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Analytical and Numerical Computation of Added Mass in Ship Vibration Analysis

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Comparison between analytical and numerical determination of the added hydrodynamic mass in ship vibration analysis is performed. Analytical analysis was done on a semi-submerged cylinder of a circular cross section. Numerical calculations were carried out by the boundary element method (BEM) implemented in NX Nastran finite element (FE) software. Fairly good agreement was found between the two methods for added mass calculation. Also, analysis of the real ship vibrations is performed. Free vertical vibrations of a 9200TEU containership are analyzed where the hull was modelled by the beam finite elements (FE), while the shell plating was modelled by the plate FE without stiffness to define boundary elements of the wetted surface. Such calculated results are compared with the ones obtained by software DYANA which uses Timoshenko's beam FE and in which the added mass is determined by traditional analytically-based method. Comparative analysis shows some moderate differences in the calculation of added mass between the two approaches, indicating the need for further research on this topic.

Keywords: *added mass, boundary element method, finite element method, ship vibrations*

Analitički i numerički proračun dodatne mase kod analize vibracije broda

Izvorni znanstveni rad

Obavljena je usporedba analitičkog i numeričkog određivanja dodatne mase okolne vode kod vibracija broda. Teorijska analiza provedena je za poluuronjeni cilindar kružnoga poprečnog presjeka. Numerički proračun obavljen je metodom rubnih elemenata pomoću programa za primjenu konačnih elemenata NX Nastran. Dodatna hidrodinamička masa određena dvjema metodama pokazuje dobro slaganje. Prikazana je također analiza vibracija realne brodske konstrukcije. Analizirane su vertikalne vibracije kontejnerskog broda 9200TEU, pri čemu je trup modeliran grednim konačnim elementima, a izvanjska oplata pločastim elementima bez krutosti kako bi se definirali rubni elementi oplakane površine. Tako dobiveni rezultati uspoređeni su s onima dobivenim pomoću programa DYANA, koji koristi konačne elemente Timošenkove grede i dodatnu masu određenu tradicionalnom metodom zasnovanom na analitičkim rješenjima. Usporedna analiza pokazala je određena odstupanja u izračunu dodatne hidrodinamičke mase dvjema metodama. Dalja istraživanja potrebna su da bi se razjasnili razlozi ovih odstupanja.

Ključne riječi: *dodatna masa, metoda konačnih elemenata, metoda rubnih elemenata, vibracije broda*

1 Introduction

Accurate evaluation of ship's vibration behaviour, natural frequencies and natural vibration modes, is important to avoid the resonance phenomenon that can be caused by periodic forces of propulsion engine, propeller and waves. Ship vibrations are greatly influenced by the inertial forces of surrounding fluid. These forces are proportional to the acceleration of wetted surface, and can consequently be represented by fictive fluid mass that vibrates along with the ship hull. This fictive mass is called "hydrodynamic added mass", which in addition to the ship structure mass represents "virtual mass". Added mass is usually significant, which in some cases reaches ship's displacement mass, and therefore must not be omitted in vibration analysis.

Ship structure is numerically most often modelled with the finite element method (FEM). Added mass is either calculated separately and explicitly included into the analysis, or implicitly coupled using numerical methods for solving problems of fluid-structure interaction. Traditionally, added mass is in ship vibration analysis determined independently of the structural model according to some available analytical solutions [1]. That procedure of added mass computation consists of two steps. Firstly, the added mass of 2D cross-sections along the ship hull is determined, and secondly, the mass is corrected with the so called J factor, which takes into account 3D flow effects. The J factor depends somewhat on the wetted surface shape, but mostly on the global vibration mode. The added mass is then added to the structural model to analyze wet vibrations of the ship hull.

Such procedure for determining the added mass is well known in shipbuilding industry, and proven to give good results for conventional ships that have pronounced strength deck and defined shapes of hull girder vibration. For ships such as passenger and RO-RO ships, the above mentioned method cannot be directly applied since the global vibration modes of the hull are not well defined and the J factor cannot be determined reliably. For unconventional ships, whose form is different from most common ship forms, the same problem arises. An example of such ships are dredgers, which amongst mentioned problems work in shallow waters for which the added mass calculation using standard procedures is rather unreliable. Vibration analysis for these ships represents tough challenge in the design phase, considering their powerful machinery with many vibration exciters [2].

