

Least Metameric Recipe Formulation

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The paper describes a variant of multi-illuminant strategy of colour match prediction calculation for the cases of CMC(*l:c*) and CIE94 colour differences. This strategy tries to minimize the colour differences (against a given standard) under several different illuminants. In case when a given standard, using the usual single-illuminant matching strategy, can not be matched non-metamerically by the colorants available, the multi-illuminant matching strategy tries to produce a more acceptable match by balancing the colour differences under several different illuminants. The theoretical concepts are illustrated by the colorimetric data of the corresponding laboratory samples produced by either strategy. The multi-illuminant-strategy regularly produced lower metamerism than the single-illuminant strategy did.

INTRODUCTION

When a colourist has to formulate the recipe to match the colour of a given standard sample, he must take into account technological limitations of the dyeing process, and on the other hand, the requests and limitations set by the customer. Some of the important requests of the customer are: the colour difference against a given standard under a reference illuminant (*e.g.* D65) should be as small as possible, low price of the dyeing, low level of metamerism, appropriate colour fastness, *etc.* Usually both the colourist and the customer have to accept some compromise among the above requests.

In the present paper we describe a possible compromise between the request for the minimal colour difference under the reference illuminant and the request for minimal metamerism of a recipe. Such a compromise could be useful in the situation, when a (limited) range of colorants available does not enable to produce a nonmetameric recipe for the colour of a given standard.

Throughout this paper the vector $\mathbf{c} = (c_1, c_2, \dots, c_N)^T$ denotes the recipe containing N colorants, c_1, c_2, \dots, c_N being their concentrations. The number of colorants (N) in the recipe is usually 3, rarely 4, 5 or 6.

DESCRIPTION OF THE PROBLEM

Let us recall the definition of the illuminant metamerism. A pair of samples is metameric if they have the same colour under one illuminant (*e.g.* under D65 and therefore the colour difference $\Delta E_{D65} = 0$), but they differ in colour under some other illuminant(s) (*e.g.* under the illuminants A and WWF the colour differences ΔE_A and ΔE_{WWF} are greater than zero). The human eye does not notice very small colour differences and consequently the customer is satisfied when the colour difference ΔE of the ordered product against the given standard is less than the appropriately set tolerance (usually the tolerance is between 0.5 and 1.5 units of colour difference).

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Let us now consider a particular combination of three (or more) colorants and examine a few different ways for optimization of the recipe (containing just these colorants) from the viewpoint of colour differences under several different illuminants. The majority of the widespread colour match prediction programs tended to achieve the equality of the tristimulus values X, Y, Z of the recipe and the standard under one particular illuminant (e.g. D65). These programs followed (in the recipe calculation and in the subsequent correction procedure) the usual single-illuminant matching strategy

$$\begin{aligned} (\Delta X, \Delta Y, \Delta Z)_{D65}^T = \\ (X_{STD} - X_{MIX}, Y_{STD} - Y_{MIX}, Z_{STD} - Z_{MIX})_{D65}^T \rightarrow (0, 0, 0)^T \quad (1) \end{aligned}$$

The indices $_{STD}$ and $_{MIX}$ in the expression (1) refer to the standard and the applied recipe respectively. As the equality of the tristimulus values (X, Y, Z) of the recipe colour and the target colour implies the zero colour difference ΔE_{D65} , the aim of the strategy (1) is equivalent to the aim of the single-illuminant strategy:

$$\Delta E_{D65} \rightarrow 0 \quad (1^*)$$

Well known and widespread algorithms of this kind are those of Allen,^{1,2} the former for the single-constant and the latter for the two-constant formulation.

The strategy (1) or (1*) when applied to a combination of appropriate colorants enables the achievement of very small colour difference ΔE_{D65} , but on the other hand, in case of a metameric recipe the resulted colour differences (e.g. $\Delta E_A, \Delta E_{WWF}$) under other important illuminants can become unacceptably high. This situation is depicted on the left-hand side of Figure 1.

