WEIGHTED KALMAN FILTER PHASE UNWRAPPING ALGORITHM BASED ON INSAR IMAGE

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Abstract:

The Kalman filter deals simultaneously with phase unwrapping and noise elimination procedure. But the errors produced by the original radar signal and post-processing can cause phase discontinuity so that the unwrapped result is not accurate. Therefore, the weighted Kalman filter phase unwrapping algorithm based on InSAR image is proposed. Through the low-quality region where the wrapped phase is masked, the Kalman filter phase unwrapping algorithm is implemented in the high-quality region. When the high-quality region is correctly unwrapped, the weighted Kalman filter phase unwrapping algorithm is implemented in masking off the low-quality region, and as a consequence a reliable result is obtained. In this paper InSAR data is chosen for performing the experiment, and for comparison with both a network flow algorithm and a quality map guided algorithm. It is subsequently verified that the proposed algorithm is effective and reliable.

1 Introduction

In relation to phase unwrapping procedure as a key step in InSAR data processing, we refer to the book by Ghiglia and Pritt [1] for an excellent overview. There are actually two general types of conventional phase unwrapping methods. The algorithms of the first group are named path-following algorithms and they isolate and/or mask problematic zones containing residues and unwrap the interferogram by avoiding these zones containing a branch-cut algorithm [2]. The techniques of the second group provide a global solution which minimizes a cost function over the whole interferogram, such as a network flow algorithm [3-4] and a quality map guided algorithm [5]. Some of these techniques do not correspond/belong to these groups, e.g., the Kalman filter algorithm [6-8].

A conventional Kalman filter phase unwrapping algorithm can simultaneously enable phase unwrapping and noise elimination/reduction. However, the original radar signal and post-processing producing a lot of unwanted errors can cause phase discontinuity and local error propagation so that an unwrapped result is not accurate. Therefore, the weighted Kalman filter phase unwrapping algorithm based on InSAR image is proposed. This algorithm masks the low-quality region in a wrapped phase, and a Kalman filter phase unwrapping strategy is implemented in the high-quality region, then a weighted Kalman filter phase unwrapping procedure is implemented in the low-

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quality region, and the error propagation is eventually reduced.

The text is organized as follows. First the weighted Kalman filter algorithm for phase unwrapping is presented in Section II. Then the performance of the new method using real data is illustrated in Section III. Next a comparison between the cited Kalman filter approach and the well-known network flow algorithm is drawn, and subsequently a quality map guided algorithm is presented. Finally, conclusions based on these approaches are drawn in Section IV.

2 Weighted Kalman filter phase unwrapping algorithm

2.1 Weighted Kalman Filter Observation Equation

The observation equation is [7]:

$$y(k)=\begin{bmatrix}\mathrm{Re}\left\{\dfrac{z(k)}{a(k)}\right\}\\ \mathrm{Im}\left\{\dfrac{z(k)}{a(k)}\right\}\end{bmatrix} \approx \begin{bmatrix}\cos(\phi(k))\\ \sin(\phi(k))\end{bmatrix}+\begin{bmatrix}v_1(k)\\ v_2(k)\end{bmatrix} \quad (1)$$
$$=h(\phi(k))+v(k)$$

where $z(k)$ represents complex interference, $a(k)$ the observed interference amplitude, $\varphi(k)$ a real phase, $h(\cdot)$ is $y(k)$ and $\varphi(k)$ is nonlinear mapping. $v_1(k), v_2(k)$ are zero-mean Gaussian white noise, the variance of which are determined by the coherence γ, that is

$$E\{v(k)\}=0 \;;$$

$$E\{v(k)v(j)^T\}=\underset{i=1,2}{diag}\left[\frac{1}{2|\gamma|^2}-\frac{1}{2}\right]\delta(k,j) \quad (2)$$

where $\delta(k,j)$ is the Kronecker function.

