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A MULTI-SERVER QUEUING MODEL STUDY OF SPECIFIC COST RATIO IN A PORT

Romeo Meštrović, Branislav Dragović, Nenad Dj. Zrnić, Maja Škurić

Original scientific paper

In this paper we establish an analytic method for determining the optimal number of berths in a port with finite waiting areas modelled by $M/M/n_b/k$ queue in the sense of minimization of the related specific cost ratio. Using the general formula for specific cost ratio, we derive a related expression of the $M/M/n_b/k$ queue. In particular, we present the optimal intervals for specific cost ratio and optimal number of berths under condition that the number of waiting areas is fixed. Related numerical results and computational experiments are presented.

Keywords: *finite waiting areas, $M/M/n_b/k$ queue, optimal interval, port, specific cost ratio, traffic intensity*

Studija specifičnog omjera troškova u luci za višekanalni model redova čekanja

Izvorni znanstveni članak

U ovom radu dajemo analitičku metodu za određivanje optimalnog broja vezova u luci s ograničenim brojem mjesta u redu čekanja predstavljenu $M/M/n_b/k$ modelom u smislu minimizacije vrijednosti pripadnog specifičnog omjera troškova. Koristeći opću formulu za specifični omjer troškova, izvodimo odgovarajući izraz za taj omjer u odnosu na $M/M/n_b/k$ model. Posebno, dajemo optimalne intervale za specifični omjer troškova i optimalan broj vezova pod uvjetom da je broj mjesta u redu čekanja ograničen. Predstavljeni su numerički rezultati i kompjuterski eksperimenti.

Ključne riječi: *intenzitet prometa, luka, $M/M/n_b/k$ model, ograničeni broj mjesta u redu čekanja, optimalan interval, specifični omjer troškova*

1 Introduction

In past decades, there were numerous investigations of queuing systems in port. The authors mainly tried to give responses about port capacity, total cost estimation, waiting time of ships in queue, etc. In order to determine the optimal port system capacity, some criteria should be taken into account such as: traffic intensity, average number of ships in queue and optimal number of berths with related optimal intervals (the so called ranges of optimal server capacities; see e.g. [1]). Similarly, some specific notion was studied in [1 ÷ 5]. They predict specific cost ratio in port and ranges of optimum berth capacity. The optimal number and capacity of servers were determined by $M^{X=\bar{a}}/M/c(\infty)$ queue in [1]. A methodology for analyzing the congestion at berths from both economic and engineering points of view is explained in [2] while in [3] convenient graphs related to optimum berth capacity curves are obtained. In [4] a method for determining the optimum allocation and size of ports considering the total inland transportation and port costs is presented. The analytical calculations of specific cost ratio for various distributions of container group arrivals at yard are presented in [5].

In [6, 7] the number of berths is determined in order to minimize the total usage cost. The handling rate is optimized by minimization of the operations cost in [8] while in [9] the assigning priorities and problem of minimization of the average cost of waiting in a port queuing system are solved. Investigations [6 ÷ 8] were extended in [10] regarding optimal cargo-handling rates. In [11] was ascertained that short-term improvements can be achieved by minimizing the total system costs of both ports and inland networks. Many evaluation indexes to estimate the initial number of berths are used in [12]. The non-dimensional evaluation indicator for various schemes of cost items at container terminal is explained in [13]. The optimal size and location of public logistics terminals is described in [14]. In [15] the improvement of container terminal operations for two stages of the operation process was studied. The determination of a density function of supply restoration duration for the application of Markov state space models is presented in [16].

In [17] the explicit expressions for specific cost ratio in a port for $M/M/1/k$ queue are derived and compared with $M/M/n_b$ queues. A short algorithm for minimization of specific cost ratio for $M/M/1/k$ queue is given in [18]. Although there have been aforementioned studies for estimating the port performances and specific cost ratio by using analytical approaches, this methodology should also be applied for some other queuing models such as $M/M/n_b/k$. In this manner, the extension of the current studies ([1 ÷ 5], [17, 18]) needs to be done in respect to the analytical modelling of ports amidst the trend of operations and ranges of optimum berth capacity. The paper tries to help decision making in port management and better understanding the relative merits of new strategy and decide whether it is applicable in ports.

