

CRITIQUE OF IMMANUEL KANT'S CRITICISM

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In recent time, in philosophical literature there is a terminological confusion with regard to the concept of “ontology” and the concept of “metaphysics.” In order to show the metaphysical nature of Kant’s “thing in itself,” first of all it is necessary to point at different meanings of these two concepts, according to the fact that they represent different philosophical disciplines whose domains only partially overlap in the respective objects of investigations.

Ontology (ὄντως — being and λόγος — science) is a science of being. The ontological science of being investigates everything that is if it is. This science about being as a being is interested in the fact that being are. Beings that are, certainly are existing beings. So ontology does not preclude epistemological investigations of how existing beings are what they are it actually renders them possible. E. g. ontological aspect of a theory determines beings that should exist if a theory should be true. It is very hard to imagine that it is possible successfully to separate the concept of truth from the concept of knowledge. So the ontology of a theory is a precondition for establishment of its truth. In Kant’s philosophy, existence is not the property or predicate of existing beings, but their subject and a precondition for investigation of their properties.

Metaphysics (μετά τά φυσικά — after the physics) is a science that treats of that which is beyond the sensible world. This science about supra-sensibility (or under-sensibility) investigates last causes, universal principles and primordial origin of the sensible world. As ontology establishes that beings are, so metaphysics establishes the foundation according to which beings are what they are. In Kant’s philosophy, metaphysics unjustifiably uses expansion of the categorical use of pure reason; it crosses the boundaries of the sensible world, and necessarily ends in transcendental illusory appearance.

Because of the partial overlapping of ontology’s and metaphysics’ respective objects (beings) of investigation, sometimes the meaning of the concept

of “ontology” is erroneously identified with the meaning of the concept of “metaphysics.”¹

However, the difference between the meaning of the concept of “ontology” and the meaning of the concept of “metaphysics” is accentuated by only partial overlapping of possible objects of investigations. The cause of this difference is the different mode of beings. Mode (τρόπος) denotes the way of existence of limited beings. The difference between the way of existence of some beings that belong to the domain of discourse of ontology, and the way of existence of some beings that belong to the domain of discourse of metaphysics, enables the differentiation between the meaning of the concept of ontology and the meaning of the concept of metaphysics. E. g. abstract mathematical entities belong to the ontological aspects of a mathematical theory. It is hard to say that these aspects are metaphysical, too. Entity (*ens, entis* — being) determines a set of qualitative properties of a being. According to the mode of existence of abstract mathematical entities, their essence is equal to their qualitative properties. Additionally, abstract mathematical entities have no unessential qualitative properties it is hard to say that with theoretical epistemological mathematical determination of qualitative properties of ontologically established existing beings, we investigate their metaphysical principles.²

The same difference is indirectly accentuated by the sameness of use of the logical procedure of inference in the theoretical determination of the qualitative properties of beings that belong to the domain of discourse of ontology (but do not belong to the domain of discourse of metaphysics), and in theoretical determination of the qualitative properties of beings that are postulated purely metaphysically.³

In the case of ontologically determined abstract mathematical entities, we investigate their properties by using the deductive procedure of inference.

- 1 A good example of this overlapping is Aristotle's hylomorphism. Existing beings are the object of ontological investigations, but their foundations are the object of metaphysical investigations.
- 2 Even in the case of a Platonically orientated philosophy of mathematics, it should not be possible consistently to claim that with theoretical epistemological mathematical determination of qualitative properties of ontologically established infinite sets of numbers we are investigating the idea of a number. Mutually different infinite sets of numbers have mutually different qualitative properties, which means that there should be as many mutually different ideas of a number, as there are mutually different infinite sets of numbers. This is the reason why in extreme variants of contemporary Platonically orientated philosophy of mathematics we can find the claim that reality is somehow bifurcated between variable sensible world and the invariable world of abstract mathematical entities.
- 3 A good example of the determination of qualitative properties of beings that are postulated purely metaphysically is Leibniz's “Monadology.” “Monadology” largely deals with a theoretical determination of qualitative properties of beings (monads) that are postulated purely metaphysically.

The deductive procedure of inference implies the use of the logically necessary forms of conclusion, e. g. *modus ponens*. Theorems of a mathematical theory are the results of the deductive procedure of inference from the set of axiomatic statements. In the case of purely metaphysically postulated entities we use conclusions from the universally postulated principles (or causes) to the particularly existing beings, or from the particularly existing beings to the universally postulated principles (or causes). During the development of logic, the conclusion from universals to particulars was called deduction, and from particulars to universals induction. This historical fact is sometimes, even now, the possible source of terminological confusion. In contemporary logic, deduction is described as a logically necessary conclusion, and induction as a logically probable conclusion.⁴

In the metaphysical conclusion from universal principles to particular beings, we are dealing with logically necessary conclusions. E. g. Plato's metaphysics of the essence of beings is founded on logically necessary conclusions. In the metaphysical conclusion from particular beings to primordial origins we are dealing with logically necessary conclusions, too. E. g. Aristotle's metaphysics of the essence of substance is founded on logically necessary conclusions.⁵

