reasons. First, it incorrectly assumes that the reinvestment rate for project cash inflows is the risk-adjusted discount rate for the project instead of the firm's opportunity rate of reinvestment (usually the firm's WACC). This leads to the assumption that the firm will be able to invest the net cash inflows from above (below) average risk project to another project of similar risk in order to generate rates of return that are above (below) firm's average marginal cost of capital (WACC). This is not always a realistic assumption and negates the reason for using risk-adjusted discount rate to account for the riskiness of the project's cash inflow. Unless the firm has a particular competitive advantage or there are barriers to entry (exit), it is unrealistic to assume higher (lower) reinvestment rates for above (below) average risk projects will be available (desirable) in the future. Dudley (1972) recommends that the firm's marginal cost of capital (WACC) is the appropriate reinvestment when selecting from competing projects. On the other hand, Meyer (1979) suggests that the correct reinvestment rate should be the average of expected rate of return on new investment, and this rate will be greater than the firm's WACC, assuming the firm's investment opportunity curve is negatively sloped. Second, the NPV method assumes that projected net cash outflows, for the time periods after the initial outlay, are also discounted at the risk-adjusted discount rate. This approach will underestimate the firm's financing cost for high-risk projects and overestimates the firm's financing cost for low-risk projects (Brigham and Gapenski 1996). McDaniel, McCarty and Jessell (1988) recommend that the marginal cost of capital (WACC) should be used to discount the net cash outflows to determine the total initial outlay (investment) of the project. Hence, we believe that projected net cash outflows should be discounted using the firm's average marginal opportunity cost of capital (WACC). Therefore, we propose to modify Beaves' (1988, 1993) generalised net present value formula to a modified net present value (MNPV) to address the issues raised above. This modification will make the computation methodology of modified NPV (MNPV) and MIRR consistent by providing a common reinvestment rate assumption and will also result in determination of a more generalised rate of return (k*). k* is the discount rate that forces the MNPV to equal zero, and it is equal to MIRR when the assumed reinvestment rate and financing rates are equal to the firm's WACC, just as IRR is the discount rate that forces the NPV to equal zero. Furthermore, when the reinvestment rate and financing rates are equal to the project's IRR, then k* is also equal to IRR. Thus, the generalised rate of return (k*) gives MIRR and IRR as a special case and it is unique for a given reinvestment rate assumption.

## Model and Discussion

NPV is defined as the present value of the cash inflows minus the present value of cash outflows and can be expressed as follows:

$$NPV = \sum_{t=1}^{n} \frac{CF_{it}}{(1+k)^t} - I_0 \qquad (1)$$

where $CF_{it}$ is net cash inflows at time $t$, $I_0$ is the total initial cash outlay (investment) and k is the appropriate discount rate. What is implicitly assumed, but not explicit stated, in equation (1) is the fact that the $CF_{it}$'s are reinvested at the discount rate k. This is also equivalent to finding the future value of the cash inflows to the assumed end of project's life, or computing the terminal value (TV) of the project, and discounting the TV. In addition, $I_0$ is the present value of all net cash outflows discounted at discount rate $k$. Therefore the NPV can also be expressed as:

$$NPV = \frac{\sum_{t=1}^{t=n} CF_{it}(1+k)^{n-t}}{(1+k)^n} - I_0 \qquad (2)$$

where the first term is equivalent to the present value of net cash inflows, the terminal value of net cash inflows discounted to time zero, and the second term is the present value of all net cash outflows.

