A-statistical approximation by Jayasri operators

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Abstract. *In this study we investigate the A- statistical approximation properties of a sequence of the Jayasri operators. Also we consider the degree of the A-statistical approximation of the sequence of these operators.*

Key words: *A-statistical convergence, positive linear operators, approximation, degree of approximation, Korovkin type theorem*

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1. Introduction

The Jayasri matrix has been introduced and studied by C. Jayasri [10]. The Jayasri matrix is used to construct a sequence of positive linear operators which are called Jayasri operators by J.P. King in [11]. King has proved a Korovkin type theorem and investigated the approximation properties of these operators in [11].

Recently the use of A- statistical convergence in approximation theory has been considered in [2], [8].

The aim of this paper is to investigate a Korovkin type approximation theorem via A-statistical convergence in the space of continuous functions. Especially, using A-statistical convergence, we deal with the approximation properties of the Jayasri operators. We also give some quantitative estimates for A-statistical convergence of approximating operators generated by the Jayasri matrix.

In order to establish the next results, we recall some definitions and notations.

Let K be a subset of $\mathbf{N}$, the set of natural numbers. The density of K is defined by $\delta(K) := \lim_n \frac{1}{n} \sum_{k=1}^{n} \chi_K(k)$ provided limit exists, where χ_K is a characteristic function of K.

Let $A := (a_{jn}),\ j, n = 1, 2, ...,$ be an infinite summability matrix. For a given sequence $x := (x_n)$, the A-transform of x, denoted by $Ax := ((Ax)_j)$, is given by

$$(Ax)_j = \sum_{n=1}^{\infty} a_{jn} x_n,$$

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provided the series converges for each j. We say that A is regular if $\lim_j (Ax)_j = L$ whenever $\lim x = L$ [9]. Suppose that A is a non-negative regular summability matrix. A sequence $x = (x_n)$ is called A-statistically convergent to L if for every $\varepsilon > 0$

$$\lim_j \sum_{n:\, |x_n - L| \geq \varepsilon} a_{jn} = 0.$$

In this case we write $st_A - \lim x = L$ [4], [7], [12], [16].

The case in which $A = C_1$, the Cesáro matrix of order one, reduces to the statistical convergence [3], [5], [6]. Also if $A = I$, the identity matrix, then it reduces to the ordinary convergence.

We note that if $A = (a_{jn})$ is a non-negative regular matrix such that

$$\lim_j \max_n \{a_{jn}\} = 0,$$

then A-statistical convergence is stronger than convergence [12].

It should be noted that the concept of A-statistical convergence may also be given in normed spaces: Assume $(X, \|.\|)$ is a normed space and $u = (u_k)$ is an X-valued sequence. Then (u_k) is said to be A-statistically convergent to $u_0 \in X$ if, for every $\varepsilon > 0$, $\delta_A \{k \in \mathbf{N} : \|u_k - u_0\| \geq \varepsilon\} = 0$ [13], [14].

2. A-statistical approximation by Jayasri operators

Let $J = (q_{nk})$ be the matrix defined by

$$q_{00} = 1,\ q_{0k} = 0 \text{ for } \quad k > 0,$$

and

$$\prod_{v=1}^{n} (f_v(z)h_v + 1 - h_v) = \sum_{k=0}^{\infty} q_{nk} z^k, \tag{1}$$

where $\{f_v\}$ is a sequence of entire functions and $\{h_v\}$ is a sequence of complex numbers. The matrix given by (1) is denoted by $J = J(f_v, h_v)$ and called the Jayasri matrix [10].

