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How to Convert the Obliquely Crossed to Non-Crossed Astigmatism? A Simple Method Using Vector Analysis

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A B S T R A C T

The authors discussed about the problem of special form in astigmatism classification. This special type of astigmatism is the form of obliquely crossed astigmatism. In which the meridians, major and minor, are not right angles. In this astigmatism is not possible to prescribing for cylindrical (toric) spectacle lens. Authors describe the Thompson formula for oblique crossed cylinder and observe that this formula is to complicate for calculation new cylinder power. In this reason, the authors create the new formula and simple procedure for this calculation. This simple formula based on vector analysis and read: DM3 = DM2 × cos² β.

***Key words**: obliquely crossed astigmatism, principal meridians, vector analysis*

Introduction

In the classification of astigmatisms, special type is presented as the *oblique lycrossed astigmatism*[1]. This type of astigmatism is regular, but the principal meridians, major and minor, do not close 90*, they are not right angles. This optical conditions is also called: *bi-oblique astigmatism*, the first determine by Roure, 1896[2]. I prefer the latin term: *astigmatismus obliquus decussatus* (This term – decussatus, decusso = crossed in form of letter »X«, which exactly represent those crossed meridians. In this way, calculation of sphero-cylindrical power is »not the function« of equation $D \times \sin^2 \alpha$ and is not possible prescribing for cylindrical spectacle lens. The value of two principal meridians, with the crossed meridian planes, must be calculated with the special equations.

One of this formulas is Thompson's formula:

$$C^2 = C_1^2 + C_2^2 + 2C_1C_2 \cos 2\gamma$$

gives the amount of the new cylinder power.

$$S = S_1 + S_2 + \frac{C_1 + C_2 - C}{2}$$

gives the amount of the new sphere power.

$$\tan 2\theta = \frac{C_2 \sin 2\gamma}{C_1 + C_2 \cos 2\gamma}$$

gives θ, the amount to be added to a_1 to give the new axes.

This Thompson's formula is objectively very complex, confusion and no clarity. This was the motive to create the new formula with the clarity of meridians power, meridians plane and direction cylinders axis. Also, must be simple for calculation and convert to usualy sphero-cylindical power for cylindrical spectacle lense. But, this calculation is also necessary in refractive surgery[3], because the front surface and back surface of the cornea in 100 percent have obliquely crossed cilinders.

Material and Methods

By 25 patients we measured (apparatus: Oculus-Pentacam) anterior and posterior corneal surface astigmatism, estemated the obliquely crossed cylinders in 100%. The means of major meridian power was from ±0.25 to

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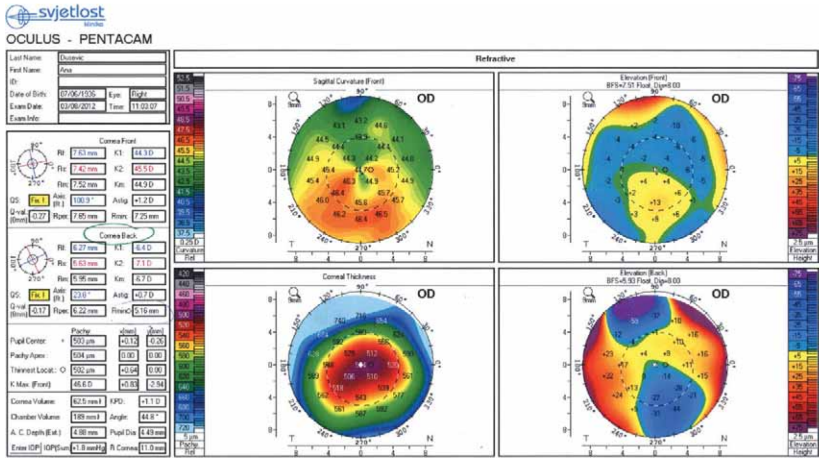

Fig. 1. Present the new method Oculus-Pentacam with possibility the meassured of front and back corneal surface. It's visible that major meridians anterior and back surface make the crossed cylinders. This method was used in our examination.

±1.50 diopters. In Figure 1. is presented the sine function of major meridian angle: D × $\sin^2 \alpha$. All cases of obliquely crossed cylinders we calculated with the vector analysis and convert to the not crossed cylinders.