2.2 Weighted Kalman Filter State Space Model

When the interferogram phase is discrete, the state space model is [7]:

$$x(k+1)=Ax(k)+w(k)$$
$$E\{w(k)\}=0$$
$$E\{w(k)w(j)\}=Q(k)\delta(k,j) \quad (3)$$

where $x(k)$ is the true phase at k point, A is the system matrix, $w(k)$ represents the state noise, and $Q(k)$ represents a state noise covariance matrix.

2.3 Weighted Kalman Filter Phase Unwrapping Algorithm

In case the noise statistics is known, we propose the weighted Kalman filter phase unwrapping algorithm. The specific process develops taking the following steps:

Step1, the masked result is best in $[-\pi/2,\pi/2]$ according to the experience based on a large number of experiments. In order to get the most accurate result, this paper masks the wrapped data in $[-\pi/3,\pi/3]$, and avoids errors propagation due to the low quality phase estimation.

Step 2, the predictive state vector value $\hat{x}_{k+1,k}$ and its covariance matrix is expressed by the equation:

$$\hat{x}_{k+1,k}=A\hat{x}_{k,k}+\hat{u}_{k,k}$$
$$P_{k+1,k}=AP_{k,k}A^T+Q_{k,k} \quad (4)$$

where $\hat{u}_{k,k}$ is the phase gradient estimation, and $Q_{k,k}$ is the state noise covariance matrix. The value of A in system matrix is the unit matrix.

Step 3, according to the predicted value $\hat{x}_{k+1,k}$ and the covariance matrix $P_{k+1,k}$ obtained from the previous step by calculating the state estimation $\hat{x}_{k+1,k+1}$ and the covariance matrix $P_{k+1,k+1}$,this phase is expressed by

$$\hat{x}_{k+1,k+1}=\hat{x}_{k+1,k}+J_{k+1}r_{k+1,k+1}$$
$$P_{k+1,k+1}=(I-J_{k+1}C_{k+1,k})P_{k+1,k} \quad (5)$$

where J_{k+1} is the filter gain matrix, $r_{k+1,k+1}$ is the residuals, and $C_{k+1,k}$ represents the linear observation matrix.

$$J_{k+1}=P_{k+1,k}C^T_{k+1,k}(C_{k+1,k}P_{k+1,k}C^T_{k+1,k}+R_{k+1,k+1})^{-1}$$
$$r_{k+1,k+1}=y_{k+1,k+1}-C_{k+1,k}\hat{x}_{k+1,k}$$
$$C_{k+1,k}=\frac{d}{dx}h(x)|_{\hat{x}_{k+1,k}} \quad (6)$$
$$=[-\sin(\hat{x}_{k+1,k}),\cos(\hat{x}_{k+1,k})]^T$$

Step 4, using the Kalman filter formulation that is to be implemented in the high-quality region, it follows that:

$$P_{k+1,k}(m,n)=\frac{1}{4}\left[P_{k,k}(m-1,n)+P_{k,k}(m,n-1)\right]+$$
$$M_{wm}Q(m-1,n)M^T_{wm}+M_{wn}Q(m,n-1)M^T_{wn}$$
$$\hat{x}_{k+1,k}(m,n)=\frac{1}{2}\left[\hat{x}_{k,k}(m-1,n)+\hat{x}_{k,k}(m,n-1)\right] \quad (7)$$

where m represents the range direction, n represents the azimuth direction, $M_{wm}=[0.5\quad 0]$ $M_{wn}=[0\quad 0.5]$.

Step 5, the weighted Kalman filter formula is implemented in the low-quality region, a predictive value is the sum of two weights corresponding to the neighbor two regions, and the error covariance matrix is the sum of two covariance matrix weights corresponding to the neighbor two regions [8].That is given by

$$P_{k+1,k}(m,n) = W_r P_{k,k}(m-1,n) + W_a P_{k,k}(m,n-1) + M_{wm} Q(m-1,n) M_{wm}^T + M_{wn} Q(m,n-1) M_{wn}^T$$

$$\hat{x}_{k+1,k}(m,n) = W_r \hat{x}_{k,k}(m-1,n) + W_a \hat{x}_{k,k}(m,n-1) \tag{8}$$

where W_r and W_a ,respectively, are weights corresponding to the range and azimuth directions, and

$$W_r = \left(\frac{(P_{k,k}(r-1,a))^{-1}}{(P_{k,k}(r-1,a))^{-1} + (P_{k,k}(r,a-1))^{-1}} \right)$$

$$W_a = \left(\frac{(P_{k,k}(r,a-1))^{-1}}{(P_{k,k}(r-1,a))^{-1} + (P_{k,k}(r,a-1))^{-1}} \right) \tag{9}$$

