ISSN 1330-3651 (Print), ISSN 1848-6339 (Online)
UDC/UDK 621.184.7:539.32.386

THEORETICAL DETERMINATION OF ELASTICITY CONSTANTS FOR STEAM BOILER MEMBRANE WALL AS THE STRUCTURALLY ORTHOTROPIC PLATE

Josip Sertić, Ivan Gelo, Dražan Kozak, Darko Damjanović, Pejo Konjatić

Preliminary notes
This paper presents theoretical determination of elasticity constants for steam boiler membrane wall as structurally orthotropic plate. Modulus of elasticity and shear modulus are determined in directions parallel and perpendicular to membrane wall pipes for both bending and membrane load. When determining modulus of elasticity for bending load, Engesser's theorem is used for obtaining deflection of membrane wall. After that, from equality of deflections of membrane wall and structurally orthotropic plate, modulus of elasticity is obtained. When determining modulus of elasticity for membrane load, Engesser's theorem is also used, and the expression is obtained from extension equation of membrane wall and structurally orthotropic plate. Shear moduli for both loading conditions are obtained using Poisson's ratios.

Keywords: *elasticity constants, membrane wall, steam boiler, structurally orthotropic plate*

Teorijsko određivanje konstanti elastičnosti membranskog zida parnog kotla kao strukturno ortotropne ploče

Prethodno priopćenje
U ovom radu prikazano je teorijsko određivanje konstanti elastičnosti membranskog zida parnog kotla kao strukturno ortotropne ploče. Modul elastičnosti i modul smika određeni su u pravcima okomitim i paralelnim s cijevima membranskog zida za savojno i membransko opterećenje. Kod određivanja modula elastičnosti za savojno opterećenje korišten je Engesserov teorem za dobivanje progiba membranskog zida. Nakon toga, iz jednakosti progiba membranskog zida i strukturno ortotropne ploče, dobiven je modul elastičnosti. Za modul elastičnosti kod membranskog naprezanja korišten je također Engesserov teorem, a izraz je dobiven iz jednakosti produljenja membranskog zida i strukturno ortotropne ploče. Moduli smika dobiveni su pomoću Poissonovih omjera.

Ključne riječi: *konstante elastičnosti, membranski zid, parni kotao, strukturno ortotropna ploča*

1 Introduction

The membrane wall of the steam boiler has the function of heat exchanger. Along with thermal stresses and stresses caused by the pressure of the working fluid, the membrane wall is a bearing construction loaded with its own mass, mass of the working fluid, mass of fouling, mass of refractory and insulation, mass of equipment, mass of buckstays and with the pressure of combustion gases. Steam boilers of modern waste incinerators often have their supports welded on membrane walls, and that results with significant increase of local stresses in such made supports. When it comes to the static calculation of the boiler, it is necessary to determine support reactions of the boiler, deflections of the boiler membrane walls and stress state in the critical zones. Because of the large number of pipes in the boiler membrane walls, it is practically impossible to discretize realistic geometry of the boiler by finite elements and perform necessary calculations. Since membrane wall structurally represents orthotropic plate, the theoretical approach of determining the elasticity modulus for membrane and bending stiffness of the equivalent orthotropic plate will be presented in this paper. After determining modulus of elasticity of the equivalent orthotropic plate, it will be possible, using finite elements for orthotropic plate, to perform the above mentioned static calculations of the steam boiler.

Significant researches in the field of structurally orthotropic plates were made by H. J. Huffington [1]. Expressions for different forms of structurally orthotropic plates can be found in the literature [2, 3, 4]. The method for stiffness homogenization of the boiler membrane wall [5, 6, 7] enables to consider the boiler as one constructional assembly on which it is possible (along with the above mentioned) to analyse the effects of thermal dilatation and buckling of membrane walls.

2 Elasticity constants of equivalent orthotropic plate for bending load

2.1 Modulus of elasticity in direction perpendicular to the axis of membrane wall pipe ($E_y^{(b)}$)

In case of bending of membrane wall around the axis parallel with the membrane wall pipe axis, approximately the membrane wall pipe, with respect to membrane wall tape, can be considered as rigid body (Fig. 1). In order to find an expression for modulus of elasticity $E_y^{(b)}$, the static beam model with clamped support on one side, loaded with continuous loading will be analysed.