An alternative to the described traditional method is the application of numerical methods for solving problems of fluid-structure interaction such as the boundary element method (BEM) or Reynolds-averaged Navier–Stokes (RANS) method [3]. Surrounding fluid can also be modelled with acoustic finite elements, but it takes huge amount of 3D finite elements to represent the fluid [4]. Therefore, it is more convenient to combine FEM for the ship structure with BEM for the fluid [5][6][7]. The advantages of such direct numerical methods are that they are not constrained with conventional ship hull shapes and that methods implicitly include 3D effects. There are different approaches of solving the added mass with BEM, such as the projection approach [5], multi-pole method [6], panel method [8], etc. Numerical methods are advancing [3][9][10], and nowadays many commercial softwares provide solution to a wide spectrum of hydroelastic engineering problems including vibrations of fully or semi-immersed elastic bodies.

However, numerical methods are not without limitations and thus should be verified and validated before their practical usage. The purpose of the present study is to compare various computational methods of the added mass of a vibrating ship, with the main aim to test practical applicability of the BEM. Although the theory is well known and implemented in many commercial FE codes, the studies demonstrating practical applicability of the BEM on hull girder vibration analysis are lacking in professional literature.

The paper begins with a short theoretical review of the added mass concept and analytical methods available for the computation of added mass. After that, a brief review of the BEM is provided. First, a comparative analysis is done on a simple example with the known analytical solution. For that purpose, vertical transverse vibration analysis of a half-immersed circular cylinder was made. Analytically, the cylinder was analyzed via Timoshenko beam theory for wet vibrations in the deep water [11]. The cylinder and its surrounding fluid were also modelled with FEM and BEM respectively, and analyzed using NX Nastran solver. The results of the numerical calculations were compared with the theoretical ones. In the second example, the free vibration analysis of the real ship was made. Vertical vibrations of a 9200TEU containership were analyzed where the hull was modelled by the beam finite elements, while the shell plating was modelled by the plate finite elements to define boundary elements of the wetted surface. Such calculated results are compared with the ones obtained by software DYANA which uses Timoshenko's beam finite elements and in which the added mass is determined by traditional method [1][12]. Finally, corresponding conclusions about different methods for added mass calculation are drawn.

2 Added mass

Whenever acceleration is imposed on a fluid flow either by acceleration of a body or by acceleration externally imposed on the fluid, additional fluid forces will act on the surfaces in contact with the fluid. The simplest engineering description of the phenomenon of added mass is that it determines the necessary work done to change the kinetic energy associated with the motion of the fluid [13]. Any motion of the fluid which occurs when a body moves through the fluid implies positive amount of kinetic energy, T, associated with the fluid motions which can be represented by

$$T = \frac{1}{2}\rho\int_{\Omega} v^2 d\Omega \tag{1}$$

where ρ is the fluid density, Ω is the entire domain of the fluid, and v is the fluid velocity. For a body steadily translating with velocity U, through a fluid at rest, T could conveniently be expressed as

$$T = \frac{1}{2}\rho I U^2 \tag{2}$$

where the integral I would be simple invariant number. If a body accelerates with dU/dt, the fluid kinetic energy will also change and additional work by the body must be done. The rate of additional work required is the rate of change of T with respect to time, dT/dt, which the body experiences as additional drag, F, such that $FU = -dT/dt$. If the flow pattern is not changing such that I remains constant, it follows that F is simply given as

$$F = -\frac{1}{U}\frac{dT}{dt} = -\rho I\frac{dU}{dt} \tag{3}$$

It is convenient to visualize the mass of a fluid, ρI, as an added mass of the fluid which is being accelerated with the body. Of course, there is no such identifiable mass; rather all of the fluid is accelerating to some degree such that total kinetic energy of the fluid is increasing.

2.1 Analytical computation of added mass

In the added mass computation, assumptions about linear potential flow and infinite excitation frequency are adopted. The latter assumption is equivalent to the assumption of no free surface effects, based on the fact that the natural frequencies of the hull girder are usually rather high. The added mass problem is usually solved in two steps. Firstly, the added mass for infinite frequency is determined for 2D sections along the hull. Then a relationship between 2D and 3D flow is established through reduction J factor, which changes considerably with the vibration mode. There are several versions of the described method for calculating added mass, differing in details how added mass and J factor are calculated [1][16]. In the present paper, only vertical vibrations are considered. However, the similar approach is applicable also for horizontal and torsional vibration [1].

