On NP - polyagroups

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Abstract. *In the present paper: 1) an NP–polyagroup is defined as a generalization of an $n-$group for $n \geq 3$; and 2) NP–polyagroups of the type $(s, n-1)$ is described as algebras of the type $< n, n-1, n-2 >$ [$=< k \cdot s+1, k \cdot s, k \cdot s-1 >; k > 1, s \geq 1$].*

Key words: $\{1, n\}$*-neutral operation, n-group, n-semigroup, n-quasigroup, Ps-associative n-groupoid, P–polyagroup, NP–polyagroup*

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1. Preliminaries

Definition 1. *Let $n \geq 2$ and let (Q, A) be an n-groupoid. We say that (Q, A) is a Dörnte n-group [briefly: n-group] iff it is an n-semigroup and an n-quasigroup as well.*

Remark 1. *A notion of an n-group was introduced by W. Dörnte in [1] as a generalization of the notion of a group. See, also [2–4].*

Proposition 1 [10]. *Let $n \geq 2$ and let (Q, A) be an n-groupoid. Then the following statements are equivalent :*

(i) (Q, A) *is an n-group;*

(ii) there are mappings $^{-1}$ *and* $\mathbf{e}$ *of the sets* Q^{n-1} *and* Q^{n-2}*, respectively, into the set Q such that the following laws hold in the algebra* $(Q, \{A, {}^{-1}, \mathbf{e}\})$ *[of the type* $\langle n, n-1, n-2\rangle$ *]*

(a) $A(x_1^{n-2}, A(x_{n-1}^{2n-2}), x_{2n-1}) = A(x_1^{n-1}, A(x_n^{2n-1})),$

(b) $A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, x) = x$ *and*

(c) $A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, a) = \mathbf{e}(a_1^{n-2})$*; and*

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(iii) there are mappings $^{-1}$ *and* $\mathbf{e}$ *of the sets* Q^{n-1} *and* Q^{n-2}*, respectively, into the set* Q *such that the following laws hold in the algebra* $(Q, \{A, ^{-1}, \mathbf{e}\})$ *[of the type* $\langle n, n-1, n-2\rangle$ *]*

$(\bar{a})$ $\quad A(A(x_1^n), x_{n+1}^{2n-1}) = A(x_1, A(x_2^{n+1}), x_{n+2}^{2n-1})$,

$(\bar{b})$ $\quad A(x, a_1^{n-2}, \mathbf{e}(a_1^{n-2})) = x$

$(\bar{c})$ $\quad A(a, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) = \mathbf{e}(a_1^{n-2})$.

Remark 2. $\mathbf{e}$ *is a* $\{1, n\}$*-neutral operation of an* n*-groupoid* (Q, A) *iff algebra* $(Q, \{A, \mathbf{e}\})$ *of the type* $\langle n, n-2\rangle$ *satisfies the laws* (b) *and* $(\bar{b})$ *from* Proposition 1 *[: [7]]. The notion of an* $\{i, j\}$*-neutral operation* $(i, j \in \{1, ..., n\},\ i < j)$ *of an* n*-groupoid is defined in a similar way [: [7]]. Every* n*-groupoid has at most one* $\{i, j\}$*-neutral operation [: [7]]. In every* n*-group* $(n \geq 2)$ *there is an* $\{1, n\}$*-neutral operation [: [7]]. There are* n*-groups without an* $\{i, j\}$*-neutral operation with* $\{i, j\} \neq \{1, n\}$ *[:[9]]. In [9],* n*-groups with* $\{i, j\}$*-neutral operations, for* $\{i, j\} \neq \{1, n\}$ *are described. Operation* $^{-1}$*from* Proposition 1 *[* $(c), (\bar{c})$*] is a generalization of the inverse operation in a group. In fact, if* (Q, A) *is an* n*-group,* $n \geq 2$*, then for every* $a \in Q$ *and for every sequence* a_1^{n-2} *over* Q

$$(a_1^{n-2}, a)^{-1} \stackrel{def}{=} \mathsf{E}(a_1^{n-2}, a, a_1^{n-2}),$$

where E *is a* $\{1, 2n-1\}$*-neutral operation of the* $(2n-1)$*-group* $(Q, \overset{2}{A})$*;* $\overset{2}{A}(x_1^{2n-1}) \stackrel{def}{=} A(A(x_1^n), x_{n+1}^{2n-1})$*[: [8]]. (For* $n = 2, a^{-1} = \mathsf{E}(a); a^{-1}$ *is the inverse element of element* a *with respect to the neutral element* $\mathbf{e}(\emptyset)$ *of the group* (Q, A)*.)*