Another special case of the Jayasri matrix is the Euler matrix $A = (q_{nk})$ given by

$$q_{nk} = \begin{cases} \binom{n}{k} r^k (1-r)^{n-k}, & 0 \leq k \leq n. \\ 0, & n < k. \end{cases}$$

where r is a complex constant. The Euler matrix appears in approximation theory as the kernel of the n^{th} Bernstein polynomial $B_n(g)$, associated with a real function g defined on $[0, 1]$. The Bernstein polynomial is defined by

$$B_n(g)(x) = \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} g\left(\frac{k}{n}\right),\ x \in [0, 1].$$

It is well known that $\{B_n(g)\}$ is uniformly convergent to g if g is continuous on $[0,1]$. Therefore the Bernstein polynomials and indirectly the Euler matrix- provide a constructive proof of the classical Weierstrass approximation theorem. Approximation properties of the Jayasri operators generated by the Jayasri matrix which is a generalization of the Euler matrix are studied by J.P. King [11].

In order to study the approximation properties of the Jayasri operators we assume the following ([11]):

Let $J(f_v, h_v) = (q_{nk})$ be the Jayasri matrix and let

i) f_v be an entire function for $v = 1, 2, ...$

ii) $f_v(1) = 1,\ v = 1, 2, ...$

iii) $f_v^{(k)}(0) \geq 0,\ v = 1, 2, ...$ and $k = 0, 1, 2, ...$

iv) $h_v = h_v(x)$ be defined on $[0,1]$, $v = 1, 2, ...$

v) $0 \leq h_v(x) \leq 1, \quad v = 1, 2, ...$ and $0 \leq x \leq 1$.

Then the generating functions in (1) will be given by

$$\prod_{v=1}^{n} (f_v(z)h_v(x) + 1 - h_v(x)) = \sum_{k=0}^{\infty} q_{nk}(x)z^k, \tag{2}$$

with $q_{nk}(x) \geq 0,\ k = 0, 1, ...,\ n = 0, 1, ...$

Let the sequences $\{f_v\}$ and $\{h_v\}$ be given as above. Fix $x \in [0,1]$ and let

$$P_n(z) = \prod_{v=1}^{n} (f_v(z)h_v(x) + 1 - h_v(x)). \tag{3}$$

The Jayasri operators are defined by

$$J_n(g)(x) = \sum_{k=0}^{\infty} q_{nk}(x) g\left(\frac{k}{n}\right), \quad n = 0, 1, ... \tag{4}$$

where $(q_{nk}(x))$ is given by (2) and g is a real valued function which is bounded on $[0,\infty)$ and continuous on $[0,1]$. It is easily seen that the Jayasri operators defined by (4) are linear and positive.

As usual $C[0,1]$ will denote the space of all continuous functions on $[0,1]$. Recall that $C[0,1]$ is a Banach space with norm

$$\|f\|_{C[0,1]} = \max_{x\in[0,1]} |f(x)|.$$

In this section we give the A-statistical approximation properties of the Jayasri operators.

Lemma 1. *Let $A = (a_{jn})$ be a non-negative regular summability matrix and let $\{J_n(g)\}$ be a sequence of the Jayasri operators defined by (4). If* *(a)* $st_A - \lim_n \left\| \frac{1}{n} \sum_{v=1}^{n} f_v^{'}(1) h_v(x) - x \right\|_{C[0,1]} = 0,$

(b) $st_A - \lim_n \left\| \frac{1}{n^2} \sum_{v=1}^{n} f_v^{''}(1) h_v(x) \right\|_{C[0,1]} = 0,$

(c) $st_A - \lim_n \left\| \frac{1}{n^2} \sum_{v=1}^{n} \left(f_v^{'}(1) h_v(x) \right)^2 \right\|_{C[0,1]} = 0$

then

$$st_A - \lim_n \|J_n(e_s)(x) - e_s(x)\|_{C[0,1]} = 0$$

where $e_s(x) = x^s$ *and* $s = 0, 1, 2$; *and* $\{f_v\}$, $\{h_v\}$ *are the sequences satisfying (i)-(iii) and (iv)-(v), respectively.*

Proof. The operators J_n defined by (4) are linear and positive because of (iii) and (iv) $J_n(g) \geq 0$ whenever $g \geq 0$.

