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REAL EXCHANGE RATE WITH NONLINEAR THRESHOLD EFFECT

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ABSTRACT

This study applies the threshold autoregressive model proposed by Caner and Hansen (2001) to examine both linearity and stationarity of Germany's real exchange rate vis-à-vis her 5 trading partner countries. Two main conclusions are drawn. Firstly, the empirical results indicate that Germany's real exchange is a nonlinear process. Secondly, a unit root in real exchange rate was rejected for most of the cases under study. This result provides strong support for purchasing power parity for Germany relative to their major trading partner countries.

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I. INTRODUCTION

Recently, there has been a growing consensus that the real exchange rate (RER) exhibits nonlinearities, and consequently, conventional unit root tests such as the Augmented Dickey Fuller (ADF) test have low power in detecting the mean reversion of exchange rate. Taylor and Peel (2000) demonstrate that the adoption of linear stationarity tests is inappropriate for the detection of mean reversion if the true process of the data generation of the exchange rate is in fact a stationary non-linear process. A number of studies have provided empirical evidence on the nonlinear adjustment of exchange rate. Reasons for the nonlinear adjustment are the presence of transactions costs that inhibit international goods arbitrage and official intervention in the foreign exchange market may be such that nominal exchange rate movements are asymmetric (see Taylor, 2004; Taylor and Peel, 2000; Juvenal and Taylor, 2008; Reitz and Taylor, 2008). Kilian and Taylor (2003) also suggest that nonlinearity may arise from the heterogeneity of opinion in the foreign exchange market concerning the equilibrium level of the nominal exchange rate: as the nominal rate takes on more extreme values, a great degree of consensus develops concerning the appropriate direction of exchange rate moves, and traders act as accordingly. However, the finding of nonlinear adjustment does not necessarily imply nonlinear mean reversion (stationarity). As such, stationarity tests based on a nonlinear framework must be applied[1].

This empirical study contributes to this line of research by determining whether Purchasing Power Parity (PPP) holds for Germany's RER relative to a sample of her major trading partner countries (Canada, France, Italy, Japan, and the United Kingdom), using the threshold autoregressive (hereafter, TAR) model and the test statistics proposed by Caner and Hansen (2001). The major advantage of this approach is that it allows us to simultaneously investigate nonstationarity and nonlinearity. With this, the current research hopes to fill the existing gap in the literature. Otherwise, Germany provides an interesting arena to research for several reasons. First, Germany is the EU's largest economy and a major power with the world's fourth largest economy by nominal GDP and the fifth largest in purchasing power parity. Second, Germany is the world's second-largest exporter and third-largest importer of goods with the foreign exchange reserves estimated at US$ 189 billion at the end of 2009. Third, Germany is the most populous country in the European Union and also home to the third-largest number of international migrants worldwide. We find that Germany's bilateral real exchange rate is a nonlinear process and not characterized by a unit root, consistent with PPP, relative to most of the trading partner countries (four out of five), with the exception of Canada/Germany.

This study is organized as follows. Section 2 presents the data used in our study. Section 3 briefly describes the TAR unit test and our empirical results. Section 4 concludes the paper.

[1] For details on previous studies, please refer to the works of Taylor (1995), Rogoff (1996), MacDonald and Taylor (1992), Taylor and Sarno (1998), Sarno and Taylor (2002), Taylor and Taylor (2004), and Lothian and Taylor (2000, 2008), who have provided in-depth information on the theoretical and empirical aspects of PPP and the RER.

II. DATA

Our empirical analysis covers a sample of 5 countries: Canada, France, Italy, Japan, and the United Kingdom. Monthly data are employed in this study, and the time span is from January 1994 to April 2010. All consumer price indices, CPI (based on 2005 = 100), and nominal exchange rates relative to the Germany Deutsche Mark (DM) data are taken from the International Monetary Fund's International Financial Statistics CD-ROM.[2] Testing for PPP against the Germany is based on the argument that Germany is the major trading partners for these 5 countries for the past decade. As shown from Figure 1, visual inspection of the real exchange rate series for these five country pairs reveals significant upward or downward trend in the real exchange rate series for most of the countries against the DM during this sample period. From these figures, for most of the series, there seem to exhibit some nonlinear adjustment patterns.

