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A FRACTIONALLY INTEGRATED MODEL FOR THE CROATIAN AGGREGATE OUTPUT (GDP) SERIES

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ABSTRACT

The general characteristics of output fluctuations in Croatia are examined under fractional integration framework. This paper evaluate the existence of long memory in real output decomposing fluctuations to transitory and permanent components. The results suggest that Croatian real output series behavior is best identified as ARFIMA model with order of integration $0.5 < d < 1.5$. This suggests that macroeconomic shocks in real output are highly persistent. Unlike other studies in Croatia that find real output to be *I(0)* or *I(1)* variable, test results from this study indicate that real output show the characteristics of long memory with mean reversion (fractional integration).

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I. INTRODUCTION

The study of real output behavior in Croatia has been a central point in the large body of literature but it is still not well understood and call for explanation. It is a well known fact that most macroeconomic and econometrics studies in Croatia identify real GDP series as I(0) or I(1) process. However, in reality real output rarely behaves like I(0) nor I(1) series. It is more likely for real output in Croatia to behave like fractionally integrated process distinct both from stationary and integrated processes. Fractionally integrated models try to identify and capture different short and long memory characteristics present in real output series. Real output macroeconomic time series are characterized by persistence and long memory with long lasting but mean reverting effects of a shock. Croatian real output nature can not be explained by the means of I(0) nor I(1) models. Correct identification and descriptions of general characteristics of fluctuation in real output is of major importance for policy makers. It is virtually impossible to determine whether fluctuations in output are transitory or permanent in nature if source of non-stationarity in real output series is explained. Nelson and Plosser (1982) following the work of Box and Jenkins (1976) try to explain the nature of fluctuations in output using I(1) models implying output fluctuations stochastic nature in contrast to the traditional view that output series in levels are stationary around a deterministic trend. Diebold and Rudebusch (1989) observes the presence of large permanent component in aggregate output conflicts with traditional economic theories. Persistence in aggregate output implies that all fluctuations in output are of a permanent nature. If this is the case, stochastic economic theory to explain output fluctuations can be developed at the best. Developing a highly complex deterministic economic model to account for these temporary deviations is not expected since in their opinion the reversion to deterministic equilibrium path is absent. Correct identification of the memory in real output is imperative for the correct model specification.

Understanding the true nature in real output fluctuation demand correct memory identification and thus model to be estimated. This is important for at least two reasons. First, implied assumption that real output series behave like I(1) and differenced real output series like I(0) processes it is likely to result in model misspecification. In real world as shown by Haubrich and Lo, 2001 real output time series neither looks like random walk nor white noise suggesting some degree of fractional integration. Thus, stochastic properties of real output series should be investigated using fractionally integrated models. Without knowing the true persistence and long memory in real output, i.e. transitory and permanent components in output fluctuations, overshooting or undershooting of the targeted real output values is certain. Miss-specified economic policies have a permanent, almost infinite impact on real output over time. For Croatia, the effects of a shock in output will dissipate after 30 quarters and real GDP will return to its equilibrium values. Faster convergence to equilibrium values (mean) can be achieved if correct set of policy measures is used. On the other hand, if wrong economic policy is used, negative effects of a shock in real output will last almost infinitely having permanent effects on GDP.

New studies trying to explain the non-stationary nature in real output turn to the fractionally integrated models characterized by persistence and long memory. Output changes decomposition to permanent and transitory component as pointed out by Michelacci and Zaffaroni, 2000, Silverberg and Verspagen, 2000, Mayoral, 2006, Caporale and Gil-Alana, 2009 requires series appropriate order determination (knife edge distinction problem between I(0) and I(1) series). Following Gil-Alana, 2004 fractionally integrated model

$$y_t = \alpha + \beta_t + x_t,\ t = 1,2... \quad (1)$$

$$(1-L)^d x_t = u_t,\ t = 1,2...,\qquad (2)$$

is used.

When d (long memory or fractional integration parameter) equals zero ($d = 0, x_t = v_t$) real output series is stationary, exhibiting short memory and mean reversion with finite variance. In this case, the effects of a shock in real output are transitory, decaying geometrically depending on the structure defining output short run dynamics. If $0 < d < 0.5$, real output series behave like fractionally integrated process implying strong dependence between past observation, long memory, mean reversion and covariance stationarity. The effects of a shock in real output lasts in the long run decaying at a slower, hyperbolic rate. When $0.5 < d < 1$, real output series is no more covariance stationary but still mean reverting. Effects of a shock in real output is long lasting and decays at an even slower rate. With $d = 1$ real output series is integrated, having unit root (nonstationary) with infinite memory and non-mean reverting. With real output as integrated series the effects of a shock in real output is permanent having a long run effect on output (forever persistent). In the case of $d > 1$, real output series is non stationary, non mean reverting with infinite memory and shocks effect diverging forever.

This paper evaluates the existence of long memory in real output in Croatia. Research studies making no distinction between fractionally integrated real output series ($0.5 < d < 1$) and simple I(0) or I(1) misidentify short/long memory characteristics violating standard classical assumptions. Ignoring this fact when computing tests and confidence intervals for the mean result in model misspecification and strong bias (Beran, 1989). This paper looks at the real output dynamics in Croatia allowing for short/long memory components and fractional integration contrasting existent literature in Croatia modeling real output series simply as I(0) or I(1) process.

Using a battery of non-parametric, semi-parametric and parametric fractional integration tests the general characteristics of fluctuations in real output is identified. Implementing various ARFIMA models on real output series this paper study how long typical recession or expansion last and whether fluctuations in real output for Croatia are mostly transitory or permanent. Study results validate real output time series in Croatia are best modeled as fractionally integrated models having strong importance for existent literature body and very large practical importance for policy makers in Croatia.

The rest of the paper is organized as follows: section 2 describes data and methodological framework. Section 3 discuss empirical results obtained from applying tests to different measures of real output time series in Croatia. Concluding remarks and possible extension for further research are presented in section 4.

II. DATA AND METHODOLOGICAL CONSIDERATIONS

Data used for this study includes real GDP, real GDP per capita and per employed person time series data. The data are quarterly with starting dates from 1996 to 2012 in all cases except for real GDP per capita with quarterly observations from 2000 – 2012. All series used in the study includes series in levels, first difference and transformed to natural logs adjusted series, seasonally adjusted input series with Census Bureau X-12-ARIMA with and without natural log transformation. Series are expressed in 2005 constant prices derived from Croatian statistical office data and publications for various years.

Table 1 provides the descriptive statistics for the series used in the analysis. Table 1 shows series are negatively skewed around zero indicating series are generally normally distributed. GDP per capita series meets all normal distribution conditions best among different measures of aggregate output used in this study. Different aggregate output measures (BDP, BDPE, BDPP) in levels generally follow normal distribution (Gaussian) while transformed input series (first-second differences, log transformation and seasonally adjusted) generally reject the null of data normal distribution (results from the Jarque-Bera, Doornik-Hansen, Shapiro-Wilk and Liliefors tests)[1].

TABLE 1 DESCRIPTIVE STATISTICS

Series	Mean	Median	Min.	Max.	Std. Dev.	C.V.	Skewness	Ex. kurtosis
BDP	51365	52373.8	37139.0	66063.7	7951.91	0.1548	-0.03680	-1.20118
BDPE	35105	35584.5	27097.3	41002.3	3432.10	0.09776	-0.38964	-0.79288
BDPP	12526	12676.8	9755.50	14866.8	1288.82	0.1028	-0.26594	-0.56844
BDP_d11	51414	52462.1	38665.4	63048.1	7597.80	0.1477	-0.15865	-1.42007
BDP_d11log	51413	52477.7	38775.4	63293.2	7600.67	0.1478	-0.15792	-1.41738
BDPE_d11	35140	36268.4	28513.4	41637.0	3196.20	0.09095	-0.49036	-0.71069
BDPE_d11log	35145	36214.5	28602.0	41780.2	3202.71	0.09112	-0.47149	-0.70979
ld_BDP	0.00634	0.0299906	-0.107442	0.106050	0.0653559	10.2944	-0.24907	-1.52209
ld_BDPE	0.00129	0.0205782	-0.136211	0.105401	0.0629246	48.4619	-0.21258	-1.33783
ld_BDPP	0.00616	0.0242789	-0.126716	0.120370	0.0743988	12.0582	-0.32735	-1.29916
BDPP_d11	12543.9	12959.4	10454.9	14269.5	1125.52	0.08972	-0.52110	-0.98515
BDPP_d11log	12544.4	12959.8	10454.1	14329.6	1127.60	0.08988	-0.51603	-0.98202
d_BDP	290.891	1413.50	-6407.44	5649.56	3476.38	11.9508	-0.31678	-1.2943
d_BDPE	49.8344	776.037	-4943.37	3642.53	2202.96	44.2056	-0.27293	-1.2386
d_BDPP	69.1541	337.626	-1627.24	1476.90	940.839	13.6050	-0.41975	-1.2075

Source: Author calculations on data provided by Croatian Statistical Office (quarterly data in 2005 constant prices)

Notes: BDP (real GDP), BDP_d11(seasonally adjusted), BDP_d11log(seasonally adjusted in log), dBDP(BDP first difference), ldBDP(BDP log differenced)

BDPE (real GDP per employed person), BDPE_d11(seasonally adjusted), BDPE_d11log(seasonally adjusted in log), dBDPE(BDPE first difference), ldBDPE(BDPE log differenced)

BDPP (real GDP per capita), BDPP_d11(seasonally adjusted), BDPP_d11log(seasonally adjusted in log), dBDP(BDPP first difference), ldBDPP(BDPP log differenced)

Figure 1 shows various aggregate output measure series (input and transformed) with their corresponding periodograms. Figure 1 displays a large peak in the periodogram of series in levels and seasonally adjusted series across zero frequency implying possible fractional integrated structure for the aggregate output series in Croatia. First and second differenced series show values close to zero at zero frequency with a large peak at 0.5 suggesting possible series overdifferencing issue. In the search of long memory in the series figure 2 shows corresponding series correlogram.

[1]This is no limitation for our analysis since we retain large sample observations.