M_{wm} and M_{wn} ,respectively, are: $M_{wm} = [0.5 \quad 0]$, $M_{wn} = [0 \quad 0.5]$.

After taking these five steps, we can get the unwrapping phase values $\hat{x}_{k+1,k+1}$ of the entire region.

3 Experimental result and analysis

In order to verify the effectiveness of the weighted Kalman filter algorithm, we use two ERS-1 satellite SAR images dating back to 1994 as the experimental primary and secondary images; the two images represent King Quebec in Canada after an interference and ground effect operation, then we select a detailed part of the interferogram as shown in Fig.1 (a) ($100pixel \times 100pixel$), Fig.1 (b) shows a coherence map and Fig.1(c) a masked interferogram in the interval of $[-\pi/3, \pi/3]$. Figs. 2（a）~（d） show , respectively, phase unwrapping results of the Kalman filter algorithm, weighted Kalman filter algorithm, network flow algorithm, and quality map guided algorithm. As observed in Fig. 2 w both the Kalman filter algorithm and network flow algorithm produce the significant error propagation and that part of the edge information has been lost. The weighted Kalman filter algorithm has almost no error propagation due to the masking effects of low-quality information, and error restrictions within a very small range. As a result, the quality map guided algorithm appears at the top and bottom of the island

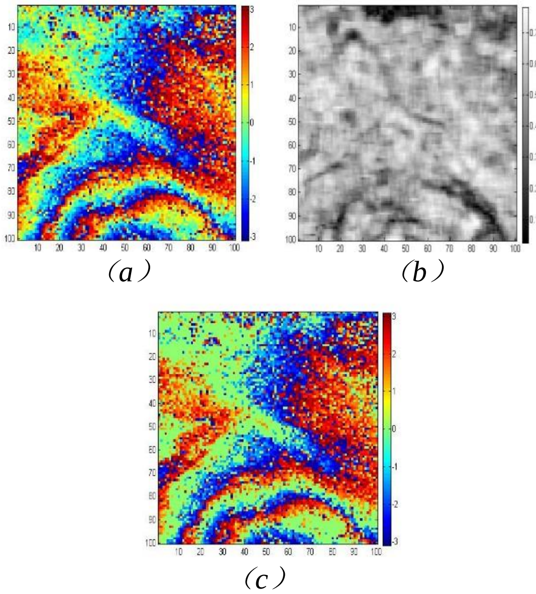

Figure 1. (a) Interferogram; (b) Coherence diagram; (c) Masked interferogram.

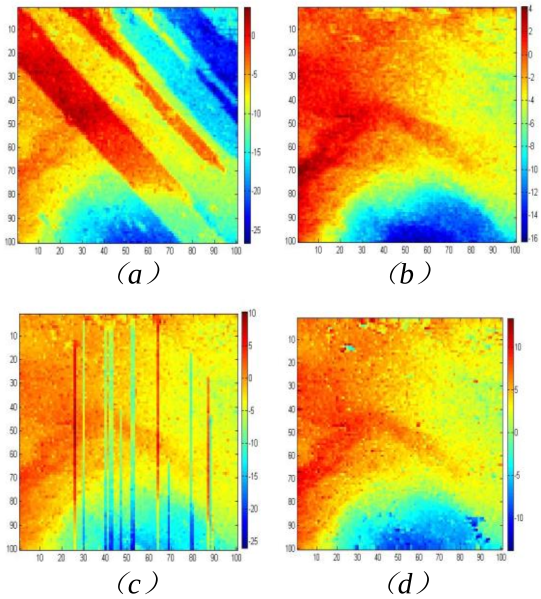

Figure 2. Unwrapping results: (a) Kalman filter algorithm ;(b) Weighted Kalman filter algorithm; (c) Network flow algorithm ; (d) Quality map guided algorithm.