It should also be pointed out, that there are a few overview concepts of new and old results of queuing models for port performance evaluation literature [19 ÷ 22], where appropriate surveys of application of queuing models to port study were done.

The total daily cost of a port queuing system with certain number of berths (servers) is defined as a sum of total service cost per day, total ship cost per day and total marginal cost per day. The *specific cost ratio* is defined as a ratio of total cost for considered period of a port queuing system and average daily cost of a ship in port. In previous investigated papers (e.g. [8 ÷ 10]), the analysis of total cost for different port systems is extensively studied while the specific cost ratio is investigated in studies mentioned before.

In this paper we establish an analytic method for determining optimal number of berths in a port with finite waiting areas modelled by $M/M/n_b/k$ queue in the sense of minimization of the related specific cost ratio. We show that the specific cost ratio $R = R_{kn_b}(\theta)$ may be written as a sum $R_{kn_b}(\theta) = rn_b + h_{kn_b}(\theta)$, where r is a ratio between daily cost of a berth and daily cost of a ship, and $h_{kn_b}(\theta)$ is a function of traffic intensity, θ, number of waiting areas, k, and number of berths in port, n_b. Furthermore, we compute "boundary" values of θ for this set of models which allow us to determine the optimal number of berths in port.

The paper is organized as follows. In Section 2 we establish a notion of specific cost ratio for arbitrary port queuing model. We also determine a boundary behaviour of functions $h_{kn_b}(\theta)$ and $R_{kn_b}(\theta)$ as $\theta \to n_b - 0$. Section 3 provides numerical examples for specific cost ratios of different $M/M/n_b/k$ queues. Using the formula for $R_{kn_b}(\theta)$, we present the optimal intervals of specific cost ratio. Also, we consider a set of $M/M/n_b/5$ queues with $n_b = 1, 2, ..., 5$. The final Section 4 gives concluding remarks.

2 Notion of specific cost ratio and related formula for $M/M/n_b/k$ queue

We consider a finite waiting areas multi-server queue with identical and independent cargo-handling capacities: $M/M/n_b/k$ where k denotes the number of waiting areas. The ships arrive in a port according to a time-homogeneous Poisson process with mean arrival rate λ. The port has the n_b berths for the service. The berths have independent, exponentially distributed service times $1/\mu$ (the mean cargo handling rate per berth is μ). The queue discipline is first come first served. In a port queuing system that does not serve each ship immediately upon arrival, a ship will attempt to arrive at a time that will minimize the expected queue length. Then the average number of ships waiting in queue, L_q, is generally a function of λ and μ.

Following [2-4] the *specific cost ratio* is defined as

$$R = \frac{AC}{c_s} = \frac{\lambda}{c_s} AC_s, \tag{1}$$

where c_s is daily cost of a ship, AC_s is a total daily cost per ship for considered port system, including a marginal cost, and $AC = \lambda \cdot AC_s$ is a total cost for considered period (in which λ ships arrive in a port).

Following [3, 4] and using a standard differential calculus with the notation $c_b/c_s = r$, where c_b is a daily cost of a berth, it can be derived that

$$R = rn_b + \left(L_q + \frac{2\lambda}{\mu}\right) + \lambda \frac{\mathrm{d}L_q}{\mathrm{d}\lambda}, \tag{2}$$

or after substitution $\lambda/\mu = \theta$ (θ is called the traffic intensity) into (1), we find that

$$R = rn_b + L_q + 2\theta + \theta \frac{\mathrm{d}L_q}{\mathrm{d}\theta}. \tag{3}$$

Generally, in port systems it can be of interest to minimize the specific cost ratio (see [1 ÷ 4, 17, 18]). Here we consider a related problem for the $M/M/n_b/k$ queue. It is well known that (see e.g. [23])

$$p_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} p_0 & \text{for } n = 1,2,...,n_b \\ \dfrac{(\lambda/\mu)^n}{n_b! n_b^{\,n-n_b}} & \text{for } n = n_b,\ n_b+1,...,k \\ & \text{for } n > k \\ 0 & \end{cases} \tag{4}$$

where p_n is a probability that there are n ships in a port. In particular, a probability that there are no ships in a port is equal to

$$p_0 = \left(\sum_{n=0}^{n_b} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^{n_b}}{n_b!} \sum_{n=n_b+1}^{k} \left(\frac{\lambda}{n_b \mu} \right)^{n-n_b} \right)^{-1}. \tag{5}$$