Equation (2) has the advantage of making explicit that the cash inflows are reinvested at the firm's opportunity rate of investment, which may be different from the appropriate risk-adjusted discount rate, to determine the terminal value (TV) and then discounted back to present. Assuming a perfect capital market, the correct reinvestment rate assumption for project cash inflows is the firm's opportunity rate of investment that may not be equal to the risk-adjusted discount rate of above (below) average risk project. On the other hand, the appropriate rate for discounting all net cash outflows should be the firm's average opportunity financing rate (WACC). The WACC would be an appropriate reinvestment rate for large and mature corporations, assuming the firm is operating at optimal level. However, there are several situations in which firms do not operate at theoretical optimums which require modification of current NPV methodology to deal with all the investment conditions actually faced by the firm. Under Myers' 'asymmetric information' conditions (Myers 1984) firms are likely to operate with reserve borrowing capacity and pass up some positive NPV projects. Under these conditions the opportunity rate of reinvestment will be different from (greater than) the firm's financing rate. In addition, firms operating under capital budget constraints (a non-optimal condition) and smaller or start-up firms with both limited investment opportunities and financial resources are likely to experience differences between the firm's financing rate and opportunity rate of reinvestment. In practice, these firms may have reinvestment rates that are greater than the risk-adjusted discount rate but less than the average IRR of the available projects with IRR > WACC (Dudley 1972, Meyer 1979).

In summary, we make the following assumptions in developing the modified net present value (MNPV):

1. Total initial outlay is the present value of all net cash outflows discounted at the firm's financing rate (WACC).
2. The appropriate reinvestment rate for net cash inflows is the firm's $k_{rr}$.
3. The risk-adjusted discount rate for high (low) risk project is WACC ± y%. (y=1%, 2%,..., etc)
4. Firm maintains its target capital structure.
5. Accepted project(s) do not affect the firm's risk characteristics.

Hence, the modified NPV (MNPV) can be expressed as follows:

$$MNPV = \frac{\sum_{t=1}^{t=n} CF_{it}(1+k_{rr})^{n-t}}{(1+k)^n} - \sum_{t=0}^{t=n} \frac{CF_{ot}}{(1+WACC)^t} \tag{3}$$

where $k_{rr}$ is the reinvestment rate, $k$ is the risk-adjusted discount rate, $CF_{ot}$ is the net cash outflow at time t and the WACC is the financing rate. The second term in equation (3) is the present value of all net cash outflows ($I_0$) discounted at the firm's WACC.

The first term in equation (3) is the present value of the terminal value of the project's net cash inflows and the second term is the present value of all net cash outflows discounted at the firm's WACC. Equation (3), the Modified Net Present Value (MNPV), will be equal to the NPV when the financing rate and the reinvestment rate are equal to the discount rate. However, when the project is above (below) average risk, the NPV methodology is biased upward (downward), because the methodology assumes that the correct reinvestment rate is the risk adjusted discount rate. This is an inappropriate assumption unless we are also willing to assume that the firm will continue to select above (below) average risk projects that will provide a high (low) return on investment that will be equal to the risk adjusted discount rate. If the firm continues to take higher (lower) than average risk projects, it would be reasonable to assume that the firm's risk characteristic and the firm's average marginal cost of capital (WACC) will increase (decrease). Eventually such projects will become the average risk project for the firm and the appropriate risk-adjusted discount rate will be equal to the firm's WACC. Furthermore, the process assumes that the firm's opportunity rate of reinvestment changes with the riskiness of each project under consideration.

Equation (3) can be simplified computationally and also clearly show the importance of reinvestment rate assumption in capital budgeting decisions. Making the usual assumption of constant discount rate, reinvestment rate and financing rate, equation (3) can be rewritten as follows (See Appendix A for derivation):

$$MNPV = \gamma^{n} \sum_{t=0}^{t=0} CF_{it}(1+k_{rr})^{-t} - \sum \frac{CF_{0t}}{(1+WACC)^{t}} \tag{3a}$$

where $\gamma = (1+k_{rr})/(1+k)$.

