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Area formulas for a triangle in the alpha plane

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Abstract. In this paper, we give three area formulas for a triangle in the alpha plane in terms of the alpha distance. The two of them are alpha versions of the standard area formula for a triangle in the Euclidean plane, and the third one is an alpha version of the well-known Heron's formula.
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Key words: alpha distance, alpha plane, area, Heron's formula, plane geometry, protractor geometry

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1. Introduction

The *taxicab metric* was given in a family of metrics of the real plane by Minkowski [12]. Later, Chen [1] developed the *Chinese Checker metric*, and Tian [14] gave a family of metrics, α-*metric* (*alpha metric*) for $\alpha \in [0, \pi/4]$, which includes the taxicab and Chinese checker metrics as special cases. Then, Gelişgen and Kaya extended the α-distance to three and n dimensional spaces in [7] and [8], respectively. Afterwards, Çolakoğlu [2] extended the α-metric for $\alpha \in [0, \pi/2)$. According to the latter, if $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are two points in $\mathbb{R}^2$, then for each $\alpha \in [0, \pi/2)$ and $\lambda(\alpha) = (\sec\alpha - \tan\alpha)$, the α-*distance* between P and Q is

$$d_\alpha(P, Q) = \max\{|x_1 - x_2|, |y_1 - y_2|\} + \lambda(\alpha)\min\{|x_1 - x_2|, |y_1 - y_2|\}, \quad (1)$$

while the well-known Euclidean distance between P and Q is

$$d_E(P, Q) = [(x_1 - x_2)^2 + (y_1 - y_2)^2]^{1/2}. \quad (2)$$

Since α-geometry has a distance function different from that of Euclidean geometry, it is interesting to study the α-analogues of topics that include the distance concept in Euclidean geometry. In this paper, we give area formulas for a triangle in the alpha plane in terms of the alpha distance.

Remark 1. *In this study, we use the usual Euclidean area notion. One can easily see that in the α-plane, there are triangles whose α-lengths of corresponding sides are the same, while areas of these triangles are different (see Figure 1).*

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This fact gives rise to a natural question: How can one compute the area of a triangle in the α-plane? It is obvious that every formula to compute the area of a triangle depends on some parameters, and using different parameters gives different formulas. Here we give three formulas to compute the area of a triangle in the α-plane, using different parameters.

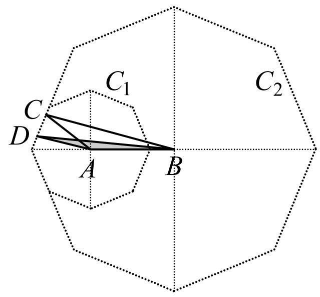


Figure 1:

Let the line AB be parallel to the x-axis, let C_1 be an α-circle with center A and radius b, C_2 an α-circle with center B and radius $b+c$, and C and D two points in $C_1 \cap C_2$. For different C and D such that C and D are not symmetric to the line AB, $Area(ABC) \neq Area(ABD)$, while $d_\alpha(A,C) = d_\alpha(A,D)$ and $d_\alpha(B,C) = d_\alpha(B,D)$.

2. Area of a triangle in α-plane

It is well-known that if ABC is a triangle with the area $\mathcal{A}$ in the Euclidean plane, and H is the point of orthogonal projection of the point A on the line BC, then standard area formula for the triangle ABC is $\mathcal{A} = \mathbf{ah}/2$, where $\mathbf{a} = d_E(B,C)$ and $\mathbf{h} = d_E(A,H)$ or $\mathbf{h} = d_E(A,BC)$ (see Figure 2). In this section, we give two α-versions of standard area formula in terms of α-distance. Clearly, an α-version of standard area formula for triangle ABC would be an equation that relates the two α-distances a and h, where $a = d_\alpha(B,C)$, $h = d_\alpha(A,H)$ or $h = d_\alpha(A,BC)$ and area $\mathcal{A}$ of triangle ABC. Here, we give two α-versions of the area formula that depend on one parameter, namely, the slope of the base segment, in addition to the other parameters. Note that the real numbers α and $\lambda(\alpha)$ are fixed.

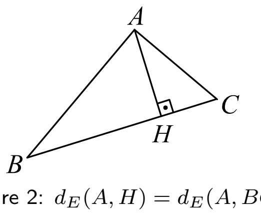


Figure 2: $d_E(A,H) = d_E(A,BC)$

The following equation, which relates the Euclidean distance to the α -distance between two points in the Cartesian coordinate plane, plays an important role in the first α-version of the area formula.