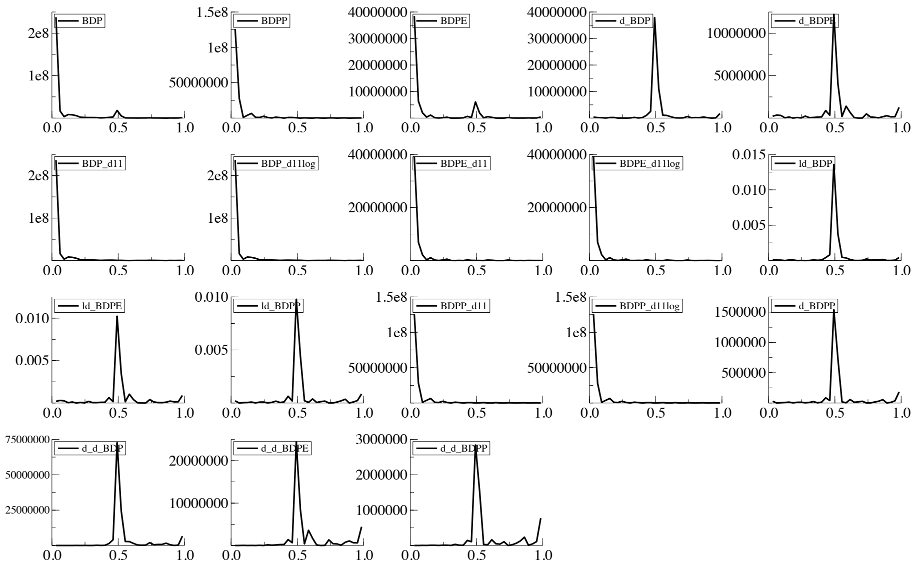

FIGURE 1—PERIODOGRAM OF INPUT AND TRANSFORMED BDP, BDPE, BDPP SERIES

Source: Author calculation on data from Croatian Statistical Office

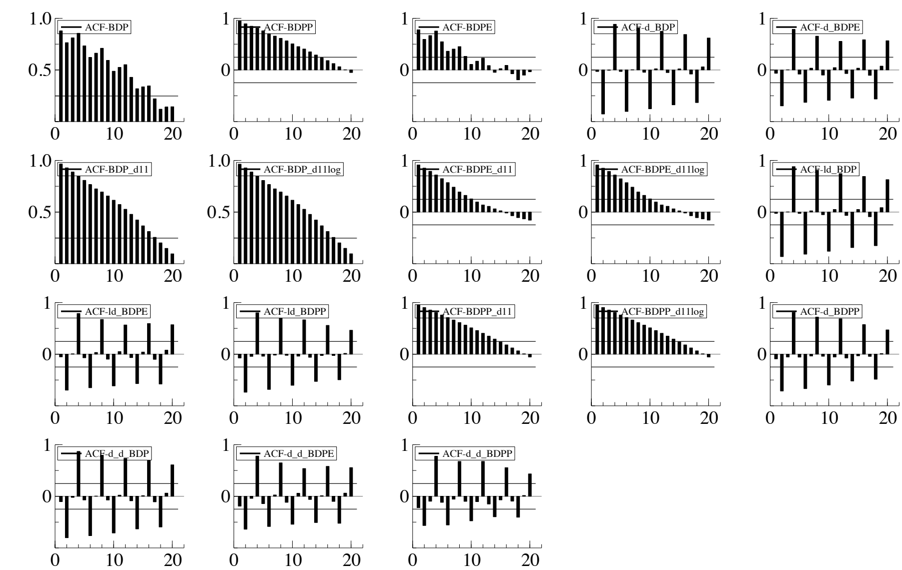

FIGURE 2—CORRELOGRAM OF INPUT AND TRANSFORMED BDP, BDPE, BDPP SERIES

Source: Author calculation on data from Croatian Statistical Office

Slow decay of in the aggregate output correlogram is another evidence of possible fractional integration in the series. Autocorrelation for the series under study decay very slowly as the lag length increases indicating aggregate output current values are dependent to own past distant values. The correlogram for the first and second differenced series still exhibit sluggish decline (first difference) or large oscillations (positive autocorrelations followed by a sequence of negative autocorrelations) in second differenced series.

Both periodogram (unbounded at zero frequency) and correlogram (slow decay with lag length) point to the possibility that aggregate output series in Croatia exhibit long memory.

To test for the series fractional integration and possible long run dependence in the series ADF and KPSS tests are used. Following Baillie et al., (1996) both ADF and KPSS tests series used in this study can be classified as stationary, unit root or fractionally integrated.

Said and Dickey, (1984) use an ARIMA model to test for the presence of the unit root in the time series of the form

$$(1+\alpha_1+\ldots+\alpha_p)^{-1}(1+\beta_1+\ldots+\beta_q)n(\hat{\rho}-1)\rightarrow\Gamma^{-1}\xi \tag{3}$$

under the null of $H_0: d = 1\ (\rho=1)$ for various values of p and q where

$d'=\rho-1,\ d_i=(\alpha+\beta)(-\beta)^{i-1},\ d'=(d_0,d_1,\ldots,d_k)$.

Lee and Schmidt, (1996) use a two sided test for $I(0) = e_t$ against fractionally integrated alternatives $I(d)$, $d < 0$, or $d > 0$ assuming ε_t are i.i.d. $N(0,\sigma^2)$ of the form

$$\omega = T^{-2}\sum_{t=1}^{T}S_t^2/\hat{\sigma}^2 \tag{4}$$

to test for the presence of long memory in stationary time series.

A. NON PARAMETRIC APPROACHES

Hurst, (1951) and Mandelbrot, (1975) introduce a long memory statistical test of the form

$$R/S(n)=\frac{1}{s_n}\left[Max\sum_{j=1}^{k}(x_j-\bar{x}_n)-Min\sum_{j=1}^{k}(x_j-\bar{x}_n)\right],1\le k\le n \tag{5}$$

with s_n being maximum likelihood estimator for x standard deviation. Long memory test R/S empirical studies show the test is valid for detecting long memory in the series but very sensitive to series length influencing standard deviation and the mean (Granero et al., 2008).

Lo, (1991) observes classical R/S test is sensitive to the short memory presence with AR(1) component in DGP seriously biasing the R/S test statistics. To account for the possible bias caused by the short memory effects, Lo use Newey-West correction to derive a rescaled adjusted range or R/S test using modified standard deviation S_q to compensate for the effects of short memory in the test. Modified Lo's R/S have the form Teverovsky et al., (1999)

$$S_a(N)=\left(\frac{1}{N}\sum_{j=1}^{N}(X_j-\bar{X}_N)^2+\frac{2}{N}\sum_{j=1}^{q}\omega_j(q)\left[\sum_{i=j+1}^{N}(X_i-\bar{X}_N)(X_{i-j}-\bar{X}_N)\right]\right)^{1/2} \tag{6}$$

$$V_q(N) = N^{-1/2} R(N) / S_q(N) \tag{7}$$

show evidence that modified R/S test is more robust than classical R/S test but still to be used within a battery of others long memory tests to investigate a true long record nature in the series. From (6) and (7) fractional differencing parameter (*d*) takes the form (Caporale and Gil-Alana, 2009)

$$d = \frac{Log(Q_T(q))}{Log(T)} \tag{8}$$

Giraitis et al., (2003) propose a rescaled variance test centering the KPSS statistics as rescaled sample second moment of partial sums. Rescaled variance test or V/S test

$$M_N = \frac{1}{\hat{s}_{N,q}^2 N^2} \left[\sum_{k=1}^{N} \left(\sum_{j=1}^{k} (X_j - \bar{X}_N) \right) - \frac{1}{N} \left(\sum_{k=1}^{N} \sum_{j=1}^{k} (X_j - \bar{X}_N \right)^2 \right] \tag{9}$$

corrected for M_n proved more robust for high volatile series (Caporale and Gil-Alana, 2009) and less sensitive to the choice of optimal bandwidth parameter q ($2 \leq q \geq 10$). Fixed bandwidth values do not perform well under short range dependence. V/S statistics show higher power in relation to the KPSS statistics but still remains weak to the shifts in the series levels (Alptekin, 2006). Under the null, V/S follows $V / S \Rightarrow \eta(t)$. Small values of test statistics reject the null hypothesis of short memory.

Lobato and Robinson, (1998) build a log periodogram nonparametric test of *I(0)* with the t-statistics

$$LM = m\hat{e}'\hat{E}^{-1}\hat{e}\,. \tag{10}$$

Under the null of series as *I(0)* process *LM* provides a test against the alternative $d > 0$ or series is a long memory process and $d < 0$ series show antipersistence, i.e. series is a mean reverting process. This would imply that increase in output would most likely be followed by a decrease in output exhibiting a highly cyclical dynamics in the economy.

Davidson and Hashimzade, (2009; Davidson, 2009) develop a bootstrap test of I(0) to overcome nonparametric tests great dependence on optimal bandwidth choice. Test performance in this case depends on how well estimated model fit DGP as well as the choice of the shock distribution (Student's t/Gaussian). In this study Gaussian shock distribution is assumed. Using Breitung, 2002 test statistics

$$\hat{\rho}_T = \frac{\sum_{t=1}^{T} U_t^2}{T \sum_{t=1}^{T} u_t^2} \tag{11}$$

and Kolmogorov-Smirnov test for equality of empirical distribution with the benchmark case a bootstrap test for I(0) is implemented with calculated *p*-values. Large test statistics show bootstrap test rejects the null of I(0) under the null distribution of Breitung's statistics.

B. SEMI-PARAMETRIC APPROACHES

Semi-parametric approaches avoid the misspecification issue emerging in fully specific parsimonious parametric models. They permit to explore the *I(d)* structure of the series constraining the long run behavior to be an *I(0)* process. By doing so, potential misspecifications and loss of power under short run effect is avoided since no short run components modeling is required (no ARMA parameter of the series clear specification). Diverse semi-parametric approaches for measuring fractional differencing parameter *(d)* exist. Geweke and Porter Hudak, (1983) propose a log periodogram estimate of fractional parameter *d* having the form

$$\log(I(\lambda_j)) = const + d\log\left(\left\{2\sin\frac{(\lambda_j}{2}\right\}^{-2}\right) + error \tag{12}$$

with

$$\hat{d} = \frac{\sum_{s=1}^{m} x_s \log I_x(\lambda_s)}{2\sum_{s=1}^{m} x_s^2} \tag{13}$$

Their approach was latter modified by Künsch, (1986), Robinson, (1995b), Hurvich and Ray, (1995; Hurvich and Ray, 2003; Hurvich et al., 2005), Shimotsu and Phillips, (2006) proposing different methods for the choice of Fourier frequency number *m* and to improve consistency and asymptotic normality when $d \geq 0$.

The appropriate choice of the frequency periodogram ordinates and regression sample length greatly influence estimated *d*. Semi-parametric methods developed prior to research of Phillips and Moon, (1999; Phillips, 1999a; Phillips, 1999b) were not based on the particular representation of the discrete Fourier transformation under the presence of unit root, i.e. $d = 1$. Phillips point out that GPH test behave poorly in the case $d > 1$ and *d* exhibiting asymptotic bias toward unity. To correct for the GPH test inconsistencies, Phillips develops a modified form of the GPH test with modified log periodogram regression estimator to reflect the distribution of *d* under the null hypothesis that $d = 1$ see Baum and Wiggins, (2000). Phillip's modified GPH test takes the form

$$\omega_x(\lambda_s) = \frac{\omega_u(\lambda_s)}{1-e^{i\lambda s}} - \frac{e^{i\lambda s}}{1-e^{i\lambda s}}\frac{X_n}{\sqrt{2\pi n}} \tag{14}$$

and

$$\tilde{d} = \frac{\sum_{s=1}^{m} x_s \log I_v(\lambda_s)}{2\sum_{s=1}^{m} x_s^2} \tag{15}$$

Unlike previous semi-parametric tests, (11) proved to be consistent for d < 1 and d > 1 fractional alternatives.

In order to improve efficiency, Robinson, (1995b) advance a pooled log periodogram estimate for a stationary and invertible time series of the form Velasco, (2006)

$$\hat{d}_m^{LP} = \left(\sum_j \Lambda_j^2\right)^{-1} \left(\sum_j \Lambda_j Y_{X,j}^{(K)}\right). \quad (16)$$

Allowing for multivariate semiparametric estimate of the long memory in the series (multivariate model under the hypothesis that different series can share a common fractional integration parameter *d*) consistency and asymptotic normality of the log periodogram estimator is achieved.

In his other study Robinson, (1995a) propose a spectral maximum likelihood estimator based on trimmed Whittle estimator of the form (Hauser, 1997)

$$\log(L_n(d,\sigma_u^2)) = -\sum_{j=1}^{m} \log(f(\lambda_j \left| d,\sigma_u^2)) - \sum_{j=1}^{m} \frac{I(\lambda_j)}{f(\lambda_j \left| d,\sigma_u^2 \right.} \right. \quad (17)$$

with (Caporale and Gil-Alana, 2009)

$$\hat{d} = \arg\min_d \left(\log \overline{C(d)} - 2d \frac{1}{m} \sum_{s=1}^{m} \log \lambda_s \right). \quad (18)$$

Robinson, (1995a) find his Gaussian semi-parametric estimate to be consistent, asymptotically normal and more efficient

$\sqrt{m}(\hat{d} - d_O) \rightarrow_d N(0,1/4), T \rightarrow \infty$ compared to other semi-parametric approaches.

Phillips and Shimotsu, (2004; Shimotsu and Phillips, 2006) find (13) to be inconsistent when approaching unity. Asymptotic properties of Robinson's Gaussian semi-parametric and local Whittle tests were limited to the $-0.5 < d < 0.5$ interval while Velasco, 1999 propose a modified log periodogram test valid even when $d > 3.75$ without differencing or detrending the data.

Harris et al., (2008) develop a test statistics for short memory process under the null hypothesis against the alternative $d > 0$ of long memory process. In relation to other fractional integration tests (testing the null of white noise against memory) here the null of short against long memory is tested ensuring statistics consistency. Test statistics takes the form

$$\hat{S}_k = \frac{\hat{N}_k + \hat{b}}{\hat{\omega}_l}. \quad (19)$$

The test is based on long-range autocovariances under N(0,1) with large values of test statistics rejecting the null of short memory.