The first term in equation (3a) is the present value of the project's net cash inflows discounted at the firm's opportunity reinvestment rate times an adjustment factor ($\gamma^{n}$) that takes into account the riskiness of the project. The adjustment factor $\gamma^{n}$, for a given project, is a function of the project's risk-adjusted discount rate, the firm's reinvestment rate and the life of the project. This formulation explicitly shows the importance of risk-adjusted discount rate ($k$), reinvestment rate ($k_{rr}$) and financing rate (WACC) in capital budgeting decision. In addition, it shows that if the reinvestment rate is greater (less) than the risk-adjusted discount rate, the NPV would under (over) estimate the true contribution of the project. Furthermore, when $\gamma$ is MNPV is equal to NPV for projects with normal cash flow.

Throughout this article we assume that corporations will maintain their target capital structure and the firm's WACC will not change as a result of each individual project taken. The recognition that the reinvestment rate, financing rate (WACC) and risk-adjusted discount rate need not all be equal provides greater flexibility in determining a more realistic NPV (i.e. MNPV) of the proposed project that reflects the reinvestment opportunities available to the firm. Furthermore, it allows computation of a generalised rate of return ($k^*$) that will give both IRR and MIRR as a special case. The computation of MNPV and $k^*$ is based on a common reinvestment rate assumption and are more consistent and comparable. The generalised rate of return ($k^*$) is a geometric compound rate of return and is based on the terminal value (TV) of cash inflows and the present value of all cash outflows, it is the rate of return that forces the MNPV to be equal to zero.

$$MNPV = 0 = \frac{\sum_{t=0}^{t=n} CF_{it}(1+k_{rr})^{n-t}}{(1+k)^n} - \sum_{t=0}^{t=n} CF_{0t}(1+WACC)^{-t}$$

Then $k^*$ can be expressed as follows:

$$k^* = \left[ \frac{\sum_{t=1}^{t=n} CF_{it}(1+k_{rr})^{n-t}}{\sum_{t=0}^{t=n} CF_{ot}(1+WAAC)^{-t}} \right]^{1/n} - 1 \tag{4}$$

Alternatively $k^*$ can also be expressed as follows (see appendix B for derivation):

$$k^* = (1+k_{rr})[PI]^{1/n} - 1 \tag{4a}$$

where $k^*$ is the average expected rate of return of the project given the firm's WACC (used as firm's financing rate) and the assumed reinvestment rate. PI is the profitability index and it is defined as the ratio of the present value of the net cash inflows discounted at the firm's reinvestment rate and the present value of the net cash outflows discounted at the firms financing rate (WACC). If the reinvestment rate is equal to the WACC, then $k^*$ reduces to the MIRR of the project. When the reinvestment rate and the financing rates are equal to project's IRR, the generalised rate of return, $k^*$, is also equal to IRR. Firms that maximise shareholders value should at the minimum generate a return that is sufficient to cover the firm's weighted average cost of capital (WACC). Hence, given a competitive capital market cash inflows from any project should be reinvested at the firms WACC. On the other hand, a more optimistic reinvestment rate assumptions should not exceed the firm's average IRR of all acceptable projects under consideration. MNPV and $k^*$ lead to a more consistent accept/reject decision. In addition, $k^*$ leads to the same accept/reject decision as *PI*.

Our proposed approach to computing the modified net present value, MNPV, is also consistent with the computation of Modified Internal Rate of Return (MIRR) (Gitman, 1994; Damodaran 1997). Unlike the NPV, the discount rate that forces the

MNPV to be equal to zero is $k^*$. MIRR is also the discount rate that forces the MNPV to equal zero only when the assumed reinvestment rate and financing rate are equal to the firm's WACC and hence equal to $k^*$. In addition, $k^*$ is equal to IRR when the reinvestment and financing rates are equal to the project's IRR. For an assumed reinvestment rate that is greater (less) than the WACC the generalised rate of return $k^*$ would be higher (lower) than MIRR and $k^*$ will be less than IRR as long as the assumed reinvestment rate is less than the project's IRR. In the following section we will use a simple numerical examples to show the computation of MNPV using equation 3(a) and generalised rate of return $k^*$ using equation 4(a). We will then compare and contrast the MNPV with NPV, $k^*$ with IRR and MIRR, and illustrate the degree of bias upward (downward) using the MNPV and NPV profiles.

