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Traffic on Motorways
Original Scientific Paper
Accepted: Oct. 19, 2012
Approved: Sep. 24, 2013

DIRECTIONAL CONTROL OF A PLATOON OF VEHICLES FOR COMFORT SPECIFICATION BY CONSIDERING PARASITIC TIME DELAYS AND LAGS

ABSTRACT

The problem of controlling a string of vehicles moving in one dimension is considered so that they all follow a lead vehicle with constant spacing between successive vehicles. The goal of this paper is to consider the effects of parasitic time delays, lags and comfort specification. The contribution of this paper is two-fold. First, negative effects of the parasitic time delays and lags on the string stability have been investigated since most physical systems usually involve parasitic time delays and lags. Second, restrictions on control parameters, to be well-suited with bounds on the dynamic variables as well as the requirement for string stability have been presented. Finally, a simulation example of multiple vehicle platoon control is used to demonstrate the effectiveness of the proposed method.

KEY WORDS

platoon of vehicles, string stability, parasitic time delays and lags, comfort specification

1. INTRODUCTION

Highway congestion is imposing an intolerable burden on urban residents. Increasing traffic density has been causing traffic congestion on the roads, which will reduce travel time, traffic safety, while increasing air pollution, and energy consumption. Congestion occurs when vehicle velocity variation propagates to the following vehicles. It is difficult for drivers to recognize small changes in the preceding vehicle velocity. There are various approaches to reduce congestion such as vehicle platoon control. One possible solution to this problem is to use the Adaptive Cruise Control (ACC) concept. The ACC system has been proposed as an enhancement over the existing cruise controllers on ground vehicles. Autonomous cruise control devices use the distances and velocity differences between vehicles by radar to calculate the required action and actuate cars accordingly [1].

The control system for a string of vehicles consists of the guidance model depicting communication between vehicles and individual vehicle control. There are many guidance models for communication structure between vehicles in a platoon. Interaction structures may be restricted by sensing, communication, or computational limitations. For example, vehicles only interact with their nearest neighbours rather than with all vehicles in the system in many implementations. The effects of either the interaction structures or information flow on the system stability were investigated using graph theory [2-4].

For multi agent systems, many properties may become significant in addition to the usual stability and robustness analysis such as string stability, and must be taken into account. The term string stability was initially introduced in 1974 [5]. String stability explains how errors propagate through the group of vehicles as result of disturbances or the reference trajectory of the formation lead. A string-stable control signifies that spacing errors between adjacent vehicles do not amplify along the vehicle string. String stability has been widely used in automated highway systems [6, 7]. The relationship between string stability and spacing policies was an important issue [9, 10]. There are two major strategies for controlling of a platoon of vehicles; these are constant time headway control and spacing distance control. In spacing distance control, the distance between two adjacent vehicles in the platoon is controlled to be the same as a specified constant that is independent of the velocity in the platoon travelling. Constant spacing strategies are usually used due to high traffic capacities. Swaroop and Hedrick investigated constant-spacing control strategy, in that the control purpose is to preserve a constant distance between adjacent vehicles in a string [6].

In this paper, the desired trajectory of the platoon is considered to be of constant-velocity type. String stability cannot be guaranteed when constant spacing policy is used [11, 12]. Also, extra information is needed. The focus of this paper is to analyze the centralized directional adaptive cruise. Directional ACC monitors the behaviour of the preceding vehicle. The centralized control receives relative distance and relative velocity information with respect to the preceding vehicle as well as velocity information of the preceding vehicle.

Time delays appear in those systems due to sensing computation and actuation. The parasitic delays and lags are the unavoidable natural characteristics of actuators and sensors in the mechanical and control systems, which may considerably affect the stability of systems in some situations. Xiao and Gao investigated string stability of a platoon of adaptive cruise control vehicles by considering the parasitic time delay and lags of the actuators and sensors [13, 14].

The passenger comfort in public ground transportation is affected by the changes in movement felt in all directions. Typically, the value of acceleration is taken as a comfort criterion [15]. However, comfort based on the movement changes in vehicle longitudinal direction (the "jerk") was used in [16]. So, the jerk is significant when assessing the discomfort caused to the passengers in a vehicle. Cook [17] investigated control of vehicle convoy with a selection of control parameters to ensure string stability while satisfying passenger comfort constraints.

