

# An application of a genetic algorithm for throughput optimization in non-broadcast WDM optical networks with regular topologies\*

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**Abstract.** *We apply a genetic algorithm from Podnar and Skorin-Kapov [5] to a virtual topology design of a Wide-Area WDM Optical Network with regular topologies. Based on a given physical topology a virtual topology consisting of optical lightpaths is constructed. The objective is to minimize the maximal throughput, which implies balancing link loads and accommodating on-growing traffic requirements in a timely fashion. The genetic algorithm is applied to benchmark instances of regular topologies.*

**Key words:** *wavelength division multiplexing, networks-graphs, network design, heuristics, genetic algorithms, regular topologies*

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## 1. Introduction

In recent years there has been an extensive interest in problem solutions devised for fiber-optic communications. Network designers have considered several different topologies and different traffic routing algorithms. Once the network has been constructed, different solution approaches have been applied to accommodate ongoing increase in bandwidth demand.

One of the most promising techniques used in designing fiber-optic networks is the wavelength-division multiplexing (WDM). Using WDM, a single optical fiber has a capability of transmitting more than one signal (channel) at the same time. The channels sharing a fiber are transmitted using laser diodes of different wavelengths. Different wavelengths are modulated, generating a superposition of the channels, which is then transmitted over the optical fiber. At each end of the fibre, separation of channels is done by a filtering procedure.

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Two nodes of an optical network are connected using a single wavelength, thus forming a one-way channel. In the so-called wavelength-routed networks a channel can cross intermediate nodes, without changing the wavelength. Every node can be source or destination of a channel, but it can also act as a transit node. In such networks optical cross-connect devices provide wavelength routing functionalities which allow reusability of wavelengths in different parts of the network. As opposed to broadcast networks in which every logical connectivity can be realized, in non-broadcast architectures some logical connections might not be feasible. However, broadcast networks would not scale to Wide Area Networks (WANs), where some wavelength routing is required. WANs would more likely be based on more complex physical infrastructures such as mesh topologies with optical cross-connect nodes. These infrastructures are not purely broadcast (e.g. multi-star implementation), or use wavelength routers that cannot support all direct logical connections. This will necessitate decisions on allocation and routing of wavelengths to embed desired logical topology into the physical infrastructure.

The work most relevant to the solution approach of this paper was done by Podnar and Skorin-Kapov [5]. They proposed a genetic algorithm for the formulation given in Mukherjee et al. [3], whereby a simulated annealing was used to obtain heuristic solutions, and some measures of the solution quality were developed.

The same problem formulation can be found in Mukherjee [4], which also serves as a good overview of fiber-optic networks. The book presents capabilities of optical devices and introduces several optical network architectures.

WDM optical networks were the main topic in Skorin-Kapov and Labourdette [6, 7, 8]. They propose mixed integer programming formulations and a tabu search heuristics to solve a design problem of a WDM *broadcast* optical network with the objective to maximize the throughput. Skorin-Kapov and Labourdette [6] consider an arbitrary virtual topology, and in [8] a virtual topology is restricted to a regular pattern. Several possible regular topologies were considered (e.g. Perfect Shuffle, de Bruijn graphs, GEMNET topologies, Manhattan Street Network topologies).

In this paper we apply a genetic algorithm for *non-broadcast* optical networks using the approach from Podnar and Skorin-Kapov [5] applied to regular topologies. For completeness of presentation the algorithm will be restated. The details of the problem formulation and algorithm definition are presented in the following Sections. In *Section 2*, a problem statement together with the formal definition is given. A heuristic solution technique is presented in *Section 3*. Computational results can be found in *Section 4*. Final remarks, conclusions, and the area of further research are summarized in *Section 5*.

## 2. Problem statement

A network of existing optical fibers forms the underlying physical topology. The traffic is supported in both directions, i.e. if there is a fiber cable from A to B, the same cable serves as a link from B to A. In our problem definition, we will assume that traffic flow through a fiber is undirected. Utilization of an optical fiber induces costs that are based on physical distances, service charges and network infrastructure. For example, the traffic can experience propagation and queuing

delays, that user wants to keep at minimum.

Each node  $i$  is assumed to be equipped with a number of wavelength routing switches. The switches enable the channel to change from one wavelength to another, or to use the node as an intermediate node without changing wavelength. The number of switches clearly depends on the number of physical fiber links entering and leaving the node. Every end of a fiber link is serviced by possibly many optical transmitters (lasers) and receivers (filters).