From the Eqs. (4) and (5) it can be deduced that [23]

$$L_q = \frac{n_b^{\,n_b} \rho^{n_b+1}}{n_b!(1-\rho)^2} (1 - \rho^{k-n_b} - (k-n_b)(1-\rho)\rho^{k-n_b}) p_0, \tag{6}$$

where $\rho = \lambda/(n_b \mu)$ is the utilization factor. Since $n_b \rho = \lambda/\mu = \theta$, so that $\rho = \theta/n_b$, then taking this into Eq. (6) immediately gives the following expression for L_q which is useful for our computational purposes:

$$L_q = \frac{\theta^{n_b+1}}{(n_b-1)!(n-\theta)^2} \left(1 - \left(\frac{\theta}{n_b}\right)^{k-n_b} - (k-n_b)\left(1 - \frac{\theta}{n_b}\right)\left(\frac{\theta}{n_b}\right)^{k-n_b}\right) p_0. \tag{7}$$

Differentiating Eq. (7) by θ, and substituting this into Eq. (3) with the notation $R = R_{kn_b}(\theta) = rn_b + h_{kn_b}(\theta)$, where

$$h_{kn_b}(\theta) = L_q + 2\theta + \theta \frac{\mathrm{d}L_q}{\mathrm{d}\theta}, \tag{8}$$

immediately gives

$$R_{kn_b}(\theta) = rn_b + L_q + 2\theta + \theta \frac{\mathrm{d}L_q}{\mathrm{d}\theta}. \tag{9}$$

In Tab. 1, we give expressions for $h_{kn_b}(\theta)$ in dependence of k, n_b and θ for single server queuing system in port and system with two berths where Eqs.

(10) and (16) are general expressions for $h_{k,1}(\theta)$ and $h_{k,2}(\theta)$ with arbitrary $k \geq 1$ and $k \geq 2$, respectively, while Eqs. (11) ÷ (15) and (17) are special cases of $h_{kn_b}(\theta)$ with one or two berths and different size of system capacity. All these formulae are derived in *Mathematica* 8.

Table 1 Expressions for $h_{kn_b}(\theta)$ for various values of k and $n_b = 1, 2$

n_b	k	Expressions for $h_{kn_b}(\theta)$
1	for arbitrary k, $k \geq 1$	$h_{k,1}(\theta) = (\theta(2-\theta+\theta^2-k\theta^k-3\theta^{1+k}+\theta^{2+k}+k\theta^{2+k}))/((1-\theta)(1-\theta^{1+k}))$ (10)
	1	$h_{1,1}(\theta) = 2\theta$ (11)
	2	$h_{2,1}(\theta) = (\theta(2+3\theta+3\theta^2))/(1+\theta+\theta^2)$ (12)
	3	$h_{3,1}(\theta) = (\theta(2+\theta+4\theta^2))/(1+\theta^2)$ (13)
	4	$h_{4,1}(\theta) = (\theta(2+3\theta+5\theta^2+7\theta^3+5\theta^4))/(1+\theta+\theta^2+\theta^3+\theta^4)$ (14)
	5	$h_{5,1}(\theta) = (\theta(1-\theta+2\theta^2)(2+3\theta+3\theta^2))/((1-\theta+\theta^2)(1+\theta+\theta^2))$ (15)
2	for arbitrary k, $k \geq 2$	$h_{k,2}(\theta) = \dfrac{\theta(2^{3+k}-2^k\theta^2+2^k\theta^3+2^k\theta^k-2^k k\theta^k-8\theta^{1+k}-2^k\theta^{1+k}+4\theta^{2+k}-2^{1+k}\theta^{2+k}+2^k k\theta^{2+k})}{(2-\theta)(2^{1+k}+2^k\theta-2\theta^{1+k})}$ (16)
	3	$h_{3,2} = (2\theta(16-2\theta^2-2\theta^3-4\theta^4+3\theta^5))/((2-\theta)^2(4+4\theta+2\theta^2+\theta^3))$ (17)