According to the Engesser's theorem, the partial derivation of body's complementary energy of deformation with respect to any force, gives the displacement at the location of the force as well as on the direction of the force. As potential energy of deformation is equal to the complementary energy for a linear-elastic body, according to the Castigliano's theorem [8] follows

$$\frac{\partial U}{\partial F} = w, \tag{1}$$

where U is the potential energy of elastic deformation of membrane wall according to Fig. 1, which has a unit width, and F is a fictive force which will be used for determining the displacement w in the direction of the z axis.

If the pipes of the membrane wall are considered as ideally rigid, total potential energy of elastic deformation is equal to potential energy of elastic deformation of membrane tapes. To simplify the problem, the part of potential energy of elastic deformation which refers to shear stresses at bending caused by the force, will be neglected. According to acquired assumptions, total potential energy of elastic deformation of membrane wall can be calculated using the expression

$$U=\frac{1}{2\cdot E\cdot I_t}\cdot\sum_{i=1}^{n}\int_0^b M_\mathrm{b}(y_i)^2\,\mathrm{d}y_i\,, \tag{2}$$

where:
E – modulus of elasticity for the material at calculating temperature, of the membrane wall, GPa
I_t – moment of inertia for bending axis of cross section of membrane tape with a unit width, m^4
$M_\mathrm{b}(x_i)$ – the distribution function of bending moments on the i-th membrane tape, N·m.

According to the expression (1), the deflection of the end of membrane wall for the place of applied force $F=0$ is

$$w=\left(\frac{\partial U}{\partial F}\right)_{F=0}=\frac{1}{E\cdot I_t}\cdot\sum_{i=1}^{n}\int_0^b M_\mathrm{b}(y_i)\cdot\frac{\partial M_\mathrm{b}(y_i)}{\partial F}\mathrm{d}y_i\,. \tag{3}$$

Bending moments are:

$$M_\mathrm{b}(y_1)=F\cdot(D+y_1)+\frac{q}{2}\cdot(D+y_1)^2\,,$$
$$M_\mathrm{b}(y_2)=F\cdot(2\cdot D+b+y_2)+\frac{q}{2}\cdot(2\cdot D+b+y_2)^2\,,$$
$$M_\mathrm{b}(y_3)=F\cdot(3\cdot D+2\cdot b+y_3)+\frac{q}{2}\cdot(3\cdot D+2\cdot b+y_3)^2\,, \tag{4}$$
$$M_\mathrm{b}(y_{n-1})=\begin{bmatrix}F\cdot((n-1)\cdot D+(n-2)\cdot b+y_{n-1})+\\+\frac{q}{2}\cdot((n-1)\cdot D+(n-2)\cdot b+y_{n-1})^2\end{bmatrix},$$
$$M_\mathrm{b}(y_n)=\begin{bmatrix}F\cdot(n\cdot D+(n-1)\cdot b+y_n)+\\+\frac{q}{2}\cdot(n\cdot D+(n-1)\cdot b+y_n)^2\end{bmatrix}.$$

Partial derivations of the bending moments (4) with respect to the force $F=0$ are:

$$\frac{\partial M_\mathrm{b}(y_1)}{\partial F}=D+y_1\,,$$
$$\frac{\partial M_\mathrm{b}(y_2)}{\partial F}=2\cdot D+b+y_2\,,$$
$$\frac{\partial M_\mathrm{b}(y_3)}{\partial F}=3\cdot D+2\cdot b+y_3\,, \tag{5}$$
$$\frac{\partial M_\mathrm{b}(y_{n-1})}{\partial F}=(n-1)\cdot D+(n-2)\cdot b+y_{n-1}\,,$$
$$\frac{\partial M_\mathrm{b}(y_n)}{\partial F}=n\cdot D+(n-1)\cdot b+y_n\,.$$

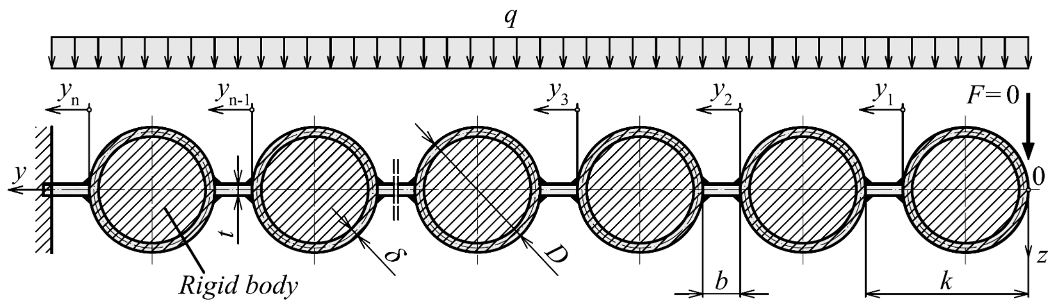


Figure 1 Geometric model for determination of equivalent modulus of elasticity $E_y^{(\mathrm{b})}$ for bending load of the boiler membrane wall