Our goal is to design a virtual topology as a network of single-wavelength channels that will support the traffic. A link in the virtual topology corresponds to an optical lightpath between a pair of nodes. The lightpath uses only a single wavelength, and might propagate through intermediate nodes. These nodes perform the switching in order to keep the continuity of the lightpath. In other words, the virtual link is routed over one of several possible paths on the physical topology. The number of virtual links leaving (resp., entering) a node cannot exceed the number of transmitters (resp., receivers) at the node. The number of lightpaths per optical fiber is limited by the prescribed maximal number of wavelengths  $M$ . If two lightpaths share an optical fiber (i.e. a physical link) they must be supported by two different wavelengths. The decision must be made which physical links shall be used for lightpaths, and what is the distribution of traffic flow using the obtained lightpaths. The formulation used in this paper was originally proposed by Mukherjee et al. [3], however, for completeness of the paper we will restate it.

First, let us list the parameters and the variables of the problem:

INPUT PARAMETERS:

- $N$  = the number of nodes
- $M$  = the maximum number of wavelengths per fiber
- $T_i, R_i$  = the number of available transmitters (receivers) per node  $i$
- Matrix  $P_{mn}$   $m, n \in \{1 \dots N\}$  indicates links in the physical topology, i.e.  $P_{mn} = 1$  if there is a physical link (a fiber) between nodes  $m$  and  $n$
- Matrix  $d_{mn}$   $m, n \in \{1 \dots N\}$  contains distances between nodes.
- Matrix  $f_{sd}$   $s, d \in \{1 \dots N\}$  contains the initial traffic between sources  $s$  and destinations  $d$ . The traffic matrix can be asymmetric.
- $C$  = data capacity of each channel (lightpath).

VARIABLES:

- $V_{ij}$ ;  $i, j \in \{1 \dots N\}$  is a binary decision variable where  $V_{ij} = 1$  if there exists a lightpath (a virtual link) from node  $i$  to node  $j$ .
- $\lambda_{ij}^{sd}$ ;  $i, j, s, d \in \{1 \dots N\}$  denotes the traffic originated at source  $s$  and with destination  $d$  that flows through the virtual link  $V_{ij}$ . Alternatively, the problem can be also defined using the 3-dimensional variables  $\lambda_{ij}^s$   $i, j, s \in \{1 \dots N\}$  denoting the traffic originated at source  $s$  that flows through the virtual link  $V_{ij}$ . The 3-dimensional variables can be obtained from 4-dimensional ones using the formula  $\lambda_{ij}^s = \sum_d \lambda_{ij}^{sd}$ .
- $p_{mn}^{ij}$ ;  $i, j, m, n \in \{1 \dots N\}$  is a binary decision variable where  $p_{mn}^{ij} = 1$  if the virtual link  $V_{ij}$  uses the physical link  $P_{mn}$ .

- $c_k^{ij}$ ;  $i, j \in \{1 \dots N\}$   $k \in \{1 \dots M\}$  is a binary decision variable where  $c_k^{ij} = 1$  if the lightpath  $V_{ij}$  uses the  $k^{\text{th}}$  wavelength ( $k = 1, \dots, M$ ).

The constraints are grouped as follows:

VIRTUAL TOPOLOGY (VT):

$$\sum_j V_{ij} \leq T_i \quad \forall i \quad (1)$$

$$\sum_i V_{ij} \leq R_j \quad \forall j \quad (2)$$

Constraint (1) limits the number of outgoing lightpaths (virtual links) by the number of available transmitters. Similarly, constraint (2) limits the number of incoming virtual links by the number of available receivers.

PHYSICAL EMBEDDING OF VIRTUAL TOPOLOGY (P):

$$p_{mn}^{ij} \leq P_{mn} \cdot V_{ij} \quad \forall i, j, m, n \quad (3)$$

$$\sum_n p_{in}^{ij} = V_{ij} \quad \forall i, j \quad (4)$$

$$\sum_m p_{mj}^{ij} = V_{ij} \quad \forall i, j \quad (5)$$

$$\sum_m p_{ml}^{ij} = \sum_n p_{ln}^{ij} \quad \forall l \neq i, j \quad (6)$$

$$p_{mn}^{ij} \geq 0, p_{mn}^{ij} \text{ binary} \quad \forall i, j, m, n$$

Constraint (3) clarifies that if there is no virtual link  $V_{ij}$ , or there is no physical link  $P_{mn}$ , then the indicator variable  $p_{mn}^{ij}$  must be 0. If the virtual link  $V_{ij}$  has been established, then the path is constructed using the flow of value 1 from origin  $i$  (constraint (4)) to destination  $j$  (constraint (5)). The conservation of this unit flow is specified in constraint (6).