Description the methods

Procedure: If we take the first major meridian M1ax 160*. than the second major meridian M2ax 20*, we must transposing in M3ax 70* (alfa3 – alfa2 = cos beta). In this new position the angle between two major meridian is ax = 90 degrees. In this picture, we must cath sight of arrow direction of the sine and cosine. In this reason, the angle of alfa3 – alfa2 is not sine function, than cosine beta.

Example: (»easier calculation«)

1. Major meridian: M1 @1 ax 160* + 3.0D
2. Major meridian: M2 @2 ax 20* + 2.0D
3. M3 @3 = 70* – alfa2 = cos beta 50*
4. DM3 = + 2.0D × $\cos^2$ (50) = 2 × 0.593
   = 1.186 dioptries

New combination for prescription:

DM1 = +3.0 D, major meridian 160*

DM3 = +1.25 D, major meridian 70*

New prescription ophthalmic lens:

I. +3.0 = –1.75 ax 160*

II. +1.25 = +1.75 ax 70*

Results

In the basic of our vectors method for convert the obliquely crossed cylinders to not crossed, we construct the trigonometric vectors tables for simple and quickly estimated the new major meridian »DM3« for prescription ophthalmic lens. This table consist the means of cosine from 0 degree to 90 degrees and corresponding new values of diopters in the new direction of second major meridian.

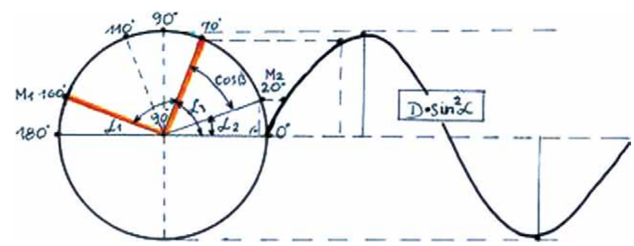

Fig. 2. Represent the diopters changeable as usual sine function (D × $\sin^2 \alpha$ of angle the major (cardinal) meridian. The cardinal meridian M1-M3 and M2-M4 are obliquely but not crossed, the axis is 90 degrees. Meridian M1-M2 are obliquely and crossed, angle alfa is 160–20=140 degrees, and this case must be convert, with the special formula, before prescription cylindrical spectacle lens.

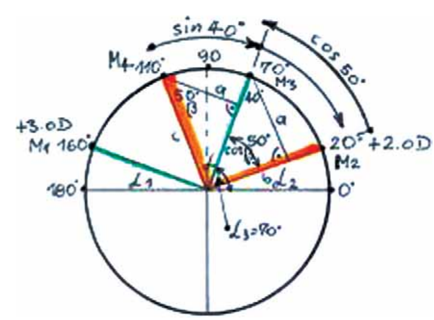

Fig. 3. Represent the vector's method for calculation of obliquely crossed cylinder to non crossed. In our example it's visible that the angle between two major meridians (M1 ax160 = +3.D and M2 ax20* = +2.0D) is 140 degrees.*

The simplification of our formula:

DM3 = DM2 × $\cos^2 \beta$

Discussion and Conclusions

Our clinical analyses of anterior et posterior corneal astigmatism estimate that the astigmatism of back cor-