In order to get a more acceptable recipe with the same combination of colorants (if possible) we will try to match predict more equally across several different illuminants. We will relax the requirement of an exact match under reference illuminant (e.g. D65) and try to improve the match under second and third illuminant (e.g. A and WWF) in turn. Let us introduce a new colour matching strategy, which will produce more balanced and, in the bounds of possibility, small colour differences under several different

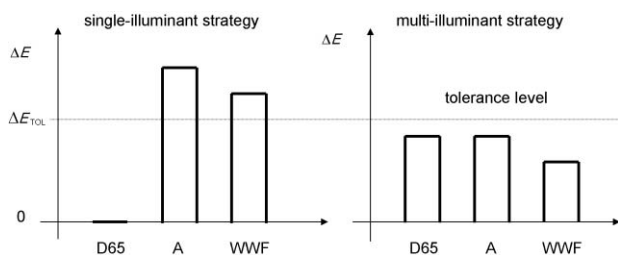


Figure 1. Possible predicted colour differences $\Delta E_{D65}, \Delta E_A, \Delta E_{WWF}$ of two recipes (against the given standard) using the same combination of colorants. The strategy $\Delta E_{D65} \rightarrow 0$ was used on the left, and the strategy $\max\{\Delta E_{D65}, \Delta E_A, \Delta E_{WWF}\} \rightarrow 0$ on the right.

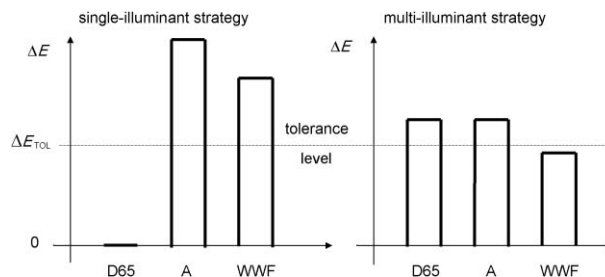


Figure 2. Possible predicted colour differences $\Delta E_{D65}, \Delta E_A, \Delta E_{WWF}$ of two recipes (against a given standard) using the same combination of colorants – like in Figure 1. Balancing the colour differences reduces the metamerism, although it cannot diminish all the colour differences under the tolerance level.

illuminants. A strategy with such properties could be the following one:

$$\max\{\Delta E_{D65}, \Delta E_A, \Delta E_{WWF}\} \rightarrow \min \quad (2)$$

In case the practical realization of this strategy gives the recipe, which produces all the colour differences $\Delta E_{D65}, \Delta E_A, \Delta E_{WWF}$ smaller than the preset tolerance ΔE_{TOL} , an acceptable recipe is obtained by the same combination of colorants as before. This situation is schematically depicted on the right-hand side in Figure 1. If the metamerism of the recipe produced by the strategy (1) is too high, the balancing of the colour differences under several different illuminants by the strategy (2) cannot bring all the colour differences under the tolerance level. Nevertheless, keeping in mind that the quantity $\max\{\Delta E_{D65}, \Delta E_A, \Delta E_{WWF}\}$ is a measure of metamerism, we can state that the metamerism of the recipe produced by the multi-illuminant strategy (2) is lower than the metamerism of the recipe produced by the single-illuminant strategy (1), see Figure 2.

NUMERICAL ALGORITHM ACCORDING TO THE MULTI-ILLUMINANT MATCHING STRATEGY

As the criterial function $\max\{\Delta E_{D65}, \Delta E_A, \Delta E_{WWF}\}$ in the matching strategy (2) is not differentiable the calculation of the »equilibrate« recipe $\mathbf{c} = (c_1, c_2, \dots, c_N)^T$ strictly respecting the strategy (2) could force us to use relatively slow optimization algorithms. It is useful to formulate an alternative matching strategy, which enables the use of faster optimization techniques, but still retains the mentioned »equilibrate« or »balancing« property of strategy (2). One way of doing this is the strategy (described in the article by Sluban):³

$$w_{D65}^2 (\Delta E_{D65})^2 + w_A^2 (\Delta E_A)^2 + w_{WWF}^2 (\Delta E_{WWF})^2 \rightarrow \min \quad (3)$$

where the weights w_{D65}, w_A, w_{WWF} reflect (possibly) different importance of agreement of the standard and the match under different illuminants. An appropriate choice of the weights w_{D65}, w_A, w_{WWF} enables to formulate the recipes with the »equilibrate« property characteristic for the results of strategy (2). An iterative algorithm fol-