The difference between the way of determination of qualitative properties of abstract mathematical entities and the way of determination of qualitative properties of purely metaphysically established entities is not in the alleged use of different forms of conclusions. It is in the respective modes of existence of mathematical entities and metaphysical entities. As suprasensible and outside of the sphere of human reason, the mode of existence of metaphysical entities is different from the mode of existence of mathematical entities established intellectually, by human reason.⁶

4 E. g. inference: " $\forall(x) (Fx \leftrightarrow Gx), \forall(x) \forall(y) x = y, Ga / \forall(x) Fx$ " is expressed by symbolical tools of the logical model of predicates. Its first two premises consist of universally quantified sentences. Its third premise is a predicatively determined individual constant. Its conclusion is a universally quantified sentence. This inference is quantificationally valid for any possible meaningful interpretation. We find universally quantified sentences among its premises, and its conclusion is a universally quantified sentence, too. So we can not say that this inference is a conclusion from universal sentences to a particular sentence. For the same reason we can not say that this is a conclusion from particular sentences to a universal sentence. Because of the presence of the third premise, which is a particular sentence, we can not say that this is a conclusion from universal sentences to a universal sentence. But, (despite this "undecidability"), we are sure that this inference is a logically necessary deductive conclusion.

5 It is possible to formalize Plato's or Aristotle's conclusions and show that their conclusions are logically necessary (validly) inferred from the sets of premises.

6 An attempt to answer the questions: "Is it possible to reduce abstract mathematical entities to sensible reality?" and "Do abstract mathematical entities result from 'inborn' abilities of human mind?" is out of the scope of interest of this article.

1. *Metaphysical character of Kant's "thing in itself"*

In "Critique of Pure Reason" Kant discusses the ground of the division of all objects into *phaenomena* and *noumena*. This discussion is presented in the third chapter of "Transcendental Doctrine of the Faculty of Judgement or, Analytic of Principles." The central topics of this discussion are the epistemological and ontological aspects of use of the categories of reason.⁷

The categories, as the most general concepts, are inferred from sensible experience by use of logical procedure of abstraction. Because they are the most general concepts, it is not possible to get their non-circular definitions. Therefore, they have meanings only under the conditions of sensibility. Only under these conditions do they correspond with any object altogether. The pure categories are merely the forms of thought which contain only the logical faculty of uniting *a priori* the manifold given in intuition. The logical function of pure categories is to classify the manifold of intuitions, to lead the manifold given in intuition under the categorical concepts. Pure categories are thus mere general concepts of the objects of intuitions which enable thoughts about sensible objects. Without formal conditions of sensibility, they have only transcendental meanings. However, in transcendental use, because of the lack of conditions of sensibility, the pure categories are not able to lead any object under the categorical concepts. Therefore, the pure categories are not for transcendental use. The categories are not particular objects given only to reason. They only serve to determine a transcendental object aided by the sensible content. This determination enables empirical epistemological recognition of phenomena under the concepts of reason.

In Kant's views we can find the logical level of discourse on the categories represented by the logical procedure of abstraction and the logical function. The categories are inferred from sensibility by reason's faculty of logical abstraction. The categories thus obtained have a logical function to classify the manifold of sensibility (phenomena) under the categorical concepts. This ontological classification of phenomenal beings enables an epistemological conceptual (theoretical) empirical inquiry. Logical non-circularity is obtained by the status of conceptual inability to define the categories. It is not possible to define the categories because of the lack of elements of definition. On the other hand, categories enable definition of any referent concept. This status of categories is the immediate consequence of the logical faculty of reason (abstraction). As abstraction is the logical faculty of reason, thus the categories

7 In Kant's philosophy, the understanding (*der Verstand*) is a faculty for the production of rules, and reason (*die Vernunft*) is a faculty for the production of unity of rules. The general principles of reason follow the particular rules of the understanding.

have only transcendental meaning. Because of the inviolable condition of sensibility, their logical function is not employed transcendently. Violation of the condition of sensibility results in transcendental illusions. Transcendental illusory appearances are present in concepts that have no content. However, the logical function of categories limited by the condition of sensibility is to produce referent empirical concepts. This logical level of discourse enables its ontological level. Ontological establishment of the existence of phenomenal beings is under the condition of sensibility. So existence is not the property or predicate of existing beings, but their subject and a precondition for investigation of its properties. The ontological level of Kant's discourse is followed by his epistemological level. The synthetical faculty of reason enables the comprehension of qualitative properties of spatial/temporal (phenomenal) beings, or ideal/mental (mathematical) entities.⁸

Additionally, the logical function of categories in their unity of use, in the sense of conceptual recognition of a transcendental object under the condition of sensibility, enables the principles of exposition of phenomena.⁹

The principles of exposition of phenomena, in the sense of a unique employment of categories of reason, underline the function of categories which is correlative to apperception: the synthesis of a manifold of sensibility.¹⁰

8 According to Kant, in natural sciences, the establishment of the truth of a theory by synthetical epistemological faculty of reason is under the directivity of a sensible reality that exists objectively and outside of reason. In mathematics, the establishment of the truth of a theory by synthetical epistemological faculty of reason is not under the directivity of the sensible reality that exist objectively and outside of reason. Otherwise, it should be possible to assert that according to Kant, natural science are grounded in the a priori field of knowledge, and mathematics in a posteriori field of knowledge. It is not difficult to show that such alleged division is not part of his philosophy. The definition of a *priori* knowledge as the knowledge that is absolutely independent of any kind of a posteriori sensible knowledge should become senseless. The concept of a *priori* synthesis should become senseless, too. Namely, the usage of a *priori* synthesis should be paradoxically present in natural sciences, but it should not be present in mathematics.