TABLE 1.
VECTOR ANALYSIS TABLES FOR CONVERT THE OBLIQUELY CROSSED CYLINDERS: BI – OBLIQUE ASTIGMATISM

Angle α3–α2	DM2 x (cosβ)² = DM3							Angle α3–α2	DM2 x (cosβ)² = DM3						
β	(cosβ)²	0,25	0,50	0,75	1,00	1,25	1,50	β	(cosβ)²	0,25	0,50	0,75	1,00	1,25	1,50
0	1.0	0.25	0.50	0.75	1.0	1,25	1,50	46	0.49	0.12	0.24	0.36	0.49	0.61	0.73
1	0.98	0.24	0.49	0.73	0.98	1,22	1,47	47	0.46	0.11	0.23	0.35	0.46	0.58	0.69
2	0.98	0.24	0.49	0.73	0.98	1,22	1,47	48	0.45	0.11	0.22	0.34	0.45	0.56	0.67
3	0.98	0.24	0.49	0.73	0.98	1,22	1,47	49	0.44	0.11	0.22	0.33	0.44	0.55	0.66
4	0.98	0.24	0.49	0.73	0.98	1,22	1,47	50	0.41	0.10	0.20	0.33	0.41	0.52	0.62
5	0.98	0.24	0.49	0.73	0.98	1,22	1,47	51	0.40	0.1	0.2	0.3	0.40	0.50	0.60
6	0.98	0.24	0.49	0.73	0.98	1,22	1,47	52	0.37	0.09	0.18	0.28	0.37	0.46	0.56
7	0.98	0.24	0.49	0.73	0.98	1,22	1,47	53	0.36	0.09	0.18	0.27	0.36	0.45	0.54
8	0.98	0.24	0.49	0.73	0.98	1,22	1,47	54	0.35	0.08	0.17	0.26	0.35	0.44	0.52
9	0.96	0.24	0.48	0.72	0.96	1,20	1,44	55	0.33	0.08	0.16	0.25	0.33	0.41	0.50
10	0.96	0.24	0.48	0.72	0.96	1,20	1,44	56	0.31	0.08	0.15	0.23	0.31	0.39	0.47
11	0.96	0.24	0.48	0.72	0.96	1,20	1,44	57	0.29	0.07	0.14	0.22	0.29	0.36	0.44
12	0.94	0.23	0.47	0.70	0.94	1,18	1,41	58	0.28	0.07	0.14	0.21	0.28	0.35	0.42
13	0.94	0.23	0.47	0.70	0.94	1,18	1,41	59	0.26	0.06	0.13	0.20	0.26	0.33	0.40
14	0.92	0.23	0.46	0.69	0.92	1,15	1,38	60	0.25	0.06	0.12	0.19	0.25	0.31	0.38
15	0.92	0.23	0.46	0.69	0.92	1,15	1,38	61	0.24	0.06	0.12	0.18	0.24	0.30	0.36
16	0.90	0.22	0.45	0.67	0.90	1,13	1,35	62	0.22	0.05	0.11	0.17	0.22	0.27	0.33
17	0.90	0.22	0.45	0.67	0.90	1,13	1,35	63	0.20	0.05	0.10	0.15	0.20	0.25	0.30
18	0.88	0.22	0.44	0.66	0.88	1,10	1,32	64	0.19	0.04	0.09	0.14	0.19	0.24	0.29
19	0.88	0.22	0.44	0.66	0.88	1,10	1,32	65	0.18	0.04	0.09	0.14	0.18	0.23	0.27
20	0.86	0.21	0.43	0.64	0.86	1,08	1,29	66	0.17	0.04	0.08	0.13	0.17	0.21	0.26
21	0.86	0.21	0.43	0.64	0.86	1,08	1,29	67	0.15	0.03	0.07	0.11	0.15	0.19	0.23
22	0.84	0.21	0.42	0.63	0.84	1,05	1,26	68	0.14	0.03	0.07	0.10	0.14	0.18	0.21
23	0.83	0.20	0.41	0.62	0.83	1,04	1,25	69	0.13	0.03	0.06	0.09	0.13	0.16	0.20
24	0.81	0.20	0.40	0.61	0.81	1,01	1,22	70	0.12	0.03	0.06	0.09	0.12	0.15	0.18
25	0.81	0.20	0.40	0.61	0.81	1,01	1,22	71	0.10	0.02	0.05	0.08	0.10	0.13	0.15
26	0.79	0.20	0.39	0.59	0.79	0.99	1,19	72	0.09	0.02	0.04	0.07	0.09	0.11	0.13
27	0.77	0.19	0.38	0.58	0.77	0.96	1,16	73	0.08	0.02	0.04	0.06	0.08	0.10	0.12
28	0.76	0.19	0.38	0.57	0.76	0.95	1,14	74	0.07	0.02	0.04	0.05	0.07	0.09	0.10
29	0.75	0.19	0.37	0.56	0.75	0.94	1,13	75	0.06	0.02	0.03	0.05	0.06	0.08	0.10
30	0.74	0.18	0.37	0.56	0.74	0.93	1,11	76	0.06	0.02	0.03	0.05	0.06	0.08	0.09
31	0.72	0.18	0.36	0.54	0.72	0.90	1,08	77	0.05	0.01	0.03	0.04	0.05	0.06	0.08
32	0.71	0.18	0.36	0.53	0.71	0.89	1,06	78	0.04	0.01	0.02	0.03	0.04	0.05	0.06
33	0.70	0.18	0.36	0.53	0.70	0.88	1,05	79	0.04	0.01	0.02	0.03	0.04	0.05	0.06
34	0.69	0.17	0.35	0.52	0.69	0.86	1,04	80	0.03	0.01	0.02	0.02	0.03	0.04	0.05
35	0.67	0.17	0.34	0.50	0.67	0.84	1,01	81	0.02	0.01	0.01	0.02	0.02	0.03	0.04
36	0.66	0.16	0.33	0.49	0.66	0.83	1.00	82	0.02	0.01	0.01	0.02	0.02	0.02	0.03
37	0.64	0.16	0.32	0.48	0.64	0.80	0.96	83	0.01	0.003	0.01	0.01	0.01	0.01	0.02
38	0.61	0.15	0.31	0.46	0.61	0.76	0.92	84	0.01	0.003	0.01	0.01	0.01	0.01	0.02
39	0.59	0.15	0.30	0.44	0.59	0.74	0.89	85	0.008	0.002	0.002	0.01	0.01	0.01	0.01
40	0.58	0.14	0.29	0.43	0.58	0.73	0.87	86	0.005	0.001	0.002	0.004	0.005	0.01	0.01
41	0.56	0.14	0.28	0.42	0.56	0.70	0.84	87	0.003	0.001	0.001	0.002	0.003	0.004	0.01
42	0.55	0.13	0.27	0.41	0.55	0.69	0.82	88	0.002	0.001	0.001	0.002	0.002	0.003	0.003
43	0.53	0.13	0.26	0.40	0.53	0.66	0.80	89	0.001	0.000	0.000	0.001	0.001	0.002	0.002
44	0.51	0.12	0.25	0.38	0.51	0.64	0.77	90	0.000	0.000	0.000	0.000	0.000	0.000	0.000
45	0.50	0.12	0.25	0.37	0.50	0.63	0.75								