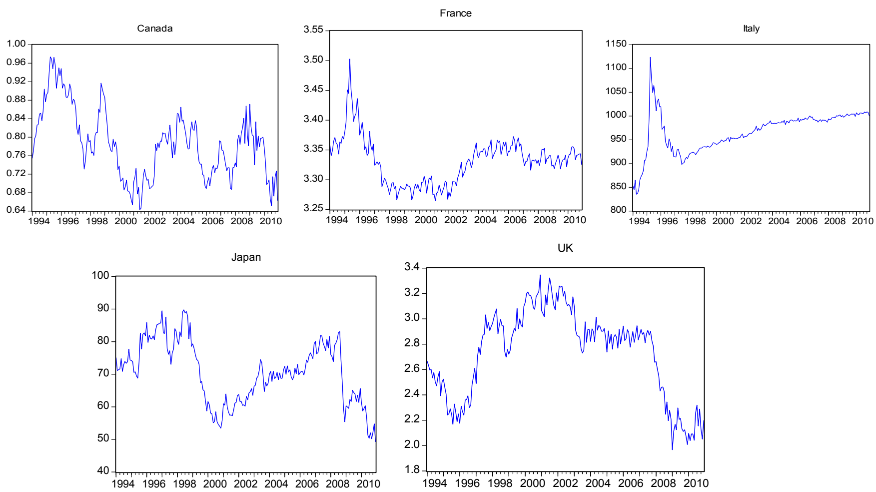

FIGURE 1. THE TENDENCY OF REAL EXCHANGE RATES

Source: Authors calculation

III. METHODOLOGY AND EMPIRICAL RESULTS

A. Caner and Hansen's (2001) Threshold Unit Root Test

Following the work of Caner and Hansen (2001), we adopt a two regime TAR(k) model with an autoregressive unit root as follow:

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[2]The real exchange rate series of a country at time t is define as $(S_t \times P_t^H) / P_t^{Germany}$, where S_t is the nominal exchange rate of home country per Mark, $P_t^{Germany}$ and P_t^H denote the consumer price indices of home country and the Germany, respectively.

$$\Delta r_t = \theta_1' x_{t-1} I_{\{Z_t<\lambda\}} + \theta_2' x_{t-1} I_{\{Z_t\geq\lambda\}} + e_t, \quad t = 1, \ldots, T \tag{1}$$

where r_t is the real exchange rate for $t = 1,2....,T$, $x_{t-1} = (r_{t-1}, v_t', \Delta r_{t-1}, \ldots, \Delta r_{t-k})'$, $I_{\{\bullet\}}$ is the indicator function, e_t is an i.i.d. disturbance, $Z_{t-1} = r_{t-1} - r_{t-m}$ is the threshold variable, m represents the delay parameter and $1 \leq m \leq k$, v_t is a vector of exogenous variables including an intercept and possibly a linear time trend. The threshold value λ is unknown and takes the values in the compact interval $\lambda \in \Lambda = [\lambda_1, \lambda_2]$, where λ_1 and λ_2 are selected according to $P(Z_t \leq \lambda_1) = 0.15$ and $P(Z_t \leq \lambda_2) = 0.85$.3 The components of θ_1 and θ_2 can be partitioned as follows:

$$\theta_1 = \begin{pmatrix} \rho_1 \\ \beta_1 \\ \alpha_1 \end{pmatrix}, \quad \theta_2 = \begin{pmatrix} \rho_2 \\ \beta_2 \\ \alpha_2 \end{pmatrix} \tag{2}$$

where ρ_1 and ρ_2 are scalar terms. β_1 and β_2 have the same dimensions as v_t, and α_1 and α_2 are k-vectors. Thus (ρ_1, ρ_2) are the slope coefficients on r_{t-1}, (β_1, β_2) are the slopes on the deterministic components, and (α_1, α_2) are the slope coefficients on $(\Delta r_{t-1}, \ldots, \Delta r_{t-k})$ in the two regimes.