Proposition 1. *For any two points P and Q in the Cartesian plane that do not lie on a vertical line, if m is the slope of the line through P and Q, then*

$$d_E(P,Q) = \rho(m) d_\alpha(P,Q) \tag{3}$$

where $\rho(m) = (1+m^2)^{1/2}/(\max\{1,|m|\} + \lambda(\alpha)\min\{1,|m|\})$. If P and Q lie on a vertical line, then by definition, $d_E(P,Q) = d_\alpha(P,Q)$.

Proof. Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ with $x_1 \neq x_2$; then $m = (y_2 - y_1)/(x_2 - x_1)$. Equation (3) is derived by a straightforward calculation with m and the coordinate definitions of $d_E(P,Q)$ and $d_\alpha(P,Q)$ given in Section 1. □

Another useful fact that can be verified by direct calculation is:

Proposition 2. *For any real number $m \neq 0$*

$$\rho(m) = \rho(-m) = \rho(1/m) = \rho(-1/m). \tag{4}$$

We first note by Proposition 1 and Proposition 2 that the α-distance between two points is invariant under all translations, rotations of $\pi/2$, π and $3\pi/2$ radians around a point, and the reflections about the lines parallel to $x = 0$, $y = 0$, $y = x$ or $y = -x$ (see [5]).

The following theorem gives an α-version of the well-known Euclidean area formula of a triangle:

Theorem 1. *Let ABC be a triangle with the area $\mathcal{A}$ in the α-plane, H orthogonal projection (in the Euclidean sense) of the point A on the line BC, m the slope of the line BC, and let $a = d_\alpha(B,C)$ and $h = d_\alpha(A,H)$.*

(*i*) *If BC is parallel to a coordinate axis, then*

$$\mathcal{A} = ah/2. \tag{5}$$

(*ii*) *If BC is not parallel to any one of the coordinate axes, then*

$$\mathcal{A} = [\rho(m)]^2 \, ah/2 \tag{6}$$

where $\rho(m) = (1+m^2)^{1/2}/(\max\{1,|m|\} + \lambda(\alpha)\min\{1,|m|\})$.

Proof. Let $\mathbf{a} = d_E(B,C)$ and $\mathbf{h} = d_E(A,H)$. Then, $\mathcal{A} = \mathbf{ah}/2$.

(i): If BC is parallel to a coordinate axis, then clearly $a = \mathbf{a}$ and $h = \mathbf{h}$. Hence, $\mathcal{A} = ah/2$.

(ii): Let BC not be parallel to any one of the coordinate axes, and let the slope of the line BC be m. Then, the slope of the line AH is $(-1/m)$. By Proposition 1 and Proposition 2, $\mathbf{a} = \rho(m)a$, $\mathbf{h} = \rho(m)h$, hence $\mathcal{A} = \rho^2(m)ah/2$. □

In the α-plane, the α-distance from a point P to a line l is defined by

$$d_\alpha(P,l) = \min_{Q \in l}\{d_\alpha(P,Q)\} \tag{7}$$

as in the Euclidean plane. It is well-known that in the Euclidean plane, Euclidean distance from a point $P = (x_0, y_0)$ to a line $l : ax + by + c = 0$ can be calculated by the following formula:

$$d_E(P, l) = \frac{|ax_0 + by_0 + c|}{(a^2 + b^2)^{1/2}}. \tag{8}$$

In Proposition 4 we give a similar formula for $d_\alpha(P, l)$, using α-circles (see [2]). It is easy to see that if $\alpha \in (0, \pi/2)$, then unit α-circle is an octagon with vertices $A_1 = (1, 0)$, $A_2 = (\frac{1}{k}, \frac{1}{k})$, $A_3 = (0, 1)$, $A_4 = (\frac{-1}{k}, \frac{1}{k})$, $A_5 = (-1, 0)$, $A_6 = (\frac{-1}{k}, \frac{-1}{k})$, $A_7 = (0, -1)$ and $A_8 = (\frac{1}{k}, \frac{-1}{k})$, where $k = 1 + \lambda(\alpha)$. If $\alpha = 0$, then α-circle is a square with vertices $A_1 = (1, 0)$, $A_3 = (0, 1)$, $A_5 = (-1, 0)$ and $A_7 = (0, -1)$ (see Figure 3). Note that A_2 and A_6 are on the line $y = x$; A_4 and A_8 are on the line $y = -x$.