Since the added mass for infinite frequency of various cross sections along a real ship is different, a relationship between added mass and cross section has to be established. This may be done either by source techniques either by conformal mapping [15]. There is no general connection between added mass and displacement, but coincidentally added mass for infinite frequency of a heaving semi-submerged circular cylinder is equal to displaced mass of water [15]. Based on that fact, the following general form of the longitudinal distribution function of the added mass of water per unit length by a vertical motion is proposed:

$$\lambda_{33}^{\infty}(x) = \frac{\pi}{8}\rho B(x)^2 C(x) \tag{4}$$

where $B(x)$ is the section beam, and $C(x)$ is the section two-dimensional added mass coefficient which is the most often solved by so-called Lewis form conformal mapping of ship sections. Section 2D added mass coefficient is available in many references dealing with ship vibration or seakeeping, e.g. [16][17].

In the second step, it is necessary to correct 2D infinite frequency added mass for three-dimensional water flow around hull girder, which leads to modified Eq.(4):

$$\lambda_{33,n}(x) = \frac{\pi}{8}\rho B(x)^2 C(x)\, J_n \tag{5}$$

Index n in Eq.(5) indicates that added mass is dependant on the mode of vibration through reduction J factor. In order to compute reduction J factor, solution for complete three-dimensional fluid flow around vibrating floating body is required. Theoretically, J changes along the vessel, as the added mass is zero at the ends of the ship. However, the effect of such variation is not significant and J is assumed to be a constant along the hull girder, representing ratio of the kinetic energy of the true three-dimensional flow around the hull and the kinetic energy obtained by integration of the two-dimensional flow along the hull girder [17]. A significant number of analyses were made in order to determine J factor for bodies of different shapes and different modes of vibration [18]. Among them, two analytically-based approaches are in practical application nowadays. In the first approach, which is suitable for ships with fine lines, reduction factors are based upon three-dimensional solutions for ellipsoidal bodies with the same overall dimensions as the ship. A good, partly empirical fit to these solutions reads [17]:

$$J_n = 1.02 - 3\left(1.2 - \frac{1}{n}\right)\frac{B}{L} \tag{6}$$

where B is the water line breadth amidships and where L is the length of the ship. The formula only applies to vertical vibrations with n=2-5.

In the second approach, Kumai proposed a formula of the J factor for a half-submerged circular cylinder of a finite length [19]:

$$J_n = \frac{16}{\pi^2}\sum_m^{\infty}\left(\frac{m}{m^2 - n^2}\right)^2\left[1 + \frac{R\pi m}{L}\frac{K_0(R\pi m/L)}{K_1(R\pi m/L)}\right]^{-1} \tag{7}$$

where K_0 and K_1 are modified Bessel functions of the 2nd kind: 0th and 1st order respectively, while m = 1,3,5,... if number of vibration nodes n is even, else m = 2,4,6,... The formula is easily extended to cross-sections defined with the Lewis transformations [17].

In Kumai's approach, assumption about global vibration mode shapes along the cylinder is adopted to derive analytical solution (7) for the reduction factor. Madsen in [20] adopted more realistic more shapes and obtained lower J factors, being in slightly better agreement with the measurements.

2.2 Numerical computation of added mass

Numerical methods for calculation of added mass are available nowadays and implemented in commercial FE packages. Added mass may be modelled by acoustic finite elements, as implemented for example in FE software ABAQUS (http://www.simulia.com/). In the present paper the boundary element method, implemented in the FE software NX Nastran [21], is considered. Both methods allow direct computation of wetted natural frequencies. In this section, a brief overview of the well known theory of the BEM for the computation of added mass is given.

To obtain fluid forces on the structure, the relationship between the fluid pressure field and the interface acceleration can be defined via potential flow theory, in which the velocity potential function, $\phi(\vec{x})$, satisfies Laplace equation, $\nabla^2\phi = 0$ (Figure 1). The boundary conditions are $\nabla\phi \cdot \vec{n} = d\vec{\delta}/dt \cdot \vec{n}$ on the solid surface, $\phi \to 0$ at infinity and on the free surface, where $\vec{n}$ is the surface normal and $\vec{\delta}$ is the surface displacement.