After combining expressions (3), (4) and (5), and if $k=b+D$, the expression for the deflection of membrane wall can be written as

$$w=\frac{q}{8\cdot E\cdot I_t}\cdot\sum_{i=1}^{n}(i\cdot k)^4-(i\cdot k-b)^4\,. \tag{6}$$

Moment of inertia for cross section of the membrane tape, with a unit width, can be calculated according to the expression

$$I_t=\frac{t^3}{12}\,. \tag{7}$$

Deflection for the equivalent orthotropic plate with the length $n\cdot k$ and a unit width, clamped on one end and loaded with continuous loading q, can be calculated according to the expression

$$w=\frac{q\cdot(n\cdot k)^4}{8\cdot E_y^{(\mathrm{b})}\cdot I_h}\,, \tag{8}$$

where:
$E_y^{(\mathrm{b})}$ – modulus of elasticity of equivalent orthotropic plate, perpendicular to the pipes of membrane wall, GPa
I_h – moment of inertia for bending axis for cross section of equivalent orthotropic plate with a unit width, m^4.

Moment of inertia for cross section of equivalent orthotropic plate can be calculated according to the expression

$$I_h=\frac{h^3}{12}\,, \tag{9}$$

where h is the thickness of the equivalent orthotropic plate which can be calculated using expression [7]

$$h=\sqrt{\frac{3\cdot\pi}{16\cdot D\cdot k}\cdot\left(D^4-(D-2\cdot\delta)^4\right)+\frac{(k-D)\cdot t^3}{D\cdot k}}\,. \quad (10)$$

From the equality of the deflections of membrane wall (6) and equivalent orthotropic plate (8), modulus of elasticity $E_y^{(\mathrm{b})}$ of equivalent orthotropic plate can be obtained for the axis perpendicular to membrane wall pipe

$$E_y^{(\mathrm{b})}=E\cdot\left(\frac{t}{h}\right)^3\cdot\frac{(n\cdot k)^4}{\sum_{i=1}^{n}(i\cdot k)^4-(i\cdot k-b)^4}\,. \quad (11)$$

It can be noticed that modulus of elasticity $E_y^{(\mathrm{b})}$ depends on the number of pipes n in membrane wall.

2.2 Modulus of elasticity in direction parallel with membrane wall pipes ($E_x^{(\mathrm{b})}$)

Modulus of elasticity of equivalent orthotropic plate in direction parallel with membrane wall pipes can be calculated using the expression [7]

$$E_x^{(\mathrm{b})}=\frac{E}{k\cdot h^3}\cdot\left[\frac{3\cdot\pi}{16}\cdot\left(D^4-(D-2\cdot\delta)^4\right)+(k-D)\cdot t^3\right]. \quad (12)$$

2.3 Poisson's ratio

Poisson's ratio ν_{xy}, for membrane and bending loading, is equated with the Poisson's ratio of membrane wall material, while the Poisson's ratio ν_{yx}, for membrane and bending loading, can be calculated from a known expression [8], which connects elasticity constants for perpendicular material axis, equation (13):

$$\nu_{yx}=\nu_{xy}\cdot\frac{E_y}{E_x}\,. \quad (13)$$

2.4 Shear modulus

Shear modulus G_{xy}, for membrane and bending loading, can be approximately calculated using expression [9]

$$G_{xy}\approx\frac{\sqrt{E_x\cdot E_y}}{2\cdot\left(1+\sqrt{\nu_{xy}\cdot\nu_{yx}}\right)}\,. \quad (14)$$