VIRTUAL TOPOLOGY TRAFFIC 3-DIMENSIONAL VERSION (T-3DIM) :

$$\sum_i \lambda_{il}^s - \sum_j \lambda_{lj}^s = f_{sl} \quad \forall s, l \quad (7)$$

$$\sum_s \lambda_{ij}^s \leq C \cdot V_{ij} \quad \forall i, j \quad (8)$$

$$\lambda_{ij}^s \geq 0 \quad \forall s, i, j$$

Constraint (7) says that the amount of flow originating at source  $s$  and staying at node  $l$ , i.e. the flow coming to  $l$  minus flow going out from  $l$ , must equal the given flow  $f_{sl}$ . Constraint (8) blocks the flow through the virtual link  $V_{ij}$ , if that virtual link has not been established. If link  $V_{ij}$  exists, then the complete flow on it cannot exceed its maximum capacity.

WAVELENGTH ASSIGNMENT (C):

$$\sum_k c_k^{ij} = V_{ij} \quad \forall i, j \quad (9)$$

$$\sum_{ij} p_{mn}^{ij} \cdot c_k^{ij} \leq 1 \quad \forall m, n, k \quad (10)$$

$$c_k^{ij} \text{ binary, } p_{mn}^{ij} \text{ binary} \quad \forall i, j, m, n, k$$

Constraint (9) asserts that if no lightpaths between  $i$  and  $j$  are used, then no wavelength is necessary. If there is a lightpath between  $i$  and  $j$ , then only one wavelength can use it. Constraint (10) assures that on the link  $(m, n)$  at most one lightpath will use wavelength  $k$ .

We will consider the objective of minimizing the maximum flow on a link. This will result with better balancing of the overall flow, providing more residual capacity for accommodating the increased flow. In this approach traffic delays are assumed to be negligible.

The objective ( $\min(\max_{i,j}(\sum_s \lambda_{ij}^s))$ ) can be modeled as:

$$\min F \quad (11)$$

s.t.

$$\sum_s \lambda_{ij}^s \leq F \quad \forall i, j \quad (12)$$

In 4-dimensional case constraint (12) reads:

$$\sum_{s,d} \lambda_{ij}^{sd} \leq F \quad \forall i, j \quad (13)$$

The presented mixed-integer formulation is not a linear one, and the non-linearity is expressed in constraint (10). In the sequel we will present a genetic algorithm for heuristically solving this formulation.

### 3. A heuristic approach to throughput optimization

In this paper we apply a genetic algorithm from Podnar and Skorin-Kapov [5] for throughput minimization in networks with regular topologies. For completeness, the main ideas behind the algorithm are restated. Details could be found in [5].

Genetic algorithms mimic “optimization” techniques from nature whereby the goal is to generate the best individual by certain standards. New individuals are expected to be fitter than their parents. For genetic algorithms’ essentials one can refer to [1].

The solutions will be based on predetermined wavelength assignments  $c_k^{ij}$ . The typical structure of  $c_k^{ij}$  entries is given in the following example.

**Example 1.** For fixed values of  $i$  and  $j$ , variables  $c_k^{ij}$   $k = 1 \dots M$  could form the following sequence:

$$\underbrace{0}_{c_1^{ij}} \underbrace{0}_{c_2^{ij}} \underbrace{0}_{c_3^{ij}} \dots \underbrace{1}_{c_k^{ij}} \dots \underbrace{0}_{c_M^{ij}}$$

The sequence consists either of all zeros meaning that no wavelength is used on the virtual link  $V_{ij}$  (i.e.  $V_{ij} = 0$ ), or there is only one  $k$  such that  $c_k^{ij} = 1$  (i.e.  $V_{ij} = 1$ ). The later case indicates that the  $k$ -th wavelength is used on the virtual link  $(i, j)$ .

Our chromosomes will represent wavelengths used on virtual links and will be coded as a matrix  $C = [c_{ij}]$   $i, j \in \{1 \dots N\}$ . The entry  $c_{ij}$  will represent the wavelength  $k$  used on the virtual link  $V_{ij}$  implying  $c_{ij}^k = 1$ . If there is no virtual link between nodes  $i$  and  $j$ , then  $c_{ij} = 0$ . Hence, the entries of the matrix  $C$  are integers from the set  $\{0, \dots, M\}$ .

$$C = \begin{bmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1N} \\ \vdots & & \vdots & & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{iN} \\ \vdots & & \vdots & & \vdots \\ c_{N1} & \cdots & c_{Nj} & \cdots & c_{NN} \end{bmatrix}; \quad c_{ij} = \text{the wavelength used on } V_{ij}$$

Once the wavelength assignments are determined, binary variables  $V_{ij}$  can be easily obtained using the following formula:  $V_{ij} = [c_{ij} \geq 1]$  where the notation  $[expression]$  means

$$[expression] = \begin{cases} 0, & \text{if the } expression \text{ is false} \\ 1, & \text{if the } expression \text{ is true} \end{cases}$$

Reconstruction of  $c_k^{ij}$  follows:  $c_k^{ij} = [c_{ij} = k]$ .