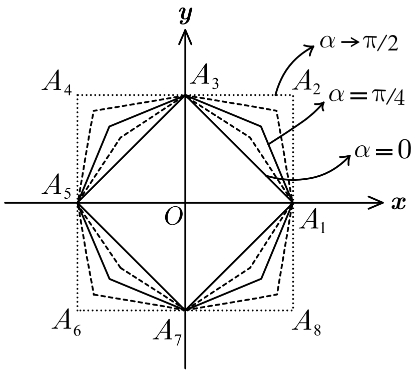


Figure 3: *Graph of the unit α-circle*

Proposition 3. *Given a point $P = (x_0, y_0)$, and a line $l : ax + by + c = 0$ in the α-plane. Then the α-distance from the point P to the line l can be calculated by the following formula:*

$$d_\alpha(P, l) = |ax_0 + by_0 + c| \,/ \max\left\{|a|, |b|, \tfrac{|a+b|}{1+\lambda(\alpha)}, \tfrac{|a-b|}{1+\lambda(\alpha)}\right\}. \tag{9}$$

Proof. Clearly, if P is on the line l, then equation (9) is true. If P is not on the line l, then we expand an α-circle with center P and radius 0 until the line l becomes a *tangent* to the α-circle, to find the minimum α-distance from the point P to the line l (Here, by *tangent* to an α-circle with center P and radius r, we mean "a line whose α-distance from P is equal to r", as in Euclidean plane.). It is easy to see that a line can only be a tangent to an α-circle at one vertex or two vertices (at one edge). Since corresponding vertices of expanding α-circle are on lines through P and parallel to lines $y = 0$, $x = 0$, $y = x$ and $y = -x$, if l is a tangent to an α-circle with center P, then a tangent point is one of points

$$P_1 = \left(\frac{-by_0 - c}{a}, y_0\right), P_2 = \left(x_0, \frac{-ax_0 - c}{b}\right), P_3 = \left(\frac{bx_0 - by_0 - c}{a + b}, \frac{-ax_0 + ay_0 - c}{a + b}\right)$$

and

$$P_4 = \left(\frac{-bx_0 - by_0 - c}{a-b}, \frac{ax_0 + ay_0 + c}{a-b} \right),$$

where P_1, P_2, P_3 and P_4 are intersection points of the line l and $y = y_0$, $x = x_0$, $-x+y+x_0-y_0 = 0$ and $x+y-x_0-y_0 = 0$, respectively (see Figure 4). Therefore, $d_\alpha(P,l) = \min\{d_\alpha(P,P_i) : i = 1,2,3,4\}$.

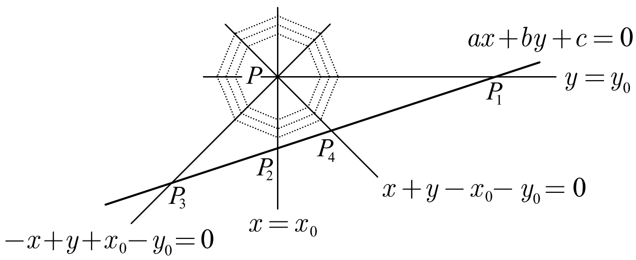


Figure 4:

Since

$$d_\alpha(P,P_1) = \frac{|ax_0+by_0+c|}{|a|}, d_\alpha(P,P_2) = \frac{|ax_0+by_0+c|}{|b|},$$
$$d_\alpha(P,P_3) = \frac{|ax_0+by_0+c|}{|a+b|/(1+\lambda(\alpha))} \text{ and } d_\alpha(P,P_4) = \frac{|ax_0+by_0+c|}{|a-b|/(1+\lambda(\alpha))},$$

we get

$$d_\alpha(P,l) = \frac{|ax_0+by_0+c|}{\max\left\{|a|, |b|, \frac{|a+b|}{1+\lambda(\alpha)}, \frac{|a-b|}{1+\lambda(\alpha)}\right\}}.$$

□

The following equation, which relates the Euclidean distance to the α-distance from a point to a line in the Cartesian coordinate plane, plays an important role in the second α-version of the area formula.

Proposition 4. *Given a point P, and a line l in the Cartesian plane that is not a vertical line, if m is the slope of the line l, then*

$$d_E(P,l) = \tau(m) d_\alpha(P,l) \tag{10}$$

where

$$\tau(m) = \max\left\{|1|, |m|, \frac{|1+m|}{1+\lambda(\alpha)}, \frac{|1-m|}{1+\lambda(\alpha)}\right\} / (1+m^2)^{1/2}.$$

If l is a vertical line, then $d_E(P,l) = d_\alpha(P,l)$.