9 "Erscheinungen, so fern sie als Gegenstände nach der Einheit der Kategorien gedacht werden, heißen Phaenomena... Also würde es, außer dem empirischen Gebrauch der Kategorien (welcher auf sinnliche Bedingungen eingeschränkt ist) noch einen reinen und doch objektivgültigen geben, und wir könnten nicht behaupten, was wir bisher vorgegeben haben: daß unsere reine Verstandeserkenntnisse überall nichts weiter wären, als Prinzipien der Exposition der Erscheinung, die auch a priori nicht weiter, als auf die formale Möglichkeit der Erfahrung gingen, denn hier stände ein ganz anderes Feld vor uns offen, gleichsam eine Welt im Geiste gedacht (vielleicht auch gar angeschaut), die nicht minder, ja noch weit edler unsern reinen Verstand beschäftigen könnte." Kant, Immanuel "Kritik der reinen Vernunft" Suhrkamp Taschenbuch Verlag, Frankfurt 1968, p.p. 278, 280.

10 It is of decisive importance to consistently adhere to Kant's establishment of a *priori* apperceptive faculty of reason for a synthesis of sensible data that enables phenomenon, and a *priori* epistemological faculty of reason for synthesis of referent concepts that enables knowledge.

The categories may serve only as the correlates of unity of apperception. The synthesis of the manifold of sensibility is the way reason unites them into a concept of an object. It is not possible to separate a transcendental object from a manifold of sensibility, because in that case there is nothing left to form a referent concept of such object. A transcendental object is merely a perception of the phenomenon under the general concept of the object. The phenomena (appearances) are but perceptions, representing under the unity of categories the objects of possible epistemological investigations.¹¹

As ontology is the study of the existence of beings, the spatial/temporal beings represented by perceptions are ontologically existing beings.¹²

However, a perception must be a perception of something, just like an appearance must be an appearance of something different from the appearance itself. The “something” Kant calls “noumena.”¹³

The concept of noumena is cognitively transcendent. This means that the concept of noumena refers to something that is absolutely beyond any faculty of reasonable understanding. In other words, the appearances (phenomena) are the sensible representations of the noumena that are absolutely independent of sensibility.¹⁴

Let us analyze the sentence which is of central importance for answering the question about the character of the noumena:

11 “*Alle unsere Vorstellungen werden in der Tat durch den Verstand auf irgend ein Objekt bezogen, und, da Erscheinungen nichts als Vorstellungen sind, so bezieht sie der Verstand auf ein Etwas, als den Gegenstand der sinnlichen Anschauung: aber dieses Etwas ist in so fern nur das transzendente Objekt.*” *ibid.* p. 280.

12 It is not difficult to refute possible objection to Kant's ontology of spatial/temporal beings from the point of view of existing beings established by natural science, e. g. photons. It is for certain reason that it is not possible to have an image of photons, but a conclusion about the existence of photons is achieved from sensible macro effects to micro causes. So, photons are in the domain of the condition of sensibility. Similar situation was in Kant's time, when in physical paradigm Newton's corpuscular and Huygens's undulatory theory of the nature of light was present.

13 “*Was aber die Ursache betrifft, weswegen man, durch das Substratum der Sinnlichkeit noch nicht befriedigt, den Phaenomenis noch Noumena zugegeben hat, die nur der reine Verstand denken kann, so beruhet sie lediglich darauf.*” *ibid.* p. 280.

14 “*Die Einteilung der Gegenstände in Phaenomena und Noumena, und der Welt in eine Sinnen— und Verstandes welt, kann daher in positiver Bedeutung gar nicht zugelassen werden, obgleich Begriffe allerdings die Einteilung in sinnliche und intellektuelle zulassen; denn man kann den letzteren keinen Gegenstand bestimmen, und sie also auch nicht für objektivgültig ausgeben... Unser Verstand bekommt nun auf diese Weise eine negative Erweiterung, d. i. er wird nicht durch die Sinnlichkeit eingeschränkt, sondern schränkt vielmehr dieselbe ein, dadurch, daß er Dinge an sich selbst (nicht als Erscheinungen betrachtet) Noumena nennt. Aber er Setzt sich auch so fort selbst Granzen, sie durch keine Kategorien zu erkennen, mithin sie nur unter dem Namen eines unbekanntes Etwas zu denken.*” *ibid.* p.p. 282–283.