Given an arbitrary wavelength assignment matrix  $C$ , our goal is to optimize the objective function (11) i.e. to minimize the maximal flow on a link subject to constraints (1)- (10).

Once matrix  $C$  is given implying that  $V_{ij}$  and  $c_k^{ij}$  are known, the remaining problem is a linear program which can be separated into two smaller sub-programs. The first sub-program (P1) solves for physical embedding paths  $p_{mn}^{ij}$  and, if not feasible, the correction is suggested. The second sub-program (T1) then finds the flow through the virtual topology, if possible.

The formulation of the physical embedding paths sub-program follows.

#### PHYSICAL EMBEDDING OF VIRTUAL TOPOLOGY (P1):

$$\max \sum_{i,j,n} p_{in}^{ij} \quad (14)$$

s.t.

$$\sum_n p_{in}^{ij} \leq V_{ij} \quad (15)$$

$$\sum_m p_{ml}^{ij} = \sum_n p_{ln}^{ij} \quad \forall l \neq i, j \quad (16)$$

$$\sum_{ij} p_{mn}^{ij} \cdot c_k^{ij} \leq 1 \quad (17)$$

$$0 \leq p_{mn}^{ij} \leq P_{mn} \cdot V_{ij} \quad (18)$$

where  $V_{ij}$  and  $c_k^{ij}$  are determined by  $V_{ij} = [c_{ij} \geq 1]$  and  $c_k^{ij} = [c_{ij} = k]$  respectively. The objective (14) is to maximize the number of outgoing paths. If there is no established virtual link  $(i, j)$  then there cannot be a physical embedding path  $(i, j)$  leaving source  $i$  (constraint (15)). However, in the case where there is a virtual link  $V_{ij}$ , there is a possibility that there is no physical embedding path either. The conservation of flow constraint (16) and the wavelength assignment constraint (17) have the same explanation as before.

Once the physical embedding paths are constructed, the correction on the matrix  $C$  can be performed if needed. If constraint (15) is a strict inequality, the entry  $c_{ij}$  will be set to zero, hence generating the equality. This will decrease the number of used wavelengths ( $c_{ij}$ ). Once we have the physical embedding of virtual topology, we can solve the second sub-problem, assigning the flow to physical embedding topology paths. We will use the 3-dim formulation which includes objective (11) and constraints (12), (7), (8). Hence, the second sub-problem formulation is:

VIRTUAL TOPOLOGY TRAFFIC (T1):

$$\min F \quad (19)$$

s.t.

$$\sum_s \lambda_{ij}^s \leq F \quad \forall i, j \quad (20)$$

$$\sum_i \lambda_{il}^s - \sum_j \lambda_{lj}^s = f_{sl} \quad (21)$$

$$\sum_s \lambda_{ij}^s \leq C \cdot V_{ij} \quad (22)$$

$$\lambda_{ij}^s \geq 0$$

Again,  $V_{ij}$  will be replaced with  $[c_{ij} \geq 1]$ .

The *initial population* of size  $N_{\text{init}}$  is generated randomly taking in consideration the VIRTUAL TOPOLOGY (VT) constraints (1) and (2). Once the initial population is established, the genetic algorithm proceeds with iterative construction of new generations. Only the best solutions will be selected for pairing. The number of selected best solutions, denoted by  $N_{\text{good}}$ , is the same through all the generations. Pairing proceeds with the generation of new solutions.

We used pairing probabilities based on relative distances between the solutions:

$$P(l = \text{a parent}) = \frac{C_l}{\sum_{p=1}^{N_{\text{good}}} C_p}, \quad \text{where } C_l = \text{cost}_l - \text{cost}_{N_{\text{good}}+1}.$$

The *mutation* will introduce a change in the wavelength assignment, but without violating the number of transmitters and/or receivers.

The *stopping criterion* will be to evaluate no more than MaxGen generations, or to stop when there is a saturation in a generation (i.e. when all solutions in the generation have the same objective value).

Once the virtual topology is generated, we will try to *rearrange* the physical embedding, in order to reduce the number of wavelengths used.