3 Elasticity constants of equivalent orthotropic plate for membrane load

3.1 Modulus of elasticity in direction perpendicular to axis of membrane wall pipe ($E_y^{(\mathrm{m})}$)

Modulus of elasticity of equivalent orthotropic plate, for membrane load in direction perpendicular to membrane wall pipes, will be determined using the Castigliano's theorem. According to Fig. 2, the displacement of point A, by a unit length of membrane wall, can be calculated by expression

$$v_\delta=\frac{\partial U}{\partial F_\mathrm{A}}\,. \quad (15)$$

Since this is statically undetermined problem (Fig. 2), firstly, it is necessary to determine inner forces in the section A. This will be solved using the theorem of minimum total potential energy of elastic deformation, using expression

$$\frac{\partial U}{\partial M_\mathrm{A}}=0\,. \quad (16)$$

Potential energy of elastic deformation of membrane wall pipe can be calculated using expression

$$U=\frac{2}{E\cdot I_\delta}\cdot\int_0^{\pi/2}M_\mathrm{b}(\varphi)^2\cdot R\,\mathrm{d}\varphi\,,\ R=\frac{D-\delta}{2}\,, \quad (17)$$

where:
I_δ – moment of inertia, for bending axis, for radial cross section of the membrane pipe with a unit width, m^4
E – modulus of elasticity for pipe and membrane wall tape material, GPa
$M_\mathrm{b}(\varphi)$ – the distribution function of bending moments, N·m.

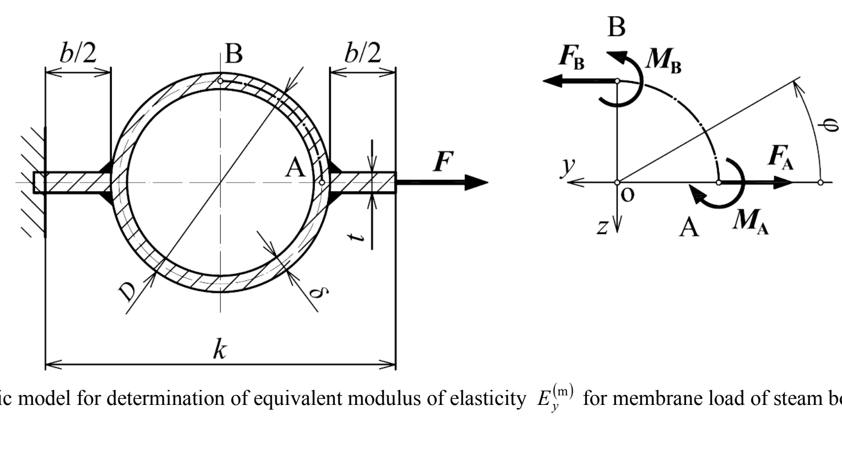


Figure 2 Geometric model for determination of equivalent modulus of elasticity $E_y^{(\mathrm{m})}$ for membrane load of steam boiler membrane wall

Bending moment with respect to coordinate φ, according to Fig. 2, is

$$M_{\mathrm{b}}(\varphi)=F_{\mathrm{A}}\cdot R\cdot\sin\varphi-M_{\mathrm{A}}, \tag{18}$$

where $F_{\mathrm{A}}=\dfrac{F}{2}$.

Combining expressions (17) and (18), it is necessary to partially derive (17) according to (16)

$$\frac{\partial U}{\partial M_{\mathrm{A}}}=-\frac{4}{E\cdot I_{\delta}}\cdot\int_{0}^{\pi/2}\left(F_{\mathrm{A}}\cdot R\cdot\sin\varphi-M_{\mathrm{A}}\right)\cdot R\,\mathrm{d}\varphi=0. \tag{19}$$

After integrating, bending moment can be obtained

$$M_{\mathrm{A}}=-\frac{2\cdot F_{\mathrm{A}}\cdot R}{\pi}=-\frac{F\cdot R}{\pi}. \tag{20}$$

The displacement of point A of the membrane wall pipe in direction of the y axis, according to the expression (15), is

$$v_{\delta}=\frac{4}{E\cdot I_{\delta}}\cdot\int_{0}^{\pi/2}M_{\mathrm{b}}(\varphi)\cdot\frac{M_{\mathrm{b}}(\varphi)}{\partial F_{\mathrm{A}}}\cdot R\,\mathrm{d}\varphi. \tag{21}$$

Bending moments are obtained if (20) is inserted into (18)

$$M_{\mathrm{b}}(\varphi)=F_{\mathrm{A}}\cdot R\cdot\left(\sin\varphi+\frac{2}{\pi}\right). \tag{22}$$

The expression (22) should be combined with expression (21), and after derivation and integration, the displacement of point A of membrane wall is obtained by unit width of membrane wall,

$$v_{\delta}=\frac{4\cdot F_{\mathrm{A}}\cdot R^{3}}{E\cdot I_{\delta}}\cdot\left(\frac{\pi}{8}-\frac{1}{\pi}\right). \tag{23}$$