Fig. 3(a)~(d) show four algorithm rewrapped results respectively, and from the results we can see the weighted Kalman filter algorithm, network flow algorithm and rewrapped maps of the quality map guided algorithm, which are approximately the same as the interferogram, which have preserved the details of the phase stripes. However, the rewrapped map of the Kalman filter algorithm cannot be recovered to the original interfergram; this arises from the part of the stripes overlapped in its rewrapped map, and shows that the stripes have been lost, and that it is seriously inconsistent with the original interferogram.

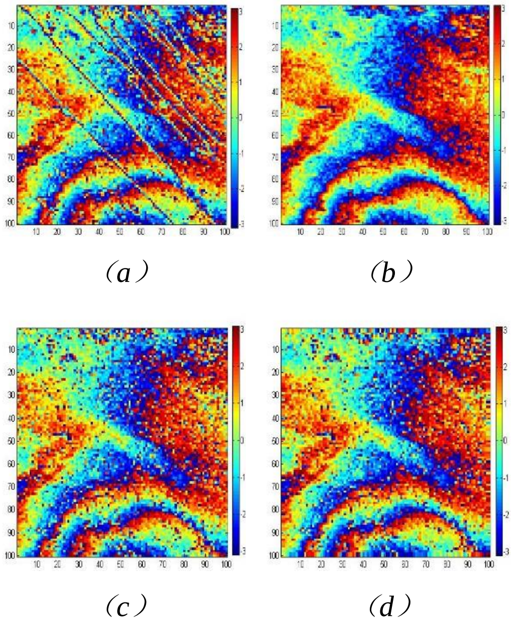

Figure 3. Rewrapped Maps: (a) Kalman filter algorithm (b) Weighted Kalman filter algorithm; (c) Network flow algorithm; (d) Quality map guided algorithm.

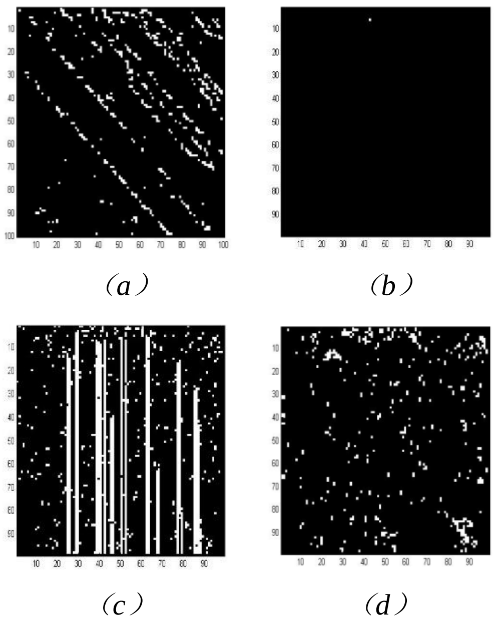

Figure 4. Discontinuous points maps: (a) Kalman filter algorithm; (b) Weighted Kalman filter algorithm; (c) Network flow algorithm; (d) Quality map guided algorithm.

From the discontinuous points maps (Fig.4) it can be seen that the Kalman filter algorithm, network flow algorithm and quality map guided algorithm exhibit a lot of discontinuous points in the low-quality region, while on the other hand, the weighted Kalman filter algorithm exhibits fewer discontinuous points, and consequently its unwrapping procedure has shown the smoothest performance results.

In this context, four unwrapping results are quantitatively analyzed employing three aspects of discontinuous points, ε value, a difference between the rewrapped phase and an original wrapping phase, respectively.