The last stage of our procedure was to obtain statistical quality measures. Given the wavelength matrix  $C$ , different measures were monitored. To reduce the propagation delay, an alternative physical embedding ( $p_{km}^{ij}$ ) was sought. Neither the feasibility nor the objective will be changed using this variant. The objective used was:

$$\sum_{i,j,m,n} p_{mn}^{ij} \cdot d_{mn} \cdot \text{Delay}$$

where 'Delay' is the average propagation delay per unit distance, i.e. the time needed to transfer one packet of data per unit of distance. Once the physical embedding was obtained, we could measure the average number of hops (links) per physical embedding path. To calculate the average number of necessary wavelength changes between source-destination pairs, we needed to use the 4-dimensional version of the Virtual Topology Traffic problem.

VIRTUAL TOPOLOGY TRAFFIC 4-DIMENSIONAL VERSION (T-4DIM) :

$$\sum_j \lambda_{sj}^{sd} = f_{sd} \quad \forall s, d \quad (23)$$

$$\sum_i \lambda_{id}^{sd} = f_{sd} \quad \forall s, d \quad (24)$$

$$\sum_i \lambda_{il}^{sd} = \sum_j \lambda_{lj}^{sd} \quad \forall l \neq s, d \quad (25)$$

$$\sum_{s,d} \lambda_{ij}^{sd} \leq C \cdot V_{ij} \quad \forall i, j \quad (26)$$

$$\lambda_{ij}^{sd} \geq 0 \quad \forall s, d, i, j$$

Constraint (23) forces the complete flow to leave source  $s$ . Constraint (24) collects all the  $(s, d)$  flow at the destination  $d$ . Conservation of flow at node  $l$  is described in constraint (25). The last constraint (26) has the same role as constraint (8) in the 3-dimensional case.

The pseudo-code of the algorithm is listed in the Appendix at the end of the paper.

## 4. Computational results

We tested our approach on different physical topologies and traffic patterns using a SUN Sparc 10. The topologies were compared and their ability to accommodate large traffic fluctuations was recorded. A number of solution quality measures were monitored to determine which physical topology performs better. For comparison purposes, we selected regular physical topologies such as: Perfect Shuffle (PS), de Bruijn graphs (dB), Generalized shuffle-exchange Multihop NETWORKS (GEMNET) and Manhattan Street Networks (MSN). Those topologies have good properties with respect to reliability and number of paths between any two nodes. The nodal degrees and diameter are usually small, implying that nodes can be connected with



a small number of transceivers per each node, and that the flow can be transferred in a small number of hops. For completeness of the presentation brief definitions of the regular topologies follow.

An  $(N, p)$  Perfect Shuffle (PS) is made of  $N = kp^k$  nodes arranged in  $k$  columns of  $p^k$  nodes. Each of the  $p^k$  nodes in a column has  $p$  links directed to different nodes in the next column, with the last column connected back to the first column, creating a circular diagram.

An  $(N, p)$  de Bruijn graph with  $N = p^D$  nodes is a directed graph with the set of nodes  $\{a_1a_2 \dots a_D : a_i \in \{0, 1, \dots, p-1\}\}$ . A link will connect nodes  $a_1a_2 \dots a_D$  and  $b_1b_2 \dots b_D$  iff the following condition is satisfied:  $b_i = a_{i-1}$  for  $i = 1, \dots, D$  (i.e.  $a_1a_2 \dots a_{D-1} = b_2b_3 \dots b_D$ ). Each node has in-degree and out-degree  $p$ . Some of the nodes may have self-loops.

In an  $(N, p)$  GEMNET topology,  $N = k \cdot M$  nodes are arranged in a cylinder of  $k$  columns and  $M$  nodes per column. For each node the in-degree and the out-degree equals  $p$ . The nodes are linked according to a generalization of the shuffle-exchange connectivity pattern. The generalization allows any number of nodes in a column. There is also no restriction on the number of nodes. They encompass both Shuffle Net and de Bruijn graphs when such graphs exist.

An  $(N, p)$  Manhattan Street Network (MSN) with  $N = M \times K$  nodes is a regular mesh structure of degree 2 with opposite sides connected into torus formation. It consists of  $M$  rows and  $K$  columns, with adjacent row (column) links alternating in direction.

We will consider all four presented regular topologies with  $N = 8$  nodes and  $p = 2$ . The physical topologies are depicted in *Figures 1,2*.