Another purpose of this paper is to take a fresh look at the string stability with these considerations in mind, with the aim of obtaining control parameters to guarantee string stability of a vehicle convoy system, whilst fulfilling restrictions compelled by considerations comfort criterion by taking into account the effect of the time delays and lags. Note that the previous analyses of homogenous string stability are conducted without using the parasitic time delays and lags to keep the comfort of passengers and take account of restrictions on the achievable performance of the driving force system.

This is inspired by the work [13, 17] that inspected the string stability analysis of 1-D vehicular platoons in a similar framework. Compared to [13], this paper makes some novel contributions. Due to practical design and implementation, the constraints imposed by the driving force system are considered. Since the comfort criterion is taken as the result of these constraints, it is important to consider it during the design of vehicles. Also, [17] did not consider parasitic time delay and lags that are the inevitable natural characteristics of the mechanical and control systems.

Vehicles require a complex high-order nonlinear model to describe the complete dynamic behaviours. The analyses are usually based on linear system theory which does not take into account the limitations of the performance of the propulsion system, such as bounds on the forces. This subject was also not considered in [13, 17]. In this paper, a hierarchical platoon controller design framework is established comprising a feedback linearization controller at the first layer and a linear controller at the second layer. By reducing the non-linear system to a linear model using the top layer feedback linearization controller, a controller is designed. Also, a platoon consisting of identical vehicles is considered that are modelled by non-linear differential equations.

The remainder of the paper is structured as follows. Section 2, briefly introduces the longitudinal vehicle model that considers the parasitic time delays and lags. Section 3 handles the development of a control law when such parasitic delays and lags are taken into account. Section 4, presents an analysis of string stability. Section 5 deals with the selection of control parameters to satisfy passenger comfort constraints. In Section 6, the simulations are presented to show the efficiency of the proposed method.

2. VEHICLE MODEL

Consider a group of vehicles in dense traffic with no overtaking and assume each vehicle uses only the information of the preceding vehicle. The formation control of $N+1$ homogeneous string of vehicles is considered so that they all follow a lead vehicle (*Figure 1*).

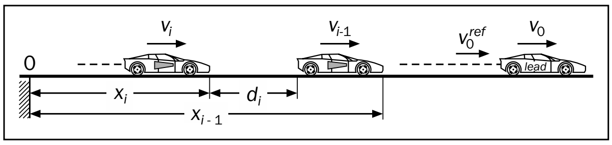

Figure 1 - 1-D of vehicle string

The position, velocity and acceleration of the lead vehicle are denoted by $x_0(t)$, $v_0(t)$, $a_0(t)$, respectively. Also, $x_i(t)$, $v_i(t)$, $a_i(t)$ denote the position, velocity and acceleration of the i-th vehicle, respectively.

A model for the motion of a vehicle must take into account the power-train, tyre forces, aerodynamic drag forces, rolling resistance forces and gravitational forces. The power-train consists of the internal combustion engine, the torque converter, the transmission and the wheels [16]. The dynamics of the i-th following vehicle can be modelled by the following non-linear differential equations ([18–19])

$$\dot{a}_i = f_i(v_i, a_i) + g_i(v_i)c_i \tag{1}$$

where c_i is the engine input and $f_i(v_i, a_i)$ and $g_i(v_i)$ are

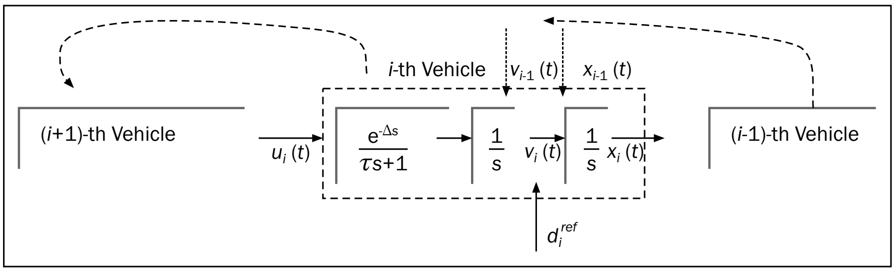

Figure 2 - Transfer Function representation of a platoon of vehicles (open loop)

$$f_i(v_i,a_i)=-\frac{1}{\tau_i}\left(a_i+\frac{\sigma A_i c_{di}}{2m_i}v_i^2+\frac{d_{mi}}{m_i}\right)-\frac{\sigma A_i c_{di} v_i a_i}{m_i}$$

$$g_i(v_i)=\frac{1}{\tau_i m_i}$$

where σ is the specific mass of the air, A_i, c_{di}, d_{mi}, m_i and τ_i are the cross-sectional area, drag coefficient, mechanical drag, mass, and engine time constant of the i-th vehicle, respectively, $\sigma A_i c_{di}/2m_i$ stands for air resistance. Note that the vehicles considered here are not necessarily identical.