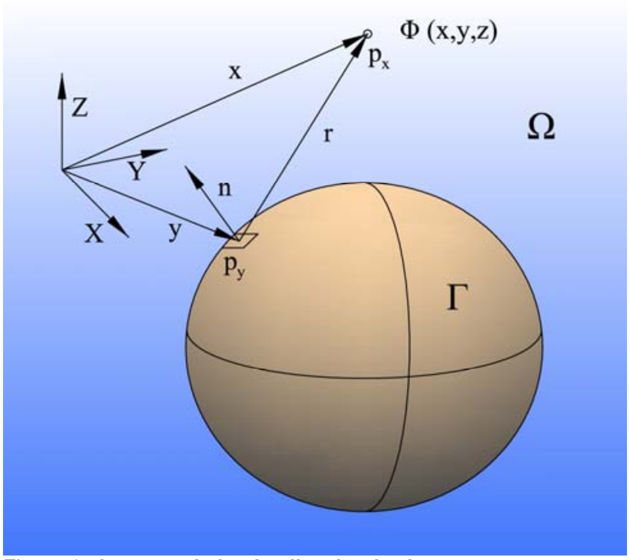

Figure 1 **Immersed elastic vibrating body**
Slika 1 **Vibrirajuće elastično tijelo uronjeno u fluid**

In the surface panel method [8], function $\phi(\vec{x})$ is defined by a distribution of singularities, i.e. sources of constant strength on panels of the wetted surface. Imaging sources of opposite sign are placed across the plane of the free surface to define a zero potential free surface. Hence free surface effects are neglected because the vibration frequencies of interest are often above the range in which free surface effects are significant. Physical interpretation is that at a very high frequency the body cannot generate any free surface waves [15]. The field potential is written in the form

$$\begin{aligned}\phi(\vec{x}) &= \int_{\Gamma} G(\vec{x},\vec{y})\sigma(\vec{y})\,d\Gamma(\vec{y}) \\ &= \sum_{j=1}^{N}\sigma_j \int_{\Gamma} G(\vec{x},\vec{y}_j)\,d\Gamma(\vec{y}_j)\end{aligned} \tag{8}$$

where $\vec{y}$ is the coordinate vector of a point on a panel as shown in Figure 1, σ is the source strength, and *G* is the fundamental solution of the Laplacian, which is for a source of strength of 4π given by

$$G(\vec{x},\vec{y}) = \frac{1}{|\vec{x}-\vec{y}|} \tag{9}$$

Since the gradient of the velocity potential, $\nabla\phi$, is forced to satisfy the boundary conditions at a number of control points (centroids of panels), the evaluation of Eq. (8) leads to the system of equations

$$\{\phi\} = [H]\{\sigma\} \tag{10}$$

$$\{U\} = [L]\{\sigma\} \tag{11}$$

where [*H*] and [*L*] are coefficient matrices that relate source strength to control point potential and control point fluid normal velocities respectively. The vector {*U*} represents fluid velocities normal to the panels which can be represented using the transformation matrix, [*T*], that contains factored direction cosines between the panel normal and global axes, $\{U\} = [T]\{\dot{\delta}\}$. For known velocities of panels, panel source strengths vector, $\{\sigma\}$, can be obtained from the Eq. (11). Now the velocity potential vector, $\{\phi\}$, can be obtained from Eq. (10).

The pressure field in fluid can be determined from the velocity potential via the Bernoulli equation $p = \rho \cdot d\phi/dt$. The pressure is integrated over the structure interface by simple product with the diagonal matrix of panel areas, [*A*], to give the control point forces

$$\{F\} = \rho[A]\{\dot{\phi}\} \tag{12}$$

Added fluid mass matrix, $[M_A]$, which can be represented in general form

$$\{F\} = [M_A]\{\ddot{\delta}\} \tag{13}$$

can now be determined from the known vector of control points forces, {*F*}, and panels accelerations, $\{\ddot{\delta}\}$.

Neglecting the damping, a structural equilibrium equation in matrix form is given by

$$([M_S]+[M_A])\{\ddot{\delta}\} + [K]\{\delta\} = 0 \tag{14}$$

where $[M_S]$ is the structural mass matrix and [*K*] is the structural stiffness matrix. Assuming harmonic solution of the problem by introducing natural frequencies of the system, $\{\delta\} = \{d\}\sin\omega t$, Eq. (14) can be reduced to

$$\left([K] - \omega^2([M_S]+[M_A])\right)\{d\} = 0 \tag{15}$$

which is known as eigenvalues problem. Non-trivial solution of the problem is obtained by solving the following equation

$$\det\left([K] - \omega^2([M_S]+[M_A])\right) = 0 \tag{16}$$

using for example the Lanczos, transformation or iteration algorithms [21].

