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SIMULATION OF RISK BASED ON ENDING ACTIVITIES OF THE DESIGN PLAN USING SPECIAL FUNCTION

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Original scientific paper

In this paper are presented the results of the theoretical - experimental research on the quantification of superimposed flow time of two local - autonomous flows in a network, on the basis of Clark's equations. The computer-based solving of this basic variant of the general flow model through the network was performed using the Monte Carlo methods of numerical simulation supplemented by the frames method. The numerical experiment was carried out using the program tool Mathcad Professional.

Keywords: *mathematical model, simulation, project plan*

Određivanje rizika realizacije aktivnosti u projektnom planu

Izvorni znanstveni članak

U radu se iznose rezultati teorijsko-eksperimentalnih istraživanja kvantifikacije superponiranog protočnog vremena dva lokalno-autonomna toka u mreži na bazi Klarkovih jednadžbi. Računarsko rješavanje ove osnovne varijante općeg modela protoka kroz mrežu izvodi se postupcima numeričke simulacije Monte-Karlo dopunjene metodom okvira. Numerički eksperiment realiziran je putem programa Mathcad Professional.

Ključne riječi: *matematički model, simulacija, projektni plan*

1 Introduction

The host model, for which Clark's equation of the equivalent (resulting) activity consists of the oriented graph, where two activities go parallel, they have the common beginning and run up to a terminal "event". In that sense activities can be local - autonomous, until they are totally realized. The results of Clark's equations in this paper are compared to the results of the Monte Carlo - frame numerical simulation. Both methods, analytical and numerical, are typical for studying different phenomena and processes, based on network models of activity flows, resources, energies and the like. These problems are, as we know from experience [1], of stochastic nature and their solution through analytical methods, without approximation, is often unfeasible. In this way, motivated by the research of Clark [1], Clemen [2], Slyke [12], Dodin [3], Fishman [4,5], Haga and O'keefe [6], Keefer and Bodily [7], Keefer and Verdini, [8], Littlefield and Randolph [11], Lock [13], Vose [14], we were able to define and solve a network model of the activity flows, and we hope that we will contribute to the development of the algorithm for solving general model of critical flows based on ordinal-parallel graph structures. In this analysis, a normal distribution of single activity ends, with average (mean) values characteristics and appropriate time deviations of their realization, is supposed.

2 Flows with critical activities

The unambiguous solution for critical activity flows, and with it for the resulting flow time, using the expecting times of the elementary activity flows, presents one of the most troublesome effects of the application of network planning based on stochastic methods. The stochastic (but also deterministic) activity networks, formed e.g. on the basis of the ADM (Arrow Diagram Method) structure can in some planning cases be very complex. There are examples of network diagrams in e.g. machine-building industry, where the number of activities amounts to several thousands, with several hundreds of identified critical and subcritical flows (paths). From the standpoint of analysis of critical flows [2], a particular problem is the variant with parallel critical flows, which are characterized in that they do not contain any common (unique) activity. The only common thing with these activities is the initial and final event and the same, approximately the same or different values of the realization time of critical and subcritical flows. The final event will be realized, if all critical flows that "run towards it" are realised. In that case, we can rightly raise the question: how great is the certainty (or probability distribution) that the resulting flow will be completed within the planned period of time T_p, taking into consideration that such an activity graph can comprise one, two or a limitless number of critical flows of parallel, ordinal or combined type. To give a correct answer to this question, it is necessary to accurately define an algorithm for the quantification of influence, primarily of critical and subcritical flows, on the forming of the resulting, i.e. superimposing flow rate time.

3 The objective of this paper

The basic aim of this paper is the quantification of the impact critical and subcritical flows have on forming the resulting, superimposed flow time. By solving it, a base for the definition of the probability distribution function, and for noticing the relativities of these flows through the application of the frames method, is being created.