2.1 Boundary behaviour of functions $h_{kn_b}(\theta)$ and $R_{kn_b}(\theta)$

In view of the fact that for a particular fixed pair (n_b, k) the function $\theta \to h_{kn_b}(\theta)$ defined by Eq. (8) is increasing on $\theta \in (0, n_b)$, it can be of interest to determine upper bound of $h_{kn_b}(\theta)$ as $\theta \to n_b - 0$. For each particular $M/M/n_b/k$ queue with any fixed $k \geq n_b$ applying L'Hospital's rule to the Eq. (10) in Tab. 1, we find that $\lim\limits_{\theta\to 1} h_{k,1}(\theta) = \dfrac{k^2+k+2}{k+1}$ for all $k \geq 1$, which is asymptotically equal to k as $k \to \infty$. This means that for $\theta \approx 1$, or equivalently $\rho \approx 1$, we have $R_{k,1}(\theta) = r + \dfrac{k^2+k+2}{k+1}$ for all $k \geq 1$, and for such a $\theta \approx 1$ there holds $R_{k1}(\theta) \approx r + k$ for sufficiently large k.

Similarly, by applying Eq. (16) we find that $\lim\limits_{\theta\to 2} h_{k,2}(\theta) = \dfrac{2(3\cdot k2^k - 2^{k+1}+4)}{2^{k+1}-1}$ for all $k \geq 2$, so that $h_{k,2} \approx 3k-1$ for $\theta \approx 2$ for sufficiently large k. It follows that when $\theta \approx 2$ there holds $R_{k,2}(\theta) \approx 2r + 3k - 1$ for all sufficiently large k. Finally, by Eqs. (13), (14) and (15), if $k = n_b$ we obtain $\lim\limits_{\theta\to n_b} h_{n_b n_b}(\theta) = 2n_b$ for all $n_b = 3, 4, 5$. It follows that when $\theta \approx n_b$ there holds $R_{n_b n_b}(\theta) \approx (r+2)n_b$ for all $n_b = 3, 4, 5$. Also, Eqs. (13), (14) and (15) for $k > n_b$ give $\lim\limits_{\theta\to n_b} h_{kn_b}(\theta) = +\infty$ for all $n_b = 3, 4, 5$ and $k > n_b$. It follows that $\lim\limits_{\theta\to n_b} R_{kn_b}(\theta) = +\infty$ for all $n_b = 3, 4, 5$ and $k > n_b$.

3 Numerical examples

In this Section, first we perform the values of specific cost ratio using Eq. (9), where the expressions for functions $h_{kn_b}(\theta)$ ($k = 1, 2, \ldots, 7$ and $n_b = 1, 2, \ldots, 6$) are calculated in *Mathematica* 8. We start considering the six cases: $(n_b, k) \in \{(1,5), (2,5), (3,5), (4,5), (5,7), (6,7)\}$ with a fixed value $r = 0{,}6$. The first four of these pairs are suitable for our further computational analysis.

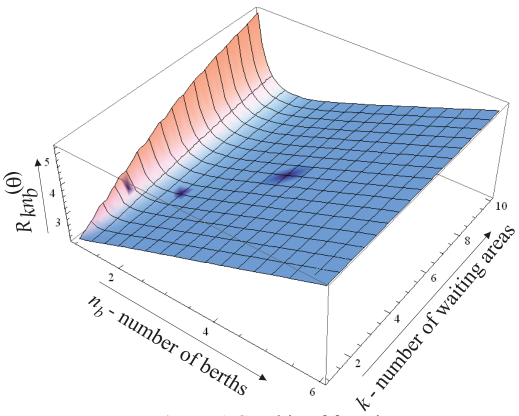

Figure 1 Graphic of function $(n_b, k) \to R_{kn_b}(\theta) = 0{,}6n_b + h_{kn_b}(\theta)$ with a fixed $\theta = 0{,}75$, $k = 1, 2, \ldots, 10$ and $n_b = 1, 2, \ldots, 6$

If we assume that value of traffic intensity θ is 0,75, $k = 1, 2, \ldots, 10$ and $n_b = 1, 2, \ldots, 6$, then Fig. 1 gives the graphic of function $(n_b, k) \to R_{kn_b}(\theta)$.

From Tab. 2 we see that the optimal number of berths with $k = 5$ for each of nine values of $\theta = 0{,}15 - 0{,}95$ with the step size 0,10 is equal to 1.