The threshold effect in Equation (1) has the null hypothesis of $H_0 : \theta_1 = \theta_2$, which is tested using the familiar Wald statistic: $W_T = W_T(\hat{\lambda}) = \sup_{\lambda\in\Lambda} W_T(\lambda)$.4 The stationarity of the process r_t can be established in two ways. The first is when there is a unit root in both regimes (a complete unit root). Here the null hypothesis is of the form $H_0 : \rho_1 = \rho_2 = 0$, which is tested against the unrestricted alternative $\rho_1 \neq 0$ or $\rho_2 \neq 0$ using the Wald statistic. The parameters of ρ_1 and ρ_2 from the Equation (1) will control the regime-dependent unit root process of the RER. If $\rho_1 = \rho_2 = 0$ holds, the RER has a unit root can be described as a rejection of PPP. This statistic is:

$$R_{2T} = t_1^2 + t_2^2 \tag{3}$$

where t_1 and t_2 are the t ratios for $\hat{\rho}_1$ and $\hat{\rho}_2$ from the ordinary least squares estimation. However, Caner and Hansen (2001) claim that this two-side Wald statistic may have less power than a one-sided version of the test. As a result, they propose the following one-sided Wald statistic as follows:

[3] According to Andrews (1993), this division provides the optimal trade-off between various relevant factors, which include the power of the test and the ability of the test to detect the presence of a threshold effect.

[4] $W_T = W_T(\hat{\lambda}) = \sup_{\lambda\in\Lambda} W_T(\lambda) = T\left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}^2(\lambda)} - 1\right)$, where $\hat{\sigma}_0^2$ and $\hat{\sigma}^2$ are residual variances from least squares estimation of the null linear and TAR models, respectively.

$$R_{1T} = t_1^2 I_{\{\hat{\rho}_1<0\}} + t_2^2 I_{\{\hat{\rho}_2<0\}} \tag{4}$$

To distinguish between the stationary case given as H_1 and the partial unit root case given as H_2, Caner and Hansen (2001) suggest using individual t_1 and t_2 statistics. If only one of $-t_1$ and $-t_2$ is statistically significant, this will be consistent with the partial unit root case. This means RER behaves like a "nonstationary process" in one regime; but exhibits a "stationary process" in the other regime, vice versa. Caner and Hansen (2001) show that both tests R_{1T} and R_{2T} will have power against both alternatives.5 To obtain maximum power form these tests, critical values are generated using bootstrap simulations with 10,000 replications, as suggested by Caner and Hansen (2001).

B. EMPIRICAL RESULTS

For the sake of comparison, we also incorporate the ADF, PP (Phillips and Perron, 1988), and KPSS (Kwiatkowski et al., 1992) tests into our study. The results of these three conventional unit root tests -- ADF, PP, and the KPSS tests, as shown in Table 1, indicate that the RERs are non-stationary for Germany relative to its major trading partners. As stated earlier, there is a growing consensus that the RER exhibits nonlinearities, and consequently, conventional unit root tests such as the ADF test, have low power in detecting the mean reversion of exchange rate. A number of studies have also provided empirical evidence on the nonlinear adjustment of exchange rate. Therefore, we proceed to test the RER by using Caner and Hansen's (2001) nonlinear TAR unit root tests.

[5] As stated by Caner and Hansen (2001) that R_{1T} has more power than that of R_{2T}, here we only report the results of R_{1T} in our study.