1）Discontinuous points: a discontinuous point is defined as the absolute value of the adjacent pixel phase difference, which is more than π . Each discontinuous point is usually calculated only once in the application. As observed in Table 1 the discontinuous point number exhibited by the weighted Kalman filter algorithm is the least, which illustrates and exemplifies that a distortion performance demonstrated by the weighted Kalman filter algorithm is also stronger.

2) ε value is defined as follows:

$$\varepsilon = \frac{1}{MN}\sum_{i=0}^{M-2}\sum_{j=0}^{N-1}\omega_{i,j}^{x}\left|\Phi_{i+1,j}-\Phi_{i,j}-\Delta_{i,j}^{x}\right|^{p} + \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-2}\omega_{i,j}^{y}\left|\Phi_{i,j+1}-\Phi_{i,j}-\Delta_{i,j}^{y}\right|^{p} \quad (10)$$

where $\omega_{i,j}^{x}$ and $\omega_{i,j}^{y}$ are weights referring to the wrapped phase gradient $\Delta_{i,j}^{x}$ and $\Delta_{i,j}^{y}$, the weight can generally be derived from the phase quality map, and transferred in binary 0 and 1. Conceptually, $p=1$ denotes the minimum of the gradient average absolute deviation, $p=2$ denotes the minimum of the radient mean square error. The smaller the value ε is, the higher the unwrapping phase quality. . The selected Quality map is a coherence map, whereas p value is selected as 1. From ε value in Table 1, it can be seen that the performance demonstrated by the weighted Kalman filter algorithm is –not as good as the one achieved by other four algorithms, which illustrates that the weighted Kalman filter algorithm exhibits the best unwrapping quality strategy in all four algorithms.

Table 1. Discontinuous Points Number and ε Value of Four Algorithms

Unwrapping algorithm	Discontinuous points' number	ε Value
Kalman filter algorithm	539	1.0998
Network flow algorithm	625	7.6310
Quality map guided algorithm	470	6.7974
Weighted Kalman filter algorithm	1	0.8969

3) A difference is drawn between a rewrapped phase and an original wrapped phase. This differentiation illustrated by using four algorithms is shown in Table 2. It can be also seen that the minimum absolute error and root mean square error of a percentage represented by the weighted Kalman filter algorithm based on InSAR image are significantly less than by the other three, which shows that reliability and validity coefficients by the weighted Kalman filter algorithm based on InSAR image are the strongest. Its absolute maximum percentage error value is slightly smaller than the one demonstrated by the Kalman filter algorithm, network flow algorithm, quality map guided algorithm. Therefore, repeated observations of the implemented weighted Kalman filter algorithm have correspondingly shown that it exhibits very high precision..

Table 2. Difference between Rewrapped Results of Four Methods and Original Wrap Phase

Unwrapping algorithm	Absolute error maximum	Root mean square error	Absolute error minimum
Kalman filter algorithm	6,2526	1,0735	0,0002
Network flow algorithm	6,2539	0,9501	0,0059
Quality map guided algorithm	6,1362	1,4434	0,0002
Weighted Kalman filter algorithm	6,1349	0,9134	0

4 Conclusion

A weighted Kalman filter solution of the phase unwrapping problem in SAR interferometry has been proposed. This method has been compared against the Kalman filter method, which shares the same philosophy of simultaneous filtering and unwrapping, the network flow algorithm, and the quality map guided algorithm. Some situations where the Kalman filter algorithm, network flow algorithm and quality guided algorithm fail and never recover have been exemplified. On the other hand, the weighted Kalman filter solution not only performs better but alsorecovers from errors better. It is worth noting that the weighted Kalman filter addresses the state of zones containing a discontinuous phase and that its unwrapping result is the best.

Our research lines are currently focused on two main issues. The first one introduces a masking technology into the weighted Kalman filter approach and the second one, combines the weighted Kalman filter approach with path-following techniques. Surely, they will be correspondingly analyzed since a better performance is expected from the synergy between both strategies. Moreover, an analysis of the performance of this algorithm in different scenarios is currently being carried out.

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