Definition 2. *Let* $k > 1, s \geq 1, n = k \cdot s + 1$ *and let* (Q, A) *be an* $n-$*groupoid. Then, we say that* (Q, A) *is a* **partially**$s-$**associative** *(briefly: Ps–associative)* $n-$**groupoid** *iff for every* $i, j \in \{t \cdot s + 1 | t \in \{0, 1, \ldots, k\}\}, i < j$*, the following law holds*

$$A(x_1^{i-1}, A(x_i^{i+n-1}), x_{i+n}^{2n-1}) = A(x_1^{j-1}, A(x_j^{j+n-1}), x_{j+n}^{2n-1})$$

[: $< i, j > -$*associative law].*

Remark 3. *For* $s = 1$ (Q, A) *is a* $(k+1)-$*semigroup;* $k > 1$*. A notion of an* s-**associative** n-**groupoid** *was introduced by F.M. Sokhatsky (for example [5]).*

Definition 3. *Let* $k > 1,\ s \geq 1,\ n = k \cdot s + 1$ *and let* (Q, A) *be an* $n-$*groupoid. Then, we say that* (Q, A) *is a* P–**polyagroup of the type** $(s, n-1)$ *iff it is a* $Ps-$*associative* $n-$*groupoid and an* $n-$*quasigroup.*

A notion of a **polyagroup** was introduced by F.M. Sokhatsky (for example [6]).

2. Auxiliary propositions

Proposition 2 [10]. *Let* $n \geq 2$ *and let* (Q, A) *be an* $n-$*groupoid. Furthermore, let the* $< 1, n > -$*associative law hold in* (Q, A)*, and let for every* $a_1^n \in Q$ *there be* **at least one** $x \in Q$ *and* **at least one** $y \in Q$ *such that the following equalities* $A(a_1^{n-1}, x) = a_n$ *and* $A(y, a_1^{n-1}) = a_n$ *hold. Then, there are mappings* $\mathbf{e}$ *and* $^{-1}$ *respectively of the sets* Q^{n-2} *and* Q^{n-1} *into the set* Q *such that the following laws*

$$A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, x) = x,\ A(x, a_1^{n-2}, \mathbf{e}(a_1^{n-2})) = x,$$

$$A((a_1^{n-2}, x)^{-1}, a_1^{n-2}, x) = \mathbf{e}(a_1^{n-2}),\ A(x, a_1^{n-2}, (a_1^{n-2}, x)^{-1}) = \mathbf{e}(a_1^{n-2}),$$

$$A((a_1^{n-2},a)^{-1},a_1^{n-2},A(a,a_1^{n-2},x))=x \text{ and}$$
$$A(A(x,a_1^{n-2},a),a_1^{n-2},(a_1^{n-2},a)^{-1})=x$$

hold in the algebra $(Q,\{A,^{-1},\mathbf{e}\})$.
(See, also [11].)

Proposition 3. *Let* $k>1,\ s\geq 1,\ n=k\cdot s+1$ *and let* (Q,A) *be an* $n-$*groupoid. Also, let*

(a) *the* $<1,s+1>-$*associative* [$<(k-1)\cdot s+1,\ k\cdot s+1>-$*associative*] *law hold in the* (Q,A); *and*

(b) *for every* $x,y,a_1^{n-1}\in Q$ *the following implication holds*

$$A(x,a_1^{n-1})=A(y,a_1^{n-1})\Rightarrow x=y$$
$$[\ A(a_1^{n-1},x)=A(a_1^{n-1},y)\Rightarrow x=y\ ].$$

Then (Q,A) *is a* $Ps-$*associative* $n-$*groupoid.*

Sketch of the proof.