Robinson and Henry, (1999) investigate properties of the semiparametric Gaussian method for estimating long memory parameters under conditional heteroscedasticity. They develop a method for estimating *d* for testing the null I(0) in the presence of conditional heteroscedasticity taking the form

$$\hat{H} = \arg\min_{\Delta_1 \le h \le \Delta_2} R(h)$$
$$R(h) = \log\left\{\frac{1}{m}\sum_{j=1}^{m}\frac{I(\lambda_j)}{\lambda_j^{1-2h}}\right\} - (2h-1)\frac{1}{m}\sum_{j=1}^{m}\log\lambda_j \quad (20)$$

They find conditional heteroscedasticity to exercise a troubling impact on the behavior of semiparametric estimates of long memory in moderate sample size.

Moulines-Soulier log-periodogram regression (Moulines and Soulier, 1999) estimate long memory time series properties using spectral density. Their semiparametric estimation of the fractional differencing coefficient *d* is defined as

$$\hat{d}_{p,n} = \arg\min_{\bar{d},\bar{\theta}_0,\ldots,\bar{\theta}_p}\sum_{k=1}^{K_n}\left(Y_{n,k} - \bar{d}g(y_k) - \sum_{j=0}^{p}\bar{\theta}_j h_j(y_k)\right)^2. \quad (21)$$

Under assumptions of Gaussian process and spectral density estimator is consistent and asymptotically normal.

C. PARAMETRIC APPROACHES

Granger and Joyeux, (1980) and Hosking, (1981) introduce a widely used parametric model

$$(1-L)^d = \sum_{k=0}^{\infty}\frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)} \quad (22)$$

to capture both long run dynamics (long memory parameter *d*) and short memory dynamics (AR/MA parameters) of the time series. For each of the aggregate output series, different ARFIMA (*p,d,q*) models with *p, q* = 3 of the form

$$\Phi(L)(1-L)^d(y_t-\mu) = \Theta(L)u_t \quad (23)$$

is used. Following Sowell, (1992) fractional ARIMA model

$$(1+\phi_1 L+\phi_2 L^2+\ldots+\phi_p L^p)(1-L)^d x_t$$
$$= (1+\theta_1 L+\theta_2 L^2+\ldots+\theta_q L^q)\varepsilon_t \quad (24)$$

a regression model of the form *(Gil-Alana, 2004)

$$y_t = \alpha + \beta_t + x_t, t = 1,2,\ldots$$
$$(1-L)^d x_t = u_t, t = 1,2\ldots, \quad (22)$$

is used. This study employs Sowell, (1992) maximum likelihood procedure in time domain

$$\phi_p(L)(1-L)^d y_t = \theta_q(L)\varepsilon_t \quad (25)$$

with L being a lag operator, *d* is a fractional differencing parameter, y_t = Croatian aggregate output measures, $\phi_p(L)$ AR and $\theta_q(L)$ MA polynomials. To explain the degree of persistence in the time series, long memory (fractional integration) parameter *d* must be correctly measured. Parametric approach lacks of flexibility when it comes to model misspecification. Spurious or incorrectly specified model (particularly short memory dependence component) significantly affects estimates of the long memory parameter *d* Gil Alana, (2001). The values of *d* describe time series dynamic behavior. With d = 0, series is a stationary *I(0)* process or a short-memory process and the effects of shocks on aggregate output ($t \rightarrow y_t$) decays geometrically to zero. Therefore, shocks in a stationary (short memory) time series have only transitory effect (short run fluctuations in aggregate output due to transitory shocks). For a nonstationary series with unit root *I(1)* and d = 1, shocks in t result in permanent output fluctuation (permanent shocks), i.e. effects on output do not disappear over time (non-mean reverting). A high value of *d* implies a long memory process with a fractionally integrated process still mean reverting as opposite to *I(1)* or *I(2)* integrated series. For -0.5 < d <0 series exhibits negative dependence between $t \rightarrow y_t$, i.e. anti persistence or short memory with summable autocovariances (fractionally over-differenced). For 0 < d < 0.5 series is covariance stationary and show long range dependence (persistence) for $t \rightarrow y_t$, revealing both short and long memory dynamics. Process is still mean-reverting (re-equilibrating) and a hybrid between white noise and random walk. The fractional integral of order 0.5 < d < 1 show y_t is no more stationary but still mean reverting with infinite variance, effects of a shock on output die out with time. Non-stationary, integrated series with infinite variance and non-returning to its original values in the future (non mean-reversion) with d = 1 the process follow a unit root behavior with both short and infinite memory dynamics. Fluctuations in output (effects of a shock) therefore are largely permanent. For d > 1 series is non stationary and exhibits perfects or infinite memory with mean having no influence on series long run evolution. Series having 1 ≤ d ≤ 1.5, Δy_t is mean reverting with finite variance. When d > 1 local Whittle *(LW)* estimator converges to unity and therefore become inconsistent Phillips and Shimotsu, (2004). Log periodogram based estimators in general are subject to inconsistency when series is nonstationary and d > 0.75. Several approach can be used to address this issue, from fractional differencing the series to tapering the data Nielsen, (2007) and the extended multivariate local Whittle is recommended Abadir et al., (2007)

$$\omega_a^x(\lambda_j) = (1-\exp(i\lambda_j))^{-p_a^0}\omega_{\Delta p_a^0 x_a}(\lambda_j) - \exp(i\lambda_j)\sum_{r_a=1}^{p_a^0}(1-\exp(i\lambda_j))^{-r_a}\omega_{\Delta r_a x_a} \tag{26}$$

$$= (1-\exp(i\lambda_j))^{-p_a^0}\omega_a^u(\lambda_j) - \exp(i\lambda_j)\sum_{r_a=1}^{p_a^0}(1-\exp(i\lambda_j))^{-r_a}\omega_{\Delta r_a x_a}, a = 1,\ldots,q,.$$

For each measure of aggregate output different ARFIMA(p,d,q) using Sowell, 1992 with p, q ≤ 3 using maximum likelihood (Gaussian). To find a best model specification and avoid model misspecification and inconsistency, each aggregate output series was tested for fractional integration using ARFIMA(p,d,q) under normality, heteroscedasticity, ARCH and Ljung-Box diagnostic test. Models that passed all diagnostic tests for each of the series were in turn subdue to model selection tests based on LR (likelihood criterions) and Akaike (AIC), Bayesian (BIC) information criterion.

In order to decide on the best model specification under test results, selected models were tested for the general characteristics of fluctuations using (IRF) impulse response function (infinite moving average representation) for each series to confirm the results. MA coefficients in ARFIMA(p,d,q) model follows Javier Contreras-Reyes

$$\psi_j \sim \left(\frac{1 + \sum_{i=1}^{q} \theta_i}{1 - \sum_{i=1}^{p} \phi_i} \right) \frac{j^{d-1}}{\Gamma(d)} \tag{27}$$

with impulse response function defined as

$$R_j \sim \sum_{i=0}^{j} \psi_i \eta_{j-i} \tag{28}$$

$\eta_t = \dfrac{\Gamma(t+d)}{(\Gamma(t+1)\Gamma(d)}$ and for large *j* using asymptotic method

$$R_j \sim \frac{j^{d-1}}{\Gamma(d)} \sum_{i=0}^{\infty} \psi_i \,. \tag{29}$$

To evaluate the responses of aggregate output to a shock an impulse response function $1-\beta(L)^{-1}\delta(L)(1+\alpha((1-L)^{d_2}-1))\partial$ is used. The effects of a shock depend on the chosen model specification making possible to describe general characteristics of fluctuation in aggregate output for Croatia.

III. EMPIRICAL RESULTS

Table 2 shows the results of ADF, KPSS and two fractional integration tests (Whittle and GPH).

TABLE 2 UNIT ROOT AND FRACTIONAL INTEGRATION TEST

Series	Unit root tests						Fractional Integration tests	
	ADF			ADF-GLS		KPSS	Whittle	GPH
	no const.	const.	const. and trend	const.	const. + t		Test *d*	Test *d*
GDP_t	1.04	-1.07	-2.21	0.14	-2.03	1.60	6.89(1.04)*	7.68(0.99)*
dGDP	-1.55	-1.99	-2.03	-1.00	-1.54	0.52*	2.09(0.32)	1.82(0.38)
dlGDP	-1.62	-2.09	-2.20	-0.64	-0.34	0.61*	2.42(0.36)*	1.53(0.41)
GDP_{d11}	1.90	-1.64	-1.10	0.10	-1.43	1.61	6.98(1.05)*	7.83(1.01)*
GDP_{d11log}	1.92	-1.56	-1.15	0.18	-1.44	1.61	7.00(1.06)*	7.86(1.02)*
$GDPC_t$	1.04	-2.02	-2.19	-0.06	-1.89	0.99	2.67(0.40)*	2.25(0.63)*
dGDPC	-1.85	-2.31	-3.22	-0.60	-1.32	0.50*	-5.80(-0.88)*	-0.30(-0.15)
dlGDPC	-1.78	-2.19	-3.25	-0.47	-1.08	0.55*	-5.75(-0.87)*	-0.26(-0.14)
$GDPC_{d11}$	1.08	-2.02	-0.73	-0.20	-1.22	0.99	6.68(1.00)*	6.03(1.02)*

$GDPC_{d11log}$	1.21	-2.26	-0.67	-0.19	-1.16	1.00	6.60(0.99)*	5.56(1.02)*
$GDPE_t$	0.56	-1.05	-2.23	-1.24	-1.58	0.72*	9.08(1.37)*	6.61(1.63)*
dGDPE	-2.37*	-2.38	-2.69	-1.36	-2.27	0.32*	2.05(0.31)*	1.68(0.40)
dlGDPE	-2.37*	-2.38	-2.63	-1.43	-1.73	0.30*	2.18(0.33)*	1.96(0.41)
$GDPE_{d11}$	0.31	-0.57	-2.03	-0.68	-0.96	0.72*	8.25(1.24)*	6.76(1.45)*
$GDPE_{d11log}$	0.30	-0.57	-2.09	-0.67	-0.93	0.72*	8.12(1.23)*	6.89(1.41)*

Source: Author calculation on data from Croatian Statistical Office

Plots of the time series are given in figures (1) and (2). Visual inspection of the series shows series have different degrees of persistence and long memory structure. Series in level and seasonally adjusted series exhibit short and long memory. Differenced series (first and second) on the other hand show the presence of anti-persistence in aggregate output.

Table (2) shows unit root tests for the aggregate output series. For most series ADF tests do not rejects the null (except for dGDPE and dlGDPE series) so there is evidence GDP series in Croatia behave as I(1) processes.

KPSS tests for most series do not reject the null of I(0). Modeling these series either as I(0) or I (1) is too restrictive and show characteristics typical for fractionally integrated processes.

Rejection by ADF and ADF-GLS and failure to reject by KPSS for difference and log differenced GDP and GDPC series is to be considered as strong evidence of stationary I(0) processes. For the GDP per employed person series (all specifications) ADF and ADF-GLS rejects the null while KPSS fail to reject the null of stationarity pointing to the conclusion that series are I(0). For GDP_t, GDPd11 and GDP d11log ADF, ADF-GLS and KPSS reject the null indicating series are following fractionally integrated alternatives. The same is valid for difference and log differenced GDP per capita series.

Table 2 summarizes long memory test results (GPH and Whittle estimator) for all output series. GPH rejects the null of stationarity for most of the series except for differenced and log differenced series of GDP, GDPC and GDPE. The evidence of long memory is found for all of the series except for the one mentioned above. The estimated values for d range from 0.63 to 1.63. GDP series in levels and seasonally adjusted do not reject the hypothesis of $d = 1$. Same holds for seasonally adjusted GDP per capita series. GDP per capita in levels show evidence of long memory with $d = 0.63$. For GDP per employed person in levels the unit root null hypothesis is rejected while for it's seasonally adjusted series the null of unit root cannot be rejected. Whittle estimator test results confirm the GPH results. Test results differs only log differenced GDP series with $d = 0.63$ implying nonstationarity but mean reversion. The null of $d = 0$ and $d = 1$ is rejected for the GDP per capita series in level and seasonally adjusted GDP per person series. Anti-persistence (cyclical fluctuation in output) is found for differenced and log differenced GDP per capita series. For all other series the null of unit root cannot be rejected with d in the range 0.99 - 1.37. Seasonally adjusted series for GDP per employed persons appear to be highly persistent but mean reverting best modeled as fractionally integrated processes.