By Vojniković-Gabrić-Dekaris: Vector Convert Crossed Cylinders – VGD formulas neal surface is constantly present[3–5], and always obliquely and crossed in correlation to the astigmatism of front corneal surface. Astigmatism anterior et posterior corneal surface both of them tends to increase the power of major meridians. Astigmatism of posterior corneal surface tended to partially compensate the astigmatism of anterior corneal surface[3,4]. Anterior corneal astigmatism is much changeable in aging than posterior astigmatism,and tend the vertical major meridian transpositioned to horizontal[2].

Today, in refractive surgery procedure, IOL implantation, laser corneal refractive treatment, can't ignoring the astigmatism of posterior corneal surface, than it's necessary estimated the total corneal astigmatism.

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KAKO KONVERTIRATI KOSI UKRŠTENI ASTIGMATIZAM U NEUKRŠTENI? PRIMJENA JEDNOSTAVNE VEKTORSKE METODE

SAŽETAK

Pojava ukrštenih ili ukrižanih (u formi slova X) cilindara, kao refrakcijska greška, tvore posebnu formu astigmatizma: ukršteni astigmatizam, kod kojeg dva glavna meridijana ne tvore međusobno pravi kut. Autori također raspravljaju sa semantičkog aspekta, terminološku ispravnost naziva »ukrižanost«, gdje u latinskoj osnovi to stanje ukrižanosti određuje naziv »decussatio«, što upravo znači ukrižanost u formi slova »X«. Najraniji naziv, za tako ukrižane cilindre, potiče od autora Roure, iz 1894. koji ih imenuje, kao: *bioblique astigmatism*. Iz tih razloga njihov odnos se više ne može tretirati kvadratom sin funkcije: $D = \sin^2 \alpha$. Ta se pojava refrakcijske greške pojavljuje, kako kod direktnog, tako i kod kosog astigmatizma, koji zajedno spadaju u grupu regularnog astigmatizma, s obzirom da su preslikavanja po čitavom meridijanu pravilna i identična. U kliničkoj refrakciji takav astigmatizam ima naziv: Astigmatism crossed i pojavljuje se na obje plohe rožnice. Autori uvode vlastitu vektorsku metodu konverzije, dovodeći ukrštene cilindre u neukrštene, sa novim vrijednostima optičke snage meridijana i nove osovine. Ova korekcija se danas mora sve više upotrebljavati, posebice u implantacijskoj refrakcijskoj kirurgiji, kada se implantiraju toričke leće. To je iz razloga, što je konstatirano, da astigmatizam stražnje plohe rožnice igra važnu ulogu u kalkulaciji implantacijske leće, i što praktički u 100% slučajeva, glavni meridijani prednje i stražnje plohe rožnice tvore ukrštene meridijane, odnosno cilindre.