Since $F_{\mathrm{A}}=\dfrac{F}{2}$, $R=\dfrac{D-\delta}{2}$ and $I_{\delta}=\dfrac{\delta^{3}}{12}$,

the expression (23) can be written as

$$v_{\delta}=\frac{3\cdot F}{E}\cdot\left(\frac{D}{\delta}-1\right)^{3}\cdot\left(\frac{\pi}{8}-\frac{1}{\pi}\right). \tag{24}$$

Extension of membrane tape by unit width of membrane wall, loaded with force F, can be calculated using expression

$$v_{t}=\frac{F\cdot b}{E\cdot t}. \tag{25}$$

Extension of membrane wall by unit width of the wall, in direction perpendicular to the axis of membrane wall pipe, can be calculated using expression

$$v=n\cdot\left(v_{\delta}+v_{t}\right)=\frac{n\cdot F}{E}\cdot\left(3\cdot\left(\frac{D}{\delta}-1\right)^{3}\cdot\left(\frac{\pi}{8}-\frac{1}{\pi}\right)+\frac{b}{t}\right), \tag{26}$$

where membrane wall consists of n pipes.

Extension of the equivalent orthotropic plate with the length $n\cdot k$ and a unit width, clamped on one end and loaded with force F, can be calculated according to the expression

$$v=\frac{F\cdot k\cdot n}{E_{y}^{(\mathrm{m})}\cdot h}, \tag{27}$$

where:

$E_{y}^{(\mathrm{m})}$ – modulus of elasticity of equivalent orthotropic plate, perpendicular to the pipes of membrane wall,

h – thickness of structurally orthotropic plate which can be calculated by expression (10).

From the extension equation of membrane wall (26) and equivalent orthotropic plate (27), the modulus of elasticity $E_{y}^{(\mathrm{m})}$, for axis perpendicular to membrane wall pipes, can be calculated

$$E_{y}^{(\mathrm{m})}=\frac{k\cdot E}{h\cdot\left[3\cdot\left(\frac{D}{\delta}-1\right)^{3}\cdot\left(\frac{\pi}{8}-\frac{1}{\pi}\right)+\frac{b}{t}\right]}. \tag{28}$$

3.2 Modulus of elasticity in direction parallel with membrane wall pipes ($E_{x}^{(\mathrm{m})}$)

Modulus of elasticity of equivalent orthotropic plate, in direction parallel with membrane wall pipes can be calculated according to the expression [7]

$$E_{x}^{(\mathrm{m})}=\frac{E}{k\cdot h}\cdot\left[\frac{\pi}{4}\cdot\left(D^{2}-(D-2\cdot\delta)^{2}\right)+(k-D)\cdot t\right]. \tag{29}$$

Poisson's ratio and shear modulus can be determined in the same way as in case of bending load [7].

4 Numerical results and discussion

In order to evaluate the accuracy of presented procedure, five models are numerically analysed (Calculations 1 to 5). Dimensions of geometric models of membrane wall and equivalent orthotropic plates are 900 × 900 mm, in all cases. Material of membrane wall pipe is P235GH and material of membrane wall tape is S235JRG2. Calculating temperature is ϑ = 275 °C. Both materials are carbon steels. Elasticity modulus, at calculating temperature, for both materials are approximately equal [10], and amount to E = 187,12 GPa. For the Poisson's ratio a value of $\nu=0{,}3$ was taken. Pitch of the pipes of membrane wall is k = 90 mm. Outlet diameter of membrane wall pipe is D = 57 mm, and its wall thickness is δ = 4 mm. Thickness of membrane wall tape is t = 6 mm. By means of earlier suggested expressions, elastic constants of equivalent orthotropic plate can be calculated:

$$E_x^{(b)} = 454{,}438\ \text{GPa},\ E_y^{(b)} = 7{,}548\ \text{GPa},\ \nu_{xy}^{(b)} = 0{,}3$$
$$\nu_{yx}^{(b)} = 0{,}00498,\ G_{xy}^{(b)} = 28{,}194\ \text{GPa}$$
$$E_x^{(m)} = 76{,}507\ \text{GPa},\ E_y^{(m)} = 1{,}367\ \text{GPa},\ \nu_{xy}^{(m)} = 0{,}3$$
$$\nu_{yx}^{(m)} = 0{,}00536,\ G_{xy}^{(m)} = 4{,}916\ \text{GPa}.$$