(A) An phenomenon must be an phenomenon of something that is different from that which is the phenomenon.

The part of the sentence “something that is different from that which is the phenomenon” (A) is not a referential concept or a description in the domain of discourse of phenomena. In the world of phenomena only referential concepts or descriptions are the concepts or descriptions that refer to manifold of sensibility. The “something that is different from than which is the phenomenon” is the abbreviation of the sentence: “There is something different from that which is the phenomeonon”. The sentence “There is something different from that which is the phenomenon” expresses the condition of a phenomenon. The meaning of the concept “phenomenon” implies the existence of a “something” that is different from that which is the phenomenon, because a phenomenon must be a phenomenon of a something. The “something” is not a property of the concept of a phenomenon because it is something different from that which is the phenomenon. Also, “phenomenon” is not the property of this “something” because the “something” is something different from that which is the phenomenon, too. It is not a case of “phenomenon” being the condition for existence of this “something” because “phenomenon” is a phenomenon only for us. The “something” that is different from the phenomenon should continue to exist even if we should vanish. It is the case of this “something” being the condition for the existence of a phenomenon because a phenomenon must be a phenomenon of something. Therefore, it is a logical necessity that a phenomenon should be conditioned by something different from that which is the phenomenon. Kant goes a step further, establishing that a phenomenon is *caused* by a transcendent noumenal “thing in itself,” which is this something different from that which is the phenomenon.¹⁵

This causation enables the following sentence:

(B) The Phenomenon exists only if there exist something different from it that causes it.

This *necessarily* conditioned compound sentence consists of the sentences “There exists a phenomenon” and “There exists something different

15 “Wenn dagegen Erscheinungen für nichts mehr gelten, als sie in der Tat sind, nämlich nicht für Dinge an sich, sondern bloße Vorstellungen, die nach empirischen Gesetzen zusammenhängen, so müssen sie selbst noch Gründe haben, die nicht Erscheinungen sind. Eine solche intelligibele Ursache aber wird in Ansehung ihrer Kausalität nicht durch Erscheinungen bestimmt, obzwar ihre Wirkungen erscheinen, und so durch andere Erscheinungen bestimmt werden können. Sie ist also samt ihrer Kausalität außer der Reihe; dagegen ihre Wirkungen in der Reihe der empirischen Bedingungen angetroffen werden. Die Wirkung kann also in Ansehung ihrer intelligibelen Ursache als frei, und doch zugleich in Ansehung der Erscheinungen als Erfolg aus denselben nach der Notwendigkeit der Natur, angesehen werden; eine Unterscheidung, die, wenn sie im allgemeinen und ganz abstrakt vorgetragen wird, äußerst subtil und dunkel scheinen muß, die sich aber in der Anwendung aufklären wird.” *ibid.* p.p. 491–492.

from it that causes it.” The sentence “There exists a phenomenon” is a simple existential statement. It can not be a synthetical statement because it does not contain the minimum two concepts that are necessary for a synthetis. The sentence “There exists something different from it that causes it” is a simple existential statement, too. However, the mutual relations of its pronouns enable the following equivalent sentence: “There exists something different from a phenomenon that causes the phenomenon.” This sentence can not be a synthetical *a posteriori* statement because there is no synthetical relation between the concept of a phenomenon and the concept (or description) of something that is different from the phenomenon and that causes it. This concept (or description) is not an *a posteriori* referential concept at all. Therefore, the entire sentence (B) is not a synthetical *a posteriori* statement. On the other hand, sentence (B) expresses a meaningful relation between a “phenomenon” and “something different from the phenomenon that causes the phenomenon.” The *a priori* faculty of reason reaches “the phenomenon.” “Something that is different from the phenomenon that causes the phenomenon” is absolutely independent of any kind of sensibility, and it is of *a priori* character. Furthermore, a meaningful relation between them is a necessary logical condition. According to the criteria of apriority and logical necessity of synthetical *a priori* statements, it seems that the sentence (B) can be a candidate for a synthetical *a priori* statement. However, the third criterion for synthetical *a priori* statements demands that its concepts be mutually logically independent. The concept of phenomenon is logically dependent (by condition of existence of phenomenon) of the concept of “something” that is different from the phenomenon. That is why sentence (B) can not be an *a priori* synthetical statement.

The remaining possibility is that sentence (B) is an analytical statement. It can be an analytical statement if the concept of phenomenon in its content somehow contains “something different from the phenomenon and that causes the phenomenon” or vice versa. However, as we have seen earlier, “phenomenon” is not a property of “something” that causes it, and vice versa. Furthermore, it is not possible to define the concept of phenomenon by using the concept of “something” different from it that causes it, because, in that case the definitions would be negative. The “something” that causes the phenomenon is something different from the phenomenon and there is no possibility for synonymy. So sentence (B) can not be an analytical statement by property or by definition or by synonymy.