Results from the table 2, testing both ADF and KPSS tests, ADF test results reject the null of unit root in general, except for the dGDPE and dlGDPE series. In general, real GDP, real GDP per capita and real GDP per employed person series do not behave as I(1) processes. For most of the series (except for real GDP per employed person) KPSS test rejects the null of I(0).

For series with ADF test rejecting the I(1) hypothesis and KPSS rejecting the I(0) indicates that series are neither I(0) nor I(1) processes but can be described as fractionally integrated processes. It is evident that real GDP series is better described as fractionally integrated process. However, for first differenced and log differenced GDP series test results show strong evidence of stationary I(0) processes. The same stands for real GDP per capita series with $GDPC_t$, $GDPC_{d11}$, $GDPC_{d11log}$ following fractionally integrated distribution while dGDPC, dlGDPC are better described as I(0). For the real GDP per employed person series, rejection of ADF test and failure to reject the KPSS test is strong evidence of stationarity I(0).

First difference and log differenced real GDP per employed persons series show insufficient information on the long dependence properties (both ADF and KPSS tests fail to reject the null). When time trend is included in the KPSS test, $GDPE_t$, $GDPE_{d11}$, $GDPE_{d11log}$ test results reject the null of stationarity indicating series are better described as fractional integrated alternatives (in accordance with Whittle and GPH test results in the table). Therefore, real GDP per employed person series under the KPSS test should be modeled with the trend inclusion in the test since more evidence is found that series follows fractionally integrated processes then being I(0). The distinction between I(d) processes for different aggregate output series is important for measuring the degree of persistence in the series. Assuming that aggregate output is a stationary processes I(0) implies no change in the output equilibrium level. If this is a case, we shall conclude that aggregate output in Croatia is a short memory processes *(d=0)*, best described as stationary and invertible ARMA (rapidly decaying autocovariances) Beran, (1994).

Supposing that aggregate output series is better described by an I(1) process infers temporary deviations from equilibrium levels in the series (unit root presence). Autocorrelation function for this case linearly declines and series are nonstationary and non-mean reverting. Perron(1989, 1990) reason that mean shifts and break in the series lead to non rejection of integrated series hypothesis while in practice only few macroeconomic series are I(1). Thus, many series can contain fractionally integrated trends. Granger and Joyeux, (1980), Granger, (1980) observe macroeconomic series usually falls between white noise *(d=0)* and random walk *(d=1)* following fractionally integrated noise *(d=0.5)*. In this case, series are stationary with long range dependence or long memory, mean reverting process with shocks affecting the series in the long run but returning to the equilibrium path some point in time (Gil Alana and Brian Henry, 2003). Autocorrelation function for such a process slowly decays (Beran, 1994). To explain the degree of persistence in Croatian macroeconomic series a set of testing procedures for fractional integration is used. Since macroeconomic series under observation are neither I(0) nor I(1) as indicated by ADF and KPSS a fractionally differenced model may be an appropriate representation.

Table 3 show test results for a battery of non-parametric and semi-parametric methods used for modeling long range dependence in aggregate output series for Croatia. I(0) tests results in table 3 show majority of the tests rejects the null of stationarity. Most tests do not reject the null of stationarity for differenced and log differenced series of real GDP and real GDP per capita. The same holds for real GDP per employed person except for the Phillips test (with power 0.5) and HML test. Two series show distinct pattern from the rest of the observation. Differenced and log differenced real GDP, real GDP per capita and real GDP per employed person do not show unit root behavior (ADF, ADF-GLS test reject the null of unit root). KPSS test for same series do not reject the null of stationarity. Therefore, results lead to the conclusion series are I(0). However, looking at the fractional integration test results, HML and Bootstrap tests strongly reject the null of stationarity. HML test do not reject the null of stationarity for difference and log differenced real GDP per capita series. Robinson/Henry, Moulines/Soulier, Phillips GPH show series exhibit anti persistence. Anti-persistence is clearly visible correlogram of the series. Whittle and GPH test show two series are anti-persistent or fractionally integrated with $d < 0.5$.

TABLE 3 FRACTIONAL INTEGRATION TESTS FOR OUTPUT SERIES

Series	R/S Hurst/ Mundelbrot	Phillips (1999) GPH			Lo R/S (1991) test	Robinson Test (1995)			*V/S test*
	t-stat	*0.5*	*0.75*	*0.9*	*t-stat*	*0.5*	*0.75*	*0.9*	*t-stat*
GDP	3.46	1.19**	0.58	0.62**	2.53	0.98**	0.86**	0.73**	0.32**
dGDP	0.56	0.44	-0.23	-0.10	0.57	0.58	-0.20	-0.08	0.14
dlGDP	0.53	0.38	-0.17	-0.14	0.54	0.51	-0.18	-0.14	0.12
GDP$_{d11}$	3.61	1.20**	1.03**	0.95**	2.57	0.99*	0.95**	0.76**	0.28**
GDP$_{d11log}$	3.61	1.21**	1.07**	1.02**	2.57	0.99*	0.94**	0.74**	0.28**
GDPC	3.63	1.42	0.45	0.61*	2.60	0.98**	0.80**	0.61**	0.22*
dGDPC	0.65	0.50	0.13	-0.19	0.68	0.55	-0.06	-0.22	0.17
dlGDPC	0.66	0.56	0.20	-0.13	0.68	0.81*	0.00	-0.23	0.15
GDPC$_{d11}$	3.65	1.43**	1.02**	0.92**	2.61	0.98**	0.88**	0.68**	0.21*
GDPC$_{d11log}$	3.65	1.40**	1.09**	0.98**	2.61	0.98**	0.88**	0.65**	0.21*
GDPE	3.22	1.67**	0.86**	1.05**	2.41	1.65**	0.89**	0.65**	0.32**
dGDPE	0.72	0.57*	0.05	0.00	0.75	0.62	-0.09	-0.06	0.11
dlGDPE	0.77	0.54*	0.11	0.03	0.79	0.54	0.01	-0.02	0.11
GDPE$_{d11}$	3.44	1.43**	1.26**	0.92**	2.49	1.64**	1.05**	0.81**	0.28**
GDPE$_{d11log}$	3.44	1.48**	1.26**	1.21**	2.49	1.63**	1.07**	0.82**	0.28**

(continued)

Robinson/ Henry (1998)	Moulines/ Soulier test	Robinson/ Lobato (1998) test	HML test (2008)	Bootstrap Test (2009)
d	d	t-stat	t-stat	t-stat
0.76**	0.93**	-	3.11**	2.27**
-0.42**	-0.14	-1.28	2.24*	7.04**
-0.44**	-0.18	-1.24	2.24*	6.96**
0.90**	1.02**	-	3.11**	2.13**
0.89**	1.01**	-	3.11**	2.91**
0.92**	0.80**	0.77	2.36**	7.03**
-0.39**	-0.22	-1.52	-1.60	7.04**
-0.36**	-0.20	-1.42	-1.60	7.04**
0.91**	1.00**	-	2.36**	7.03**

0.91**	0.97**	-	2.36**	7.03**
0.65**	0.88**	0.35	3.07**	2.98**
-0.28**	-0.11	-1.39	1.81*	6.96**
-0.26**	-0.03	-1.30	1.86*	7.03**
0.95**	1.34**	-	3.08**	2.20**
0.96**	1.16**	-	3.08**	2.27**

*Source: Author calculations on data provided by Croatian Statistical Office (quarterly data in 2005 constant prices), Notes: * , ** denotes significance at 5, 1% level. Future values of agregate output have a tendency to return to a long-term mean and an increase in aggregate output is more likely to be followed by an output decrease or vice versa. Additional fractional integration tests should be implemented on the two series (fractional integration under structural breaks, white noise, AR parameters) before final judgment on the behavior of the two series can be made. However, this study test results firmly put forward two series are not pure stationary process with d = 0.*

Parametric test results for the two series (see table 4) confirms above statements. Table 4 shows fractional integration parameter *d* value strongly depends on the appropriate model specification. For parametric test in table 4 estimated values of d ranges from -0.80 to 0.84. To determine the correct model specification for difference and log differenced series across different model presented table 4 diagnostic test (no serial correlation, functional form, normality, homoscedasticity) on the residuals and AIC (Akaike), SIC (Schwarz), Log likelihood information criteria along with LR (Langrange multiplier) test were used. Test results show differenced and log differenced aggregate output series in Croatia can be best described as fractionally integrated ARFIMA model.

The order of integration strongly depends on the short run components modeling of the series. Differenced series for real GDP is best modeled according to residuals diagnostic and LR tests as ARFIMA(0,-0.55,0), for real GDP per capita ARFIMA(0,-0.57,0) and for real GDP per person employed ARFIMA(0,-0.35,0). Using information criteria, best model specification for real GDP is ARFIMA(3,0.44,2), for real GDP per capita ARFIMA(3,0.47,1) and for real GDP per employed person ARFIMA(3,0.68,1).

Log differenced series for real GDP (passing diagnostic and LR tests) is best described as ARFIMA(2,0.44,3), for real GDP per capita ARFIMA(2,0.09,3) and for real GDP per employed person ARFIMA(3,0.14,3).Turning to the information criteria selection procedure, best model specification for real GDP is ARFIMA(3,0.63,1), for real GDP per capita ARFIMA(3,-1.12,2) and for real GDP per employed person ARFIMA(3,0.72,1).According to the residual diagnostic and LR tests best model specifications for differenced real GDP reject the null of unit root while the null of stationarity can not be rejected. Results are confirmed by information criteria model selection since AIC, BIC and LL best model selection rejects the null of unit root in favor of stationary series with long memory (fractional model).The order of integration for log differenced aggregate output series are greater in relation to differenced output series. Based on residual diagnostic and LR tests the null of stationarity cannot be rejected for log difference real GDP per capita and real GDP per employed person series. On the other hand, information criteria support non stationary and long memory models for real GDP and real GDP per employed person. For real GDP per capita, information criteria tests show stationary, anti-persistence and fast mean reversing behavior. Table 4 Best model specification for individual aggregate output series

TABLE 4 BEST MODEL SPECIFICATION

Series	ARFIMA (p, d, q)	t- tests'			AR parameters			MA parameters		
		$t_{d=0}$	$t_{d=1}$	$t_{d=2}$	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
GDP_t	(0, 0.75, 0) (0.09)	67.8	7.48	187.3	-	-	-	-	-	-
	(3, 1.51, 2) (0.25)	36.8	4.20	3.89	-0.74	-1.03	-0.74	-0.03	-0.68	-
dGDP	(0, -0.55, 0) (0.11)	24.9	195.3	527.3	-	-	-	-	-	-
	(3, 0.44, 2) (0.31)	2.02	3.16	24.8	-0.74	-1.03	-0.74	-0.08	-0.61	-
dlGDP	(2, 0.44, 3) (0.36)	1.55	2.45	19.2	-0.01	-1.01	-	0.62	-0.85	0.42
	(3, 0.63, 1) (0.26)	5.74	1.90	26.2	-0.89	-1.01	-0.86	-0.11	-	-
GDP_{d11}	(0, 1.29, 3) (0.20)	39.8	2.02	11.9	-	-	-	0.27	-0.16	-0.03
	--	--	--	--	--	--	--	--	--	--
GDP_{d11log}	(0, 2.33, 3) (0.24)	97.7	31.9	2.00	-	-	-	1.33	-0.48	0.15
	--	--	--	--	--	--	--	--	--	--
$GDPC_t$	(0, 1.14, 3) (0.16)	52.1	0.77'	29.3	-	-	-	0.08	0.17	0.12
	--	--	--	--	--	--	--	--	--	--
dGDPC	(0. -0.57, 0) (0.11)	29.0	218.5	584.5	-	-	-	-	-	-
	(3, 0.47, 1) (0.27)	2.99	3.73	31.3	-0.88	-0.98	-0.85	-0.05	-	-
dlGDPC	(2, 0.09, 3) (0.29)	0.09'	9.55	42.1	-0.00	-1.00	-	0.42	-0.67	0.29
	(3, -1.12, 2) (0.15)	56.6	202.1	437.4	0.96	-1.01	0.95	0.17	-0.67	-
$GDPC_{d11}$	(0, 1.07, 3) (0.07)	218.8	0.82'	168.4	-	-	-	0.08	0.02	-0.0
	--	--	--	--	--	--	--	--	--	--