3 Comparison of analytical and numerical methods

Verification of the BEM for obtaining the added mass is performed on a semi-submerged steel circular pontoon having the characteristics given in Table 1. The 3D FE model of a cylinder was completely made of plate finite elements, which is shown in Figure 2. The mesh size was about 100x100 mm, so the model consists of 6584 shell finite elements. Although the analytical representation of the pontoon is a hollow cylinder that has no caps, they had to be modelled with the material of negligible density to enclose and stiffen the floating vibrating structure. Numerical analysis was done with NX Nastran FE software [21].

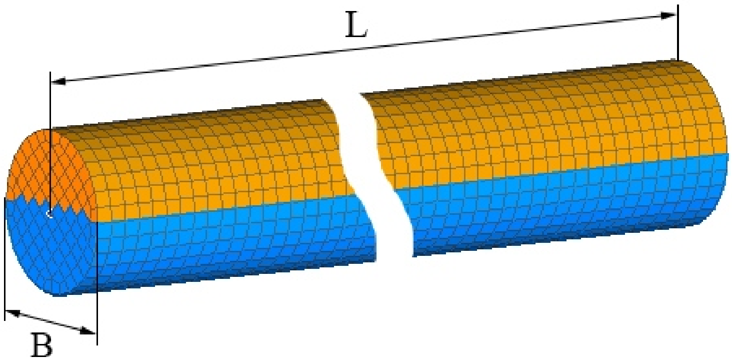

Figure 2 **3D FE model of the pontoon with blue BE of wetted surface**
Slika 2 **3D FE model pontona s plavim rubnim elementima oplakane površine**

Table 1 **Pontoon characteristics**
Tablica 1 **Karakteristike pontona**

Length	L = 20.0 m
Beam	B = 1.0 m
Draft	T = 0.5 m
Material density	ρ_s = 7800 kg/m^3
Young's modulus	E = 200×10^9 N/m^2
Poisson's coefficient	ν = 0.26
Plating thickness	t = 16.0 mm
Fluid density	ρ_w = 1000 kg/m^3

Added mass values for vertical cylinder heaving oscillations in the deep water are given in Table 2. The rigid body oscillations of the cylinder are simulated by suspending the cylinder on a spring with stiffness equal to the restoring force. The value of the added mass agrees reasonably with the analytical solution.

Table 2 **Added mass of heaving pontoon, in kg**
Tablica 2 **Dodatna masa pontona pri poniranju, u kg**

Analytic	BEM	% Difference
7854	7615	-3.0

Natural frequencies of vertical dry vibrations were checked and were found to be identical with the analytical results. Natural frequencies of vertical wet vibrations of the pontoon in the deep water are shown in Table 3, while mode shapes are presented in Figure 3. The values obtained numerically are somewhat higher than the analytical computation. Analytical results are obtained by Timoshenko beam theory [11], while the added mass is computed using the J factor from Eq. (7).

Table 3 **Natural frequencies of pontoon for wet vibrations, in Hz**
Tablica 3 **Prirodne frekvencije pontona za mokre vibracije, u Hz**

Mode	Analytic	3D FEM + BEM	% Difference
1	11.471	11.561	+0.78
2	30.842	31.185	+1.11
3	58.376	59.210	+1.43
4	92.427	93.939	+1.64

Through the ratio of dry and wet natural frequencies, one can calculate the added mass value for a specific vibration mode as

$$m_{33,n} = \Delta\left[\left(\frac{\omega_{n,dry}}{\omega_{n,wet}}\right)^2 - 1\right] \qquad (17)$$

where Δ is the ship displacement. Using Eq. (17), added masses are extracted and comparison of results is given in Table 4.

Table 4 **Added mass of pontoon for wet vibrations, in kg**
Tablica 4 **Dodatna masa pontona za mokre vibracije, u kg**

Mode	Analytic	3D FEM + BEM	% Difference
1	6807	6738	-1.01
2	6609	6459	-2.27
3	6378	6156	-3.48
4	6134	5868	-4.33

Small differences in added mass and natural frequencies in Tables 2-4 may be attributed to small inaccuracies in the *J* factor calculated by Eq. (7). Namely, it was shown in [17] that *J* reduction factors obtained by simplified analytical approach slightly overestimate measured values. Increasing discrepancy of analytical and numerical results by the vibration mode may also be noticed.