TABLE 1. UNIVARIATE UNIT ROOT TESTS

	Level			1st difference		
	ADF	PP	KPSS	ADF	PP	KPSS
Canada	-1.563(0)	-1.680(2)	0.914[11]***	-13.53(0)***	-13.53(2)***	0.113[0]
France	-1.592(3)	-1.911(9)	1.279[11]***	-16.60(0)***	-18.74(17)***	0.055[19]
Italy	-2.088(8)	-1.416(6)	1.287[11]***	-4.91(10)***	-12.87(5)***	0.052[5]
Japan	-1.529(0)	-1.669(3)	0.956[11]***	-13.09(0)***	-13.12(5)***	0.073[4]
UK	-1.308(0)	-1.373(4)	0.582[11]**	-13.48(0)***	-13.49(3)***	0.306[3]

Note: *** indicates significance at the 1% level. The number in parenthesis indicates the lag order selected based on the recursive t-statistic, as suggested by Perron (1989). The number in the brackets indicates the truncation for the Bartlett Kernel, as suggested by the Newey-West test (1994).
Source: Authors calculation

First, we use the Wald test W_T to examine whether or not we can reject the linear autoregressive model in favor of a threshold model. The results of the Wald test along with the bootstrap critical values generated at conventional levels of significance are reported in Table 2. The bootstrap p-value for threshold variables of the form $Z_{t-1} = r_{t-1} - r_{t-m}$ for delay parameters m varies from 1 to 12. Since the parameters m is generally unknown, there is no reason to think the optimal delay parameter will be the same across countries. To circumvent this, Caner and Hansen (2001) suggest making m endogenous by selecting the least squares estimate of m that minimizes the residual variance. This amounts to selecting m at the value that maximizes the W_T statistic. We find that the W_T statistic is maximized for Italy/Germany when $m = 3$, for both Canada/Germany and Japan/Germany when $m = 5$, for France/Germany when $m = 7$, and for UK/Germany when $m = 10$. Taken together, these results imply strong statistical evidence against the null hypothesis of linearity at least at the 5% significance level for all the cases indicating that simple linear models are inappropriate and the TAR model is our preference.

TABLE 2. THRESHOLD UNIT ROOT TEST

Countries	Wald Statistic	Bootstrap p-value	m	Threshold $\hat{\lambda}$	Number of observations in Regime 1 and its percentage
Canada	68.993	0.003***	5	-0.0366	59(32.24%)
France	145.713	0.000***	7	0.0049	155(84.69%)
Italy	541.249	0.000***	3	0.0019	155(84.69%)
Japan	62.302	0.048**	5	-0.0378	41(22.40%)
UK	83.979	0.032**	10	0.0348	129(70.49%)

Note: ** and *** indicate significance at the 5% and 1% level, respectively. Following much of the existing empirical literature on monthly real exchange rates and PPP, we set a maximum lag of 12 and base all our bootstrap tests on 10,000 replications. Most of the statistics are significant, which supports the presence of threshold effects.

Source: Authors calculation

Next, we explore the threshold unit root properties of RER based on the R_{1T} statistic for each delay parameter m, ranging from 1 to 12, paying particular attention to the results obtained for our preferred model. The R_{1T} test results, together with the bootstrap critical value at the conventional levels of significance and the bootstrap p-value, are reported in Table 3. We are able to reject the unit root null hypothesis for all of cases at the 10% significance level, with the exception of Canada/Germany. Taken together our results provide strong support for PPP for most of the Germany's trading partner countries and point that the RERs of these countries are non-linear stationary, implying that deviations of exchange rate is mean reverting towards the PPP equilibrium. As we mentioned earlier that trade barriers, transaction costs, as well as interventions in the exchange market, could be behind this nonlinear behavior. The one-sided test statistic of R_{1T}, however, is not able to distinguish the complete and partial unit root in real exchange rate, we examine further evidence on the unit root hypothesis (partial unit root) by examining the individual t statistics, t_1 and t_2. The results are reported in Table 4. These statistics are associated with delay parameters chosen from the linearity test, and their p-values are also obtained using 10,000 bootstrap. This means that the unit root null is strongly rejected in favor of $\rho_1 < 0$ in the outer regime. In contrast, the p-values for the t_2 statistic indicate that we are unable to reject the unit root null hypothesis in the inner regime for all countries at any conventional level of significance. Moreover, the magnitudes of ρ_1 and ρ_2 suggest that in the outer regime, the real exchange rate displays mean reversion, while in the inner regime it is highly persistent and is best described as a random walk. Note that for France, Italy, Japan and UK, the p-values of t_1 for these four countries are less than 10%, we can find while a partial unit root exists for the PPP may be characterized as a stationary TAR.