(continued)

$GDPC_{d11log}$	(0, 1.08, 3) (0.07)	223.9	0.97'	168.7	--	--	--	0.08	0.01	0.00
	--	--	--	--	--	--	--	--	--	--
$GDPE_t$	(0, 0.67, 0) (0.10)	40.8	11.3	172.1	-	-	-	-	-	-
	--	--	--	--	--	--	--	--	--	--
dGDPE	(0, -0.35, 0) (0.09)	12.08	182.2	553.2	-	-	-	-	-	-
	(3, 0.68, 1) (0.30)	4.99	1.05'	18.3	-0.89	-0.97	-0.82	-0.14	-	-
dlGDP	(3, 0.14, 3) (0.18)	0.57'	21.1	99.0	-0.75	-1.01	-0.74	-0.51	-0.87	-0.49
	(3, 0.72, 1) (0.31)	5.14	0.81'	16.5	-0.88	-0.96	-0.80	-0.11	-	-
$GDPE_{d11}$	(0, 1.19, 3) (0.51)	5.42	0.15'	2.41	-	-	-	0.17	0.00	-0.09
	(2, 0.94, 3) (0.67)	1.96	0.00'	2.49	0.74	0.00	-	0.68	-0.03	-0.12
$GDPE_{d11log}$	(0, 1.16, 3) (0.52)	4.93	0.09'	2.57	-	-	-	0.11	-0.02	-0.10
	(2, 1.29, 2) (0.23)	31.2	1.56'	9.53	-0.58	-0.12	-	-0.33	0.07	-

Source: Author calculations

Tables (5,6,7,8,9,10,11,12 see appendix) summarize selected models for level and seasonally adjusted (X-12 ARIMA) aggregate output series according to both selections criteria used. Tables show that for level and seasonally adjusted series order of integration varies.

For seasonally adjusted real GDP order of integration ranges from 0.99 - 1.81 while for real GDP in levels 0.57 - 1.81. Resulting model from the established model selection criterion based on residual diagnostics and LR tests is an ARFIMA(0,0.75,0).Identified model implies nonstationarity but mean reversion. This result is in contradiction with a large body of empirical studies in Croatia identifying real GDP series in levels as I(1).

This study however shows that a fractional model with d smaller then one (d = 0.75) better describes series behavior. For the same series, AIC, SIC and LC model selection procedure chooses an ARFIMA(3,1.51,2) model rejecting both I(1) and I(2) hypothesis are rejected. Once again, results of this study contradict various other study for Croatia. Correct model specification for real GDP per capita series in levels appears to be ARFIMA(0,1.14,3) supporting other studies finding this series to be an I(1) variable. For the real GDP per employed person series identified correct model is an ARFIMA(0,0.67,0) implying nonstationarity, mean reversion and rejection of I(1) assumption. For the X-12 ARIMA adjusted GDP per capita series best identified model follows ARFIMA(0,1.29,3) thus not rejecting the I(1) hypothesis. Therefore, X-12 ARIMA adjusted real GDP series in Croatia is best modeled as I(1) process. Same holds for X-12 ARIMA real GDP per capita modeled as ARFIMA(0,1.07,3) clearly not rejecting I(1) hypothesis. For X-12 ARIMA real GDP per employed person series a identified plausible model follows ARFIMA(0,1.19,3) confirming once again the null of I(1). Tests results are confirmed by the LR tests except for X-12 ARIMA real GDP series.

To describe general characteristics of fluctuation in aggregate output series in Croatia impulse response functions for all series are identified (see figures 3,4 and 5).

Figures (3,4 and 5) show real GDP series have long memory in levels and that the hypothesis of $d = 1$ is strongly rejected. Thus, using this series as unit root process in modeling is inappropriate. Series clearly show long memory or long range dependence as a mean reverting process with corresponding impulse responses convergent to zero. We observe that the effect of a shock on real GDP die out slowly in the long run (30 periods later). The same is valid for real GDP per employed person series while real GDP per capita series show the effects of a shock are divergent hyperbolically ($d > 1$). For first differenced series of real GDP/GDP per capita/GDP per employed person effects of a shock die out quickly. Right side graphs display impulse response function for model selected under information criteria. Anti-persistence and $d < 0$ is clearly visible with effect of a shock in aggregate output shifting from positive to negative period after period. A shock in output is followed by an increase in output in the following period after which another decline appear with series tending to revert to the mean. Anti-persistence is quite strong still present after 30 periods showing that differenced and log differenced aggregate output series are highly anti-persistent characterized by short memory and IR coefficients that die out fast. However, memory characteristics for the anti-persistent series varies from a standard short memory process so differenced and log differenced aggregate output series in Croatia should be modeled as intermediate memory process and not as I(0) contradicting other studies for Croatia implicitly assuming that differenced and log differenced real output is an I(0) process. Seasonally adjusted series for real output (GDP, GDP per capita, GDP per employed persons) clearly show impulse response coefficients diverge hyperbolically with $d > 1$. Thus the effects of a shock on real output seasonally adjusted series tend to be permanent. Series are nonstationary and non mean reverting. Not only, effect of shocks in real output permanent component in the series intensify over time as showed by the impulse response function graphs.