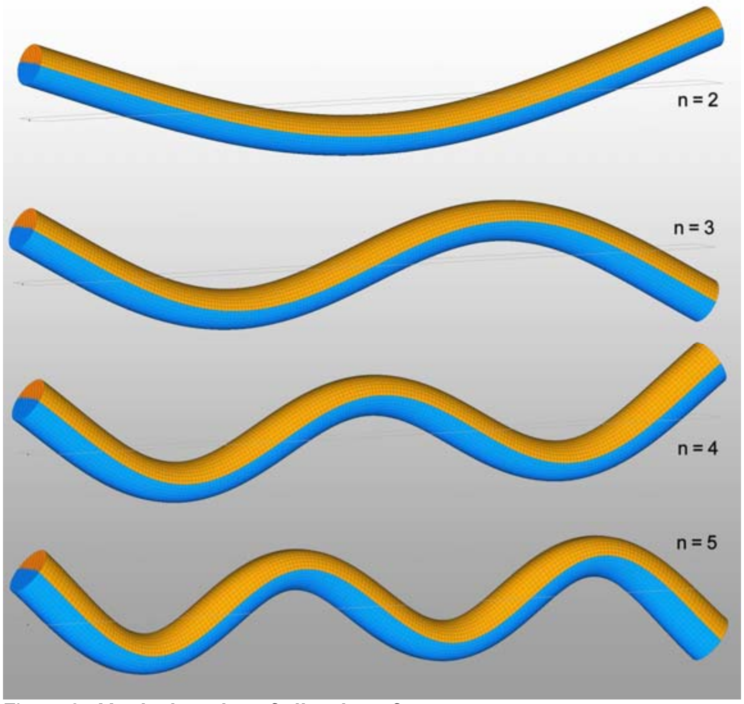

Figure 3 **Vertical modes of vibration of pontoon**
Slika 3 **Vertikalni oblici vibriranja pontona**

4 Practical Application

Numerical analysis of vibrations of a real ship was performed. Vertical vibrations of a 9200 TEU containership were analyzed using a 1D FEM + BEM model. Hull stiffness was modelled with beam finite elements whose nodes were rigidly connected to the corresponding wetted shell plating nodes. The plating was modelled using shell finite elements to define wetted boundary elements. Plate elements had no stiffness, as their purpose was only to transfer fluid inertial loads to the beam elements. The model of the 349 m long containership is shown in Figure 4. The results calculated using NX Nastran solver are compared with the ones obtained with software DYANA which uses Timoshenko's beam finite elements and in which the added mass is determined by the method described in section 2.1 [12]. Natural frequencies for vertical dry and wet vibrations of the ship are presented in Tables 5 and 6 respectively, while dry natural modes are shown in Figure 5.

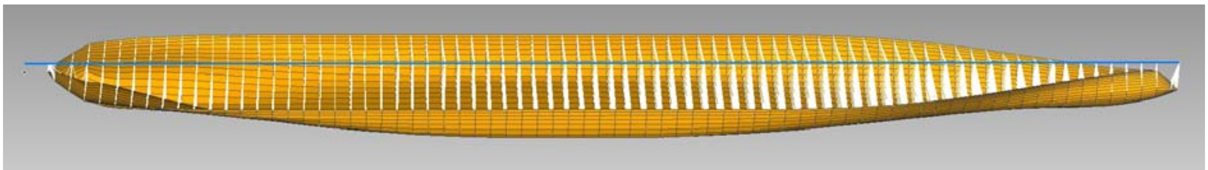

Figure 4 **FE beam model of ship with BE of wetted surface**
Slika 4 **Gredni model broda s rubnim elementima oplakane površine**

Table 5 **Natural frequencies of ship for dry vibrations, in Hz**
Tablica 5 **Prirodne frekvencije broda za suhe vibracije, u Hz**

Mode	DYANA	1D FEM	% Difference
1	0.681	0.681	0.00
2	1.446	1.457	0.76
3	2.235	2.284	2.19
4	2.940	3.096	5.31

Table 6 **Natural frequencies of ship for wet vibrations, in Hz**
Tablica 6 **Prirodne frekvencije broda za mokre vibracije, u Hz**