Obviously that $P_n(1) = 1$ from (3). By (2) and (3) we get

$$J_n(e_0)(x) = 1 = e_0(x).$$

Hence we have

$$st_A - \lim_n \|J_n(e_0)(x) - e_0(x)\|_{C[0,1]} = 0.$$

Considering (3) we write

$$\log P_n(z) = \sum_{v=1}^{n} \log \left(f_v(z) h_v(x) + 1 - h_v(x) \right)$$

so that

$$P_n^{'}(z) = P_n(z) \sum_{v=1}^{n} \frac{f_{\ v}^{'}(z) h_v(x)}{f_v(z) h_v(x) + 1 - h_v(x)} \tag{5}$$

when the differentiation is with respect to z.

From (2) we have

$$P_n^{'}(z) = \sum_{k=0}^{\infty} k q_{nk}(x) z^{k-1}$$

and

$$J_n(e_1)(x) = \sum_{k=0}^{\infty} q_{nk}(x) \frac{k}{n} = \frac{1}{n} P_n^{'}(1)$$

or

$$J_n(e_1)(x) = \frac{1}{n} \sum_{v=1}^{n} f_{\ v}^{'}(1) h_v(x).$$

By condition (a) we obtain

$$st_A - \lim_n \|J_n(e_1)(x) - e_1(x)\|_{C[0,1]} = st_A - \lim_n \left\| \frac{1}{n} \sum_{v=1}^{n} f_v^{'}(1) h_v(x) - x \right\|_{C[0,1]} = 0.$$

Since

$$J_n(e_2)(x) = \frac{1}{n^2}\sum_{k=0}^{\infty} k^2 q_{nk}(x)$$

and

$$\sum_{k=0}^{\infty} k^2 q_{nk}(x) = P_n^{''}(1) + P_n^{'}(1)$$

we get

$$J_n(e_2)(x) = \frac{1}{n^2}\left(P_n^{''}(1) + P_n^{'}(1)\right).$$

Now (5) yields

$$P_n^{''}(1) = P_n^{'}(1)\sum_{v=1}^{n} f_v^{'}(1)h_v(x) + \sum_{v=1}^{n} f_v^{''}(1)h_v(x) - \sum_{v=1}^{n}\left(f_v^{'}(1)h_v(x)\right)^2.$$

Hence

$$|J_n(e_2)(x) - e_2(x)| \le \left|\left(\sum_{v=1}^{n} f_v^{'}(1)h_v(x)\right)^2 - x^2\right| + \frac{1}{n^2}\sum_{v=1}^{n} f_v^{''}(1)h_v(x)$$

$$+\frac{1}{n^2}\sum_{v=1}^{n}\left(f_v^{'}(1)h_v(x)\right)^2 + \frac{1}{n}\left|\frac{1}{n}\sum_{v=1}^{n} f_v^{'}(1)h_v(x) - x\right| + \frac{1}{n}x$$

$$= S_1(n) + S_2(n) + S_3(n) + S_4(n) + \frac{1}{n}x, \text{ say.} \tag{6}$$

Now, for a given $\varepsilon > 0$ define

$$U = \left\{n : S_1(n) + S_2(n) + S_3(n) + S_4(n) + \frac{1}{n}x \ge \varepsilon\right\},$$

$$U_1 = \left\{n : S_1(n) \ge \frac{\varepsilon}{5}\right\}, \quad U_2 = \left\{n : S_2(n) \ge \frac{\varepsilon}{5}\right\},$$

$$U_3 = \left\{n : S_3(n) \ge \frac{\varepsilon}{5}\right\}, \quad U_4 = \left\{n : S_4(n) \ge \frac{\varepsilon}{5}\right\},$$

$$U_5 = \left\{n : \frac{1}{n} \ge \frac{\varepsilon}{5}\right\}.$$

It is easy to see that $U \subseteq U_1 \cup U_2 \cup U_3 \cup U_4 \cup U_5$. Therefore by (6) we have