Graphics of functions $R_{kn_b}(\theta) = 0{,}6n_b + h_{kn_b}(\theta)$ with $(n_b, k) \in \{(1,5), (2,5), (3,5), (4,5), (5,7), (6,7)\}$ presented in Fig. 2 suggest a simple analytic method to determine the optimal intervals for specific cost ratio calculating intersections of consecutive curves $R_{kn_b}(\theta)$.

Table 2 Values of specific cost ratio $R = R_{kn_b}(\theta) = 0{,}6n_b + h_{kn_b}(\theta)$ for the $M/M/n_b/k$ queue ($r = 0{,}6$)

	$(n_b, k) \in \{(1,5),(2,5),(3,5),(4,5),(5,7),(6,7)\}$					
θ	(1,5)	(2,5)	(3,5)	(4,5)	(5,7)	(6,7)
0,15	0,930	1,501	2,100	2,700	3,300	3,900
0,25	1,202	1,705	2,300	2,900	3,500	4,100
0,35	1,541	1,913	2,500	3,100	3,700	4,300
0,45	1,968	2,126	2,702	3,300	3,900	4,500
0,55	2,497	2,343	2,903	3,500	4,100	4,700
0,65	3,131	2,555	3,104	3,700	4,300	4,900
0,75	3,859	2,756	3,304	3,900	4,500	5,100
0,85	4,658	2,938	3,503	4,100	4,700	5,300
0,95	5,502	3,108	3,701	4,300	4,900	5,500

Each of fifteen values of θ in Tab. 3 is obtained in *Mathematica* 8 as a positive solution of algebraic equations $R_{kn_b}(\theta) = R_{k'n_b'}(\theta)$ obtained by using Eqs. (7) and (9), with all different pairs $(n_b, k), (n_b', k')$ that belong to the set $\{(1,5),(2,5),(3,5),(4,5),(5,7),(6,7)\}$. A positive solution of each of these equations is unique under condition that it is less than or equal to $\min\{n_b, n_b'\}$ (see Fig. 2). In fact, these solutions can be considered as "boundary" values for determining the optimal ranges (intervals) for specific cost ratio which correspond to the set of the $M/M/n_b/5$ queue with $n_b = 1, 2, ..., 5$.

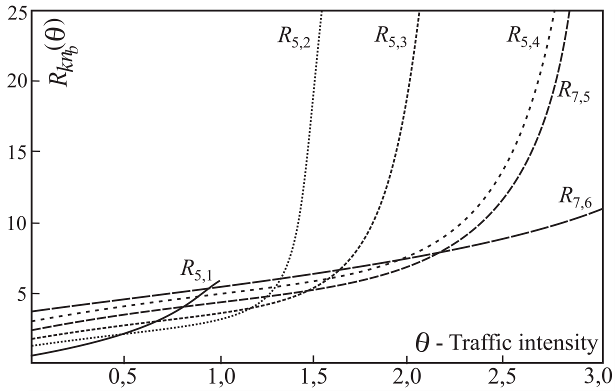

Figure 2 Graphics of functions $R_{kn_b}(\theta) = 0{,}6n_b + h_{kn_b}(\theta)$ with $(n_b, k) \in \{(1,5),(2,5),(3,5),(4,5),(5,7),(6,7)\}$ and $\theta \in (0,3)$

Table 3 Positive solutions of equations $R_{kn_b}(\theta) = R_{k'n_b'}(\theta)$ with pairs $(n_b, k), (n_b', k')$

	$R_{1,5}(\theta)$	$R_{2,5}(\theta)$	$R_{3,5}(\theta)$	$R_{4,5}(\theta)$	$R_{5,7}(\theta)$	$R_{6,7}(\theta)$
$R_{1,5}(\theta)$		**0,505**	0,644	0,757	0,857	0,950
$R_{2,5}(\theta)$			**1,218**	1,264	1,297	1,321
$R_{3,5}(\theta)$				**1,504**	1,596	1,649
$R_{4,5}(\theta)$					3,267	2,182
$R_{5,7}(\theta)$						2,010

From Fig. 2 we see that for any fixed value of traffic intensity θ (on related range), $R_{5n_b}(\theta) < R_{5(n_b+1)}(\theta)$ for each $n_b = 1, 2, ..., 4$. A computation shows that the analogous inequalities are probably satisfied for all values $k \geq 2$ and $n_b \geq 1$ with $n_b \leq k$, that is, $R_{kn_b}(\theta) < R_{k(n_b+1)}(\theta)$ for each $n_b = 1, 2, ..., k-1$ and $\theta \in (0, n_b)$.