## Numerical Examples

Suppose we have three projects, projects X, and Y, require total initial cash outlay (investment) of $100,000 each, and project Z requires total initial outlay (PV of investment) of $118,628. Project X shows a normal cash flow stream following initial outlay and has a four years life. Project Y also shows a normal cash flow stream, similar to project X, except all the cash inflows of the project occur at the end of the project's four years life. Project Z shows a non-normal cash flow, time zero cash outflow followed by positive net cash inflows but has negative net cash outflow in the last year and the project has a 5 year life.

Table 1 shows the projects' net after tax cash flows (Panel A); NPV, IRR and MIRR (Panel B); Panel C shows the MNPV and $k^*$ when the projects are considered high risk and the reinvestment rates ranged from 10% to 15%. Panel D shows the MNPV and $k^*$ when the projects are considered low risk and the reinvestment rates ranged from 7% to 12%. In Panel C and D, the financing rate for all cash outflow is assumed to be the firm's WACC. The comparison of NPV and MNPV will focus primarily on the cases where the reinvestment rate is equal to the firm's WACC and the discount rate is equal to the appropriate risk-adjusted discount rate.

Examination of the Table 1 shows that the NPV (Panel B) and MNPV (Panel C & D) lead to accepting project X and Y when both projects are considered high, average and low risk projects. MNPV (Panel C & D) are computed using the WACC as a financing rate for various reinvestment rates assumptions. However, the RADR method of computing the NPV (Panel B) of project X, for example, overestimated by 30.7% (25611/19597-1) the MNPV (Panel C) when the project is considered a high risk and the reinvestment rate is the firm's WACC. In contrast the RADR methodology of computing NPV underestimated project X's MNPV by 14.8% (41572/48765-1) when the project is considered low risk (Panel D) and the reinvestment rate is the firm's WACC. When the project is considered an average risk and the reinvestment rate is also the WACC, the NPV and the MNPV are equal. In addition, the IRR of 25.95% and MIRR of 18.17%, which is also equals to $k^*$, also lead to the decision to accept project X under all three scenarios when the reinvestment rate is the WACC. Although both the NPV and MNPV lead to the same accept decision, the expectation of how much the project will add to shareholder value are significantly different due to differing reinvestment rate assumption.

Table 1.: Computation of MNPV and NPV

| | | | |
|---|---|---|---|
| | ***Panel A: CASH FLOWS*** | | |
| t | ***Project X*** | ***Project Y*** | ***Project Z*** |
| 0 | ($100,000) | ($100,000) | ($100,000) |
| 1 | $ 50,000 | $0 | $50,000 |
| 2 | $ 45,000 | $0 | $45,000 |
| 3 | $ 40,000 | $0 | $40,000 |
| 4 | $ 30,000 | $197,500 | $30,000 |
| 5 | | | ($30,000) |
| PV of $CF_{out}$ | ($100,000) | ($100,000) | ($118,628) |
| | ***Panel B: NPV*** | | |
| NPV(K=13%) | $ 25,611 | 21,130 | $ 9,328 |
| NPV(K=10%) | $ 33,188 | $ 34,895 | $ 14,560 |
| NPV(K=7%) | $ 41,572 | $ 50,672 | $ 20,183 |
| IRR | 25.95% | 18.55% | 18.94% |
| MIRR | 18.17% | 18.55% | 12.58% |
| | ***Panel C: Modified NPV (MNPV) of High Risk Project*** | | |
| | *(Discount Rate = 13%, Financing rate=WACC=10%)* | | |
| *Reinvestment Rate:* | | | |
| 15.0% | $ 29,751 | $ 21,130 | $ 13,420 |
| 14.0% | $ 27,668 | $ 21,130 | $ 10,170 |
| 13.0% | $ 25,611 | $ 21,130 | $ 6,983 |
| 12.0% | $ 23,580 | $ 21,130 | $ 3,859 |
| 11.0% | $ 21,576 | $ 21,130 | $ 796 |
| 10.0% | $ 19,597 | $ 21,130 | $ (2,206) |
| $K^*$ (when $K_{rr}$ = 10%) | 18.17% | 18.55% | 12.58% |
| $K^*$ (when $K_{rr}$ = 13%) | 19.63% | 18.55% | 14.30% |
| | ***Panel D: Modified NPV (MNPV) of Low Risk Project*** | | |
| | *(Discount Rate = 7%, Financing rate=WACC=10%)* | | |
| *Reinvestment Rate:* | | | |
| 12.0% | $ 53,719 | $ 50,672 | $ 42,275 |
| 11.0% | $ 51,226 | $ 50,672 | $ 38,251 |