It is a logical condition that a phenomenon must be a phenomenon of something. However, the metaphysical and/or ontological framework of various theoretical systems variously define this logical condition. E. g. we can suppose that existing beings are phenomenal imperfect beings that are conditioned by perfect ideas.¹⁶

16 In Plato's philosophy, a “phenomenon” (φαίνεσθαι) means an imperfect appearance of sensible things. By demiurg intermediary activity this appearance is conditioned by perfect ideas. Kant refutes Plato's metaphysics of the essence of beings, as he refutes Aristotle's

Logically, metaphysically and/or ontologically, we can equally suppose that existing beings are the phenomenal beings that are conditioned by a transcendent noumenal “thing in itself.”¹⁷

It is a logical condition that a phenomenon should be the phenomenon of something, but it is only a matter of theoretical choice to define this something as a transcendent noumenal “thing in itself.” There is no logical equivalence between sentences (A) and (B).¹⁸

metaphysics of substance of beings. Kant's theoretical choice is a transcendent “thing in itself,” which does not allow its positive cognitive establishment.

17 With regard to the “thing in itself,” there is sometimes in texts about Kant's philosophy, an inadmissible identification of the meaning of the concept “transcendental” with the meaning of the concept “transcendent.” It is of decisive importance to use the concept “transcendental” as *a priori* general thought about the suprasensible “thing in itself,” and the concept “transcendent” as something that is totally out of any of epistemological examination. In another words, the “thing in itself” means an undetermined thought about the suprasensible something that exists outside of the possible reach of reason's cognitive power.

18 It is possible to demonstrate in the logical model of predicates that sentence (A) and sentence (B) are not mutually quantificationally equivalent by application of the following symbolic interpretation:

D = {everything that exists}

Px: x is a phenomenon

Sy: y is something

Dxy: x is different from y

Cyx: y causes x

Symbolization:

(A) An appearance must be an appearance of something that is different from that which is the appearance.

(B) An appearance exists only if there exists something different from it that causes it.

(A) $\forall (x) (Px \rightarrow \exists y (Sy \wedge Dxy))$

(B) $\forall (x) (Px \rightarrow \exists y (Sy \wedge Dxy \wedge Cyx))$

Sentences (A) and (B) are quantificationally equivalent if and only if there is no interpretation according to which (A) and (B) have different truth-values. Sentence (A) and sentence (B) are different only in “Cyx” which is part of sentence (B), but it is not part of sentence (A). Suppose that (A) and (B) are both true in some interpretation. Then “ $\forall(x) Px$ ” is either true or false for this interpretation. If “ $\forall(x) Px$ ” is false then both (A) and (B) are true. If “ $\forall(x) Px$ ” is true, then “ $\exists y (Sy \wedge Dxy)$ ” and “ $\exists y (Sy \wedge Dxy \wedge Cyx)$ ” are true. If the part “ $\exists y (Sy \wedge Dxy \wedge Cyx)$ ” of (B) is true then the part “ $\exists y (Sy \wedge Dxy)$ ” of (A) is true. If the part “ $\exists y (Sy \wedge Dxy)$ ” of (A) is true, then the part “ $\exists y (Sy \wedge Dxy)$ ” of (B) is true and the part “Cyx” of (B) is true. Suppose that both (A) and (B) are false in some interpretation. Then “ $\forall(x) Px$ ” is true for this interpretation. If “ $\forall(x) Px$ ” is true then “ $\exists y (Sy \wedge Dxy)$ ” and “ $\exists y (Sy \wedge Dxy \wedge Cyx)$ ” are false. If the part “ $\exists y (Sy \wedge Dxy \wedge Cyx)$ ” of (B) is false then the part “ $\exists y (Sy \wedge Dxy)$ ” of (A) is false, and (A) is false. If the part “ $\exists y (Sy \wedge Dxy)$ ” of (A) is false, then the part “ $\exists y (Sy \wedge Dxy)$ ” of (B) is false, and (B) is false. Suppose that (A) is false and (B) is true in some interpretation. Then “ $\forall(x) Px$ ” as part of (A) is true, and “ $\forall(x) Px$ ” as part of (B) is either true or false. Therefore “ $\forall(x) Px$ ” as part of (B) can be only true for a consistent interpretation. If “ $\forall(x) Px$ ” is true then “ $\exists y (Sy \wedge Dxy)$ ” is false, and “ $\exists y (Sy \wedge Dxy \wedge Cyx)$ ” true. This is not a consistent interpretation. Suppose that (A) is true and (B) is false in some interpretation. Then “ $\forall(x) Px$ ” as part of (A) is either true or false, and “ $\forall(x) Px$ ” as part of (B) is true. Therefore “ $\forall(x) Px$ ” as part of (A) can be only true for a consistent interpretation. If “ $\forall(x) Px$ ”

Therefore, there is no logical necessity in the establishment of the “thing in itself” by sentences (A) and (B). In other words, sentence (B) can not be:

- an *a posteriori* synthetical statement because the “thing in itself” is of suprasensible character;
- an *a priori* synthetical statement because the concept of the phenomena is logically dependent of the concept of the noumenal “thing in itself”; and
- an analytical statement by property, definition or synonymy.