Expressions for members of elasticity matrix [8] of equivalent orthotropic plate for membrane stiffness are

$$A_{11} = \frac{E_x^{(m)} \cdot h}{1-\nu_{xy}^{(m)} \cdot \nu_{yx}^{(m)}},\quad A_{12} = \frac{E_x^{(m)} \cdot h \cdot \nu_{yx}^{(m)}}{1-\nu_{xy}^{(m)} \cdot \nu_{yx}^{(m)}},$$
$$A_{22} = \frac{E_y^{(m)} \cdot h}{1-\nu_{xy}^{(m)} \cdot \nu_{yx}^{(m)}},\quad A_{33} = G_{xy}^{(m)} \cdot h, \tag{30}$$

and for bending stiffness are as follows

$$D_{11} = \frac{E_x^{(b)} \cdot h^3}{12 \cdot \left(1-\nu_{xy}^{(b)} \cdot \nu_{yx}^{(b)}\right)},\quad D_{12} = \frac{E_x^{(b)} \cdot h^3 \cdot \nu_{yx}^{(b)}}{12 \cdot \left(1-\nu_{xy}^{(b)} \cdot \nu_{yx}^{(b)}\right)},$$
$$D_{22} = \frac{E_y^{(b)} \cdot h^3}{12 \cdot \left(1-\nu_{xy}^{(b)} \cdot \nu_{yx}^{(b)}\right)},\quad D_{33} = \frac{G_{xy}^{(b)} \cdot h^3}{6}. \tag{31}$$

By introducing of elasticity constants in expressions above, values of elasticity matrix members can be calculated:

$$A_{11} = 1799279\ \text{N/mm},\ A_{12} = 9633\ \text{N/mm},$$
$$A_{22} = 32111\ \text{N/mm},\ A_{33} = 115367\ \text{N/mm},$$
$$D_{11} = 490875910\ \text{N}\cdot\text{mm},\ D_{12} = 2200049\ \text{N}\cdot\text{mm},$$
$$D_{22} = 7333497\ \text{N}\cdot\text{mm},\ D_{33} = 57798594\ \text{N}\cdot\text{mm}.$$

Constitutive equations of equivalent orthotropic plate/shell, according to the Kirchhoff-Love theory [8, 9], can be presented as matrix equation as follows

$$\boldsymbol{\sigma} = \boldsymbol{D} \cdot \boldsymbol{\varepsilon} \tag{32}$$

or

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 & 0 & 0 \\ & A_{22} & 0 & 0 & 0 & 0 \\ & & A_{33} & 0 & 0 & 0 \\ & & & D_{11} & D_{12} & 0 \\ & sym. & & & D_{22} & 0 \\ & & & & & D_{33} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ 2\kappa_{xy} \end{bmatrix}, \tag{33}$$

where $\boldsymbol{\sigma}$ is the vector of internal forces, $\boldsymbol{D}$ is elasticity matrix and $\boldsymbol{\varepsilon}$ is deformations vector. Elasticity matrix $\boldsymbol{D}$ applies in stiffness matrix of finite element

$$\boldsymbol{k} = \int_A \boldsymbol{B}^{\mathrm{T}} \boldsymbol{D} \boldsymbol{B}\, \mathrm{d}A. \tag{34}$$