4 Definition of basic time parameters for autonomous critical flows

According to the research [1, 14], superimposing intervals of critical and subcritical flow times and their deviations and reducing them to an equivalent flow can be performed by:

- Analytical methods: with Clark's equations for solving parallel flows, and on the basis of the central limit theorem, for solving ordinal flows and
- Numerical methods: - Monte Carlo - frame-based simulation for parallel - ordinal flows. To illustrate how the above-mentioned fundamental algorithms are applied, we shall take an ADM network with two parallel flows π_1 and π_2 (Fig. 1).
- Fuzzy modelling.

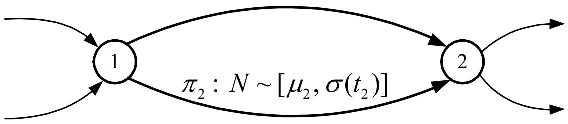

Figure 1 The flow network with two local autonomous activity flows

4.1 The superposed time and the flow variance

In structuring an algorithm for the analytical solving of this critical flow variance, we start from Clark's authentic equations. With these equations the flow parameters are solved as follows: the superimposed flow time $t_{1,2}$ and its variances $\sigma^2(t_{1,2})$ (or $\sigma_{1,2}{}^2$). For a basic oriented graph with two parallel flows, from the initial ① to the terminal ② event (Fig. 2), the corresponding flow time values $t_{1,2}$, are:

- The superimposed flow time:

$$t_{1,2} = t_1 \cdot \Phi(\xi_{1,2}) + t_2 \cdot \Phi(-\xi_{1,2}) + \lambda_{1,2} \cdot \Psi(\xi_{1,2}), \tag{1}$$

where: $\Phi(\xi) = (2\pi)^{-1/2} \cdot \int_{-\infty}^{\xi} \exp(-u^2/2)\mathrm{d}u$ – Laplace integral, $\Psi(\xi) = (2\pi)^{-1/2} \exp(-\xi^2/2)$ – the density function of the centred normal distribution and $\lambda_{1,2} = \sqrt{\sigma_1{}^2 + \sigma_2{}^2}$, i.e. $\xi_{1,2} = \frac{1}{2}(t_1 - t_2)$ -the parameter ($\lambda_{1,2}$) and variables ($\xi_{1,2}$) of Clark's functions. In addition, usually the predicted or mean values of time intervals

$$\overline{t_1} = \mu_1 \text{ and } \overline{t_2} = \mu_2 \tag{2}$$

are used, and according to [1], it follows that:

- The mean superimposed flow time is:

$$\mu_{1,2} = \mu_1 \cdot \Phi(\overline{\xi}_{1,2}) + \mu_2 \cdot \Phi(-\overline{\xi}_{1,2}) + \lambda_{1,2} \cdot \Psi(\overline{\xi}_{1,2}). \tag{3}$$

- The superimposed dispersion is presented by Clark's second equation:

$$\begin{aligned}\sigma_{1,2}^2 &= (\mu_1^2 + \sigma_1^2) \cdot \Phi(\overline{\xi}_{1,2}) + (\mu_2^2 + \sigma_2^2) \cdot \Phi(-\overline{\xi}_{1,2}) + \\ &+ (\mu_1 + \mu_2) \cdot \lambda_{1,2} \cdot \Psi(\overline{\xi}_{1,2}) - \mu_{1,2}{}^2\end{aligned} \tag{4}$$

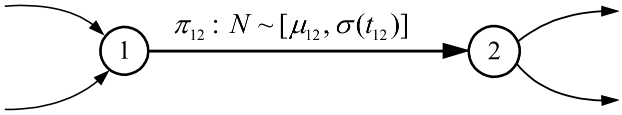

Figure 2 The equivalent – superimposed activity flow

where $\overline{\xi}_{1,2} = \frac{1}{2}(\mu_1 - \mu_2)$ is the parameter difference in the mean expected activity time. With these equations we can describe the characteristics of one equivalent flow instead of the previous two flows (Fig. 2).