The following control law has been adopted

$$c_i = u_i m_i + 0.5\sigma A_i c_{di} v_i^2 + d_{mi} + \tau_i \sigma A_i c_{di} v_i a_i \tag{2}$$

where u_i is additional input signal to be designed. Obviously, this control law can be achieved by feedback linearization, since after introducing (2), equation (1) becomes

$$\tau_i \dot{a}_i + a_i = u_i \tag{3}$$

The feedback linearization controller in (2) plays the role of the first layer controller in our architecture. It helps to simplify the system model by excluding some characteristic parameters of the vehicle from its dynamics.

Due to the pure information delay or sensing delay, the controller can access just the delayed information but not the instant information to achieve the desired acceleration or deceleration. It means that the expression of the control law is $u_i(t)$ replaced by $u_i(t-\Delta)$ [20-21]. Therefore, equation (3) has to be rewritten in the following form

$$\tau_i \dot{a}_i + a_i = u_i(t-\Delta_i) \tag{4}$$

One may treat Δ_i and τ_i as "lumped" parasitic time delay and lag, respectively, which are usually caused by several sources such as the pure time delay and lag in the engine response, of the throttle actuator, brake actuator and bandwidth of low-pass filters used for other sensors such as engine manifold pressure sensor, wheel speed sensor [22-23]. Because of the parasitic time delays and lags, the commanded acceleration may not be accessible instantly.

For simplicity, all the vehicles are supposed to be identical, although this assumption may be neither entirely necessary for the discussion nor satisfied in practice. Therefore,

$$\tau_i = \tau,\ \Delta_i = \Delta \quad \forall i = 1,\dots,N$$

Assuming that a model of single vehicle is described by equation (4), the resultant model for a platoon is shown in *Figure 2*.

In this paper, only linear models will be used for the dynamics, but constraints will be forced on the magnitudes of the variables, representing bounds on what can be achieved in practice. Distinctively, the acceleration will be assumed to satisfy a boundary

$$|a_i(t)| \le \alpha \tag{5}$$

This is for the worst case scenario in an urgent situation. While the jerk is required to be restricted by

$$|\dot{a}_i(t)| \le \beta \tag{6}$$

Because the imposition of constraints does not contradict the assumption of linear dynamics, it is supposed that the accelerations are always maintained within the boundary (5), and the investigation is concerned with the conditions which should be imposed on the control parameters in order for the bounds such as (6) then to remain satisfied.

3. LOOK-AHEAD CONTROL LAW

There are many feasible control strategies for vehicle following in a platoon, depending on the information about the motion of other vehicles. In this paper, the controller on each vehicle utilizes only the information about itself and the one directly ahead. For the development of a control law, it will be supposed that there is a lead vehicle, indexed by 0, that executes manoeuvres and the following vehicles are controlled to keep a desired following distance. Constant spacing strategy is used for the desired following distance of any following vehicle.

The spacing error can be defined as:

$$\begin{aligned}\xi_i(t) &= x_{i-1}(t) - x_i(t) - l_{i-1}\\ \delta_i(t) &= \xi_i(t) - d_i^{ref}\end{aligned} \tag{7}$$

Let $\xi_i(t)$ indicate spacing (the following distance) between the i-th vehicle and the $(i-1)$-th vehicle, l_{i-1} indicates the length of the $(i-1)$-th vehicle, d_i^{ref} indicates the spacing distance reference, $\delta_i(t)$ indicates the spacing error of the i-th vehicle which is deviation between the spacing and the desired spacing.

It is to be noted that the desired distance in between is not necessarily identical for vehicles. However, it is assumed to be constant in this study.

It is clear then that

$$\xi_i(t-\Delta) = x_{i-1}(t-\Delta) - x_i(t-\Delta) - l_{i-1},$$
$$\delta_i(t-\Delta) = \xi_i(t-\Delta) - d_i^{ref}.$$

The objective of the controller is to asymptotically drive the spacing error to zero. With the aim of extracting the control law, the following equation is considered:

$$\ddot{\delta}_i + K_v\dot{\delta}_i + K_c\delta_i = 0 \tag{8}$$

where K_c and K_v are constant proportional and derivative gains, respectively. If gains are positive, the spacing error will converge to zero. Then the control input can be generated from Eq. (8) using the simplified model of a vehicle.

$$u_i(t) = \dot{v}_{i-1}(t) + K_v\dot{\delta}_i(t) + K_c\delta_i(t) \tag{9}$$

Because of the parasitic time delays of the actuators and sensors, the control law (9) must be replaced with

$$u_i(t-\Delta) = \dot{v}_{i-1}(t-\Delta) + K_v\dot{\delta}_i(t-\Delta) + K_c\delta_i(t-\Delta) \tag{10}$$

The goal of the controller as defined above is to follow the lead vehicle with a constant spacing between successive vehicles.