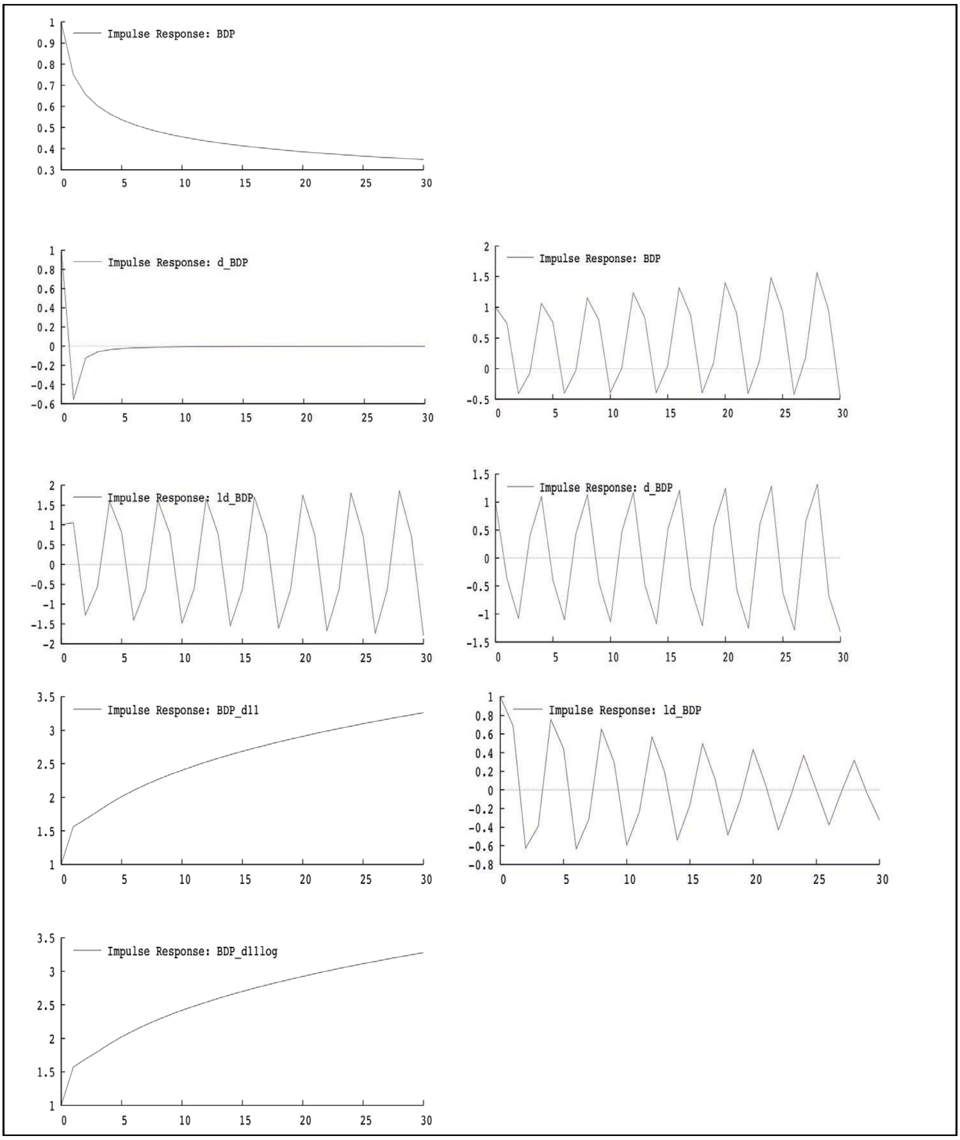

FIGURE 3 IMPULSE RESPONSE FUNCTION FOR REAL GDP

Source: Author calculations

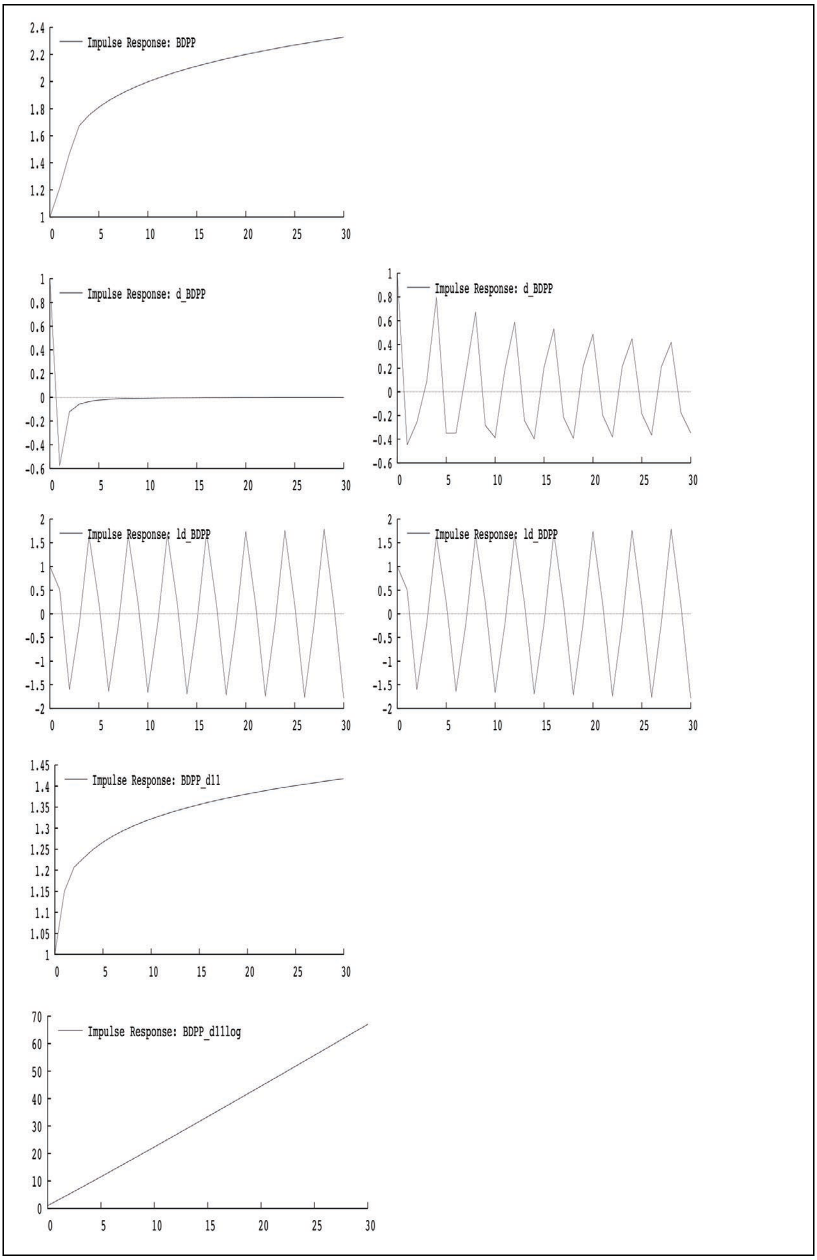

FIGURE 4 IMPULSE RESPONSE FUNCTION FOR REAL GDP PER CAPITA

Source: Author calculation

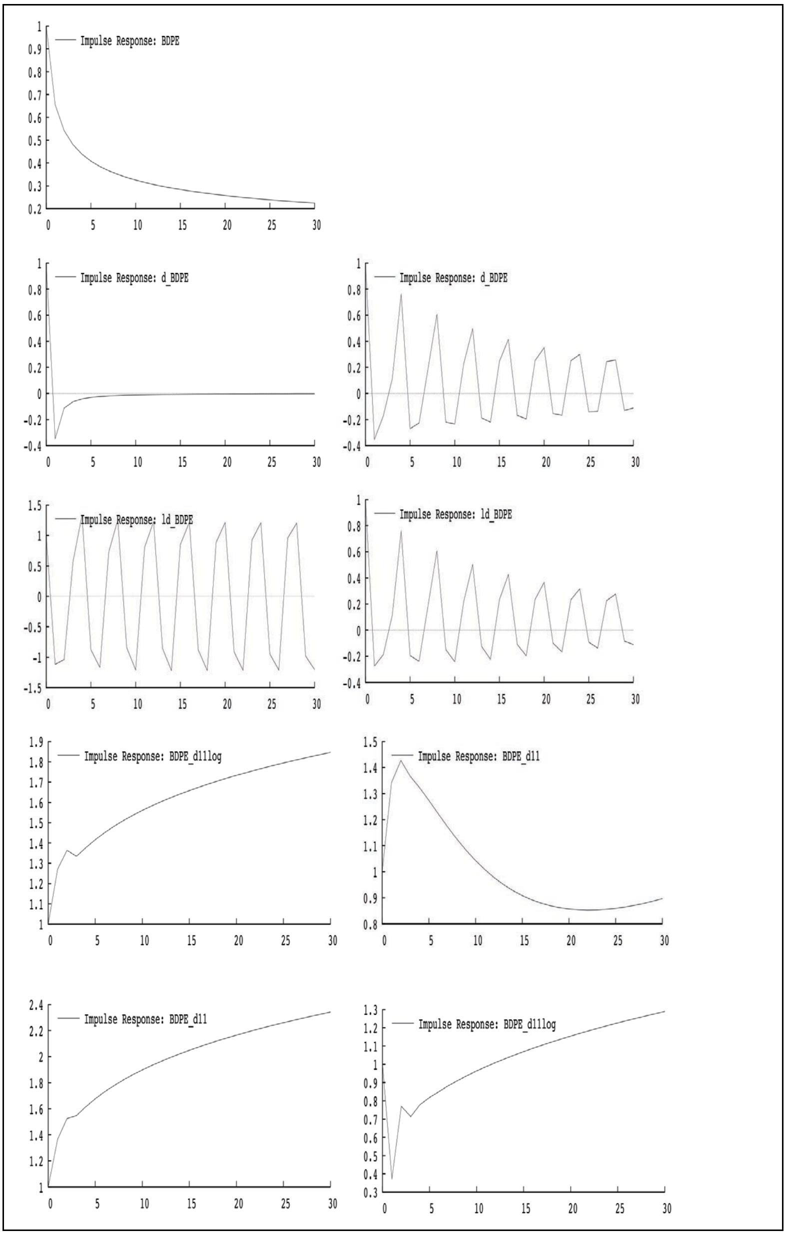

FIGURE 5: IMPULSE RESPONSE FUNCTION FOR REAL GDP PER EMPLOYED PERSON

Source: Author calculation

IV. CONCLUSION

The quarterly structure of real GDP, real GDP per capita and real GDP per employed person in Croatia has been examined using fractional integration methods. Using a battery of non-parametric, semi-parametric and parametric tests order of integration of aggregate output series was investigated. The tests statistics clearly reject the null of trend stationary process I(0) in favor of near-integration (intermediate memory) or long memory processes. This study results clearly contradict other studies results for aggregate output in Croatia implicitly assuming different GDP series as I(0) or I(1) variables. Using fractional integration techniques evidence of both anti-persistence (intermediate memory) and persistence (long memory) is found. ARFIMA models turn out to best the best model for describing real output dynamics in Croatia over 1996-2012. Presented test results indicate real output in Croatia is a highly persistent process. Differenced series show anti-persistence or long range dependence demanding correct order of integration estimation rather than constraining the order of integration to zero or one. For the real output seasonally adjusted series (except for real GDP and real GDP per capita log differenced/differenced series) the null of unit root cannot be rejected.

This paper describe in details the general characteristics of real output fluctuations in Croatia. Evidence succinctly and clearly shows ARFIMA models best capture short and long memory characteristics in real output series. Fractionally integrated models for Croatian GDP series obviously show that real output series behave distinctly from stationary or integrated processes most often assumed in studies related to output dynamics in Croatia. This fact has two important implications for further study and policy implementation. Further research on real output dynamics for Croatia should focus on estimating the correct order of integration rather then assuming stationary, unit root behavior or cointegration. Second, even more important, shocks affecting real output in Croatia have long lasting and even increasing effects over time. Thus, fluctuations in real output in Croatia are largely permanent. Consequently, this implies that appropriate policy actions are required to reestablish output equilibrium. However, consequences of a miss-specified economic policy can be even more devastating than no policy action. Effects of shocks in Croatian real output can last up to 8 years but outcomes of using badly designed economic policy can last almost infinitely. Only a small part of output fluctuations in Croatia are of transitory nature. This indicates cycles have minor effects on economy in relation to devastating consequence of wrong policy implementation that appears to be a case in Croatia.

V. REFERENCES

***Abadir, K.M., W. Distaso, and L. Giraitis**. 2007. Nonstationarity-extended local Whittle estimation. Journal of Econometrics 141, no. 2: 1353-1384.*

***Alptekin, N.** 2006. Long memory analysis of USD/TRL exchange rate. International Journal of Social Sciences 1, no. 2:*

***Baillie, R.T., C.F. Chung, and M.A. Tieslau.** 1996. Analysing inflation by the fractionally integrated ARFIMA-GARCH model. Journal of applied econometrics 11, no. 1: 23-40.*

***Baum, C.F., and VL Wiggin**s. 2000. sts16: Tests for long memory in a time series. Stata Technical Bulletin 57, 39-44.*

***Beran, J.** 1989. A test of location for data with slowly decaying serial correlations. Biometrika 76, no. 2: 261-269.*

***Beran, J.** 1994. Statistics for long-memory processes. Chapman & Hall/CRC.*

***Box, G.E.P., and G.M. Jenkins.** 1976. Time series analysis: Forecasting and Control, revised ed. Holden-Day San Francisco.*

Breitung, J. *2002. Nonparametric tests for unit roots and cointegration. Journal of econometrics 108, no. 2: 343-363.*

Caporale, G.M., and L.A. Gil-Alana. *2009. Long memory in US real output per capita.*

Davidson, J. *2009. When is a time series I(0)? In The Methodology and Practice of Econometrics, ed. Jennifer Castle, Shepherd, Neil, Oxford University Press.*

Davidson, J., and N. Hashimzade. *2009. Type I and type II fractional Brownian motions: A reconsideration. Computational Statistics & Data Analysis 53, no. 6: 2089-2106.*

Diebold, F.X., and G.D. Rudebusch. *1989. Long memory and persistence in aggregate output. Journal of monetary economics 24, no. 2: 189-209.*

Geweke, J., and S. Porter-Hudak. *1983. The estimation and application of long memory time series models. Journal of time series analysis 4, no. 4: 221-238.*

Gil-Alana, LA. *2004. A fractionally integrated model for the Spanish real GDP. Economics Bulletin*

Gil-Alana, L.A. *2001. A fractionally integrated exponential model for UK unemployment. Journal of Forecasting 20, no. 5: 329-340.*

Gil-Alana, L.A., and SG Brian Henry. *2003. Fractional Integration and the Dynamics of UK Unemployment*. oxford Bulletin of Economics and statistics 65, no. 2: 221-239.*

Giraitis, L., P. Kokoszka, R. Leipus, and G. Teyssiere. *2003. Rescaled variance and related tests for long memory in volatility and levels. Journal of econometrics 112, no. 2: 265-294.*

Granero, S., JE Trinidad Segovia, and J. García Pérez. *2008. Some comments on Hurst exponent and the long memory processes on capital markets. Physica A: Statistical Mechanics and its Applications 387, no. 22: 5543-5551.*

Granger, C.W.J. *1980. Long memory relationships and the aggregation of dynamic models. Journal of econometrics 14, no. 2: 227-238.*

Granger, C.W.J., and R. Joyeux. *1980. An introduction to long-memory time series models and fractional differencing. Journal of time series analysis 1, no. 1: 15-29.*

Harris, D., B. McCabe, S. Leybourne, PJ Brockwell, R. Davis, MA Delgado, PM Robinson, EJ Hannan, B. Hobijn, and PH Franses. 2008. Testing for long memory. Econometric Theory 24, no. 1: 143.

Haubrich, J.G., and A.W. Lo. *2001. The sources and nature of long-term memory in aggregate output. Economic Review-Federal Reserve Bank of Cleveland 37, no. 2: 15-30.*

Hauser, M.A. *1997. Semiparametric and nonparametric testing for long memory: A monte carlo study. Empirical Economics 22, no. 2: 247-271.*

Hosking, J.R.M. *1981. Fractional differencing. Biometrika 68, no. 1: 165-176.*

Hurst, H.E. *1951. Long-term storage capacity of reservoirs}. Trans. Amer. Soc. Civil Eng. 116, 770-808.*

Hurvich, C.M., E. Moulines, and P. Soulier. *2005. Estimating long memory in volatility. Econometrica 73, no. 4: 1283-1328.*

Hurvich, C.M., and B.K. Ray. *1995. Estimation of the memory parameter for nonstationary or noninvertible fractionally integrated processes. Journal of time series analysis 16, no. 1: 17-41.*

Hurvich, C.M., and B.K. Ray. *2003. The local Whittle estimator of long-memory stochastic volatility. Journal of Financial Econometrics 1, no. 3: 445-470.*

Javier Contreras-Reyes, *GMG, Wilfredo Palma. "afmtools: Estimation, Diagnostic and Forecasting Functions for ARFIMA models", R package available at CRAN.*

Javier Contreras-Reyes, *GMG, Wilfredo Palma (2011) "afmtools: Estimation, Diagnostic and Forecasting Functions for ARFIMA models", R package available at CRAN.*

Estimation*, Diagnostic and Forecasting Functions for ARFIMA models. R package available at CRAN*

Künsch, H. *1986. Discrimination between monotonic trends and long-range dependence. Journal of applied Probability 1025-1030.*

Lee, D., and P. Schmidt. *1996. On the power of the KPSS test of stationarity against fractionally-integrated alternatives. Journal of econometrics 73, no. 1: 285-302.*

Lo, A.W*. 1991. Long-term memory in stock market prices. Econometrica 59, 1279-1313.*

Lobato, I.N., and P.M. Robinson*. 1998. A nonparametric test for I (0). The Review of Economic Studies 65, no. 3: 475-495.*

Mandelbrot, B.B. *1975. Limit theorems on the self-normalized range for weakly and strongly dependent processes. Probability Theory and Related Fields 31, no. 4: 271-285.*