4.2 The increase of the superimposed flow time in relation to the critical flow

On the basis of the new superimposed time distribution function $t_{1,2}$ with the characteristics $N \sim [\mu_{1,2}, \sigma_{1,2}]$ one can quantify the increase of time $t_{1,2}$ in relation to the single time t_1 or t_2, dependent on which of them has a critical feature. For an elementary network with the autonomous flows π_1 and π_2, that increase or "the superimposed extract", after a non-complicated calculation yields:

$$\Delta\mu_{1,2} = \lambda_{1,2} \cdot \Psi(\overline{\xi}_{1,2}) + (\mu_2 - \mu_1) \cdot \Phi(-\overline{\xi}_{1,2}). \tag{5}$$

Meanwhile, in the case of a different choice, the following applies:

$$\Delta\mu_{2,1} = \lambda_{2,1} \cdot \Psi(\overline{\xi}_{2,1}) + (\mu_1 - \mu_2) \cdot \Phi(-\overline{\xi}_{2,1}). \tag{6}$$

In addition, these values are by nature always negative, i.e.: $\Delta\mu_{1,2} \geq 0$ and $\Delta\mu_{2,1} \geq 0$.

4.3 Equations for superimposed flow time in the form of a series

If expressions (3) and (4) are used, then the known expressions for the cumulative distribution function can be used in the form of an error function, or an alternative series

$$\Phi(\xi) = (2\pi)^{-1/2} \cdot \int_{-\infty}^{\xi} \exp(-u^2/2)\mathrm{d}u = \frac{1}{2}\operatorname{erfc}(-\xi/\sqrt{2}), \tag{7}$$

where the basic error function can be defined as $\operatorname{erf}(z) = \frac{2}{\sqrt{2}} \sum_{k=0}^{\infty} \frac{(-1)^k \cdot z^{1+2k}}{(1+2k) \cdot k!}$, while its complementary value equals $\operatorname{erfc}(z) = 1 - \frac{2}{\sqrt{2}} \sum_{k=0}^{\infty} \frac{(-1)^k \cdot z^{1+2k}}{(1+2k) \cdot k!}$. It is known that an exponential function in the form of a series is primarily expressed as $\exp(z) = \sum_{k=0}^{\infty} \frac{z^k}{k!}$. Taking into account these complementary relations $\operatorname{erf}(z) + \operatorname{erfc}(z) = 1$

and the previous relations (2), Clark's equation for the superimposed time can be written as:

$$\mu_{1,2} = \frac{1}{2}\cdot\left(\mu_1 \cdot \mathrm{erfc}(-\overline{\xi}_{1,2}/\sqrt{2}) + \mu_2 \cdot \mathrm{erfc}(\overline{\xi}_{1,2}/\sqrt{2})\right) + \\ + \lambda_{12}\cdot\Psi(\overline{\xi}_{1,2}). \tag{8}$$

These expressions can be rearranged through their short forms, in the form of an (alternative) series of superimposed time for two parallel flows of activities,

$$\mu_{1,2} = \frac{\mu_1+\mu_2}{2} + \frac{1}{\sqrt{2\pi}}\sum_{k=0}^{\infty}\frac{(\mu_1-\mu_2)^{2k}}{(-2)^k k!\sqrt{(\sigma_1^2+\sigma_2^2)^{2k-1}}}\cdot \\ \cdot\left(1+\frac{1}{1+2k}\cdot\frac{(\mu_1-\mu_2)^2}{\sigma_1^2+\sigma_2^2}\right) \tag{9}$$

or in a summary form, and taking into account relations (2), we come to the following expression:

$$\mu_{1,2} = \frac{\mu_1+\mu_2}{2} + \frac{\lambda_{12}}{\sqrt{2\pi}}\sum_{k=0}^{\infty}\frac{\overline{\xi}_{1,2}^{\,2k}}{(-2)^k k!}\left(1+\frac{\overline{\xi}_{1,2}^{\,2}}{1+2k}\right). \tag{10}$$