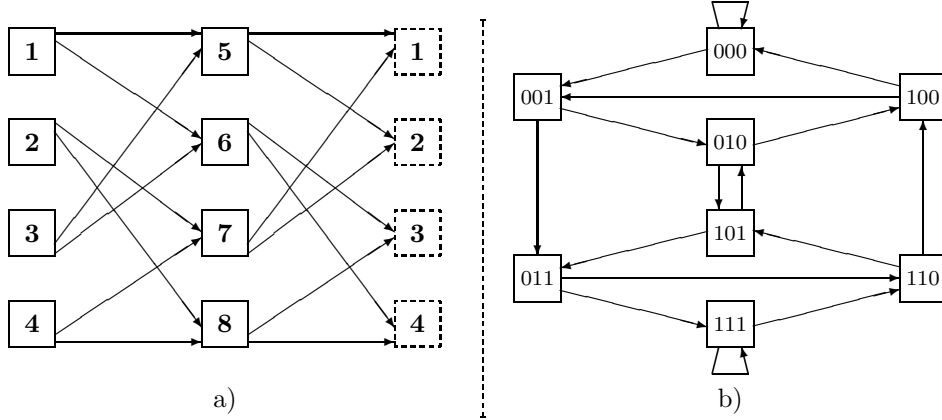


Figure 1. a)  $(8,2)$  Perfect Shuffle, b)  $(8,2)$  de Bruijn graph

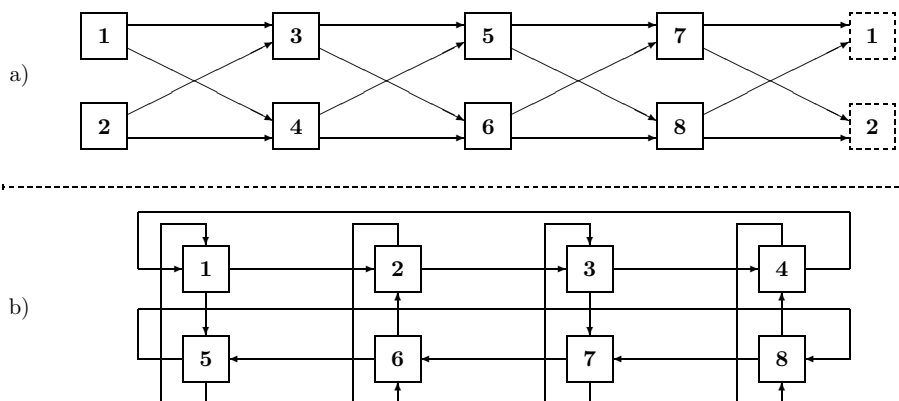


Figure 2. a) (8,2) GEMNET, b) (8,2) Manhattan street network

We will use the traffic matrices as described in Skorin-Kapov and Labourdette [7, 8] for throughput optimization in *broadcast* networks. Several different traffic matrices were generated based on different flow patterns (uniform, quasi-uniform, ring, disconnected).

Several preliminary tests were performed in order to adjust the GA parameters. We started with the initial population of size  $N_{\text{init}} = 60$ . The subsequent generations were of size  $N_{\text{pop}} = 35$ . The subset of best members of the generation which were selected for paring was set to  $N_{\text{good}} = 15$ . The mutation rate of  $\mu = 0.20$  was used. After the mutation, a correction procedure was performed in order to make the mutated solution feasible. Because of the correction, the exact mutation rate could in fact be smaller than  $\mu$ . Only 60 generations were created.

Table 1 summarizes the experimental results based on four different physical topologies (PS, dB, GEMNET, MSN) and five different traffic patterns (uniform, quasi-uniform 1&2, ring, disconnected). The data sets were tested with  $T_i = R_j = 2$ . For test purposes, the capacity of the links was set to be 35 (units/time).

Several determinations of solution were monitored, including the objective function, the number of hops per virtual link and the number of intermediate nodes per node pair. The objective function measures the minimal maximum flow through a link ( $\max \text{Load} \equiv \min \max \text{flow/link}$ ). The number of hops per virtual link is calculated as the average length of a virtual link in terms of physical links. The number of intermediate nodes per pair measures the average number of times that a source-destination flow changes its wavelength. Every time the flow needs to change its wavelength, it must be received and transmitted again.

From the tests performed, one can observe that the length of a virtual path (hops per virtual links) is between 2 and 2.75, and in average it never reaches 3. The majority of flow from a node must reach nodes which are not immediate neighbors of the node. This is a reason that the virtual paths with length more than 2 are prevailing.

The average number of wavelength switches varies between 0.96 and 1.57. This means that the majority of the traffic flow  $f_{ij}$  will switch wavelengths at least once.

We proceed with a performance comparison of the presented model against the model proposed by Skorin-Kapov and Labourdette [8]. The objective in both models is to minimize the throughput through an optical network.