$$\sum_{n:\,|J_n(e_2)(x)-e_2(x)|\ge\varepsilon} a_{jn} \le \sum_{n\in U} a_{jn} \le \sum_{n\in U_1} a_{jn} + \sum_{n\in U_2} a_{jn}$$

$$+ \sum_{n\in U_3} a_{jn} + \sum_{n\in U_4} a_{jn} + \sum_{n\in U_5} a_{jn}.$$

Taking limit as $j \to \infty$, conditions (a)-(c) give the result. We note that since $\frac{1}{n} \to 0$ $(n \to \infty)$, $st_A - \lim_n \frac{1}{n} = 0$. □

Now using *Lemma 1* we have the following Korovkin type theorem for the sequence $\{J_n\}$ of the operators given by (4). Recall that some results on approximation properties of positive linear operators may be found in [1], [15].

Theorem 1. *Let $A=(a_{jn})$ be a non-negative regular summability matrix. If*

$$st_A - \lim_n \|J_n(e_s)(x) - e_s(x)\|_{C[0,1]} = 0,\ \ s=0,1,2 \tag{7}$$

then

$$st_A - \lim_n \|J_n(g)(x) - g(x)\|_{C[0,1]} = 0$$

for every function $g \in C[0,1]$ which is bounded on $[0,\infty)$.

Proof. From *Lemma 1* we have conditions (7). So the result follows from Theorem 1 in [8] (see also [2]). We note that Theorem 1 in [8] is given for statistical convergence but the proof also works for A-statistical convergence. □

If we take $A=I$, the identity matrix, then we have Theorem 2.1 in [11]. We recall that Theorem 2.1 deals with pointwise convergence of $\{J_n(g)\}$ to g but Theorem 2.1 also gives uniform convergence provided the convergence hypotheses hold uniformly.

Corollary 1. *If $0 \le f_v^{''}(1) \le f_v^{'}(1) \le 1,\ v=1,2,...,$ in addition to (i), (ii), (iii) and (iv), then*

$$st_A - \lim_n \|J_n(g)(x) - g(x)\|_{C[0,1]} = 0$$

for $x\in[0,1]$ provided only

$$\text{(a) } st_A - \lim_n \left\| \frac{1}{n}\sum_{v=1}^{n} f_v^{'}(1)h_v(x) - x \right\|_{C[0,1]} = 0.$$

Proof. Since $0 \le f_v^{''}(1) \le f_v^{'}(1) \le 1,\ v=1,2,...$ we write

$$0 \le \frac{1}{n^2}\sum_{v=1}^{n} f_v^{''}(1)h_v(x) \le \frac{1}{n^2}\sum_{v=1}^{n} f_v^{'}(1)h_v(x) = \frac{1}{n}\Big(\frac{1}{n}\sum_{v=1}^{n} f_v^{'}(1)h_v(x) - x\Big) + \frac{1}{n}x. \tag{8}$$

For a given $\varepsilon>0$ define

$$\begin{aligned} U &= \left\{ n : \frac{1}{n^2}\sum_{v=1}^{n} f_v^{''}(1)h_v(x) \ge \varepsilon \right\} \\ U_1 &= \left\{ n : \frac{1}{n}\left(\frac{1}{n}\sum_{v=1}^{n} f_v^{'}(1)h_v(x) - x\right) \ge \varepsilon/2 \right\} \\ U_2 &= \left\{ n : \frac{1}{n}x \ge \varepsilon/2 \right\}. \end{aligned}$$

Since $U \subset U_1 \cup U_2$, (8) implies that

$$\sum_{n\in U} a_{jn} \le \sum_{n\in U_1} a_{jn} + \sum_{n\in U_2} a_{jn}.$$

Taking limit as $j \to \infty$ we obtain

$$st_A - \lim_n \left\| \frac{1}{n^2} \sum_{v=1}^{n} f_v^{''}(1) h_v(x) \right\|_{C[0,1]} = 0.$$