4. STRING STABILITY ANALYSIS

The question of string stability/error amplification is usually answered by looking at the transfer functions that relate the spacing errors between two successive vehicle pairs. The string-stability ensures that errors range decreases as they propagate along the vehicle stream. The string stability performance is mainly determined by the structure of the controller and headway policy. In other words, the string stability performance is mostly dependent on the information received and used by the longitudinal controllers.

The velocity and spacing error dynamics models can be derived based on the vehicle dynamics model (4).

$$\tau\dot{a}_i(t) + a_i(t) = \dot{v}_{i-1}(t-\Delta) + K_v\dot{\delta}_i(t-\Delta) + K_c\delta_i(t-\Delta) \tag{11}$$

By differentiating both sides of (11), we obtain

$$\tau\ddot{a}_i(t) + \dot{a}_i(t) = \ddot{v}_{i-1}(t-\Delta) + K_v(\dot{v}_{i-1}(t-\Delta) - \dot{v}_i(t-\Delta)) + K_c(v_{i-1}(t-\Delta) - v_i(t-\Delta)) \tag{12}$$

The Laplace transforms can be used for analyzing the conventional notation and the regular assumption of zero initial conditions for the derivation of transfer functions. Therefore, by taking the Laplace transformation on both sides of (12), we obtain

$$(\tau s^3 + s^2)(t-\Delta)V_i(s) = (s^2 + K_v s + K_c)e^{-\Delta s}V_{i-1}(s) - (K_v s + K_c)e^{-\Delta s}V_i(s)$$

Therefore,

$$G(s) = \frac{V_i(s)}{V_{i-1}(s)} = \frac{(s^2 + K_v s + K_c)e^{-\Delta s}}{\tau s^3 + s^2 + (K_v s + K_c)e^{-\Delta s}} \tag{13}$$

This describes the velocity dynamics model of two successive vehicles in the string of vehicles.

It is clear that the range error output must be smaller than or equal to the range error input to avoid range errors from propagating indefinitely along the string. For this uniform vehicle string, a string-stability definition is widely used [24-25] and described as follows:

$$|G(i\omega)| \le 1 \quad \forall\omega > 0$$

where $G(i\omega)$ is derived from the spacing error transform function (13) by substituting $s = i\omega$.

Theorem 1: The condition $|G(i\omega)| \le 1$ is satisfied for $\forall\omega > 0$ if the following condition holds

$$K_c \ge K_v\frac{\tau + \Delta}{\tau\Delta}$$

Proof: $|G(i\omega)|$ can be expressed as

$$|G(i\omega)| = \sqrt{a/(b+c)}$$

where

$$a = K_v^2\omega^2 + K_c^2 + \omega^4 - 2K_c\omega^2$$
$$b = K_v^2\omega^2 + K_c^2 + \omega^4 - 2K_c\omega^2\cos(\Delta\omega)$$
$$c = \tau^2\omega^6 - 2K_v\omega^4\tau\cos(\Delta\omega) + 2K_c\omega^3\tau\sin(\Delta\omega) - 2K_v\omega^3\sin(\Delta\omega)$$

Because of $\cos(\Delta\omega) \le 1$, then $b > a$. Apparently, if $c \ge 0$ for $\forall\omega > 0$, $a/(b+c) \le 1 \rightarrow |G(i\omega)| \le 1$ will be obtained.

The spacing errors have most of their energy in the region of low frequencies, which is also called the key region of the string stability [26].

Knowing that $\cos(\Delta\omega) \le 1$, $\sin(\Delta\omega) \le \Delta\omega$ and at low frequencies $\sin(\Delta\omega) \cong \Delta\omega$

$$c > \tau^2\omega^6 - 2K_v\omega^4\tau + 2K_c\omega^3\tau\sin(\Delta\omega) - 2K_v\omega^3\sin(\Delta\omega)$$

at low frequency

$$\cong \tau^2\omega^6 + 2\omega^4(\Delta(K_c\tau - K_v) - K_v\tau)$$

If inequality $(K_c\tau - K_v)\Delta - K_v\tau \ge 0$ holds then $c > 0$. Here, the proof of the theorem is completed.