Proof. Let $P = (x_0, y_0)$ be a point, and $l : ax+by+c = 0$ be a line in the Cartesian plane. If l is not a vertical line, then $b \neq 0$ and $m = -\frac{a}{b}$. Using m in equation (8) and equation (9), one gets $d_E(P,l) = |ax_0+by_0+c| / |b| (1+m^2)^{1/2}$ and

$$d_\alpha(P,l) = |ax_0+by_0+c| / |b| \max\left\{|1|, |m|, \frac{|1+m|}{1+\lambda(\alpha)}, \frac{|1-m|}{1+\lambda(\alpha)}\right\}.$$

Hence, $d_E(P,l) = \tau(m) d_\alpha(P,l)$ where

$$\tau(m) = \max\left\{|1|, |m|, \tfrac{|1+m|}{1+\lambda(\alpha)}, \tfrac{|1-m|}{1+\lambda(\alpha)}\right\} / (1+m^2)^{1/2}.$$

If l is a vertical line, then $b = 0$ and $a \neq 0$. Therefore, $d_E(P,l) = |ax_0 + c| / |a|$ and $d_\alpha(P,l) = |ax_0 + c| / |a|$, hence $d_E(P,l) = d_\alpha(P,l)$. □

The following theorem gives another α-version of the well-known Euclidean area formula of a triangle:

Theorem 2. *Let ABC be a triangle with area $\mathcal{A}$ in the α-plane, m the slope of the line BC, and let $a = d_\alpha(B,C)$ and $h = d_\alpha(A,BC)$.*

(*i*) *If BC is parallel to a coordinate axis, then*

$$\mathcal{A} = ah/2. \tag{11}$$

(*ii*) *If BC is not parallel to any one of the coordinate axes, then*

$$\mathcal{A} = \sigma(m) ah/2 \tag{12}$$

where

$$\sigma(m) = \max\left\{|1|, |m|, \frac{|1+m|}{1+\lambda(\alpha)}, \frac{|1-m|}{1+\lambda(\alpha)}\right\} / (\max\{1,|m|\} + \lambda(\alpha)\min\{1,|m|\}).$$

Proof. Let $\mathbf{a} = d_E(B,C)$ and $\mathbf{h} = d_E(A,BC)$. Then, $\mathcal{A} = \mathbf{ah}/2$.

(i): If BC is parallel to a coordinate axis, then clearly $a = \mathbf{a}$ and $h = \mathbf{h}$. Hence, $\mathcal{A} = ah/2$.

(ii): Let BC not be parallel to any one of the coordinate axes, and let the slope of the line BC be m. Then, by Proposition 1 and Proposition 5, $\mathbf{a} = \rho(m)a$, $\mathbf{h} = \tau(m)h$, hence $\mathcal{A} = \rho(m)\tau(m)ah/2$. Since $\rho(m)\tau(m) = \sigma(m)$, we get $\mathcal{A} = \sigma(m)ah/2$. □

3. Alpha version of Heron's formula

It is well-known that if ABC is a triangle with the area $\mathcal{A}$ in the Euclidean plane, and $\mathbf{a} = d_E(B,C)$, $\mathbf{b} = d_E(A,C)$, $\mathbf{c} = d_E(A,B)$, and $\mathbf{p} = (\mathbf{a}+\mathbf{b}+\mathbf{c})/2$, then $\mathcal{A} = [\mathbf{p}(\mathbf{p}-\mathbf{a})(\mathbf{p}-\mathbf{b})(\mathbf{p}-\mathbf{c})]^{1/2}$, which is known as the *Heron's formula*. In this section, we give an α-version of this formula in terms of the α-distance. Clearly, an α-version of the Heron's formula for triangle ABC would be an equation that relates the three α-distances a, b and c, where $a = d_\alpha(B,C)$, $b = d_\alpha(A,C)$, $c = d_\alpha(A,B)$, and the area $\mathcal{A}$ of triangle ABC. Here, we give an α-version of the Heron's formula that depend on three new parameters in addition to a, b, c and $\mathcal{A}$.

We need the following two definitions given in [13] and [11] respectively, to give an α-version of the Heron's formula:

Definition 1. *Let ABC be any triangle in the α-plane. Clearly, there exists a pair of lines passing through every vertex of the triangle, each of which is parallel to a coordinate axis. A line l is called a base line of ABC if and only if*

(1) *l passes through a vertex;*

(2) *l is parallel to a coordinate axis;*

(3) *l intersects the opposite side (as a line segment) of the vertex in* (1).