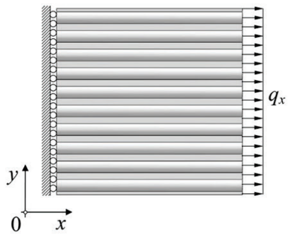
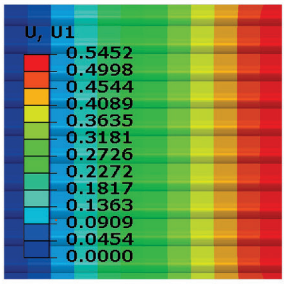
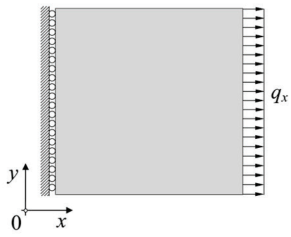
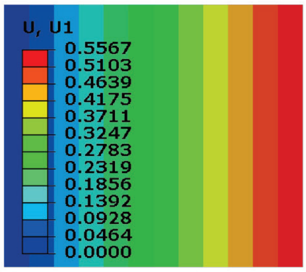
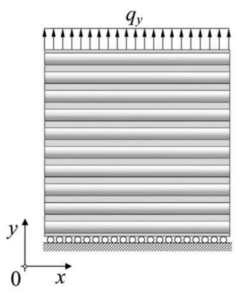
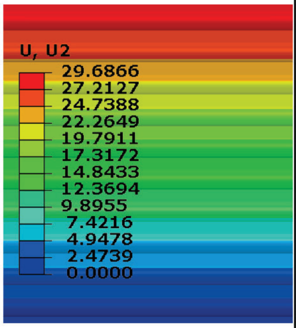
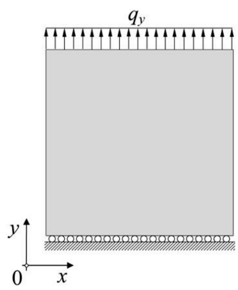
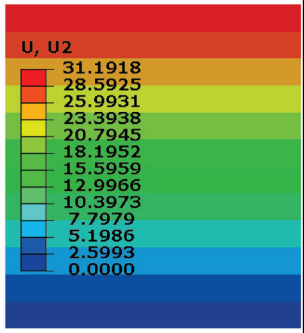
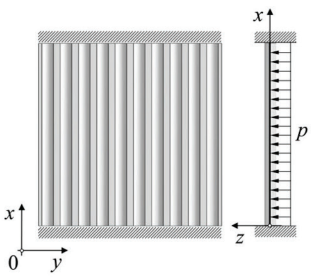
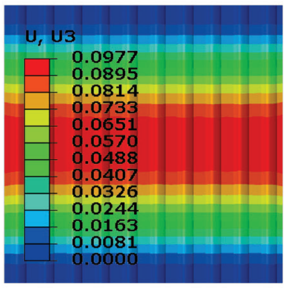
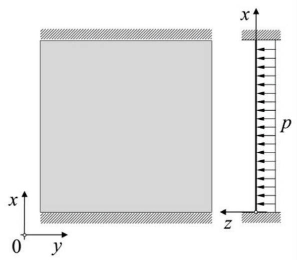
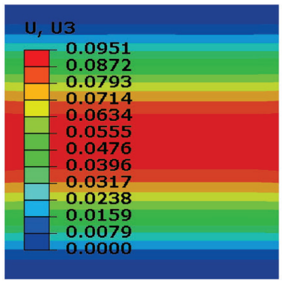
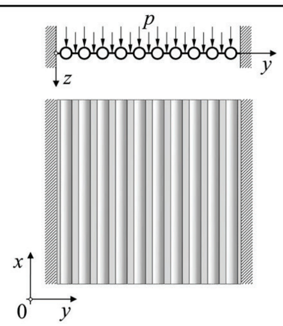
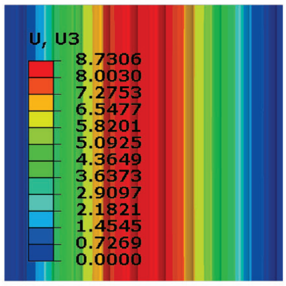
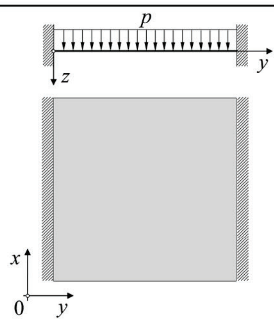
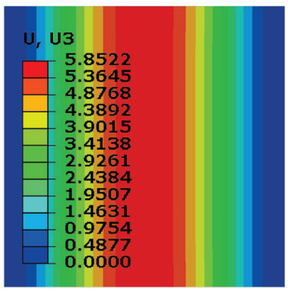
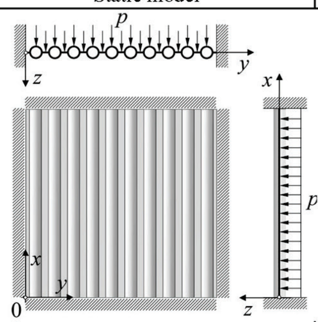
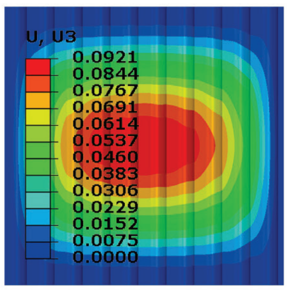
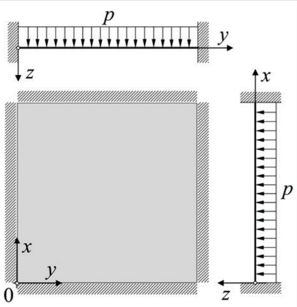
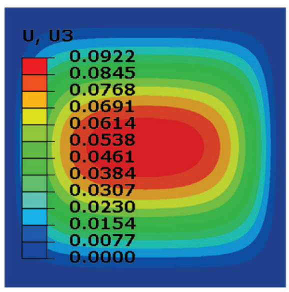
Numerical calculations are done by appliance of Abaqus software. Double curved S4R5 shell finite element with four nods was used. This element has five degrees of freedom per each nod. Geometry of membrane wall was meshed by 463 500 finite elements. Numerical model of membrane wall contains 2 762 826 degrees of freedom. Geometry of equivalent orthotropic plate was meshed by 202 500 finite elements. Numerical model of equivalent orthotropic plate contains 1 220 406 degrees of freedom.