4.4 Superimposed variance equation in the form of a series

In a similar way, by substitution we can obtain an expression for the superimposed variance in the form of an alternative series

$$\upsilon_{1,2} = \frac{1}{2}(\mu_1^2+\sigma_1^2+\mu_2^2+\sigma_2^2) - \mu_{1,2}^{\;2} + \\ + \frac{1}{\sqrt{2\pi}}\sum_{k=0}^{\infty}\frac{(\mu_1-\mu_2)^{2k+1}}{(-2)^k k!\sqrt{(\sigma_1^2+\sigma_2^2)^{2k-1}}}\cdot \\ \cdot\left(\frac{\mu_1+\mu_2}{\mu_1-\mu_2}+\frac{\mu_1^2+\sigma_1^2-\mu_2^2-\sigma_2^2}{(1+2k)\cdot(\sigma_1^2+\sigma_2^2)}\right) \tag{11}$$

or in an abbreviated form, as

$$\upsilon_{1,2} = \frac{1}{2}\cdot(\mu_1^2+\mu_2^2+\lambda_{1,2}^2) + \frac{\lambda_{1,2}}{\sqrt{2\pi}}\cdot \\ \cdot\sum_{k=0}^{\infty}\frac{\overline{\xi}_{1,2}^{\,2}}{(-2)^k k!}\left(\mu_1+\mu_2+\frac{(\mu_1^2+\sigma_1^2-\mu_2^2-\sigma_2^2)\cdot\overline{\xi}_{1,2}}{(1+2k)\cdot\lambda_{1,2}}\right). \tag{12}$$

Here the term $(-2)^k k! = (-1)^k 2^k k!$ i $(-1)^k = 1/(-1)^k$ can be replaced with $(-1)^k/k!!$, because $2^k k! = (2k)!!$ [10]. The principles of invariance lead us to the following solution $\mu_{1,2} = \mu_{2,1}$

$$\mu_{2,1} = \frac{\mu_2+\mu_1}{2} + \frac{\lambda_{2,1}}{\sqrt{2\pi}}\sum_{k=0}^{\infty}\frac{\overline{\xi}_{2,1}^{\,2k}}{(-2)^k k!}\left(1+\frac{\overline{\xi}_{2,1}^{\,2}}{1+2k}\right) \tag{13}$$

and the variance

$$\upsilon_{2,1} = \frac{1}{2}(\mu_2^2+\mu_1^2+\lambda_{2,1}^{\;2}) - \mu_{2,1}^{\;2} + \frac{\lambda_{1,2}}{\sqrt{2\pi}}\cdot \\ \cdot\sum_{k=0}^{\infty}\frac{\overline{\xi}_{2,1}^{\,2}}{(-2)^k k!}\left(\mu_2+\mu_1+\frac{(\mu_2^2+\sigma_2^2-\mu_1^2-\sigma_1^2)\cdot\overline{\xi}_{2,1}}{(1+2k)\cdot\lambda_{2,1}}\right). \tag{14}$$

4.5 Testing the invariance testing of a flow model

The invariance ought to prove whether the obtained values remained unchanged and uniquely fixed when the flow order is changed in the calculation process. It is well-known that with only two flows having two parameters for each flow, we can have nine relations for each flow. Analysing the following possible relations between the expected times and the deviations of single flows, as:

$$\mu_1 \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \mu_2 \quad \text{and} \quad \sigma_1 \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \sigma_2, \tag{15}$$

we can conclude that a total of nine different combinations can be formed here. Taking into account the following:

$$\lambda_{1,2} = \lambda_{2,1},\ \xi_{1,2} = -\xi_{2,1},\ \Psi(\xi_{1,2}) = \Psi(\xi_{2,1}) \text{ and}$$

$$\Phi(-\xi_{1,2}) = \Phi(\xi_{2,1}), \tag{16}$$

we can obtain the invariant relations of the basic tested values that are connected to the superposed flow, i.e.:

$$\mu_{1,2} = \mu_{2,1}\ ;\ \Delta\mu_{1,2} = \Delta\mu_{2,1} \text{ and } \sigma_{1,2}^{\;2} = \sigma_{2,1}^{\;2}. \tag{17}$$

It can be concluded that it is irrelevant which of the two observed flows we proclaim to be critical, and which one subcritical. This property of the invariance of model [9] is very important and in addition to the analytical verification, a numerical verification can also be performed: [4, 6], using the Monte Carlo simulation.