Thus, from the logical necessity of sentence (A) does not follow any logical necessity to establish the “thing in itself” by sentence (B). Logically, there remains only that the establishment of the “thing in itself” by sentence (B) is a logical possibility as a matter of theoretical choice. This theoretical possibility of the establishment of the “thing in itself” can be only ontological or metaphysical. Ontology is a science of the existence of beings. In Kant’s philosophical theory existing beings are spatial/temporal sensible beings and ideal/mental (mathematical) entities. Noumenal transcendent “thing in itself” is not of the spatial/temporal or mathematical character. Metaphysics is a science that treats of that which is beyond the sensible world, investigating the last causes. Noumenal transcendent “thing in itself” is beyond the sensible world and it is the cause of sensibility. Thus, “thing in itself” is of the metaphysical character.¹⁹

Px ” is true, then “ $\exists y (Sy \wedge Dxy)$ ” is true, and “ $\exists y (Sy \wedge Dxy \wedge Cyx)$ ” false. *This is a consistent interpretation if “ Cyx ” is false.*

- 19 Let us briefly comment Kant’s critique of Leibniz’s philosophy concerning the fact that the “thing in itself” is established purely metaphysically.

“Als Objekt des reinen Verstandes muß jede Substanz dagegen innere Bestimmungen und Kräfte haben, die auf die innere Realität gehen. Allein was kann ich mir für innere Akzidenzen denken, als diejenigen, so mein innerer Sinn mir darbietet? nämlich das, was entweder selbst ein Denken, oder mit diesem analogisch ist. Daher machte Leibniz aus allen Substanzen, weil er sie sich als Noumena vorstellte, selbst aus den Bestandteilen der Materie, nachdem er ihnen alles, was äußere Relation bedeuten mag, mithin auch die Zusammensetzung, in Gedanken genommen hatte, einfache Subjekte mit Vorstellungskräften begabt, mit einem Worte, Monaden.” *ibid.* p. 289.

Kant established the “thing in itself” as *noumena*. He did not declare that the “thing in itself” was a simple substantial subject with powers of representation. He declared that the “thing in itself” was a suprasensible transcendent object, which is in the foundations of any subjective sensible phenomenal reality. He did not deny to the “thing in itself” anything like external relation. He warranted external relation to the “thing in itself” by its establishment as the foundation of any perception and its external composition or combination. The difference in logical steps of Kant and Leibniz’s procedure of establishment of their mutually different philosophical theories is of irreconcilable importance. The only exception is in the logical step of conclusion to suprasensibility. This step is a metaphysical step, too.

2. *Viability of a priori synthesis*

In the second chapter of “Critique of Pure Reason” entitled “Of the Supreme Principle of All Analytical Judgements,” the supreme principle of all analytical judgements is established.²⁰

The content of this supreme principle consists of the logical principle of contradiction. It is not possible to accept any kind of simple statement, whose concepts include mutually exclusive properties in respect to referential objects.²¹

It is not possible to accept any kind of compound statement, whose elements are mutually exclusive simple statements.²²

In the same chapter, under the title “Of the Supreme Principle of all Synthetical Judgements,” the supreme principle of all synthetical judgements is established. It demands that any synthetical statement be under the directivity of the condition of sensibility.²³

In accordance with this principle Kant established synthetical *a priori* statements. Synthetical *a priori* statements are necessary true statements if on the one hand reason achieved them by means of a connection between the formal condition of the *a priori* intuition, the synthesis of imagination and the necessary unity of that synthesis in a transcendental apperception and on the other hand, by means of a possible cognitive empirical *a priori* knowledge. Because the conditions of the possibility of experience in general are at the

20 “Der Satz nun: Keinem Dinge kommt ein Prädikat zu, welches ihm widerspricht, heißt der Satz des Widerspruchs, und ist ein allgemeines, obzwar bloß negatives, Kriterium aller Wahrheit, gehört aber auch darum bloß in die Logik, weil er von Erkenntnissen, bloß als Erkenntnissen überhaupt, unangesehen ihres Inhalts gilt, und sagt: daß der Widerspruch sie gänzlich vernichte und aufhebe.” *ibid.* p. 196.

21 “Von welchem Inhalt auch unsere Erkenntnis sei, und wie sie sich auf das Objekt beziehen mag, so ist doch die allgemeine, obzwar nur negative Bedingung aller unserer Urteile überhaupt, daß sie sich nicht selbst widersprechen; widrigenfalls diese Urteile an sich selbst (auch ohne Rücksicht aufs Objekt) nichts sind. Wenn aber auch gleich in unserm Urteile kein Widerspruch ist, so kann es demohngeachtet doch Begriffe so verbinden, wie es der Gegenstand nicht mit sich bringt, oder auch, ohne daß uns irgend ein Grund weder *a priori* noch *a posteriori* gegeben ist, welcher ein solches Urteil berechtigte, und so kann ein Urteil bei allem dem, daß es von allem innern Widerspruche frei ist, doch entweder falsch oder grundlos sein.” *ibid.* p. 196.