By comparing $|G(i\omega)|_{\Delta=0}$ and $|G(i\omega)|_{\tau=0}$ at low frequencies, where $|G(i\omega)|_{\Delta=0}$ denotes value of $|G(i\omega)|$ with $\Delta = 0$, and value of $|G(i\omega)|_{\tau=0}$ denotes value of $|G(i\omega)|$ with $\tau = 0$, the most negative effect of the parasitic time delays and lags can be recognized. Greater values denote higher negative effect.

Theorem 2: The parasitic time delay has more negative effect on the string stability if the lumped parasitic time delay and the lumped parasitic time lag are the same.

Proof: $|G(i\omega)| = \sqrt{a/(b+c)}$ numerator is independent of parasitic time delays and lags. Therefore each one has a smaller denominator which has the most negative effect on the string stability.

$$(b+c)|_{\Delta=0} - (b+c)|_{\tau=0} = \tau^2\omega^6 - 2K_c\omega^2(1-\cos(\Delta\omega)) - 2K_v\omega^3(\tau\omega - \sin(\Delta\omega)) \tag{14}$$

At low frequencies, when the lumped parasitic time delay and the lumped parasitic time lag are the same, Eq. (14) can be rewritten as follows

$(b+c)\big|_{\Delta=0} - (b+c)\big|_{\tau=0} = \tau^2\omega^6 > 0$

Then the $\left|G(i\omega)\right|_{\tau=0}$ is greater than $\left|G(i\omega)\right|_{\Delta=0}$.

Here, the proof of the theorem is completed.

Also, theorem 2 is true for a string of vehicles by considering constant time headway [13].

5. CONSTRAINT FORCE

For the sake of practical restrictions on dynamic variables, it is essential to consider the restrictions in order to be compatible with bounds on the dynamic variables and requirement of the string stability.

In order to assess the implications of bounds on acceleration and jerk, it is necessary to examine the relation between the acceleration of the $(i-1)$-th vehicle and the jerk of the i-th vehicle, given by the transfer function [17]

$$\Gamma(s) = \frac{s^2 V_i(s)}{s V_{i-1}(s)} = sG(s) \tag{15}$$

which becomes

$$\Gamma(s) = \frac{s(s^2 + K_v s + K_c)e^{-\Delta s}}{\tau s^3 + s^2 + (K_v s + K_c)e^{-\Delta s}} \tag{16}$$

Then, if the jerk is to remain within acceptable limits, for any allowable acceleration of the preceding vehicle, it is required that [17]

$$\int_0^\infty |\gamma(t)|dt \le \frac{\beta}{\alpha} \tag{17}$$

where $\gamma(t)$ is impulse response of transfer function $\Gamma(s)$.

$$\begin{aligned} &\Gamma(s) = \int_0^\infty \gamma(t)e^{-st}dt \Rightarrow \Gamma(j\omega) = \int_0^\infty \gamma(t)e^{-i\omega t}dt \\ &\Rightarrow \sup_\omega |\Gamma(j\omega)| \le \int_0^\infty |\gamma(t)|dt \le \frac{\beta}{\alpha} \end{aligned} \tag{18}$$

Theorem 3: Comfort criterion is satisfied if the following condition is met:

$$K_v \le \frac{\Delta + \tau}{2}$$

Proof: $|\Gamma(i\omega)|$ can be expressed as

$$|\Gamma(i\omega)| = \sqrt{a_1/(b+c)}$$

The nominator $|\Gamma(i\omega)|$ by setting $\omega = K_v$ will be:

$$a_1\big|_{\omega=K_v} = K_v^6 + K_c^2K_v^2 + K_v^6 - 2K_cK_v^4$$

Using the lower boundary of

$$K_c \ge K_v\frac{\tau+\Delta}{\tau\Delta}$$

then

$$\begin{aligned} a_1\big|_{\omega=K_v} &\ge K_v^6 + K_v^2\left(K_v\frac{\tau+\Delta}{\tau\Delta}\right)^2 + K_v^6 - 2K_v^4\left(K_v\frac{\tau+\Delta}{\tau\Delta}\right) \ge \\ &\ge K_v^4\left(\left(\left(\frac{\tau+\Delta}{\tau\Delta}\right) - K_v\right)^2 + K_v^2\right) \end{aligned} \tag{19}$$