| | | | |
|---|---|---|---|
| 10.0% | $ 48,765 | $ 50,672 | $ 34,308 |
| 9.0% | $ 46,335 | $ 50,672 | $ 30,443 |
| 8.0% | $ 43,938 | $ 50,672 | $ 26,656 |
| 7.0% | $ 41,572 | $ 50,672 | $ 22,945 |
| K* (when Krr = 7%) | 16.72% | 18.55% | 10.85% |
| High Risk Discount Rate | 13% | | |
| Average Risk (WACC) | 10% | | |
| Low Risk Discount Rate | 7% | | |

Similarly, the NPV (Panel B) and MNPV (Panel C & D) lead to same accept/reject decision for project Y under all three scenarios of high, average and low risk project. Both the NPV and MNPV are equal under all three risky scenarios and various reinvestment rate assumptions. This occurs because project Y's cash inflows occur at the end of the project life in year four and there are no intermediate cash inflows and hence the reinvestment rates are irrelevant. Furthermore, the IRR, MIRR and $k^*$ are all equal to 18.55% and lead to accepting project Y when it is considered high, average and low risk project.

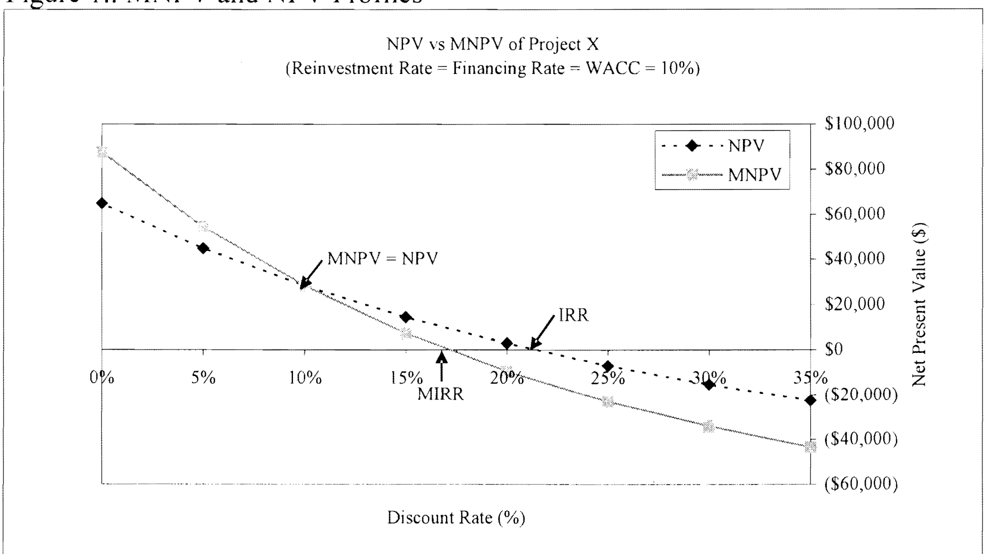
Third, the NPV (Panel B) and MNPV (Panel C & D) do not lead to same accept/reject decision for project Z under all three scenarios of high, average and low risk project. When project Z is considered a high-risk project the NPV (Panel B) and MNPV (Panel C) would lead to conflicting decisions. The NPV and IRR criterion lead to accepting project Z, while the NPV and MIRR criterion leads to a conflicting decision. On the other hand, the MNPV and k*, when the reinvestment rate is the WACC, criterion lead to rejecting project Z. The NPV methodology resulted in NPV of +$9,328 while the MNPV (panel C) leads to ($2206) when the project is considered high risk. Thus, the RADR methodology overestimated project Z's NPV over MNPV methodology by 522.8% (9328/-2206 - 1) for two reasons. First, the RADR method of computing NPV incorrectly uses the risk-adjusted discount rate as reinvestment rate instead of the firm's reinvestment opportunity rate, and second is the discounting of net cash outflows that occur after time zero by the risk-adjusted discount rate instead of the firm's financing rate (WACC). When project Z is considered below average risk both the NPV and MNPV (Panel D) criterion lead to the same accept decision. However, the RADR approach of computing NPV underestimated the NPV relative to MNPV by 58.8% (20183/34308 – 1) when the project is considered below average risk (Panel D).