5 Application of the simulation models

5.1 The use of the Monte Carlo method for solving Clark's flow model

Because the elementary activities of the flow time have a normal distribution with parameters $N \sim [\mu_j, \sigma_j]$, ($j = 1,2$), then the method of inverse functions is accepted as a convenient method for the modelling of the observed random variable. In Fig. 3 presented are the results of the numeric simulation of $n = 5\times10^5$ replications for the chosen values $N \sim [\mu_1 = 120, \sigma_1 = 8]$ and $N \sim [\mu_1 = 115, \sigma_1 = 6{,}5]$.

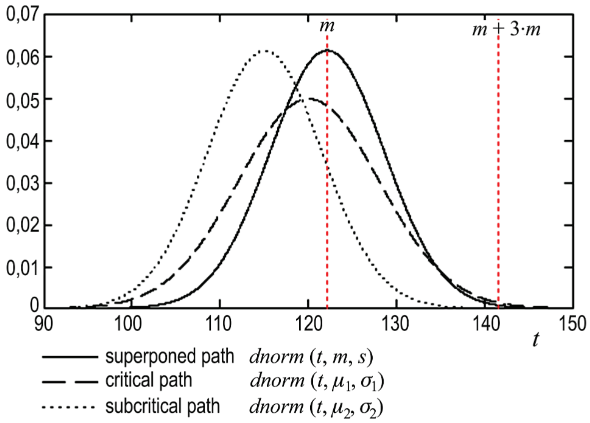

Figure 3 The distributions of probability for critical, subcritical and superimposed flow time

Also obtained hereby are:

- theoretical values:

$$N\sim[\mu_{12}=122{,}086724024,\ \ \sigma_{12}=6{,}510493997]\ \text{and}$$

- simulation values:

$$N\sim[m_{12}\approx 122{,}08322398,\ \ s_{12}\approx 6{,}492407778].$$

However, as this algorithm is automatically established by computers, now the main problem is the scope of simulation. In this case, in one session of a simulation of n = 5×10^5 replications, the testing of only one chosen variance was done, when it is $\mu_1>\mu_2$ and $\sigma_1<\sigma_2$ of the possible nine.

5.2 The application of the Monte Carlo - frame methods in solving Clark's flow model

The extension of the scope of the Monte Carlo method is possible by using frames (15). Using computer frames, by changing the fixed parameter, or through a vector value μ_1, the given Monte Carlo algorithm makes it easier to understand and visualize a broader class of phenomena, than it was the case in previously wide-used stereotyped static view regarding the process and simulation results. In this sense, up to three variants can be solved in one simulation session:

$$\mu_1\begin{Bmatrix}<\\=\\>\end{Bmatrix}\mu_2\quad\text{and}\quad\sigma_1<\sigma_2\,. \tag{18}$$

The frame number depends on the complexity of the problem, i.e. the studied process. One should not neglect the aesthetic moment of the frames presentation, and therefore the integrated Monte Carlo method - frame has also very educative role.

Here the frames very naturally connect to the simulation process, and in this way the Monte Carlo simulation is expanded to "a new dimension", what can be partially presented by a series of selected frames (Figs. 4 ÷ 13).

Example 1:

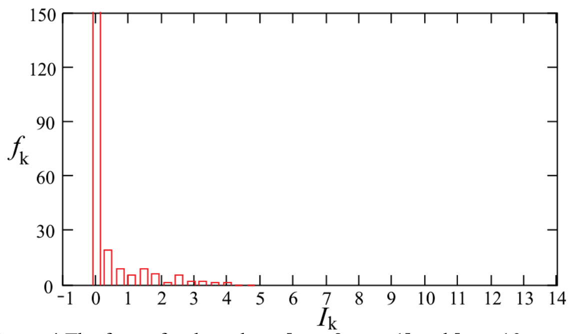

Figure 4 The frame for the values: [μ_1 = 8, σ_1 = 1] and [μ_2 = 10, σ_2 = 2]