Mayoral, L. *2006. Further Evidence on the Statistical Properties of Real GNP*. Oxford Bulletin of Economics and Statistics 68, 901-920.*

Michelacci, C., and P. Zaffaroni. *2000. (Fractional) beta convergence. Journal of Monetary Economics 45, no. 1: 129-153.*

Moulines, E., and P. Soulier. *1999. Broadband log-periodogram regression of time series with long-range dependence. The Annals of Statistics 27, no. 4: 1415-1439.*

Nelson, C.R., and C.R. Plosser. *1982. Trends and random walks in macroeconmic time series:: Some evidence and implications. Journal of monetary economics 10, no. 2: 139-162.*

Nielsen, F. *2007. Local polynomial Whittle estimation covering non-stationary fractional processes. CREATES Research Paper No. 2008-28*

Perron, P. *1989. The great crash, the oil price shock, and the unit root hypothesis. Econometrica: Journal of the Econometric Society 1361-1401.*

Perron, P. *1990. Testing for a unit root in a time series with a changing mean. Journal of Business & Economic Statistics 153-162.*

Phillips, P.C.B. *1999a. Discrete Fourier transform of fractional processes. Cowles Foundation for Research in Economics, Unpublished working paper No. 1243,*

Phillips, P.C.B. *1999b. Unit root log periodogram regression. Cowles foundation discussion paper*

Phillips, P.C.B., and H.R. Moon*. 1999. Linear regression limit theory for nonstationary panel data. Econometrica 1057-1111.*

Phillips, P.C.B., and K. Shimotsu. *2004. Local Whittle estimation in nonstationary and unit root cases. The Annals of Statistics 32, no. 2: 656-692.*

Robinson, P.M*. 1995a. Gaussian semiparametric estimation of long range dependence. The Annals of statistics 23, no. 5: 1630-1661.*

Robinson, P.M. *1995b. Log-periodogram regression of time series with long range dependence. The annals of Statistics 1048-1072.*

Robinson, P.M., and M. Henry. *1999. Long and short memory conditional heteroskedasticity in estimating the memory parameter of levels. Econometric theory 15, no. 03: 299-336.*

Said, S.E., and D.A. Dickey. *1984. Testing for unit roots in autoregressive-moving average models of unknown order. Biometrika 71, no. 3: 599-607.*

Shimotsu, K., and P.C.B. Phillips. *2006. Local Whittle estimation of fractional integration and some of its variants. Journal of Econometrics 130, no. 2: 209-233.*

Silverberg, G., and B. Verspagen*. 2000. A note on Michelacci and Zaffaroni, long memory, and time series of economic growth. MERIT, Maastricht Economic Research Institute on Innovation and Technology.*

Sowell, F. *1992. Modeling long-run behavior with the fractional ARIMA model. Journal of Monetary Economics 29, no. 2: 277-302.*

Teverovsky, V., M.S. Taqqu, and W. Willinger. *1999. A critical look at Lo's modified R/S statistic. Journal of statistical Planning and Inference 80, no. 1-2: 211-227.*

Velasco, C. *1999. Non-stationary log-periodogram regression. Journal of Econometrics 91, no. 2: 325-371.*

Velasco, C. *2006. Semiparametric estimation of long-memory models.*

APPENDIX

TABLE 5 MAXIMUM LIKELIHOOD ESTIMATES OF D IN ARFIMA(P,D,Q) MODELS FOR THE CROATIAN GDP

ARMA (p,q)	*GDP_t*	*dGDP*	*dlGDP*	*$GDPC_t$*	*dGDPC*	*dlGDPC*	*$GDPE_t$*	*dGDPE*	*dlGDPE*
(0,0)	0.75	-0.55	-0.51	0.99	-0.57	-0.55	0.65	-0.35	-0.33
(1,0)	0.62	-0.67	-0.61	0.90	-0.66	-0.64	0.56	-0.42	-0.41
(0,1)	0.63	-0.79	-0.73	0.93	-0.77	-0.75	0.46	-0.53	-0.52
(1,1)	0.57	-0.80	-0.72	0.94	-0.73	-0.72	0.50	-0.48	-0.47
(2,0)	1.13	0.01	0.04	1.06	-0.19	-0.15	1.08	0.05	0.08
(0,2)	1.05	-0.30	-0.24	1.13	-0.34	-0.31	0.89	-0.12	-0.11
(2,1)	1.45	0.33	0.37	1.14	0.15	0.19	1.35	0.37	0.39
(1,2)	1.10	-0.33	-0.27	1.15	-0.33	-1.26	0.97	-1.07	-0.11
(2,2)	-	0.02	0.07	-	-0.17	-1.25	-	0.02	0.04
(3,0)	1.81	0.84	0.74	1.20	0.50	-1.13	-0.02	0.78	0.79
(0,3)	0.99	-0.22	-0.17	1.13	-0.20	-0.66	0.59	-0.40	0.09
(3,1)	1.71	0.75	0.63	1.18	0.47	-0.76	0.38	0.68	0.71
(3,2)	1.51	0.44	-	-	0.22	-1.12	-	-	-
(1,3)	-	-0.42	-0.24	1.15	-0.83	-0.72	0.69	-0.49	-0.23
(2,3)	-	0.34	0.44	1.07	0.07	0.09	-0.86	0.35	0.31
(3,3)	-	-	-	-	-0.02	-0.80	-0.98	-	0.14

Source: Author calculations

TABLE 6 MAXIMUM LIKELIHOOD ESTIMATES OF D IN ARFIMA(P,D,Q) MODELS FOR THE CROATIAN GDP (SEASONALLY ADJUSTED X-12 ARIMA)

ARMA(p,q)	*GDP_{d11}*	*GDP_{d11log}*	*$GDPC_{d11}$*	*$GDPC_{d11log}$*	*$GDPE_{d11}$*	*$GDPE_{d11log}$*
(0,0)	1.18	-0.55	0.99	-0.57	0.65	-0.35
(1,0)	1.35	-0.67	0.90	-0.66	0.56	-0.42
(0,1)	1.40	-0.79	0.93	-0.77	0.46	-0.53
(1,1)	1.33	-0.80	0.94	-0.73	0.50	-0.48
(2,0)	1.13	0.01	1.06	-0.19	1.08	0.05
(0,2)	1.05	-0.30	1.13	-0.34	0.89	-0.12
(2,1)	1.45	0.33	1.14	0.15	1.35	0.37
(1,2)	1.10	-0.33	1.15	-0.33	0.97	-1.07
(2,2)	-	0.02	-	-0.17	-	0.02
(3,0)	1.81	0.84	1.20	0.50	-0.02	0.78
(0,3)	0.99	-0.22	1.13	-0.20	0.59	-0.40
(3,1)	1.71	0.75	1.18	0.47	0.38	0.68
(3,2)	1.51	0.44	-	0.22	-	-
(1,3)	-	-0.42	1.15	-0.83	0.69	-0.49
(2,3)	-	0.34	1.07	0.07	-0.86	0.35
(3,3)	-	-	-	-0.02	-0.98	-

Source: Author calculations

TABLE 7 PARAMETER ESTIMATES OF ARFIMA MODELS FOR GDP_T

	Long memory	*AR parameters*			*MA parameters*			*Log-likelihood*	*Lik. criterions*	
ARMA	*d*	ϕ1	ϕ2	ϕ3	θ1	θ2	θ3		AIC	SIC
(0,0)	0.75	-	-	-	-	-	-	618.5	621.5	624.7
	(0.09)									
(1,0)	0.62	0.15	-	-	-	-	-	607.5	611.5	615.9
	(0.08)	(0.12)								
(0,1)	0.63	-	-	-	-0.61	-	-	612.5	618.2	620.8
	(0.09)				(0.05)					
(1,1)	0.57	-0.02	-	-	-0.62	-	-	602.2	607.1	612.5
	(0.10)	(0.18)			(0.07)					
(2,0)	1.13	-0.08	-0.91	-	-	-	-	557.2	562.2	567.5
	(0.06)	(0.07)	(0.06)							
(0,2)	1.05	-	-	-	0.09	0.51	-	605.9	610.8	616.3
	(0.13)				(0.06)	(0.05)				
(2,1)	1.45	-0.05	-0.93	-	0.48	-	-	552.0	558.0	564.5
	(0.14)	(0.06)	(0.06)		(0.11)					
(1,2)	1.09	-0.07	-	-	0.13	0.56	-	595.3	601.3	607.8
	(0.04)	(0.05)			(0.07)	(0.05)				
(2,2)	--	--	--	--	--	--	--	--	--	--
(3,0)	1.81	-0.93	-1.02	-0.92	-	-	-	523.6	529.6	536.0
	(0.20)	(0.09)	(0.03)	(0.09)						
(0,3)	0.99	-	-	-	0.61	0.26	-0.68	596.8	602.8	609.3
	(0.11)				(0.09)	(0.08)	(0.09)			
(3,1)	1.71	-0.95	-1.02	-0.93	-0.13	-	-	523.4	530.4	537.8
	(0.22)	(0.10)	(0.03)	(0.09)	(0.13)					
(3,2)	1.51	-0.74	-1.03	-0.74	-0.03	-0.68	-	517.9	525.9	534.4
	(0.25)	(0.29)	(0.01)	(0.29)	(0.25)	(0.32)				
(1,3)	--	--	--	--	--	--	--	--	--	--
(2,3)	--	--	--	--	--	--	--	--	--	--
(3,3)	--	--	--	--	--	--	--	--	--	--

*Source: Author calculations. Notes: Standard errors in parenthesis, *model pass diagnostic tests on the residuals (no serial correlation, functional form, normality and homoscedasticity) at the 5% significance level.*

TABLE 8 PARAMETER ESTIMATES OF ARFIMA MODELS FOR $GDPC_T$

	Long memory	*AR parameters*			*MA parameters*			*Log-likelihood*	*Lik. criterions*	
ARMA	*d*	$\phi1$	$\phi2$	$\phi3$	$\theta1$	$\theta2$	$\theta3$		AIC	SIC
(0,0)	0.99	-	-	-	-	-	-	564.6	567.6	570.8
	(0.06)									
(1,0)	0.90	0.11	-	-	-	-	-	556.0	560.0	564.3
	(0.10)	(0.11)								
(0,1)	0.93	-	-	-	-0.13	-	-	564.4	568.4	572.7
	(0.08)				(0.09)					
(1,1)	0.94	-0.22	-	-	-0.33	-	-	556.1	561.1	566.5
	(0.09)	(0.26)			(0.25)					
(2,0)	1.06	-0.01	-0.18	-	-	-	-	547.3	552.3	557.7
	(0.10)	(0.10)	(0.17)							
(0,2)	1.13	-	-	-	0.11	0.17	-	563.9	568.9	574.3
	(0.20)				(0.22)	(0.17)				
(2,1)	1.14	0.23	-0.20	-	0.33	-	-	547.1	553.1	559.6
	(0.13)	(0.22)	(0.26)		(0.26)					
(1,2)	1.15	0.18	-	-	0.30	0.17	-	555.6	561.6	568.1
	(0.19)	(0.25)			(0.30)	(0.14)				
(2,2)	--	--	--	--	--	--	--	--	--	--
(3,0)	1.20	-0.17	-0.26	-0.18	-	-	-	538.5	544.5	550.0
	(0.24)	(0.24)	(0.17)	(0.15)						
(0,3)	1.14	-	-	-	0.08	0.17	0.12	563.6	569.6	576.1
	(0.16)				(0.16)	(0.13)	(0.09)			
(3,1)	1.18	-0.38	-0.27	-0.23	-0.24	-	-	538.3	545.3	552.8
	(0.20)	(0.34)	(0.20)	(0.17)	(0.22)					
(3,2)	--	--	--	--	--	--	--	--	--	--
(1,3)	1.15	--	--	--	-0.09	0.19	0.17	555.2	562.2	569.7
	(0.16)				(0.18)	(0.15)	(0.11)			
(2,3)	1.07	0.00	-1.01	-	0.01	-1.06	0.16	538.9	546.9	555.4
	(0.04)	(0.02)	(0.02)		(0.09)	(0.01)	(0.10)			
(3,3)	--	--	--	--	--	--	--	--	--	--