Mode	DYANA	1D FEM + BEM	% Difference	% Total difference
1	0.519	0.503	-3.08	-3.08
2	1.103	1.062	-3.72	-4.48
3	1.728	1.666	-3.59	-5.78
4	2.367	2.289	-3.30	-8.60

As may be seen from Table 5, there are some discrepancies in dry natural frequencies as a consequence of details of the stiffness and mass distribution of two different beam models. Such discrepancies are considered to be within acceptable limits, as they are lower than discrepancies obtained in benchmarking study performed by ISSC Committee for Dynamic Response [3]. Comparison of wet natural frequencies, presented in Table 6, indicates differences between two methods of about 3-4% for all vibration modes. However, the actual difference in natural frequencies caused by the added mass calculation is larger, as discrepancies in dry natural frequencies should also be taken into account. This is presented in the last column of Table 6. The BEM directly takes into consideration 3D effects of ship aft and fore parts, which are of more complex form than the simplified analytical method can account for, and consequently the BEM is expected to give more accurate values of the added mass. However, more comparisons are to be done in order to further verify that statement. It may also be concluded from Table 6 that discrepancies in added mass are increasing by the vibration mode. The similar conclusion is drawn also for the cylinder in Section 3 of the paper.

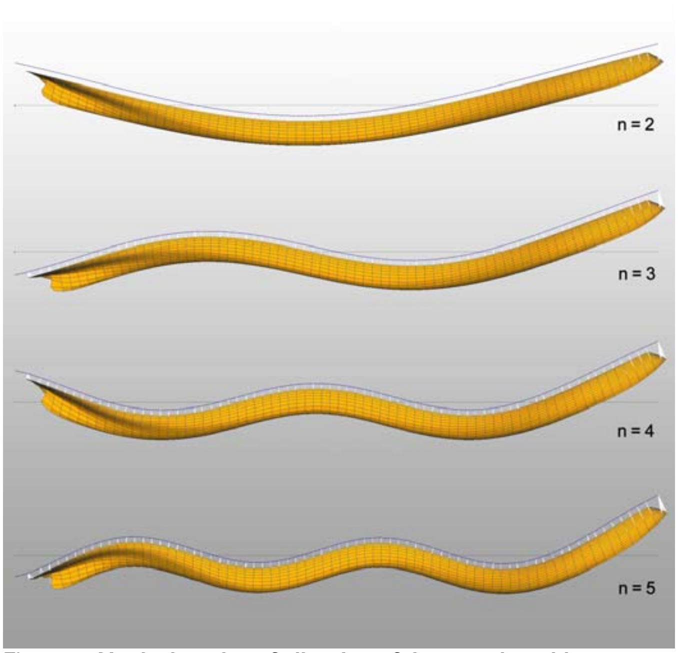

Figure 5 **Vertical modes of vibration of the containership**
Slika 5 **Vertikalni oblici vibriranja kontejnerskog broda**

5 Conclusions

Analytical and numerical analysis of the added hydrodynamic mass was made for the needs of the ship vibrations analysis. Numerical solving of the problem was done with combination of FEM for the structure, and BEM for the fluid. NX Nastran FEM solver was employed for that purpose.

The comparison was performed on a half-immersed cylinder, for which analytical solution is available. The results of the numerical model were in fairly good agreement with the results obtained analytically.

An example of the practical application of the BEM for obtaining the added mass of the real ship structure was shown. Free vertical vibrations of a 9200 TEU containership were analyzed using a 1D FEM + BEM model and the results were compared with those obtained by software DYANA. Added mass calculations showed moderate differences, probably arising from the fact that the BEM more consistently takes into account 3D effects of ship aft and fore parts, compared to the simplified analytically-based method. However, more comparisons are required to get general conclusions about relative accuracy of the BEM and the traditional method for added mass calculation.

There are at least two great advantages of the application of numerical methods for modelling of the added mass compared to analytical methods. Firstly, uncertainties related to the calculation of the J factor are avoided. Secondly, all types of vibrations are considered together, facilitating appropriate modelling of the added mass for coupled horizontal and torsional vibrations that is a difficult task with the analytical method.

The drawback of using the BEM for the calculation of the added mass, which is noticed during this study, is the need of substantial computing resources in order to acquire numerical solutions. Therefore, special care should be paid to the computational cost when analyzing vibration of complex structures with a large number of degrees of freedom. Further research of the problem is necessary to analyze the reasons for that and to improve computational efficiency.

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