However, conditions on other port performances demand for a number of berths to have some lower bound, and also for a related number of waiting areas to be sufficiently large. In other words, under additional conditions, for many port systems must be satisfied

$$n_b \geq n_{b0} \text{ and } k \geq k_0 . \tag{18}$$

Hence, in order to study a minimization of specific cost ratio with respect to $M/M/n_b/k$ queues, it is natural to assume that both conditions of Eq. (18) are satisfied with some fixed n_{b0} and k_0.

For example, using numerical results from Tab. 3 and the additional result that the positive root of the equation $R_{4,5}(\theta) = R_{5,5}(\theta)$ is 2,020, we can determine the optimal intervals for optimal numbers of berths in the previous sense, under condition that $k = 5$. This is given in Tab. 4.

Table 4 The optimal intervals for specific cost ratio $R_{5n_b}(= 0{,}6n_b + h_{5n_b}(\theta))$ for the set of the $M/M/n_b/5$ queues, $n_b = 1,2,3,4,5$

The optimal intervals for θ	The optimal number of berths
(0; 0,505)	1
(0,505; 1,218)	2
(1,218; 1,504)	3
(1,504; 2,020)	4
(2,020; 5)	5

Remark: We see from Fig. 2 that all the bounds of optimal intervals given in Tab. 4 are obtained as intersections of pairs of consecutive curves $R_{5,n_b}(\theta)$ and $R_{5,n_b+1}(\theta)$ with $n_b = 1, 2, ..., 4$. The values of $R_{kn_b}(\theta)$ from Tab. 3 are obtained by using Eqs. (7) and (9) in the dependence of different values for the traffic intensity $\theta \in \{0{,}15; 0{,}25; ...; 0{,}95\}$. It is evident that optimal values of $R_{kn_b}(\theta)$ always correspond to the first column of this Tab. 4, that is, to the minimal number of berths, $n_b = 1$.

Table 5 Values of specific cost ratio $R = R_{kn_b}(\theta) = 0{,}6n_b + h_{kn_b}(\theta)$ for the $M/M/n_b/k$ queue ($r = 0{,}6$)

θ	$(n_b,k) \in \{(1,2),(2,3),(3,4),(4,5),(5,6),(6,7)\}$					
	(1,2)	(2,3)	(3,4)	(4,5)	(5,6)	(6,7)
0,15	0,922	1,501	2,100	2,700	3,300	3,900
0,25	1,156	1,703	2,300	2,900	3,500	4,100
0,35	1,412	1,906	2,500	3,100	3,700	4,300
0,45	1,678	2,111	2,701	3,300	3,900	4,500
0,55	1,953	2,314	2,901	3,500	4,100	4,700
0,65	2,236	2,516	3,101	3,700	4,300	4,900
0,75	2,526	2,714	3,302	3,900	4,500	5,100
0,85	2,820	2,908	3,501	4,100	4,700	5,300
0,95	3,117	3,101	3,700	4,300	4,900	5,500

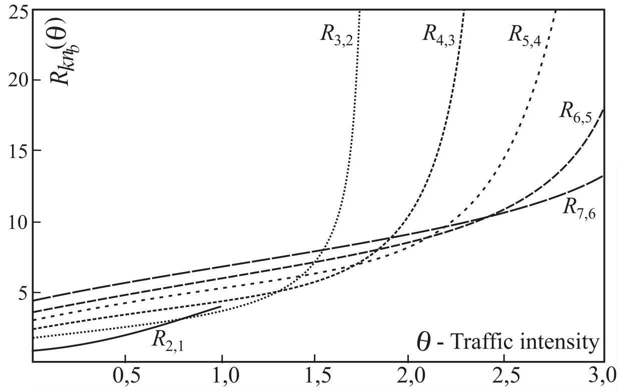

Figure 3 Graphics of functions $R_{kn_b}(\theta) = 0{,}6n_b + h_{kn_b}(\theta)$ with $(n_b,k) \in \{(1,2),(2,3),(3,4),(4,5),(5,6),(6,7)\}$ and $\theta \in (0,3)$

Furthermore, it can be of interest to compare specific cost ratios for various $M/M/n_b/k$ queues satisfying $k - n_b = 1$, that is, models with one waiting space in queue. In this case it can be shown that by increasing n_b (and consequently, also k), related values $R_{kn_b}(\theta)$ will also increase (see Tab. 5 and Fig. 3). Similarly, using Fig. 3 and computing positive solutions of five equations $R_{n_b+1,n_b}(\theta) = R_{n_b+2,n_b+1}(\theta)$ with $n_b = 1, 2, ..., 5,$ we get the following results in Tab. 6.