In numerical calculations continuous loadings $q_x = 491{,}39$ N/m and $q_y = 1111{,}1$ N/m, and also the pressure p = 0,025 N/mm^2 were applied. Calculation results are presented in Tab. 1.

5 Conclusion

Acquired expressions enable fast and simple determination of elasticity constants of equivalent orthotropic plate which represent stiffness of steam boiler membrane wall. There are also opened possibilities of a more detailed structural analysis of steam boiler, e.g. determining the reactions in supports, dimensioning of buckstays, determination of thermal extension influence, determination of deflection of the membrane walls as well as buckling of the membrane walls. Finite elements method calculations were done on example of membrane wall. Elasticity constants of equivalent orthotropic plate, also as members of elasticity matrix, were calculated. According to the values of elasticity matrix members D_{11} and D_{22} it can be concluded that the calculating stiffness for bending by the y axis of membrane wall is approximately 67 times greater than the stiffness for bending by the x axis. Displacements of membrane wall and equivalent orthotropic plate for the first three calculations are nearly equal. At the third calculation significant deviation has occurred. Because of the above mentioned difference of membrane wall stiffness, this deviation will not have a significant influence on deflection values of steam boiler membrane walls. That can be noticed in deflections given in the fifth calculation. The following research should be going in direction of corrections of expressions values for $E_y^{(b)}$ elasticity modulus in order to increase the accuracy of approximation of D_{22} elasticity matrix member. It is needed to make additional numerical calculations which can evaluate the accuracy of approximation of steam boiler membrane wall stiffness considering the membrane shear deformations and torsion deformations.

Table 1 Calculation results

	Membrane wall		Equivalent orthotropic plate	
Calculation 1	 Static model	 Displacements in *x* direction / mm	 Static model	 Displacements in *x* direction / mm
Calculation 2	 Static model	 Displacements in *y* direction / mm	 Static model	 Displacements in *y* direction / mm
Calculation 3	 Static model	 Deflection / mm	 Static model	 Deflection / mm
Calculation 4	 Static model	 Deflection / mm	 Static model	 Deflection / mm
Calculation 5	 Static model	 Deflection / mm	 Static model	 Deflection / mm

6 References

[1] Huffington, H. J. Theoretical determination of rigidity properties of orthotropic stiffened plates. // Journal of Applied Mechanics. 23, 1(1956), pp. 15–20.

[2] Timoshenko, S.P.; Woinowski-Krieger S. Theory of Plates and Shells, 2nd ed. McGraw-Hill, New York, 1959.

[3] Boyarshinov S. V. The Fundamentals of Structurally Mechanics of Machines. Mashinostroenie, Moscow, 1973.

[4] Troitsky, M. S. Orthotropic Bridges – Theory and Design. The James Lincoln Arc Welding Foundation, Cleveland, 1987.

[5] Sertić, J.; Kozak, D. Static calculation of reaction in boiler supports. Boiler project Mailand, Technical report, Mechanical Engineering Faculty in Slavonski Brod, Slavonski Brod, 2011.

[6] Milošević-Mitić, V.; Gaćeša, B.; Anđelić, N.; Maneski, T. Numerical Calculation of the Water-Tube Boiler Using Finite Element of the Orthotropic Plate. // Structural Integrity and Life. 12, 3(2012), pp. 185-190.

[7] Sertić, J.; Kozak, D.; Damjanović, D.; Konjatić, P. Homogenization of steam boiler membrane wall. // Proceedings of 7th International Congress of Croatian Society of Mechanics, Zadar, 2012.

[8] Solecki, R.; Conant, R. J. Advanced Mechanics of Materials. New York and Oxford, Oxford University Press, 2003.

[9] Ventsel, E.; Krauthammer, T. Thin Plates and Shells. Theory, Analysis and Applications. Marcel Dekker, Inc., New York and Basel, 2001.

[10] ASME B31.1-1995

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