$$\begin{bmatrix}
 w_{D65} \frac{1}{lS_L} \frac{\partial L^*}{\partial c_1} & w_{D65} \frac{1}{lS_L} \frac{\partial L^*}{\partial c_2} & \dots & w_{D65} \frac{1}{lS_L} \frac{\partial L^*}{\partial c_N} \\
 w_{D65} \frac{1}{cS_C} \frac{\partial C^*}{\partial c_1} & w_{D65} \frac{1}{cS_C} \frac{\partial C^*}{\partial c_2} & \dots & w_{D65} \frac{1}{cS_C} \frac{\partial C^*}{\partial c_N} \\
 w_{D65} \frac{C^*}{S_H} \frac{\partial h}{\partial c_1} & w_{D65} \frac{C^*}{S_H} \frac{\partial h}{\partial c_2} & \dots & w_{D65} \frac{C^*}{S_H} \frac{\partial h}{\partial c_N} \\
 w_A \frac{1}{lS_L} \frac{\partial L^*}{\partial c_1} & w_A \frac{1}{lS_L} \frac{\partial L^*}{\partial c_2} & \dots & w_A \frac{1}{lS_L} \frac{\partial L^*}{\partial c_N} \\
 w_A \frac{1}{cS_C} \frac{\partial C^*}{\partial c_1} & w_A \frac{1}{cS_C} \frac{\partial C^*}{\partial c_2} & \dots & w_A \frac{1}{cS_C} \frac{\partial C^*}{\partial c_N} \\
 w_A \frac{C^*}{S_H} \frac{\partial h}{\partial c_1} & w_A \frac{C^*}{S_H} \frac{\partial h}{\partial c_2} & \dots & w_A \frac{C^*}{S_H} \frac{\partial h}{\partial c_N} \\
 w_{WWF} \frac{1}{lS_L} \frac{\partial L^*}{\partial c_1} & w_{WWF} \frac{1}{lS_L} \frac{\partial L^*}{\partial c_2} & \dots & w_{WWF} \frac{1}{lS_L} \frac{\partial L^*}{\partial c_N} \\
 w_{WWF} \frac{1}{cS_C} \frac{\partial C^*}{\partial c_1} & w_{WWF} \frac{1}{cS_C} \frac{\partial C^*}{\partial c_2} & \dots & w_{WWF} \frac{1}{cS_C} \frac{\partial C^*}{\partial c_N} \\
 w_{WWF} \frac{C^*}{S_H} \frac{\partial h}{\partial c_1} & w_{WWF} \frac{C^*}{S_H} \frac{\partial h}{\partial c_2} & \dots & w_{WWF} \frac{C^*}{S_H} \frac{\partial h}{\partial c_N}
 \end{bmatrix}_{9 \times N} \begin{bmatrix} \Delta c_1 \\ \Delta c_2 \\ \vdots \\ \Delta c_N \end{bmatrix} = \begin{bmatrix} w_{D65} \frac{\Delta L_{D65}^*}{lS_L} \\ w_{D65} \frac{\Delta C_{D65}^*}{cS_C} \\ w_{D65} \frac{\Delta H_{D65}^*}{S_H} \\ w_A \frac{\Delta L_A^*}{lS_L} \\ w_A \frac{\Delta C_A^*}{cS_C} \\ w_A \frac{\Delta H_A^*}{S_H} \\ w_{WWF} \frac{\Delta L_{WWF}^*}{lS_L} \\ w_{WWF} \frac{\Delta C_{WWF}^*}{cS_C} \\ w_{WWF} \frac{\Delta H_{WWF}^*}{S_H} \end{bmatrix} \quad (4)$$

lowing the strategy (3) which takes into account the CIE $L^*a^*b^*$ (1976) colour difference is proposed in the article.³ In the present paper the above mentioned algorithm is modified for the case of colour differences CMC($l:c$) and CIE94.

The modification for the case of the CMC($l:c$) colour difference is as follows. In a particular step of this algorithm (and also in the subsequent correction procedure for test dyeings) the particular correction of concentrations – the vector $\Delta c = (\Delta c_1, \Delta c_2, \dots, \Delta c_N)^T$ – is the least squares solution of the overdetermined system of linear equations (4).