$A(A(x_1^n),x_{n+1}^{2n-1})=A(x_1^s,A(x_{s+1}^{s+n}),x_{s+n+1}^{2n-1})\Rightarrow A(y_1^s,A(A(x_1^n),x_{n+1}^{2n-1}),y_{s+1}^{n-1})$
$=A(y_1^s,A(x_1^s,A(x_{s+1}^{s+n}),x_{s+n+1}^{2n-1}),y_{s+1}^{n-1})\Rightarrow A(A(y_1^s,A(x_1^n),x_{n+1}^{2n-1-s}),x_{2n-s}^{2n-1},y_{s+1}^{n-1})$
$=A(A(y_1^s,x_1^s,A(x_{s+1}^{s+n}),x_{s+n+1}^{2n-1-s}),x_{2n-s}^{2n-1},y_{s+1}^{n-1})\Rightarrow A(y_1^s,A(x_1^n),x_{n+1}^{2n-1-s})$
$=A(y_1^s,x_1^s,A(x_{s+1}^{s+n}),x_{s+n+1}^{2n-1-s})$.
(See, also [10,11].) □

3. Results

Definition 4. *Let* $k>1,s\geq 1,\ n=k\cdot s+1$ *and let* (Q,A) *be a* $Ps-$*associative* $n-$*groupoid. We shall say that* (Q,A) *is a* **near–P–polyagroup (briefly: NP–polyagroup) of the type** $(s,n-1)$ *iff for every* $i\in\{t\cdot s+1|t\in\{0,1,\ldots,k\}\}$ *and for all* $a_1^n\in Q$ *there is exactly one* $x_i\in Q$ *such that the equality*

$$A(a_1^{i-1},x_i,a_i^{n-1})=a_n$$

holds.

Remark 4. *Every P–polyagroup of the type* $(s,n-1)$ *is an NP–polyagroup of the type* $(s,n-1)$.

Example 1. *Let* $(Q,\cdot)$ *be a group and let* α *be a mapping of the set* Q *into the set* Q. *Let, also, for each* $x_1^5\in Q$

$$A(x_1^5)\overset{def}{=}x_1\cdot\alpha(x_2)\cdot x_3\cdot\alpha(x_4)\cdot x_5.$$

Then (Q,A) *is an NP–polyagroup of the type (2,4). Moreover, if* α *is not a permutation of the set* Q, *then* (Q,A) *is not a* $5-$*quasigroup.*

Theorem 1. *Let* $k>1,\ s\geq 1,\ n=k\cdot s+1$ *and let* (Q,A) *be an* $n-$*groupoid. Then,* (Q,A) *is an* **NP–polyagroup of the type** $(s,n-1)$ *iff there are mappings* $^{-1}$ *and* $\mathbf{e}$ *respectively of the sets* Q^{n-1} *and* Q^{n-2} *into the set* Q *such that the following laws hold in the algebra* $(Q,\{A,^{-1},\mathbf{e}\})$ [*of the type* $<n,n-1,n-2>$]:

(i) $A(A(x_1^n), x_{n+1}^{2n-1}) = A(x_1^s, A(x_{s+1}^{s+n}), x_{s+n+1}^{2n-1})$,

(ii) $A(x, a_1^{n-2}, \mathbf{e}(a_1^{n-2})) = x$ *and*

(iii) $A(a, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) = \mathbf{e}(a_1^{n-2})$.

[See, also Proposition 1*,* Remark 2 *and* Theorem 2*]*

Proof. 1)$\Rightarrow$: Let (Q, A) be an NP–polyagroup of the type $(s, n-1)$. Then, by *Proposition 2*, there is an algebra $(Q, \{A, ^{-1}, \mathbf{e}\})$ of the type $< n, n-1, n-2 >$ in which the laws $(i) - (iii)$ hold.