Thus hypothesis (b) of *Lemma 1* holds. Also we have

$$0 \le \frac{1}{n^2} \sum_{v=1}^{n} \left(f_v^{'}(1) h_v(x) \right)^2 \le \frac{1}{n^2} \sum_{v=1}^{n} f_v^{'}(1) h_v(x).$$

So hypothesis (c) of *Lemma 1* also holds and the corollary is proved. □

Corollary 2. *If $f_v^{'}(1) = 1$, $v = 1, 2, ...,$ $\left\{ f_v^{''}(1) \right\}$ is a bounded sequence and if (i), (ii), (iii) and (iv) hold then*

$$st_A - \lim_n \|J_n(g)(x) - g(x)\|_{C[0,1]} = 0$$

provided

$$st_A - \lim_n \frac{1}{n} \sum_{v=1}^{n} h_v(x) = x, \ x \in [0,1]. \tag{9}$$

Proof. From (9) and $f_v^{'}(1) = 1, \ v = 1, 2, ...$ we get condition (a) of *Lemma 1*. Since $\left\{ f_v^{''}(1) \right\}$ is a bounded sequence there exists some M such that $\left| f_v^{''}(1) \right| \le M$ so that by (9)

$$0 \le st_A - \lim_n \frac{1}{n^2} \sum_{v=1}^{n} f_v^{''}(1) h_v(x) \le st_A - \lim_n M \frac{1}{n^2} \sum_{v=1}^{n} h_v(x) = 0.$$

Hence (b) and similarly (c) hold. Therefore the hypotheses of *Lemma 1* hold and so *Corollary 2* is proved. □

Remark 1. *We now present an example of a sequence of positive linear operators satisfying the conditions of* Theorem 1 *but that does not satisfy the conditions of Theorem 2.1 of King [11].*

Assume now that $\{u_n\}$ is an A-statistically null sequence but not convergent. Notice that, if $A = (a_{jn})$ is a non-negative regular matrix such that $\lim_j \max_n \{a_{jn}\} = 0$, then A-statistical convergence is stronger than convergence [12]. Without loss of generality we may assume that $\{u_n\}$ is non-negative; otherwise we would replace $\{u_n\}$ by $\{|u_n|\}$. Now define $\{P_n\}$ on $C[0,1]$ by

$$P_n(g)(x) = (1 + u_n) J_n(g)(x)$$

where $\{J_n\}$ is the sequence of Jayasri operators. Now observe that $\{J_n\}$ being convergent and $\{u_n\}$ being A-statistical null, their product will also be A-statistical null. Hence $\{P_n\}$ will not be convergent to g but A-statistically convergent to g.

3. Degree of A-statistical approximation

The modulus of continuity of the function f in $C[0,1]$ is defined as

$$\omega(f,\delta)=\sup_{|x-y|<\delta}|f(x)-f(y)|\,,\ x,y\in[0,1]\,.$$

It is well known that a necessary and sufficient condition for a function $f\in C[0,1]$ is

$$\lim_{\delta\to 0}\omega(f,\delta)=0.$$

It is also well known that for any constant $\lambda>0,\ \delta>0$

$$\omega(f,\lambda\delta)\leq(1+\lambda)\omega(f,\delta). \tag{10}$$

Let $A=(a_{nk})$ be a non-negative regular summability matrix and let (a_n) be a positive non-increasing sequence. Following [2] we say that the sequence $x=(x_k)$ is A-statistical convergent to number L with the rate of $o(a_n)$ if for every $\varepsilon>0$,

$$\lim_n\frac{1}{a_n}\sum_{k:\,|x_k-L|\geq\varepsilon}a_{nk}=0.$$

In this case we write

$$x_k-L=st_A-o(a_n),\quad (\text{as } k\to\infty).$$

The following Lemma may be found in [2], but it could also be proved directly.