*Source: Author calculations. Notes: Standard errors in parenthesis, *model pass diagnostic tests on the residuals (no serial correlation, functional form, normality and homoscedasticity) at the 5% significance level.*

TABLE 9 PARAMETER ESTIMATES OF ARFIMA MODELS FOR GDPET

	Long memory	*AR parameters*			*MA parameters*			*Log-likelihood*	*Lik. criterions*	
ARMA	*d*	ϕ1	ϕ2	ϕ3	θ1	θ2	θ3		AIC	SIC
(0,0)	0.65	-	-	-	-	-	-	587.5	591.8	593.8
	(0.10)									
(1,0)	0.56	0.17	-	-	-	-	-	578.5	582.5	586.8
	(0.11)	(0.13)								
(0,1)	0.46	-	-	-	-0.51	-	-	583.7	587.6	592.0
	(0.10)				(0.08)					
(1,1)	0.50	-0.11	-	-	-0.55	-	-	575.0	580.0	585.4
	(0.11)	(0.17)			(0.08)					
(2,0)	1.08	-0.14	-0.74	-	-	-	-	550.8	555.8	561.2
	(0.09)	(0.10)	(0.09)							
(0,2)	0.89	-	-	-	0.04	0.35	-	581.5	586.5	591.9
	(0.18)				(0.12)	(0.05)				
(2,1)	1.37	-0.09	-0.77	-	0.39	-	-	548.2	554.2	560.6
	(0.16)	(0.09)	(0.10)		(0.13)					
(1,2)	0.97	-0.07	-	-	0.06	0.38	-	573.0	579.0	585.6
	(0.10)	(0.15)			(0.12)	(0.08)				
(2,2)	--	--	--	--	--	--	--	--	--	--
(3,0)	-0.01	0.90	-0.62	0.72	-	-	-	541.3	547.3	553.7
	(0.02)	(0.09)	(0.15)	(0.10)						
(0,3)	0.59	-	-	-	-0.66	0.08	0.42	577.3	583.4	589.9
	(0.12)				(0.14)	(0.09)	(0.06)			
(3,1)	0.38	0.88	-0.69	0.74	-	-	-	538.7	545.7	553.2
	(0.14)	(0.09)	(0.14)	(0.09)						
(3,2)	--	--	--	--	--	--	--	--	--	--
(1,3)	0.69	0.52	--	--	0.72	0.16	-0.54	566.3	573.3	580.8
	(0.35)	(0.51)			(0.30)	(0.09)	(0.07)			
(2,3)	-0.86	1.87	-0.86	-	0.59	0.23	-0.50	557.1	565.1	**573.7**
	(0.18)	(0.07)	(0.07)		(0.15)	(0.09)	(0.12)			
(3,3)	-0.98	2.18	-1.43	0.26	0.66	0.22	-0.42	549.3	558.3	567.9
	(1.12)	(0.49)	(0.90)	(0.41)	(0.37)	(0.29)	(0.30)			

Source: Author calculations

TABLE 10 PARAMETER ESTIMATES OF ARFIMA MODELS FOR GDPD11

	Long memory	*AR parameters*			*MA parameters*			*Log-likelihood*	*Lik. criterions*	
ARMA	*d*	ϕ1	ϕ2	ϕ3	θ1	θ2	θ3		AIC	SIC
(0,0)	1.18	-	-	-	-	-	-	528.3	531.3	534.6
	(0.12)									
(1,0)	1.35	0.35	-	-	-	-	-	518.4	522.4	526.7
	(0.17)	(0.15)								
(0,1)	1.40	-	-	-	0.34	-	-	526.8	530.8	535.2
	(0.18)				(0.19)					
(1,1)	1.33	-0.37	-	-	-0.05	-	-	518.3	523.3	528.7
	(0.14)	(0.15)			(0.22)					
(2,0)	1.38	-0.38	-0.07	-	-	-	-	510.5	515.5	520.9
	(0.17)	(0.22)	(0.22)							
(0,2)	1.32	-	-	-	0.30	-0.16	-	526.1	531.1	536.5
	(0.21)				(0.27)	(0.14)				
(2,1)	1.39	-0.00	0.01	-	0.36	-	-	510.5	516.5	522.8
	(0.28)	(1.60)	(0.27)		(1.05)					
(1,2)	1.29	-0.32	-	-	-0.02	-0.06	-	518.3	524.3	530.8
	(0.24)	(0.22)			(0.22)	(0.24)				
(2,2)	0.88	0.96	-0.04	0.83	-0.21	--	--	510.0	517.0	524.5
	(0.18)	(0.10)	(0.08)	(0.22)	(0.18)					
(3,0)	1.13	-0.13	0.17	0.00	-	-	-	501.5	507.5	513.9
	(2.01)	(2.03)	(1.37)	(0.09)						
(0,3)	1.29	-	-	-	0.27	-0.16	-0.03	526.0	532.0	538.6
	(0.20)				(0.29)	(0.14)	(0.14)			
(3,1)	1.16	0.03	0.17	0.01	0.21	-	-	501.5	508.4	515.8
	(0.40)	(0.65)	(0.25)	(0.02)	(0.22)					
(3,2)	1.11	1.16	-0.58	-0.04	1.30	-0.85	-	499.5	507.6	516.0
	(0.51)	(0.45)	(0.28)	(0.21)	(0.42)	(0.27)				
(1,3)	1.24	-0.29	-	-	-0.05	-0.09	-0.07	518.2	525.2	532.7
	(0.26)	(0.27)			(0.21)	(0.26)	(0.12)			
(2,3)	--	--	--	--	--	--	--	--	--	--
(3,3)	--	--	--	--	--	--	--	--	--	--

*Source: Author calculations. Notes: Standard errors in parenthesis, *model pass diagnostic tests on the residuals (no serial correlation, functional form, normality and homoscedasticity) at the 5% significance level.*

TABLE 11 PARAMETER ESTIMATES OF ARFIMA MODELS FOR $GDPC_{D11}$

	Long memory	*AR parameters*			*MA parameters*			*Log-likelihood*	*Lik. criterions*	
ARMA	*d*	ϕ1	ϕ2	ϕ3	θ1	θ2	θ3		AIC	SIC
(0,0)	1.01	-	-	-	-	-	-	559.8	562.8	566.0
	(0.02)									
(1,0)	1.03	-0.03	-	-	-	-	-	551.5	555.5	559.8
	(0.07)	(0.08)								
(0,1)	1.06	-	-	-	0.08	-	-	559.7	563.7	568.1
	(0.04)				(0.03)					
(1,1)	1.02	-0.02	-	-	0.02	-	-	551.5	556.5	561.9
	(0.06)	(0.06)			(0.01)					
(2,0)	1.03	-0.06	-0.02	-	-	-	-	543.3	548.3	550.0
	(0.02)	(0.02)	(0.01)							
(0,2)	1.07	-	-	-	0.09	0.02	-	559.7	564.7	570.1
	(0.05)				(0.05)	(0.02)				
(2,1)	1.07	-0.95	-0.10	-	-0.86	-	-	543.4	549.4	555.8
	(0.02)	(0.03)	(0.03)		(0.02)					
(1,2)			--	--			--	--	--	--
	--	--			--	--				
(2,2)	--	--	--	--	--	--	--	--	--	--
(3,0)	--	--	--	--	--	--	--	--	--	--
(0,3)	1.07	-	-	-	0.08	0.02	-0.00	559.7	565.7	572.2
	(0.07)				(0.08)	(0.03)	(0.03)			
(3,1)	1.05	-0.90	-0.07	0.02	-	-	-	535.3	542.2	549.7
	(0.03)	(0.06)	(0.05)	(0.03)						
(3,2)	--	--	--	--	--	--	--	--	--	--
(1,3)	1.00	-0.00	--	--	0.02	-0.00	-0.02	551.5	558.5	566.0
	(0.12)	(0.12)			(0.01)	(0.06)	(0.03)			
(2,3)	--	--	--	--	--	--	--	--	--	--
(3,3)	--	--	--	--	--	--	--	--	--	--

*Source: Author calculations. Notes: Standard errors in parenthesis, *model pass diagnostic tests on the residuals (no serial correlation, functional form, normality and homoscedasticity) at the 5% significance level.*

TABLE 12 PARAMETER ESTIMATES OF ARFIMA MODELS FOR $GDPE_{D11}$

	Long memory	*AR parameters*			*MA parameters*			*Log-likelihood*	*Lik. criterions*	
ARMA	d	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3		AIC	SIC
(0,0)	1.10	-	-	-	-	-	-	535.7	542.0	535.7
	(0.10)									
(1,0)	1.09	-0.01	-	-	-	-	-	527.1	531.1	527.1
	(0.11)	(0.13)								
(0,1)	1.28	-	-	-	0.25	-	-	535.3	539.3	543.7
	(0.34)				(0.40)					
(1,1)	1.23	0.06	-	-	0.26	-	-	526.8	531.8	537.2
	(0.31)	(0.16)			(0.33)					
(2,0)	1.29	-0.27	-0.12	-	-	-	-	518.8	523.8	529.1
	(0.23)	(0.25)	(0.18)							
(0,2)	1.30	-	-	-	0.26	0.01	-	535.3	542.5	545.7
	(0.39)				(0.41)	(0.15)				
(2,1)	1.27	-0.40	-0.14	-	0.16	-	-	518.8	524.8	531.2
	(0.24)	(0.35)	(0.15)		(0.49)					
(1,2)	1.23	0.11	-	-	0.31	-0.03	-	526.8	532.8	539.2
	(0.33)	(0.50)			(0.58)	(0.27)				
(2,2)	1.27	-0.27	-0.18	--	-0.03	-0.07	--	518.7	525.7	533.2
	(0.21)	(0.40)	(0.20)		(0.52)	(0.26)				
(3,0)	1.19	-0.16	-0.04	0.07	-	-	-	511.0	517.0	523.4
	(0.65)	(0.70)	(0.49)	(0.30)						
(0,3)	1.20	-	-	-	0.17	0.00	-0.09	535.1	541.1	547.6
	(0.52)				(0.53)	(0.22)	(0.14)			
(3,1)	0.32	0.67	0.17	0.12	-	-	-	510.3	517.3	524.7
	(0.41)	(0.83)	(0.47)	(0.33)						
(3,2)	--	--	--	--	--	--	--	--	--	--
(1,3)	0.98	-0.05	--	--	-0.04	-0.06	-0.13	526.8	533.8	541.3
	(0.02)	(0.03)			(0.18)	(0.14)	(0.09)			
(2,3)	0.94	0.74	-0.00	-	0.67	-0.02	-0.12	518.4	526.4	**535.0**
	(0.67)	(0.41)	(0.03)		(0.28)	(0.21)	(0.18)			
(3,3)	--	--	--	--	--	--	--	--	--	--

*Source: Author calculations. Notes: Standard errors in parenthesis, *model pass diagnostic tests on the residuals (no serial correlation, functional form, normality and homoscedasticity) at the 5% significance level*

FRAKCIONIRANO INTEGRIRANI MODEL ZA NIZ HRVATSKOG UKUPNOG OUTPUTA (BDP)

SAŽETAK

Opće karakteristike fluktuacije outputa u Hrvatskoj proučene su u okviru frakcionarne integracije. Ovaj rad procjenjuje postojanje dugoročne memorije u fluktuacijama realne dekompozicije outputa prema tranzitornim i stalnim komponentama. Rezultati upućuju na to da je ponašanje niza hrvatskog realnog outputa najlakše identificirati s ARFIMA modelom s redom integracije 0.5< d <1.5. To ukazuje na činjenicu da su makroekonomski šokovi u realnom outputu visoko prisutni. Za razliku od drugih studija u Hrvatskoj po kojima je realni output I(0) ili I(1) varijabla, rezultati ispitivanja koji proizlaze iz ove studije ukazuju na to da realni output pokazuje karakteristike dugoročne memorije sa srednjom reverzijom (frakcionarnom integracijom).

Ključne riječi: frakcionarna integracija, ARFIMA, odgovor na impuls, realni BDP Hrvatska, dugoročna memorija