2)$\Leftarrow$: Let $(Q, \{A, ^{-1}, \mathbf{e}\})$ be an algebra of the type $< n, n-1, n-2 >$ in which the laws $(i) - (iii)$ hold. We prove respectively that in that case the following statements hold:

1° For every $x, y, a_1^{n-1} \in Q$ the following implication holds

$$A(x, a_1^{n-1}) = A(y, a_1^{n-1}) \Rightarrow x = y.$$

2° (Q, A) is a $Ps-$associative $n-$groupoid.

3° $(\forall a_i \in Q)_1^{n-2}(\forall x \in Q)A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, x) = x$.

4° $(\forall a_i \in Q)_1^{n-2}(\forall x \in Q)A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, a) = \mathbf{e}(a_1^{n-2})$.

5° For every $x, y, a_1^{n-1} \in Q$ the following implication holds

$$A(a_1^{n-1}, x) = A(a_1^{n-1}, y) \Rightarrow x = y.$$

6° For every $x, y, a_1^{n-1} \in Q$ and for all $t \in \{1, \dots, k-1\}$ the following implication holds

$$A(a_1^{t\cdot s}, x, a_{t\cdot s+1}^{n-1}) = A(a_1^{t\cdot s}, y, a_{t\cdot s+1}^{n-1}) \Rightarrow x = y.$$

7° For every $i \in \{t \cdot s + 1 | t \in \{0, 1, \dots, k\}\}$ and for all $a_1^n \in Q$ there is **at least one** $x_i \in Q$ such that the following equality holds $A(a_1^{i-1}, x_i, a_i^{n-1}) = a_n$.

The proof of the statement of 1° :

By $n \geq 3$ $(: n = k \cdot s + 1, k > 1, s \geq 1)$, we conclude that the following series of implications holds:

$A(x, a_1^{s-1}, a, a_s^{n-2}) = A(y, a_1^{s-1}, a, a_s^{n-2}) \Rightarrow$

$A(A(x, a_1^{s-1}, a, a_s^{n-2}), a_1^{s-1}, \mathbf{e}(a_s^{n-2}, a_1^{s-1}), \overset{n-2-s}{a}, \mathbf{e}(a_1^{s-1}, \overset{n-2-s+1}{a})) =$

$A(A(y, a_1^{s-1}, a, a_s^{n-2}), a_1^{s-1}, \mathbf{e}(a_s^{n-2}, a_1^{s-1}), \overset{n-2-s}{a}, \mathbf{e}(a_1^{s-1}, \overset{n-2-s+1}{a})) \Rightarrow$

$A(x, a_1^{s-1}, A(a, a_s^{n-2}, a_1^{s-1}, \mathbf{e}(a_s^{n-2}, a_1^{s-1})), \overset{n-2-s}{a}, \mathbf{e}(a_1^{s-1}, \overset{n-2-s+1}{a})) =$

$A(y, a_1^{s-1}, A(a, a_s^{n-2}, a_1^{s-1}, \mathbf{e}(a_s^{n-2}, a_1^{s-1})), \overset{n-2-s}{a}, \mathbf{e}(a_1^{s-1}, \overset{n-2-s+1}{a})) \Rightarrow$

$A(x, a_1^{s-1}, a, \overset{n-2-s}{a}, \mathbf{e}(a_1^{s-1}, \overset{n-2-s+1}{a})) =$

$A(y, a_1^{s-1}, a, \overset{n-2-s}{a}, \mathbf{e}(a_1^{s-1}, \overset{n-2-s+1}{a})) \Rightarrow x = y.$

The proof of the statement of 2° :

By (i), $1°, n \geq 3$ and by *Proposition 3*, we conclude that (Q, A) is a $Ps-$associative $n-$groupoid.

The proof of the statement of 3° :

$A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, a) = b \Rightarrow A(A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, a), a_1^{n-2}, (a_1^{n-2}, a)^{-1})$

$= A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Rightarrow A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, A(a, a_1^{n-2}, (a_1^{n-2}, a)^{-1}))$

$= A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Rightarrow A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, \mathbf{e}(a_1^{n-2}))$

$= A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Rightarrow \mathbf{e}(a_1^{n-2}) = A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Rightarrow$

$A(a, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) = A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Rightarrow a = b$

[: $2°, (iii), (ii), (iii), 1°$*]*.