Also, the denominator $|\Gamma(i\omega)|$ by setting $\omega = K_v$ is

$$\begin{aligned} b + c\big|_{\omega=K_v} &= 2K_v^4 + K_c^2 + \tau^2K_v^6 + \\ &+(-2K_cK_v^2 - 2K_v^5\tau)\cos(\Delta K_v) + \\ &+(2K_cK_v^3\tau - 2K_v^4)\sin(\Delta K_v) \end{aligned} \tag{20}$$

Knowing that

$$-\sqrt{a_2^2 + b_2^2} \le a_2\sin x + b_2\cos x \le \sqrt{a_2^2 + b_2^2}$$

and the fact that $\Delta/(\tau+\Delta) < 1$, we will obtain

$$\begin{aligned} &(-2K_cK_v^2 - 2K_v^5\tau)\cos(\Delta K_v) + (2K_cK_v^3\tau - 2K_v^4)\sin(\Delta K_v) \le \\ &\le \sqrt{4K_v^4K_c^2\left(1 + K_c^2\tau^4\left(\frac{\Delta}{\tau+\Delta}\right)^2\right)^2} \end{aligned}$$

Then, equation (20) can be written as the following inequality

$$\begin{aligned} (b+c)\big|_{\omega=K_v} &\le 2K_v^4 + K_c^2 + \tau^2K_v^6 + \\ &+2K_v^2K_c\left(1 + K_c^2\tau^4\left(\frac{\Delta}{\tau+\Delta}\right)^2\right) \end{aligned} \tag{21}$$

By taking the upper boundary of $K_v \le K_c\tau\Delta/(\tau+\Delta)$ inequality (21) can be written in the following form

$$\begin{aligned} (b+c)\big|_{\omega=K_v} &\le 2K_c^4\left(\frac{\tau\Delta}{\tau+\Delta}\right)^4 + \tau^2K_c^6\left(\frac{\tau\Delta}{\tau+\Delta}\right)^6 + K_c^2 + \\ &+2K_c^3\left(\frac{\tau\Delta}{\tau+\Delta}\right)^2\left(1 + K_c^2\tau^4\left(\frac{\tau\Delta}{\tau+\Delta}\right)^2\right) = \\ &= K_c^2\left(\tau^2K_c^4\left(\frac{\tau\Delta}{\tau+\Delta}\right)^6 + 4K_c^3\tau^2\left(\frac{\tau\Delta}{\tau+\Delta}\right)^4 + \right. \\ &\left. +6K_c^2\left(\frac{\tau\Delta}{\tau+\Delta}\right)^4 + 4K_c\left(\frac{\tau\Delta}{\tau+\Delta}\right)^2 + 1\right) - \\ &-K_c^2\left(2K_c^3\tau^2\left(\frac{\tau\Delta}{\tau+\Delta}\right)^4 + 4K_c^2\left(\frac{\tau\Delta}{\tau+\Delta}\right)^4 + 2K_c\left(\frac{\tau\Delta}{\tau+\Delta}\right)^2\right) \end{aligned} \tag{22}$$

With some simplifications, inequality (22) can be written as follows:

$$\begin{aligned} &(b+c)\big|_{\omega=K_v} \le \\ &\le K_c^2\left(\left(K_c\tau\left(\frac{\tau\Delta}{\tau+\Delta}\right) + 1\right)^4 - \left(K_c\left(\frac{\tau\Delta}{\tau+\Delta}\right)^2 + 1\right)^2\right) \end{aligned} \tag{23}$$

By considering inequalities (18),(19) and (23)

$$\sqrt{\frac{K_v^4\left(\left(\left(\frac{\tau+\Delta}{\tau\Delta}\right) - K_v\right)^2 + K_v^2\right)}{K_c^2\left(\left(K_c\tau\left(\frac{\tau\Delta}{\tau+\Delta}\right) + 1\right)^4 - \left(K_c\left(\frac{\tau\Delta}{\tau+\Delta}\right)^2 + 1\right)^2\right)}} \le \sqrt{\frac{\beta}{\alpha}} \tag{24}$$

The jerk is significant when evaluating the discomfort brought about to passengers in a vehicle. Then, an acceptable criterion is that the restricted longitudinal accelerations and jerks can satisfy a definite degree of comfort in longitudinal control. Because of comfort criteria, the bounded jerk is maintained at less than $3\ m/s^3$ i.e. $(\beta = 3)$ and α is the deceleration of the i-th vehicle under the maximum brake action which can be obtained from the vehicle manufacturers or by experiments. In this paper the value of α has been considered as equal to $7\ m/s^2$.