Given a traffic matrix, the model presented in Skorin-Kapov and Labourdette [8] searches for the best possible allocation of traffic nodes (sources and destinations) to the existing physical nodes. The allocated nodes will follow one of the regular topology patterns (Perfect Shuffle, de Bruijn, GEMNET or MSN). Once the allocation has been performed, direct optical fiber links are placed following the underlying regular topology. In other words, the model finds the regular topology embedding with the optimal throughput. For the regular topologies introduced (PS, dB, GEMNET, MSN) the number of transmitters and receivers equals two. The optical fiber link network corresponds to our notion of the virtual topology. The physical topology is assumed to be able to support any embedding of the given regular topology. Concisely, the problem is to find the best physical topology (embedding) given a regular graph as a virtual topology. The summary of the results from Skorin-Kapov and Labourdette [8] for the case of 8 nodes is presented in *Table 2*.

Our model starts with the nodes already placed on a given regular physical topology. The goal is to find a virtual topology with the smallest throughput. Concisely, the problem is to find the best virtual topology given a regular graph as a physical topology.

Comparison of the two tables (*Tables 1* and *2*) leads to the conclusion that our solution method yields better optimal solutions in majority of cases. Direct comparison of the regular topologies shows that our solution networks consistently provide better throughput than the solutions presented by Skorin-Kapov and Labourdette [8]. For example, for uniform traffic pattern the maximum flow on a link is always less than 7, compared to the values of 8 and 9 in *Table 2*. Even when no restrictions are imposed on the topology in Skorin-Kapov and Labourdette's model, some of our solutions reach the same objective (6.67). Similar observations hold for the quasi-uniform and disconnected traffic matrices. Only for the ring traffic pattern using either PS or GEMNET regular topology, Skorin-Kapov and Labourdette provide a better throughput. The majority of traffic in the ring traffic matrix flows through the ring  $1 \rightarrow 2 \rightarrow 3 \dots \rightarrow 8 \rightarrow 1$ . One can notice the underlying cycles in the PS traffic pattern ( $1 \rightarrow 6 \rightarrow 4 \rightarrow 8 \rightarrow 3 \rightarrow 5 \rightarrow 2 \rightarrow 7 \rightarrow 1$ ) and in the GEMNET traffic pattern ( $1 \rightarrow 3 \rightarrow 5 \rightarrow 7 \rightarrow 2 \rightarrow 4 \rightarrow 6 \rightarrow 8 \rightarrow 1$ ). These embeddings can easily support the ring traffic pattern, generating, in turn, better solutions for Skorin-Kapov and Labourdette's model.

In all other cases the model and the solution technique presented in this paper generate improved results.

We proceed with the conclusion and the areas of further research.

	$T_i = R_j = 2$	uni	quasiuni1	quasiuni2	ring	disconn
PS	max flow/link	6.83	6.48	7.04	14.40	32.00
	hops/V Links	2.13	2.00	2.25	2.50	2.13
	intermed. nodes/pair	1.14	1.23	1.16	1.30	1.39
dB	max flow/link	6.67	6.23	7.10	14.53	32.00
	hops/V Links	2.19	2.19	2.56	2.25	2.38
	intermed. nodes/pair	1.09	1.18	1.21	1.44	1.39
GEMNET	max flow/link	7.00	6.48	6.93	14.36	32.00
	hops/V Links	2.00	2.50	2.50	2.50	2.75
	intermed. nodes/pair	0.96	1.23	1.16	1.14	1.39
MSN	max flow/link	6.67	6.38	7.20	13.80	32.00
	hops/V Links	2.38	2.00	2.00	2.25	2.00
	intermed. nodes/pair	1.05	1.16	1.16	1.57	1.39

Table 1. *Four different physical topologies (PS,dB,GEMNET,MSN); uniform, quasi-uniform, ring and disconnected traffic matrices*

Results by Skorin-Kapov and Labourdette [8]					
topology	uni	quasiuni1	quasiuni2	ring	disconn
PS	8.00	7.23	7.75	13.95	33.35
dB	9.00	8.25	8.90	16.30	41.45
GEMNET	8.00	7.23	7.75	12.70	33.60
MSN	8.00	7.23	7.75	13.95	33.35
Arbitrary	6.67	6.06	6.57	12.40	27.54

Table 2. *Minimal maximum flow; PS, dB, GEMNET and MSN physical topologies; uniform, quasi-uniform, ring and disconnected traffic matrices*

## 5. Conclusions and further research

In this paper the genetic algorithm of Podnar and Skorin-Kapov from [5] was applied to throughput optimization of non-broadcast fiber optic networks with wavelength-division multiplexing and regular topologies.