Furthermore, Table 1 also shows that the modified net present value (MNPV) is equal to NPV when both the discount rate and reinvestment rate are equal to the firm's weighted average cost of capital for all projects. When the reinvestment rate is equal to the appropriate risk-adjusted discount rate (but not equal to the firm's WACC) MNPV (Panel C &D) and NPV are equal for projects with normal cash flows but not equal for projects with non-normal cash flows. For example, when project X and Y are considered high (low) risk and the appropriate discount rates are 13% (7%) and the financing rate is the firm's WACC, NPV and MNPV are equal. However, when a project has a non-normal cash flow, such as project Z, NPV will overestimate (underestimate) MNPV by 33.58% (12.04%) when the project is considered high (low) risk. Lastly, the NPV will overestimate (underestimate) the MNPV when the reinvestment rate is less (greater) than the appropriate risk-adjusted discount rate consistently.

Table 1 also shows that when the reinvestment and financing rates are equal to the WACC (Panel C) $k^*$ is equal to MIRR for all three projects. On the other hand, when the reinvestment rate is greater than the financing rate (Panel C & D) $k^*$ is greater than MIRR but less than IRR, as long as the reinvestment rate is greater than the firm's WACC but less than the project's IRR. Furthermore, for each reinvestment rate assumption there is a unique generalised rate of return and is directly related to the reinvestment rate and $n^{th}$ root of the profitability index (PI) where $n$ is the project's life. The table also shows that the MNPV is a function of the discount rate, reinvestment rate, financing rate and the cash flow pattern. Notice that only in case of a project with no intermediate cash flows, such as project Y, will the reinvestment rates be irrelevant. Finally, one can observe from Panel C and Panel D that MNPV and $k^*$ lead to a consistent accept/reject decisions.

Figure 1, illustrated below, shows graphically the relationship between Net Present Value (NPV) and Modified Net Present Value (MNPV) of project X when the discount rates are varied from 0% to 35%.

Figure 1.: MNPV and NPV Profiles



Examination of Figure 1, the MNPV and NPV profile of project X's cash flow, shows that the NPV is equal to zero when the discount rate is equal to the project's IRR. In contrast, the MNPV is equal to zero when the discount rate is equal to the project's $k^*$ (in this case $k^*$ = MIRR). The MNPV profile is drawn with assumption of constant reinvestment rate at WACC throughout, but the discount rate varies as in the NPV profile.

When the risk-adjusted discount rate of a project is less than the firm's average marginal cost of capital (WACC), the NPV computation consistently underestimates the MNPV of the project's cash flow. The MNPV and NPV profiles, Figure 1, shows that this underestimation increases with increase in the spread between the firm's WACC and risk-adjusted discount rate. This occurs because the NPV computation implicitly assumes that the project's cash flows are reinvested at the lower risk-adjusted discount rate rather than the firm's WACC. This underestimation could lead to frequent rejection of low- risk projects that may add value to the firm's shareholders.