Clearly, at least one of vertices of the triangle always has one or two base lines. Such a vertex of the triangle is called a basic vertex. A base segment is a line segment on a base line, which is bounded by a basic vertex and its opposite side.

Definition 2. *A line with slope m is called a steep line, a gradual line and a separator if $|m| > 1$ or $m \to \infty$, $|m| < 1$ and $|m| = 1$, respectively.*

The following theorem gives an α-version of the Heron's formula:

Theorem 3. *Let ABC be a triangle with area $\mathcal{A}$ in the α-plane, such that C is a basic vertex, $a = d_\alpha(B, C)$, $b = d_\alpha(A, C)$ and $c = d_\alpha(A, B)$. Let D be the intersection point of a base line and AB, the opposite side of the basic vertex C. Let H_1 and H_2 be orthogonal projections (in the Euclidean sense) of A and B on the base line CD, respectively. Then,*

$$\mathcal{A} = \begin{cases} \frac{l}{2}\left[2p - c - \lambda(\alpha)(l_1 + l_2)\right] & ; \textit{if } C_1 \textit{ is valid} \\ \frac{l}{2\lambda(\alpha)}\left[2p - c - (l_1 + l_2)\right] & ; \textit{if } C_2 \textit{ is valid} \\ \frac{l}{2\lambda(\alpha)}\left[2p - c + (\lambda(\alpha) - 1)b - (\lambda^2(\alpha)l_1 + l_2)\right] & ; \textit{if } C_3 \textit{ is valid} \\ \frac{l}{2\lambda(\alpha)}\left[2p - c + (\lambda(\alpha) - 1)a - (l_1 + \lambda^2(\alpha)l_2)\right] & ; \textit{if } C_4 \textit{ is valid} \end{cases} \tag{13}$$

where

$$p = (a + b + c)/2, l = d_\alpha(C, D), l_1 = d_\alpha(C, H_1), l_2 = d_\alpha(C, H_2),$$

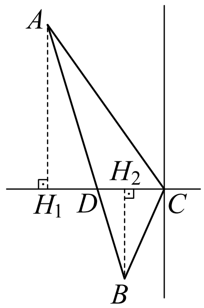
C_1 *: lines AC and BC are not gradual and base line CD is horizontal, or lines AC and BC are not steep and base line CD is vertical;*

C_2 *: lines AC and BC are not steep and base line CD is horizontal, or lines AC and BC are not gradual and base line CD is vertical;*

C_3 *: line AC is not gradual, line BC is not steep and base line CD is horizontal, or line AC is not steep, line BC is not gradual and base line CD is vertical;*

C_4 *: line AC is not steep, line BC is not gradual and base line CD is horizontal, or line AC is not gradual, line BC is not steep and base line CD is vertical.*

Proof. Let ABC be a triangle with area $\mathcal{A}$ in the α-plane, such that C is a basic vertex, $a = d_\alpha(B, C)$, $b = d_\alpha(A, C)$ and $c = d_\alpha(A, B)$. Let D be the intersection point of a base line and AB, the opposite side of the basic vertex C. Let H_1 and H_2 be orthogonal projections of A and B on the base line CD, respectively. And let $p = (a + b + c)/2$, $l = d_\alpha(C, D)$, $l_1 = d_\alpha(C, H_1)$, $l_2 = d_\alpha(C, H_2)$, $h_1 = d_\alpha(A, H_1)$, $h_2 = d_\alpha(B, H_2)$. Since the α-distance between two points is invariant under all translations, rotations of $\pi/2$, π and $3\pi/2$ radians around a point, and reflections about the lines parallel to $x = 0$, $y = 0$, $y = x$ or $y = -x$, Figure 5 and Figure 6 represent all triangles for which C_1 holds, Figure 7 and Figure 8 represent all triangles for which C_2 holds, Figure 9 and Figure 10 represent all triangles for which C_3 holds, and finally Figure 11 and Figure 12 represent all triangles for which C_4 holds. In Figure 5 and Figure 6, $a = h_2 + \lambda(\alpha)l_2$ and $b = h_1 + \lambda(\alpha)l_1$ by the α-distance defi-