We tested the genetic algorithm against a benchmark data set from Skorin-Kapov and Labourdette [7, 8]. This data set includes different underlying physical topologies (Perfect Shuffle, de Bruijn graphs, Generalized Shuffle-Exchange Multihop Networks and Manhattan Street Networks) combined with different traffic matrices of a specific flow pattern. The physical topologies consist of 8 nodes with every node of the (bidirectional) degree 2. Based on a fixed traffic matrix, the physical topologies expressed very similar behavior.

Our solutions to the non-broadcast WDM problem with  $T_i = R_j = 2$  can be compared with the solutions to the broadcast problem in Skorin-Kapov and Labourdette [8]. The starting point in their model is a given virtual topology. Their objective is to find the optimal embedding of the physical topology with respect to the network throughput. In contrast to their model, we initialized our physical topology to be one of the given benchmark regular topologies. We then proceed with the task of finding the best virtual topology. Our model shows an improvement

in the networks throughput in majority of regular topology cases. The arbitrary topology generates a better solution, but the assumption of availability of fiber optic cables in-between every pair of nodes might not be practical.

Further work will include several modifications to the problem. Specifically, other objective functions can be considered, for example minimizing the propagation and/or queuing delay.

In addition, we will explore ways to modify the heuristic approach in order to achieve solvability of larger problems.

## Appendix

The genetic algorithm and the wavelength reduction pseudo-code is presented.

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### INPUT

- Read the INPUT PARAMETERS ( $N, M, T_i, R_i, P_{mn}, d_{mn}, f_{sd}, C$ )
- Increase  $M$  by a suitable increment.

### MAIN ALGORITHM

- Perform the GA to obtain a good heuristic solution. OUTPUT: Wavelength matrix  $C$ .
- Perform WAVELENGTH REDUCTION procedure on  $C$

### OUTPUT

- Calculate STATISTICS to test the solution quality.

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### GENETIC ALGORITHM (GA)

- Initialize the (GA) parameters:  $N_{\text{init}}, N_{\text{pop}}, N_{\text{good}}, N_{\text{bad}}, \mu, \text{MaxGen}$
- Generate initial population ( $N_{\text{init}}$ ) that satisfies VIRTUAL TOPOLOGY (VT) constraints (1,2).
- Sort the initial population in increasing order using members' fitness values. Calculate the cumulative pairing probabilities. Print the best individual and the statistics.  
REPEAT UNTIL STOPPING CRITERION IS SATISFIED
  - Perform the pairing procedure based on the current population and the cumulative pairing probabilities. Make it feasible ((VT) constraints).
  - Perform the mutation. Make it feasible ((VT) constraints).
  - Determine the PHYSICAL EMBEDDING. Correct the wavelength matrix.
  - Evaluate the VIRTUAL TOPOLOGY TRAFFIC and the objective.
  - Sort the current population.
  - Calculate new cumulative pairing probabilities.
  - Print the best individual and the statistics (average fitness, standard deviation).
  - If the stopping criteria is not satisfied redo the loop.
- Save the best solution found ( $C$ ) and record the total CPU time.

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**WAVELENGTH REDUCTION**

- Read the INPUT: the wavelength matrix  $C = [c_{ij}]$ .
  - Obtain the wavelength frequencies.
  - Sort the wavelengths according to the frequencies, reassign the wavelengths.
  - for  $i = 1$  to  $N$
  - for  $j = 1$  to  $N$
- REPEAT UNTIL STOPPING CRITERION IS SATISFIED  
(Stop if either  $c_{ij} = 1$  or the (PHYSICAL EMBEDDING) not feasible.)
- Decrease  $c_{ij}$  by 1.
  - Test the (PHYSICAL EMBEDDING) feasibility.
  - If the stopping criteria is not satisfied redo the loop.
- Print the obtained wavelength matrix and the number of wavelengths needed.
- 

**STATISTICS**

- Based on the obtained wavelength matrix  $C$ , determine the PHYSICAL EMBEDDING ( $p_{km}^{ij}$ ) according to the smallest propagation delay objective ( $\sum_{i,j,m,n} p_{mn}^{ij} \cdot d_{mn} \cdot \text{Delay}$ ).
  - Based on the obtained wavelength matrix  $C$  determine the VIRTUAL TOPOLOGY TRAFFIC  $\lambda_{ij}^{sd}$ .
  - Calculate the statistics for various scale-up factors, such as: Average packet delay, Queuing delay, Average propagation delay, Average number of intermediate nodes, Average virtual link hop distance (hops per virtual link), Minimal and Maximal Link loading.
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