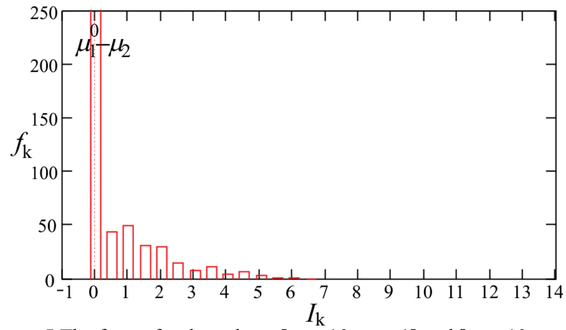

Figure 5 The frame for the values: [μ_1 = 10, σ_1 = 1] and [μ_2 = 10, σ_2 = 2]

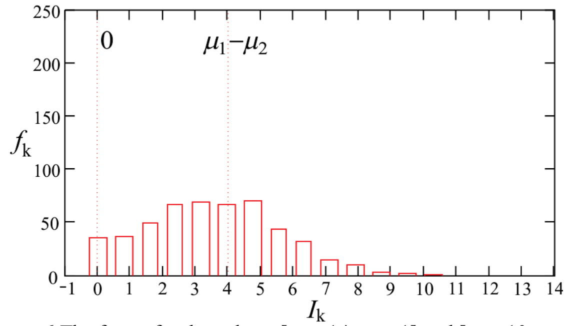

Figure 6 The frame for the values: [μ_1 = 14, σ_1 = 1] and [μ_2 = 10, σ_2 = 2]

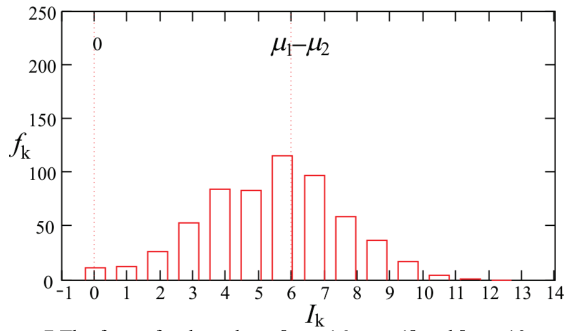

Figure 7 The frame for the values: [μ_1 = 16, σ_1 = 1] and [μ_2 = 10, σ_2 = 2]

Example 2:

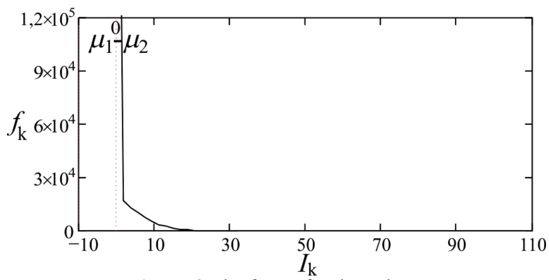

Figure 8 The frame for the values: [μ_1 = 120, σ_1 = 8] and [μ_2 = 130, σ_2 = 6,5]

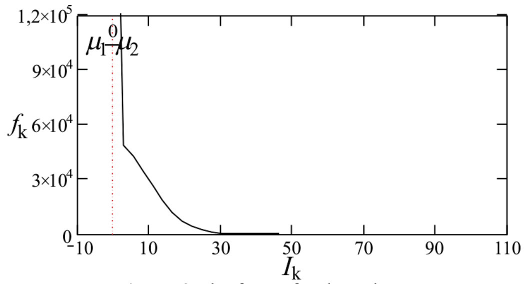

Figure 9 The frame for the values: [μ_1 = 120, σ_1 = 8] and [μ_2 = 120, σ_2 = 6,5]

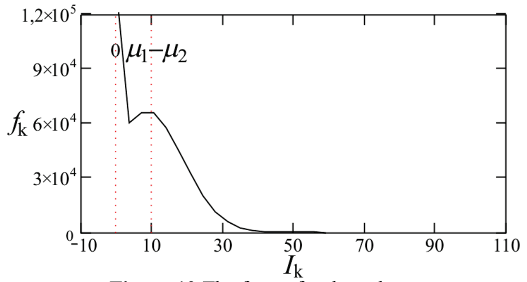

Figure 10 The frame for the values: [μ_1 = 120, σ_1 = 8] and [μ_2 = 110, σ_2 = 6,5]