The proof of the proof of 4° :

$A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, a) = b \Rightarrow A(A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, a), a_1^{n-2}, (a_1^{n-2}, a)^{-1})$

$= A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Rightarrow A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, A(a, a_1^{n-2}, (a_1^{n-2}, a)^{-1}))$

$= A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Rightarrow A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, \mathbf{e}(a_1^{n-2}))$

$= A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Rightarrow A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, (a_1^{n-2}, a)^{-1})$

$= A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Rightarrow \mathbf{e}(a_1^{n-2}) = b$

[: $2°, (iii), (ii), 3°, 1°$*]*.

The proof of the statement of 5° :

Since the $< 1, n > -$associative law *[:2°]* as well as the statements 4° and 3° hold in (Q, A), we conclude that for every $x, y, a \in Q$ and for every sequence a_1^{n-2} over Q the following series of implication holds:

$A(a, a_1^{n-2}, x) = A(a, a_1^{n-2}, y) \Rightarrow A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, A(a, a_1^{n-2}, x))$

$= A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, A(a, a_1^{n-2}, y)) \Rightarrow A(A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, a), a_1^{n-2}, x)$

$= A(A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, a), a_1^{n-2}, y) \Rightarrow A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, x)$

$= A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, y) = x = y.$

The proof of the proof of 6° :

$A(a_1^{t\cdot s}, x, a_{t\cdot s+1}^{k\cdot s}) = A(a_1^{t\cdot s}, x, a_{t\cdot s+1}^{k\cdot s}) \Rightarrow A(b_1^{(k-t)\cdot s}, A(a_1^{t\cdot s}, x, a_{t\cdot s+1}^{k\cdot s}), b_{(k-t)\cdot s+1}^{k\cdot s})$

$= A(b_1^{(k-t)\cdot s}, A(a_1^{t\cdot s}, y, a_{t\cdot s+1}^{k\cdot s}), b_{(k-t)\cdot s+1}^{k\cdot s}) \Rightarrow$

$A(A(b_1^{(k-t)\cdot s}, a_1^{t\cdot s}, x), a_{t\cdot s+1}^{k\cdot s}, b_{(k-t)\cdot s+1}^{k\cdot s}) = A(A(b_1^{(k-t)\cdot s}, a_1^{t\cdot s}, y), a_{t\cdot s+1}^{k\cdot s}, b_{(k-t)\cdot s+1}^{k\cdot s})$

$\Rightarrow A(b_1^{(k-t)\cdot s}, a_1^{t\cdot s}, x) = A(b_1^{(k-t)\cdot s}, a_1^{t\cdot s}, y) \Rightarrow x = y$

[: $2°, 1°, 5°$*]*.

The proof of the proof of 7° :

a) $t = 0$: $A(x, a_1^{n-2}, a) = b \Leftrightarrow$

$A(A(x, a_1^{n-2}, a), a_1^{n-2}, (a_1^{n-2}, a)^{-1}) = A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Leftrightarrow$

$A(x, a_1^{n-2}, A(a, a_1^{n-2}, (a_1^{n-2}, a)^{-1})) = A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Leftrightarrow$

$A(x, a_1^{n-2}, \mathbf{e}(a_1^{n-2})) = A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1}) \Leftrightarrow$

$x = A(b, a_1^{n-2}, (a_1^{n-2}, a)^{-1})$

[: $2°, (iii), (ii)$*]*.

b) $t = k:$ $A(a, a_1^{n-2}, x) = b \Leftrightarrow$
$A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, A(a, a_1^{n-2}, x)) = A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, b) \Leftrightarrow$
$A(A(a_1^{n-2}, a)^{-1}, a_1^{n-2}, a), a_1^{n-2}, x)) = A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, b) \Leftrightarrow$
$A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, x) = A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, b) \Leftrightarrow$
$x = A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, b)$