REMARK 1. Both the number and the type of the illuminants used in strategy (3) and in the corresponding iterative algorithm (4) were chosen arbitrarily. In case of some other choice of these two items the principal structure of the related »correction system« remains completely analogical to the structure of system (4).

REMARK 2. In order to better understand the construction of the overdetermined system (4), let us take a closer look at its first three equations. If the weights w_{D65} are omitted, these three equations form the matrix equation:⁴

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{S_C} & 0 \\ 0 & 0 & \frac{1}{S_H} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cosh & \sinh \\ 0 & -\sinh & \cosh \end{bmatrix} \begin{bmatrix} 0 & \frac{\partial L^*}{\partial Y} & 0 \\ \frac{\partial a^*}{\partial X} & \frac{\partial a^*}{\partial Y} & 0 \\ 0 & \frac{\partial b^*}{\partial Y} & \frac{\partial b^*}{\partial Z} \end{bmatrix} \begin{bmatrix} \frac{\partial X}{\partial c_1} & \frac{\partial X}{\partial c_2} & \dots & \frac{\partial X}{\partial c_N} \\ \frac{\partial Y}{\partial c_1} & \frac{\partial Y}{\partial c_2} & \dots & \frac{\partial Y}{\partial c_N} \\ \frac{\partial Z}{\partial c_1} & \frac{\partial Z}{\partial c_2} & \dots & \frac{\partial Z}{\partial c_N} \end{bmatrix} \begin{bmatrix} \Delta c_1 \\ \Delta c_2 \\ \vdots \\ \Delta c_N \end{bmatrix} = \begin{bmatrix} \Delta L^* \\ \frac{\Delta C^*}{S_C} \\ \frac{\Delta H^*}{S_H} \end{bmatrix} \quad (7)$$

$$\mathbf{J}_{CMC} \mathbf{B}(\Delta c_1, \Delta c_2, \dots, \Delta c_N)^T = (\Delta L^*/(lS_L), \Delta C^*/(cS_C), \Delta H^*/S_H)^T \quad (5)$$

which transforms the small concentration changes $\Delta c_1, \Delta c_2, \dots, \Delta c_N$ into $\Delta L^*/(lS_L), \Delta C^*/(cS_C), \Delta H^*/S_H$ – the components of the CMC($l:c$) colour difference. In this way, the first three equations of the overdetermined system (4) refer to the illuminant D65. Similarly, the second three equations of the system (4) refer to the illuminant A and the last three equations of the system (4) refer to the illuminant WWF.

In case the CIE94 colour difference formula⁵ is preferred the iterative algorithm (4) should be only slightly modified. Due to similarities of CMC($l:c$) and CIE94 colour difference equations the requested »correction system« is easily obtained by inserting the following expressions for the scaling factors S_L, S_C, S_H, l , and c into Eq. (4):

$$S_L = 1, S_C = 1 + 0,45 C^*, S_H = 1 + 0,15 C^*, \\ l = 1, c = 1. \quad (6)$$

REMARK 3. In case of CIE 94 colour difference the Eq. (5) modifies to Eq. (7) where the parameters S_C and S_H are set to the values presented in Eq. (6).

TABLE I. The CMC(2:1) colour differences ΔE against a given standard, exhibited by two matches. The first of them was prepared according to the usual strategy (1*) – Recipe 1 in each particular block, and the second one according to the multi-illuminant strategy (3) – Recipe 2 in each particular block of the table. In recipes there are short names of colorants followed by their concentrations in percentages.