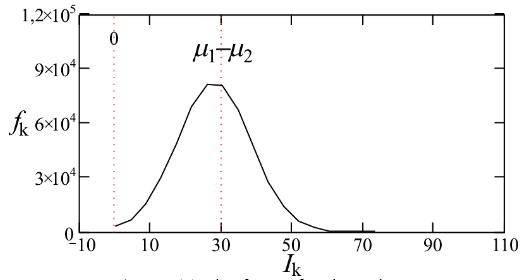

Figure 11 The frame for the values: [μ_1 = 120, σ_1 = 8] and [μ_2 = 90, σ_2 = 6,5]

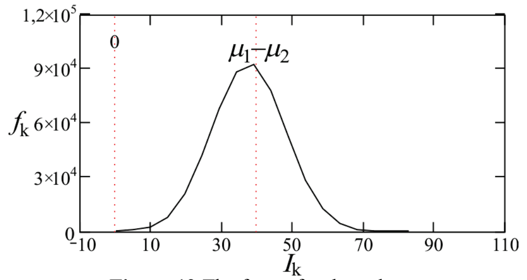

Figure 12 The frame for the values: [μ_1 = 120, σ_1 = 8] and [μ_2 = 80, σ_2 = 6,5]

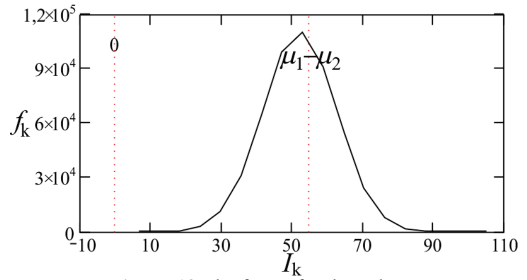

Figure 13 The frame for the values: [μ_1 = 120, σ_1 = 8] and [μ_2 = 65, σ_2 = 6,5]

6 Conclusion

The most important advantage of the Monte Carlo simulation method in solving this flow problem through a network, is the possibility to model the probability distribution function for the superimposed flow time of the basic network model presented in Fig. 1. However, the advantage of the Monte Carlo frame simulation method is substantially increased because of the possibility of dynamic flow modelling through a network.

The frames provide a more reliable basis to acquire and expand our knowledge in this field, especially in relation to the relativity of critical flow activities. Using a combination of Monte Carlo and frame procedures, time planning for critical flows can be carried out better than by standard procedures of network planning and network management, e.g. through PERT (Program Evaluation and Review Technique). With classical PERT, the flow time planning is carried out using expected values of elementary flow times, and by this a considerable planning error is done, since, in principle, the influence of subcritical flows on the forming of the total superimposing flow time is neglected. Slyke indicated [14] that in a flow network with ten (sub)critical flows of autonomous type, the resulting flow time increases by 11 % in regard to the time we should obtain by calculation using the PERT method. This "planning error", as a theoretical result, also verified by the simulation of two parallel flows, is $\Delta \approx 1{,}67-6{,}09\,\%$. The result obtained through the frames method is not unique, but depends on the chosen value pairs $N \sim [\mu_j, \sigma_j]$, $(j=1{,}2)$. In addition, we can clearly see, looking from frame to frame, that the domination of the critical flow diminishes in favour of the subcritical flow, if the mean value of the latter is increased in regard to the first one. Of course, it is also possible to test the remaining cases using the Monte Carlo frame method, e.g. when inserting a deviation vector, instead of the mean value vector, as in (10):

$$\mu_1 = \mu_2 \quad \text{and} \quad \sigma_1 \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \sigma_2 . \tag{19}$$

These influences (18) and (19) can be explicitly perceived by simulation in more complex ADM (Activity On the Node) networks [9]. The essential consequences of ignoring the obtained results can be very negative, especially in cases where complex stochastic flow activities are planned and controlled using a network.

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