22 “Sage ich aber, kein ungelehrter Mensch ist gelehrt, so ist der Satz analytisch, weil das Merkmal (der Ungelahrtheit) nunmehr den Begriff des Subjekts mit ausmacht, und alsdenn erhellet der verneinende Satz unmittelbar aus dem Satze des Widerspruchs, ohne das die Bedingung: zugleich, hinzu kommen darf.” *ibid.* p. 198.

23 “Das oberste Principium aller synthetischen Urteile ist also: ein jeder Gegenstand steht unter den notwendigen Bedingungen der Synthetischen Einheit des Mannigfaltigen der Anschauung in einer möglichen Erfahrung.” *ibid.* p. 201.

same time the conditions of the possibility of the objects of experience, synthetic *a priori* statements are of objective validity.²⁴

As synthetical *a priori* statements represent examples of necessary *a priori* knowledge, and as *a priori* knowledge is absolutely independent of any kind of sensible experience, the possibility of *a priori* knowledge is founded on the unity of the synthesis of *a priori* intuitions and the synthesis of concepts of pure reason. This unity is applied to mathematical and “dynamical” research.²⁵

Since mathematics rests on *a priori* knowledge and on a synthetical faculty of reason, it is the necessary consequence that all mathematical statements are synthetical *a priori* statements.²⁶

As geometry represents the mathematics of extension founded on a successive synthesis of the productive imagination in the generation of figures, its axioms express the condition of sensuous intuition *a priori*.²⁷

24 “Auf solche Weise sind synthetische Urteile *a priori* möglich, wenn wir die formalen Bedingungen der Anschauung *a priori*, die Synthesis der Einbildungskraft, und die notwendige Einheit derselben in einer transzendentalen Apperzeption, auf ein mögliches Erfahrungserkenntnis überhaupt beziehen, und sagen: die Bedingungen der Möglichkeit der Erfahrung überhaupt sind zugleich Bedingungen der Möglichkeit der Gegenstände der Erfahrung, und haben darum objektive Gültigkeit in einem synthetischen Urteile *a priori*.” *ibid.* p. 201.

25 “Alle Versuche, jene reine Verstandesbegriffe von der Erfahrung abzuleiten, und ihnen einen bloß empirischen Ursprung zuzuschreiben, sind also ganz eitel und vergeblich. Ich will davon nichts erwähnen, da z. E. der Begriff einer Ursache den Zug von Notwendigkeit bei sich führt, welche gar keine Erfahrung geben kann, die uns zwar lehrt: da auf eine Erscheinung gewöhnlicher Maen etwas andres folge, aber nicht, da es notwendig darauf folgen müsse, noch da *a priori*, und ganz allgemein daraus als einer Bedingung auf die Folge könne geschlossen werden.” *ibid.* p. 171.

“Es gibt aber reine Grundsätze *a priori*, die ich gleichwohl doch nicht dem reinen Verstande eigentümlich beimessen möchte, darum, weil sie nicht aus reinen Begriffen, sondern aus reinen Anschauungen (obgleich vermitteltst des Verstandes) gezogen sind; Verstand ist aber das Vermögen der Begriffe. Die Mathematik hat dergleichen, aber ihre Anwendung auf Erfahrung, mithin ihre objektive Gültigkeit, ja die Möglichkeit solcher synthetischen Erkenntnis *a priori* (die Deduktion derselben) beruht doch immer auf dem reinen Verstande.” *ibid.* p. 202.

26 “Denn da Gleiches zu Gleichem hinzugetan, oder von diesem abgezogen, ein Gleiches gebe, sind analytische Sätze, indem ich mir der Identität der einen Grönerzeugung mit der andern unmittelbar bewut bin; Axiomen aber sollen synthetische Sätze *a priori* sein, Dagegen sind die evidenten Sätze der Zahlverhältnis zwar allerdings synthetisch, aber nicht allgemein, wie die der Geometrie, und eben um deswillen auch nicht Axiomen, sondern können Zahlformeln genannt werden. Da $7 + 5 = 12$ sei, ist kein analytischer Satz. Denn ich denke weder in der Vorstellung von der Zusammensetzung beider die Zahl 12 (da ich diese in der Addition beider denken solle, davon ist hier nicht die Rede; denn bei dem analytischen Satze ist nur die Frage, ob ich das Prädikat wirklich in der Vorstellung des Subjekts denke.” *ibid.* p. 206

27 “Auf diese sukzessive Synthesis der produktiven Einbildungskraft, in der Erzeugung der Gestalten, gründet sich die Mathematik der Ausdehnung (Geometrie) mit ihren Axiomen, welche die Bedingungen der sinnlichen Anschauung *a priori* ausdrücken, unter denen allein das Schema eines reinen Begriffs der äueren Erscheinung zu Stande kommen kann; z. E. zwischen zwei Punkten ist nur eine gerade Linie möglich; zwei gerade Linien schliessen keinen Raum ein etc.” *ibid.* p. 206.