				$\Delta E(\text{CMC}(2:1))$			
<i>Standard 1</i>	$L^* = 65.90$	$C^* = 13.52$	$h = 56.85$ (ill. D65)	D65	A	WWF	
Recipe 1	RED 0.0413		YEL 0.0939	BL2 0.0218	0.40	1.34	0.82
Recipe 2	RED 0.0397		YEL 0.0935	BL2 0.0222	1.20	0.69	0.22
<i>Standard 2</i>	$L^* = 63.62$	$C^* = 52.05$	$h = 61.94$ (ill. D65)	D65	A	WWF	
Recipe 1	BL1 0.0067		RED 0.1116	YEL 0.5828	0.26	3.00	2.73
Recipe 2	BL1 0.0100		RED 0.1033	YEL 0.5604	1.97	2.02	1.23
<i>Standard 3</i>	$L^* = 62.06$	$C^* = 11.82$	$h = 84.58$ (ill. D65)	D65	A	WWF	
Recipe 1	RED 0.0376		YEL 0.1394	BL2 0.0404	0.30	2.33	1.53
Recipe 2	RED 0.0346		YEL 0.1419	BL2 0.0380	1.38	1.09	0.93
<i>Standard 4</i>	$L^* = 65.56$	$C^* = 13.06$	$h = 197.47$ (ill. D65)	D65	A	WWF	
Recipe 1	DB 0.0206		BL1 0.1085	YEL 0.0323	0.30	1.34	1.22
Recipe 2	DB 0.0170		BL1 0.1156	YEL 0.0349	0.86	0.80	0.61
<i>Standard 5</i>	$L^* = 42.87$	$C^* = 25.04$	$h = 73.40$ (ill. D65)	D65	A	WWF	
Recipe 1	RED 0.2318		YEL 1.0735	BL2 0.1463	0.35	1.22	0.64
Recipe 2	RED 0.2069		YEL 0.9826	BL2 0.1335	0.86	0.82	0.29
<i>Standard 6</i>	$L^* = 44.57$	$C^* = 21.03$	$h = 103.84$ (ill. D65)	D65	A	WWF	
Recipe 1	RED 0.0907		YEL 0.7966	BL2 0.1800	0.11	2.04	1.37
Recipe 2	RED 0.0814		YEL 0.7979	BL2 0.1820	1.10	1.04	0.87
<i>Standard 7</i>	$L^* = 47.81$	$C^* = 25.14$	$h = 130.80$ (ill. D65)	D65	A	WWF	
Recipe 1	DB 0.1423		BL1 0.2854	YEL 0.5262	0.31	1.96	1.60
Recipe 2	DB 0.1569		BL1 0.2553	YEL 0.5007	0.42	1.28	1.09
<i>Standard 8</i>	$L^* = 44.63$	$C^* = 22.41$	$h = 250.05$ (ill. D65)	D65	A	WWF	
Recipe 1	BL1 0.2878		YEL 0.0828	BL2 0.2274	0.16	1.11	1.36
Recipe 2	BL1 0.3654		YEL 0.0791	BL2 0.1986	0.60	0.85	0.87
<i>Standard 9</i>	$L^* = 44.50$	$C^* = 19.84$	$h = 284.78$ (ill. D65)	D65	A	WWF	
Recipe 1	RED 0.0881		YEL 0.0574	BL2 0.2272	0.18	2.05	1.59
Recipe 2	RED 0.0799		YEL 0.0604	BL2 0.2399	0.88	1.39	1.17
<i>Standard 10</i>	$L^* = 41.30$	$C^* = 22.57$	$h = 317.23$ (ill. D65)	D65	A	WWF	
Recipe 1	DB 0.0920		BL1 0.2475	RED 0.2887	0.20	1.49	1.35
Recipe 2	DB 0.0756		BL1 0.2342	RED 0.2944	0.62	0.81	1.05
<i>Standard 11</i>	$L^* = 27.48$	$C^* = 10.34$	$h = 57.40$ (ill. D65)	D65	A	WWF	
Recipe 1	RED 0.6051		YEL 1.6653	BL2 0.5012	0.61	1.80	1.20
Recipe 2	RED 0.5741		YEL 1.6914	BL2 0.5084	1.09	1.38	0.59
<i>Standard 12</i>	$L^* = 35.83$	$C^* = 5.75$	$h = 308.15$ (ill. D65)	D65	A	WWF	
Recipe 1	RED 0.2208		YEL 0.3499	BL2 0.3572	0.38	1.51	1.37
Recipe 2	RED 0.2060		YEL 0.3412	BL2 0.3527	0.88	1.07	1.28

EXPERIMENTAL RESULTS

The practical effects of the usual single-illuminant matching strategy (1) or (1*) and the multi-illuminant matching strategy (3) have been compared in the matching of 12 standard samples prepared on polyester fabric. The colours of these standards were from different parts of the colour solid. In match prediction calculations the following colorants have been used:

C.I. Disperse Black	– in the table shortly: DB (Palanil Schwarz GTS)
C.I. Disperse Blue 56	– in the table shortly: BL1 (Palanil Blau R)
C.I. Disperse Red 167:1	– in the table shortly: RED (Palanil Rot 3BLS)
C.I. Disperse Yellow 198	– in the table shortly: YEL (Palanil Gelb GL)
C.I. Disperse Blue 148	– in the table shortly: BL2 (Palanil Dunkel Blau 3RT)

For the colour of each particular standard we chose the recipe with the moderate level of metamerism predicted by the computer program following strategy (1*). In addition, an alternative recipe according to the multi-illuminant strategy (3) was calculated using the same combination of colorants. After test samples had been prepared and the correction procedure carried out, the resulted colour differences of each such pair of samples against the corresponding standard were measured. They are displayed in Table I.

DISCUSSION

In Table I it can be seen that neither the colour difference ΔE_{D65} for the samples prepared according to single-illuminant strategy Recipe 1 is zero, nor are the colour differences ΔE_{D65} and ΔE_A for the samples prepared according to multi-illuminant strategy Recipe 2 perfectly balanced. This is due to small random inaccuracies and variations in the dyeing process which prevent to (re)produce the desired colour exactly. In repeated dyeings according to a particular recipe the colour of the produced samples scatters around the target colour. The amount of this scattering can also vary when the position of the target colour and/or the combination of dyes is changed.^{6,7} In spite of the above inaccuracies the subsequent correction procedures for each particular treated target colour resulted in:

(i) The sample prepared according to the Recipe 1 (strategy (1*)) had smaller colour difference ΔE_{D65}

against the target than the sample prepared according to the Recipe 2 (strategy (3)).

(ii) The sample according to the Recipe 2 (strategy (3)) had lower $\max\{\Delta E_{D65}, \Delta E_A, \Delta E_{WVF}\}$ than the sample according to the Recipe 1. Therefore, the multi-illuminant-strategy (3) regularly produced lower metamerism than the single-illuminant strategy (1*).

If the tolerance would be set e.g. on the value $\Delta E_{TOL} = 1.0$ and this tolerance level would be equal for all the illuminants considered then the outcome of the above experiment would be as follows. In three cases (out of 12) the unacceptable metamerism of Recipe 1 was brought within tolerance of 1.0 unit of colour difference under all three illuminants considered. In further three cases the $\max\{\Delta E_{D65}, \Delta E_A, \Delta E_{WVF}\}$ was brought under 1.1 unit of colour difference – close to the tolerance level. In the other six cases the metamerism was either slightly or substantially reduced.

CONCLUSIONS

The multi-illuminant colour match prediction algorithm is extended to the cases of CMC(*l:c*) and CIE94 colour differences. The multi-illuminant matching strategy produces the recipes with more balanced predicted colour differences under several different illuminants than the usual single-illuminant strategy does. If the metamerism of the single-illuminant recipe is not too high, the multi-illuminant strategy (and the related correction procedure) applied on the same dye combination can substantially reduce the metamerism of the recipe colour and thus it can produce a more acceptable match.

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SAŽETAK**Formulacija receptura s najmanjom metamerijom****Boris Sluban i Olivera Šauperl**

Opisana je inačica »višesvjetlosne« strategije za predviđanje receptura za bojanje uz pretpostavku da su razlike u boji opisane s tzv. CMC (*l:c*) i CIE 94 jednadžbama. Ova strategija pokušava za različita osvjetljenja istovremeno smanjiti razlike u boji u odnosu na zadani standard. U slučajevima kada se raspoloživa bojila primjenom »jednosvjetlosne« strategije ne daju nemetamerijski složiti u recepturu usporedivu sa zadanim standardom, »višesvjetlosna« strategija, uravnoteživanjem razlika u boji pod različitim osvjetljenjima, nudi prihvatljivu formulaciju. Obje su strategije objašnjene i ilustrirane s kolorimetrijskim podacima niza laboratorijskih uzoraka, pri čemu je pokazano da »višesvjetlosna« strategija u pravilu daje manje metamerijske recepture.