Inequality (24) should be maintained in addition to string stability requirement. Then, for obtaining comfort criteria, by substituting variable as $x = K_v$ and

$y = K_c - K_v(\tau+\Delta)/(\tau\Delta)$ inequality (24) can be written as follows:

$$\sqrt{\frac{x^4\left(\left(\left(\frac{\tau+\Delta}{\tau\Delta}\right)-x\right)^2+x^2\right)}{H}} \le \frac{\beta}{\alpha} \tag{25}$$

where

$$H = \left(y + x\frac{\tau+\Delta}{\tau\Delta}\right)^2\left(\left(\left(y+x\frac{\tau+\Delta}{\tau\Delta}\right)\tau\left(\frac{\tau\Delta}{\tau+\Delta}\right)+1\right)^4 - \right.$$
$$\left. -\left(\left(y+x\frac{\tau+\Delta}{\tau\Delta}\right)\left(\frac{\tau\Delta}{\tau+\Delta}\right)^2+1\right)^2\right)$$

and $x, y > 0$

We can obtain that inequality (23) is true for $K_v \le 0.5(\Delta+\tau)$.

Here, the proof of the theorem is completed.

6. SIMULATION

In order to validate the performance of the proposed control algorithm, computer simulations have been carried out for the platoon system containing nine followers, i.e., N = 9. As stated before, the ability of maintaining the distance between vehicles is important for the safety of the vehicle platoon system. The most important disturbances in a platoon control system include the lead vehicle acceleration/deceleration. The disturbance is defined as any source that causes the vehicle string to lose maintaining constant velocity. In the simulations, the desired vehicle spacing was set as $D_{i-1,i} = 8m$ and the length of vehicle as $L_i = 4m$; other parameters used in the simulations are the same as parameters mentioned in [27], that is, $\sigma_i = 1\,kg/m^3$, $A_i = 2.2\,m^2$, $c_{di} = 0.35$, $m_i = 1{,}500\,kg$, and $d_{mi} = 150\,N$.

In the simulations, the desired acceleration for the lead vehicle is given by

$$a_{des} = \begin{cases} 0 & t \le 20s \\ 2\,m/s^2 & 20s \le t \le 30s \\ 0 & t \ge 30s \end{cases}$$

The initial velocity is $v_{initial} = 0\,m/s$ and the final desired velocity is $v_{final} = 20\,m/s$ (equal to 72 km/h). Two groups of parameters have been applied to reveal the conditions of Theorem 1. Table I presents the relationship between K_c and $K_v(\tau+\Delta)/(\tau\Delta)$ by two cases when $\Delta = 0.2s$ and $\tau = 0.2s$. Both cases have been simulated to express the stable and unstable states, respectively.

Figure 3 illustrates Case1 in *Table 1* which represents the string stable condition $K_c > K_v(\tau+\Delta)/(\tau\Delta)$. *Figure 3* shows tracking of velocity and it also shows the spacing errors of the vehicles smoothly reduce upstream in the string.

Figure 5 illustrates Case2 in *Table 1* which represents the string unstable condition of $K_c < K_v(\tau+\Delta)/(\tau\Delta)$. *Figure 5* illustrates worse instability in tracking velocity, as well as in keeping the spacing.

Table 1 - Parameters for the platoon

Parameters	Case 1	Case 2
Δ	0.2	0.2
τ	0.2	0.2
K_v	0.15	0.15
K_c	2	1
Condition	$K_c > K_v\frac{\tau+\Delta}{\tau\Delta}$	$K_c < K_v\frac{\tau+\Delta}{\tau\Delta}$

Figures 6 and 7 illustrate acceleration and jerk of the vehicles along the string, respectively. The figures