[:$2°, 4°, 3°$*].*

c) $0 < t < k:$ $A(a_1^{t\cdot s}, x, a_{t\cdot s+1}^{k\cdot s}) = b \Leftrightarrow A(b_1^{(k-t)\cdot s}, A(a_1^{t\cdot s}, x, a_{t\cdot s+1}^{k\cdot s}), b_{(k-t)\cdot s+1}^{k\cdot s})$
$= A(b_1^{(k-t)\cdot s}, b, b_{(k-t)\cdot s+1}^{k\cdot s}) \Leftrightarrow$
$A(A(b_1^{(k-t)\cdot s}, a_1^{t\cdot s}, x), a_{t\cdot s+1}^{k\cdot s}, b_{(k-t)\cdot s+1}^{k\cdot s})$
$= A(b_1^{(k-t)\cdot s}, b, b_{(k-t)\cdot s+1}^{k\cdot s})$

[:$6°, 2°$*].* □

By a simple imitation of the proof of *Theorem 1* it is possible to prove that the following proposition holds:

Theorem 2. *Let* $k > 1,\ s \geq 1,\ n = k \cdot s + 1$ *and let* (Q, A) *be an* $n-$*groupoid. Then,* (Q, A) *is an* **NP–polyagroup of the type** $(s, n-1)$ *iff there are mappings* $^{-1}$ *and* $\mathbf{e}$ *respectively of the sets* Q^{n-1} *and* Q^{n-2} *into the set* Q *such that the following laws hold in the algebra* $(Q, \{A, ^{-1}, \mathbf{e}\})$ *[of the type* $< n, n-1, n-2 >$*]:*

$(\overline{i})$ $A(x_1^{(k-1)\cdot s}, A(x_{(k-1)\cdot s+1}^{(k-1)\cdot s+n}), x_{(k-1)\cdot s+n+1}^{2n-1}) = A(x_1^{k\cdot s}, A(x_{k\cdot s+1}^{2n-1})),$

$(\overline{ii})$ $A(\mathbf{e}(a_1^{n-2}), a_1^{n-2}, x) = x$ *and*

$(\overline{iii})$ $A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, a) = \mathbf{e}(a_1^{n-2}).$

Similarly, it is possible to prove that the following proposition holds. (See, also [10,11].)

Theorem 3. *Let* $k > 1$ $s \geq 1,\ n = k \cdot s + 1$ *and let* (Q, A) *be an* $n-$*groupoid. Then,* (Q, A) *is an* **NP–polyagroup of the type** $(s, n-1)$ *iff there are mappings* $^{-1}$ *and* $\mathbf{e}$ *respectively of the sets* Q^{n-1} *and* Q^{n-2} *into the set* Q *such that the following laws hold in the algebra* $(Q, \{A, ^{-1}, \mathbf{e}\})$ *[of the type* $< n, n-1, n-2 >$*]:*

(1) (i) *from* Theorem 1 **or** $(\overline{i})$ *from* Theorem 2*;*

(2) (ii) *from* Theorem 1*; and*

(3) $A((a_1^{n-2}, a)^{-1}, a_1^{n-2}, A(a, a_1^{n-2}, x)) = x.$

Theorem 4. *Let* $k > 1$ $s \geq 1,\ n = k \cdot s + 1$ *and let* (Q, A) *be an* $n-$*groupoid. Then,* (Q, A) *is an* **NP–polyagroup of the type** $(s, n-1)$ *iff there are mappings* $^{-1}$ *and* $\mathbf{e}$ *respectively of the sets* Q^{n-1} *and* Q^{n-2} *into the set* Q *such that the following laws hold in the algebra* $(Q, \{A, ^{-1}, \mathbf{e}\})$ *[of the type* $< n, n-1, n-2 >$*]:*

$(\overline{1})$ (i) *from* Theorem 1 **or** $(\overline{i})$ *from* Theorem 2*;*

$(\overline{2})$ $(\overline{ii})$ *from* Theorem 2*; and*

$(\overline{3})$ $A(A(x, a_1^{n-2}, a), a_1^{n-2}, (a_1^{n-2}, a)^{-1}) = x.$

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