The scheme of a pure conception of external intuition can exist only under the condition of sensuous *a priori* intuition. Therefore, Kant's example, the sentence:

C) "Between two points only a straight line is possible,"

is an example of a necessary synthetical *a priori* statement.²⁸

This sentence is an expression of the axiom of Euclidean geometry. Sentence (C) under the conditions of Kant's philosophy is: aprioristic — because the axioms of geometry express the conditions of sensuous *a priori* intuition, necessary — because all mathematical statements, according to the usage of mathematical principles, are apodictically necessary, and synthetical — because all geometrical statements are generally synthetical. Neither in the perception of two points, nor in the perception of a straight line can we non-synthetically (analytically) find the property of drawing only one straight line. We can only synthetically, from the concept of two points and from the concept of a straight line, achieve the necessity of sentence (C).

This attitude may be criticized by means of some facts from the history of geometry. The fact is that mathematicians tried to infer the fifth Euclidean axiom from the remaining four. Motivation for this trial is founded on the fact that it is possible deductively to infer the sentence: "Whenever a straight line intersects two straight lines so that the sum of two inner angles on the same side of the intersecting line is equal to the sum of two right angles, then the two straight lines shall not meet," from the first four Euclidean axioms.²⁹

As a proof for this sentence, fifth Euclidian axiom: "Whenever a straight line intersects two straight lines such that the sum of two inner angles on the same side of the intersecting line is equal to the sum of two inner angles on the same side of the intersecting line is smaller than the sum of two right angles, then the two straight lines shall meet each other on that side of the intersecting line" is not needed at all. Namely, from the property of possibility of drawing only one straight line (which is of infinite length) between two points, logically necessarily follows the property of possibility of drawing only one parallel straight line through the point that is out of that straight line. This sentence, namely a theorem, tells us as much about geometrical spatial relations as the fifth Euclid's axiom does. So a trial to infer the fifth Euclid's

28 "Die Synthesis der Räume und Zeiten, als der wesentlichen Form aller Anschauung, ist das, was zugleich die Apprehension der Erscheinung, mithin jede äüere Erfahrung, folglich auch alle Erkenntnis der Gegenstände derselben, möglich macht, und was die Mathematik im reinen Gebrauch von jener beweiset, das gilt auch notwendig von dieser." *ibid.* p. 207.

29 In the proof of this theorem one must appeal to an axiom not stated by Euclid, but tacitly applied: "Every straight line divides the plane into two separate parts."

axiom from the remaining four is understandable. But only the trial to infer a counterproof to the fifth Euclid's axiom, by construction of a counterexample according to which it is possible to draw more than one straight line through the point that is out of that straight line, brought about some advance.³⁰

The construction of the counterexample showed the possibility of the establishment of non-Euclidean geometrical models which are equally consistent, coherent, complete and internally axiomatically independent as the Euclidean geometrical model. Furthermore, a theorem which is provable from the first four axioms of the Euclidean geometrical model, is logically adequate to the theorems that are provable from the first four axioms of non-Euclidean geometrical models. Therefore, it is possible to state an axiom of non-Euclidean model which is logically adequate to the axiom of the Euclidean geometrical model:

D) "Between two points at least two straight lines are possible".

It is not possible to find any reason against the view that sentence (D) satisfies all *a priori*, necessary, and synthetical conditions as sentence (C) does. For the same reasons that are previously mentioned in connection with the view that sentence (C) satisfies all *a priori*, necessary, and synthetical conditions. Furthermore, by the supreme principle of all synthetical judgements it is established that "every object is subject to the necessary conditions of the synthetical unity of the manifold of intuition in a possible experience." Therefore, logically follows the possibility that synthetical judgement (C) and synthetical judgement (D), taken together, by the synthetical unity of the manifold of intuition in a possible experience, express the following property of an object:

E) "Between two points only one straight line is possible and between two points at least two straight lines are possible."

This sentence is in contradiction with the logical principle of contradiction, because of the attribution of mutually excluding properties to the concept which is referent in relation to the object of possible experience. Consequently, it contradicts the supreme principle of all synthetical judgements, because of the attribution of mutually excluding predicates to the necessary conditions of the synthetical unity of the manifold of intuition in a possible experience. On the other hand, sentence (C) and sentence (D) are the result of a synthesis *a priori*. Thus, taken together, sentence (C) and sentence (D) are necessary, mutually contradictory, *a priori* judgements. It is not the case that

30 This advance is achieved by Girolamo Saccheri. His intention was to prove the inner non-contradiction of the fifth Euclid's axiom by means of *reductio ad absurdum*.

these circumstances can really shatter Kant's theory of space, but they make the necessity of *a priori* synthetic judgements dependent of various geometrical models. *A priori* synthetic judgements are necessary only within the frame of a geometrical model.

* * *

There is no logical theoretical necessity in establishing a "thing in itself." The establishment of the "thing in itself" is only a logical possibility as a matter of theoretical choice. Furthermore, the necessity of *a priori* synthetic judgements is limited by the inner structure of theoretical models. In other words, they depend on various theoretical models. That is why Kant's theoretical solutions are not a matter of logical necessity, but only a matter of his theoretical choice.