For a firm that maximises shareholder's wealth, investment of cash inflows at a rate below the firm's WACC is inconsistent with shareholder's wealth maximisation. On the other hand, when the risk-adjusted discount rate is greater than the firm's WACC (see MNPV vs NPV profile), NPV overestimates the MNPV of the project's cash flows. This overestimation will lead to frequent acceptance of high-risk projects that may decrease the firm's shareholders value. Again, this overestimation occurs because the computation of NPV assumes the cash flows from high-risk project will be reinvested in other similar high risky projects. Furthermore, the estimation biases are not symmetrical and varies by the pattern of cash flows of the project(s), the riskiness of the project(s) under consideration, the reinvestment rate and the financing rate. Although the biases are higher for high-risk projects and lower for low-risk projects these biases still lead to frequent acceptance of high-risk projects and rejection of low risk projects.

Finally, the NPV and MNPV are equal when the discount rate, financing rate and reinvestment rates are equal. When the discount rate is not equal to the reinvestment rates, the MNPV formulation provides a more realistic decision making tool, because it takes into account the reinvestment rate opportunities available to the firm. There is no reason to believe that the firm's reinvestment rate will change simply because of the risk of the particular project being considered. It is our belief that the appropriate reinvestment rate should normally be the firm's opportunity cost of capital (WACC) not the risk-adjusted discount rate. The assumption that the NPV approach uses the 'correct' reinvestment rate is only true when all of the project's cash flows are discounted at the firm's WACC. Otherwise, the NPV shares one of the shortcomings of IRR, that is assumption of incorrect reinvestment rate. Furthermore, the generalised rate of return, k*, gives MIRR and IRR as a special case and leads to the same conclusion as profitability index.

## Conclusion

In this paper we have shown that, when the firm's opportunity rate of reinvestment is different from its financing rate, the risk-adjusted discount rate method (RADR) of computing Net Present Value (NPV) leads to biased results. We proposed a new methodology, the Modified NPV (MNPV), to eliminate the RADR methodology bias and in the process derived a generalised rate of return under all combinations of reinvestment and financing rates for a firm. Each reinvestment rate assumption has its own unique generalised rate of return. In addition we derive and demonstrate the linkages and consistency between the new MNPV, k*, PI and MIRR methodologies.

The RADR methodology of computing the NPV uses two incorrect assumptions when the project is above (below) average risk and the discount rate is risk-adjusted. We have first pointed out that the cash inflows are reinvested at the risk-adjusted discount rate instead of the firms reinvestment rate (WACC) thus overestimating (underestimating) the present value of the cash inflows. We also pointed out that when the project's cash flows are irregular and negative cash flows (cash outflows) occur after period zero, these cash flows are discounted at the same risk adjusted discount rate instead of the firm's financing rate and hence underestimating (overestimating) the firm's financing cost. The combination of the two leads to the overestimation (underestimation) of the above (below) average risk project's NPV. Using simple examples, we have shown that these over (under) estimations could be large. We have shown that the MNPV corrects these problems and will lead to a more accurate selection of above (below) average risk projects. We have developed a generalised rate of return (k*) that gives MIRR and IRR as a special case and leads to a consistent accept/reject decision. We have shown that the project's rate of return (k*) is a function of the opportunity reinvestment rate, the firm's financing rate, the project's life and the profitability index (PI), and it leads to the same accept/reject decision as PI. Furthermore, MNPV and k* are modelled using the same reinvestment rate assumption and hence are more consistent measures than was the case previously.