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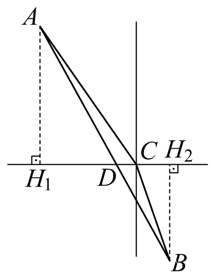
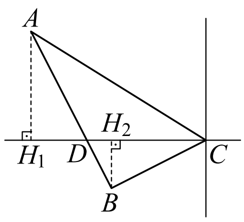


Figure 6:

nition. Since $A(ABC) = A(ADC) + A(BDC) = \frac{l}{2}(h_1 + h_2)$, using h_1 and h_2 values, one gets $\mathcal{A} = \frac{l}{2}\left[2p - c - \lambda(\alpha)(l_1 + l_2)\right]$. In Figure 7 and Figure 8, $a = l_2 + \lambda(\alpha)h_2$



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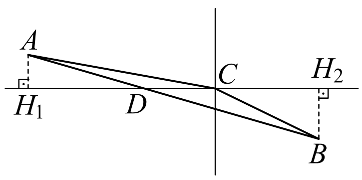

Figure 8:

and $b = l_1 + \lambda(\alpha)h_1$ by the α-distance definition. Since $A(ABC) = A(ADC) + A(BDC) = \frac{l}{2}(h_1 + h_2)$, using h_1 and h_2 values, one gets $\mathcal{A} = \frac{l}{2\lambda(\alpha)}\left[2p - c - (l_1 + l_2)\right]$. In Figure 9 and Figure 10, $a = l_2 + \lambda(\alpha)h_2$ and $b = h_1 + \lambda(\alpha)l_1$ by the α-distance

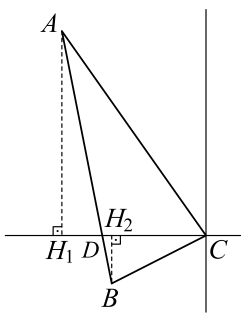

Figure 9:

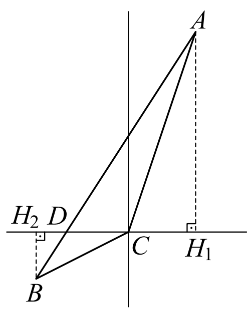

Figure 10:

definition. Since $A(ABC) = A(ADC) + A(BDC) = \frac{l}{2}(h_1 + h_2)$, using h_1 and h_2 values, one gets $\mathcal{A} = \frac{l}{2\lambda(\alpha)}\left[2p - c + (\lambda(\alpha) - 1)b - (\lambda^2(\alpha)l_1 + l_2)\right]$. In Figure 11 and

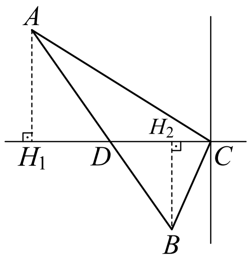

Figure 11:

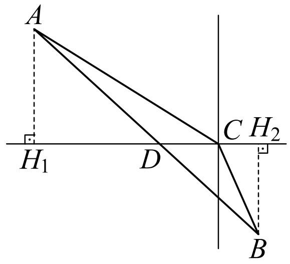

Figure 12:

Figure 12, $a = h_2 + \lambda(\alpha)l_2$ and $b = l_1 + \lambda(\alpha)h_1$ by the α-distance definition. Since $A(ABC) = A(ADC) + A(BDC) = \frac{l}{2}(h_1 + h_2)$, using h_1 and h_2 values, one gets $\mathcal{A} = \frac{l}{2\lambda(\alpha)}\left[2p - c + (\lambda(\alpha) - 1)a - (l_1 + \lambda^2(\alpha)l_2)\right]$. □

Remark 2. *Since well-known taxicab and Chinese Checker distances are special cases of the α-distance for $\alpha = 0$ and $\alpha = \pi/4$, respectively, Theorem 3, Theorem 6 and Theorem 7 also give taxicab and Chinese Checker versions of area formulas for a triangle, when $\alpha = 0$ and $\alpha = \pi/4$, respectively (see [10], [13] and [9]). Note that if $\alpha = 0$, then $\lambda(\alpha) = 1$, and we get simple equations for taxicab plane: $\rho(m) = (1 + m^2)^{1/2}/(1 + |m|), d_0(P, l) = d_T(P, l) = |ax_0 + by_0 + c| / \max\{|a|, |b|\}, \tau(m) = \max\{|1|, |m|\}/(1+m^2)^{1/2}, \sigma(m) = \max\{|1|, |m|\}/(1+|m|), \mathcal{A} = \frac{l}{2}[2p - c - (l_1 + l_2)]$.*

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