Table 6 The optimal intervals for specific cost ratio $R_{n_b+1,n_b}(= 0{,}6n_b + h_{n_b+1,n_b}(\theta))$ for the set of the $M/M/n_b/(n_b+1)$ queues, $n_b = 1, 2, ..., 5$

The optimal intervals for θ	The optimal number of berths
(0; 0,935)	1
(0,935; 1,392)	2
(1,392; 1,732)	3
(1,732; 2,073)	4
(2,073; 4,502)	5
(4,502; 6)	6

The results about finding optimal intervals for traffic intensity depending on proposed number of berths in Tab. 6 serve as a good foundation in developing a strategy and practical application in ports. In fact, traffic intensity of ships is presented here as one of the most important parameters in defining the specific cost ratio as well as getting supposed for optimal number of berths.

Port operators should take into account information about the ships arrivals and service time to give further analysis about the number of finite waiting areas. It is very important because for predicted ships arrivals the number of occupied berths in port can be determined. Meanwhile, applying this methodology, it gives the possibility for more accurate approach in employing berths for already known traffic intensity.

4 Conclusion

Queuing modelling approach to analyse the specific cost ratio has been and would be very useful for understanding various kinds of port operations and analysing them in a view to improve the port performance. Considered model provides the possibility to assume and solve more realistic problems (incorporating real data from port) in an acceptable way.

In this paper, we show that specific cost ratio is a function of daily cost of a ship and berth ratio, number of berths, average number of ships waiting in queue and traffic intensity. First we determine positive solutions of equations $R_{kn_b}(\theta) = R_{k'n_b'}(\theta)$ with all different pairs $(n_b,k),(n_b',k')$ that belong to the set $\{(1,5),(2,5),(3,5),(4,5),(5,7),(6,7)\}$. The obtained results are in fact "boundary" values of optimal intervals of traffic intensity with respect to the minimization of specific cost ratio. Computational experiments of optimal intervals for minimizing specific cost ratio with related optimal values of berths solve a problem of minimization the specific cost ratio under the condition that the number of finite waiting areas is equal to 5. Furthermore, we compared values for specific cost ratio for six models satisfying $k - n_b = 1$, that is, models with one waiting space in queue. Finally, appropriate "boundary" values of five equations $R_{i+1,i}(\theta) = R_{i+2,i+1}(\theta)$ with $i = 1, 2, ..., 5,$ represent the optimal intervals for $R_{i+1,i}(= 0{,}6i + h_{i+1,i}(\theta))$.

This study is significant for solving decision making processes in port such as to determine the optimal number of berths in relation to appropriate traffic intensity. The analytical approach together with computational results can serve as a good start for dealing with simulation modelling of related problems. Also, this methodology should be applied for some other queuing models and performance evaluation expanding it in the sense of optimizing port capacity.

5 References

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Authors' addresses

Romeo Meštrović, Prof. Dr.
University of Montenegro
Maritime Faculty
Dobrota 36, 85330 Kotor, Montenegro
Tel: 38269478140, Fax: 38232303185
E-mail: romeo@ac.me

Branislav Dragović, Prof. Dr.
University of Montenegro
Maritime Faculty
Dobrota 36, 85330 Kotor, Montenegro
E-mail: branod@ac.me

Nenad Dj. Zrnić, Assoc. Prof. Dr.
University of Belgrade,
Faculty of Mechanical Engineering
Department of Material Handling, Construction and Logistics
Kraljice Marije 16, 11000 Belgrade, Serbia
E-mail: nzrnic@mas.bg.ac.rs

Maja Škurić, MSc
University of Montenegro
Maritime Faculty
Dobrota 36, 85330 Kotor, Montenegro
E-mail: mskuric@ac.me