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## Appendix A

The computation of MNPV, given as equation (3) in the text, can be can be simplifies as follows:

$$MNPV = \frac{\sum_{t=1}^{t=n} CF_{it}(1+k_{rr})^{n-1}}{(1+k)^n} - \sum_{t=0}^{t=n} \frac{CF_{ot}}{(1+WACC)^t} \tag{3}$$

Simplification of equation (3) is based on assumption that the reinvestment rate (krr) and the discount rate (k) are constant for the life of the project. The PV of the terminal value (TV), the first term in the right, can be simplified algebraically by factor $(1+k_{rr})^n$ from the numerator of equation (3) to arrive at the following:

$$\frac{\sum_{t=1}^{t=n} CF_{it}(1+k_{rr})^{n-t}}{(1+K)^n} = \frac{1}{(1+k)^n}(CF_{i1}(1+k_{rr})^{n-1} + CF_{i2}(1+k_{rr})^{n-2} + \ldots + CF_{in}(1+k_{rr})^{n-n})$$

$$= \frac{(1+k_{rr})^n}{(1+k)^n}(CF_{i1}(1+k_{rr})^{-1} + CF_{i2}(1+k_{rr})^{-2} + \ldots + CF_{in}(1+k_{rr})^{-n})$$

The expression in the parenthesis is the PV of the cash inflows discounted at $k_{rr}$ and can write $n$ as:

$$= \left(\frac{1+k_{rr}}{1+k}\right)^n \sum_{t=1}^{t=n} CF_{it}(1+k_{rr})^{-1}$$

Now equation (3) can be re-written as:

$$MNVP = \gamma^n \sum_{t=1}^{t=n} CF_{it}(1+k_{rr})^{-1} - \sum_{t=1}^{t=n} \frac{CF_{ot}}{(1+WACC)^t} \tag{3a}$$

where $\gamma = (1+k_{rr})/(1+k)$ is the adjustment factor. The adjustment factor $\gamma^n$ depends on the spread (ratio) between the firm's reinvestment rate, the risk-adjusted discount rate, and the life of the project. In fact equation (3a) says that the MNPV is a factor ($\gamma^n$) times the present value of the cash inflows discounted at the firm's reinvestment rate minus the present value of the cash outflow discounted at the WACC. This formulation provides a very simple and familiar way to compute the MNPV.

## Appendix B

A simpler computation method of generalised rate of return k* can be derived by algebraic manipulation of equation (4) in the text.

$$k^* = \left[ \frac{\sum_{t=1}^{t=n} CF_{it}(1+k_{rr})^{n-t}}{\sum_{t=0}^{t=n} CF_{ot}(1+WACC)^{-t}} \right]^{1/n} - 1 \qquad (4)$$

If we assume that the reinvestment rate ($k_{rr}$) is constant for the life of the project, we can factor out $(1+k_{rr})^n$ from the numerator of the expression in bracket:

$$k^* = \left[ (1+k_{rr})^n * \frac{\sum_{t=1}^{t=n} CF_{it}(1+k_{rr})^{-t}}{\sum_{t=0}^{t=n} CF_{ot}(1+WACC)^{-t}} \right]^{1/n} - 1$$

$$k^* = (1+k_{rr}) \left[ \frac{\sum_{t=1}^{t=n} CF_{it}(1+k_{rr})^{-t}}{\sum_{t=0}^{t=n} CF_{ot}(1+WACC)^{-t}} \right]^{1/n} - 1$$

The numerator of the terms in bracket is the present value of cash inflows discounted at the firms reinvestment rate and the denominator is the present value of the cash outflows discounted at the firms financing rate (in this case the WACC). We know that the ratio of the present value of the projects cash inflow divided by the present value of the projects cash outflow is by definition the profitability index (PI). Therefore, $k^*$ can now be expressed more simply as:

Note that $k^*$ depends on the reinvestment rate, the PI and the projects life. When the reinvestment rate and the financing rate are equal to the firm's WACC, $k^*$ is equal to the MIRR of the project.

$$k^* = (1+k_{rr})[PI]^{1/n} - 1 \qquad (4a)$$