Lemma 2 [2]. *Let $x=(x_k)$ and $y=(y_k)$ be two sequences. Assume that $A=(a_{nk})$ is a non-negative regular summability matrix. Let (a_n) and (b_n) be positive non-increasing sequences. If for some real numbers L_1, L_2, we have $x_k-L_1=st_A-o(a_k)$ and $y_k-L=st_A-o(b_k)$ as $k\to\infty$, then the following holds:*

(I) $(x_k-L_1)\pm(y_k-L_2)=st_A-o(c_k)$

(II) $(x_k-L_1)\,(y_k-L_2)=st_A-o(c_k)$, *where* $c_n=\max\{a_n,b_n\}$.

Now we find the degree of A-statistical approximation for the sequence of positive linear operators $\{J_n\}$ given by (4).

Theorem 2. *Let $A=(a_{jn})$ be a non-negative regular summability matrix. If the sequence of positive linear operators $\{J_n\}$ satisfies the conditions*

(a) $J_n(e_0)(x)-e_0(x)=st_A-o(a_n(x))$ *with* $e_0(x)=1$,

(b) $\omega(g;\alpha_n(x))=st_A-o(b_n(x))$ *with* $\alpha_n(x)=\sqrt{J_n(\varphi_x(y))}$ *and* $\varphi_x(y)=(y-x)^2$,

where $(a_n(x))$ and $(b_n(x))$ are non-increasing sequences, then

$$J_n(g)(x)-g(x)=st_A-o(c_n(x))$$

where $c_n(x)=\max\{a_n(x),\,b_n(x)\}$.

Proof. Considering (10) we can write

$$\begin{aligned}|J_n(g)(x)-g(x)| &\le \sum_{k=0}^{\infty} q_{nk}(x)\left|(g)\left(\frac{k}{n}\right)-g(x)\right| \\ &\le \omega(g;\delta_n)\sum_{k=0}^{\infty} q_{nk}(x)\left[1+\frac{\left|\frac{k}{n}-x\right|}{\delta_n}\right] \\ &= \omega(g;\delta_n)\left[J_n(e_0)(x)+\frac{1}{\delta_n}\sum_{k=0}^{\infty} q_{nk}(x)\left|\frac{k}{n}-x\right|\right].\end{aligned}$$

Applying the Cauchy-Schwartz inequality to $\sum\limits_{k=0}^{\infty} q_{nk}(x)\left|\frac{k}{n}-x\right|$ we obtain

$$\begin{aligned}|J_n(g)(x)-g(x)| &\le \omega(g;\delta_n)\left[J_n(e_0)(x)+\frac{1}{\delta_n}\left(\sum_{k=0}^{\infty} q_{nk}(x)\left(\frac{k}{n}-x\right)^2\right)^{1/2}\right] \\ &= \omega(g;\delta_n)\left[J_n(e_0)(x)+\frac{1}{\delta_n}\sqrt{J_n((y-x)^2)(x)}\right].\end{aligned}$$

Choosing $\delta_n=\sqrt{J_n((y-x)^2)(x)}=\alpha_n(x)$ we have

$$\begin{aligned}|J_n(g)(x)-g(x)| &\le \omega(g;\alpha_n(x))\left[J_n(e_0)(x)+1\right] \\ &\le 2\omega(g;\alpha_n(x))+\omega(g;\alpha_n(x))\left|J_n(e_0)(x)-(e_0)(x)\right|.\end{aligned}$$

This implies that

$$\begin{aligned}\frac{1}{c_n(x)}\sum_{n:\,|J_n(g)(x)-g(x)|\ge\varepsilon} a_{jn} \le \frac{1}{b_n(x)}&\sum_{n:\,2\omega(g;\alpha_n(x))\ge\varepsilon/2} a_{jn} \\ +\frac{1}{c_n(x)}&\sum_{n:\,\omega(g;\alpha_n(x))|J_n(e_0)(x)-(e_0)(x)|\ge\varepsilon/2} a_{jn}.\end{aligned}$$

Now conditions (a), (b) and *Lemma 2* yield the proof. □

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