Also, with the exception of the Canada/Germany, the statistics for t_1 are smaller than the critical value at the 10% level of significance, and this leads us to the conclusion that RERs in most of the Germany's trading partner countries are nonlinear process that are not characterized by a unit root process, consistent with the PPP. Therefore, it is possible to claim that deviations in the short-run form the PPP are not prolonged for Germany and there are some forces which are capable of bringing the exchange rate back to its PPP values in the long-run.

TABLE 3. ONE SIDED UNIT ROOT TESTS

Countries	m	R_{1T}	Bootstrap critical values			Bootstrap p-value
			10%	5%	1%	
Canada	5	4.918	9.775	11.886	16.972	0.371
France	7	11.074	10.893	14.139	22.235	0.091*
Italy	3	14.226	12.496	17.378	32.518	0.075*
Japan	5	13.075	9.266	11.653	16.743	0.033**
UK	10	11.558	10.659	13.111	18.405	0.075*

Note: * and ** indicate significance at the 10% and 5% level, respectively.

Source: Authors calculation

TABLE 4. PARTIAL UNIT ROOT RESULTS

Countries	m	t_1^2	Bootstrap p-value	t_2^2	Bootstrap p-value
Canada	5	0.916	0.555	2.019	0.187
France	7	3.659	0.091*	-0.951	0.941
Italy	3	3.771	0.033**	-4.306	0.994
Japan	5	3.432	0.016**	1.137	0.474
UK	10	3.218	0.039**	1.094	0.500

Note: * and ** indicate significance at the 10% and 5% level, respectively.

Source: Authors calculation

The major policy implication that emerges from our study is that the government in Germany can use PPP to determine the equilibrium exchange rate and the unbounded gains from arbitrage in traded good are not possible in Germany.

The findings of PPP hold between Germany and its major trading partners implied that the Germany economy is integrated with these countries. Hence, these had important policy implication on cross-border agreement for international trade and investment with these countries. Given the goods and services markets appeared quite integrated, future liberalization will be likely pronounced in financial markets. If we envision this process of integration continuing, in particular in the European region, and to the extent that this process requires even more political engagement, we believe the prospects for cooperation along a variety of dimensions are good.

IV. CONCLUSIONS

This study applies the TAR model proposed by Caner and Hansen (2001) to examine both linearity and stationarity of Germany's real exchange rate vis-à-vis her 5 trading partner countries. Two main conclusions are drawn. Firstly, the empirical results indicate that Germany's real exchange is a nonlinear process. Secondly, a unit root in real exchange rate was rejected for most of Germany's trading partner countries while a partial unit root exists for the PPP under study. This provides strong support for purchasing power parity for Germany relative to their major trading partner countries.

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REALNA TEČAJNA STOPA S NELINEARNIM EFEKTOM PRAGA

SAŽETAK

Ovaj rad koristi TAR autoregresijski model kojeg su osmislili Caner i Hansen (2001) za istraživanje linearnosti i stacionarnosti njemačke realne tečajne stope u odnosu na njenih 5 zemalja trgovinskih partnera. Donose se dva glavna zaključka. Prvo, empirijski rezultati ukazuju na to da je njemački realni tečaj nelinearni proces. Drugo, jedinični korijen u realnoj tečajnoj stopi je odbačen za većinu istraženih slučajeva. Takav rezultat snažno podupire paritet kupovne moći za Njemačku u odnosu na njene glavne trgovinske partnere

Ključne riječi: test jediničnog korijena nelinearnog efekta praga, linearnost i stacionarnost, paritet kupovne moći,