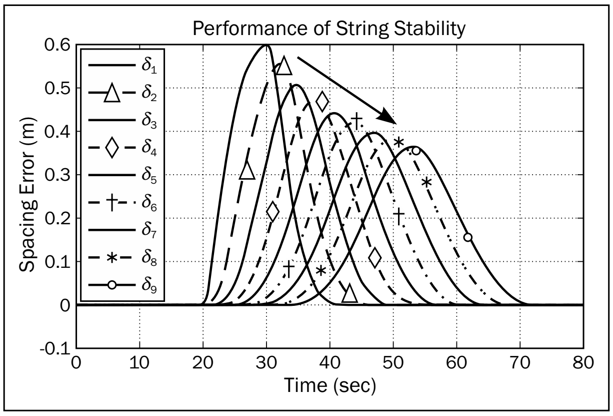

Figure 3 - Performance of string stability of ten vehicles

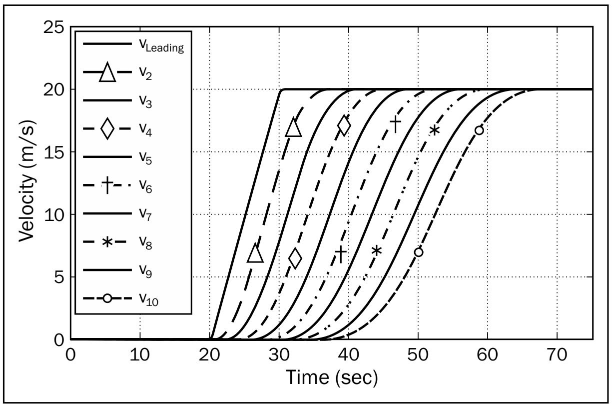

Figure 4 - Performance of velocity tracking of ten vehicles

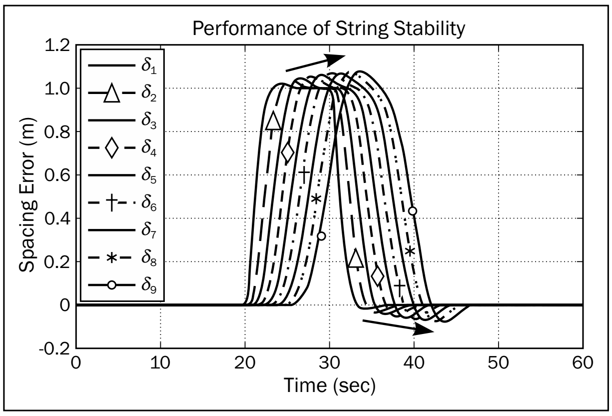

Figure 5 - Performance of string instability of ten vehicles

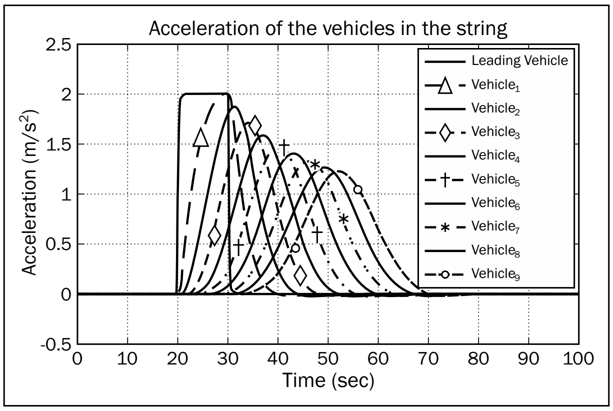

Figure 6 - Acceleration of vehicles along the string

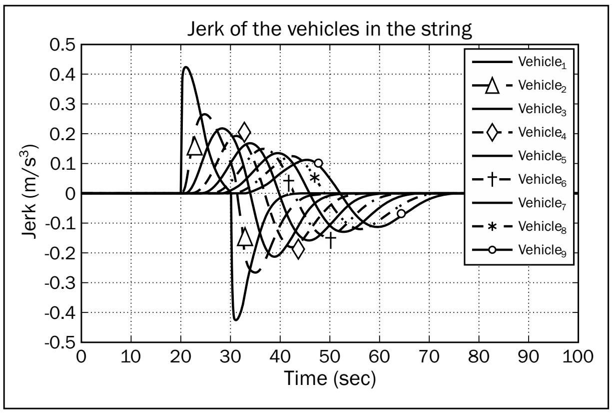

Figure 7 - Jerk of vehicles along the string

also show the performances of the vehicle platoon control system so that the bounds on acceleration and jerk are preserved.

The ability of maintaining the distance between vehicles regardless of the measurement signal noises is important for the vehicle platoon. In this paper, apart from the measurement signal noises the robustness is considered. Even though the information is measurable via sensors, the extraneous noises were unavoidably included in the measurement process, and the overall performance of the string of vehicle control system can be worse as result of the noises. In this simulation, in order to take into account the robustness against the noise in the measurement signals, it is assumed that the measurement signals of the position and the velocity are added into the random number with a mean of 0 and a variance of 0.01 for the position and a mean of 0 and a variance of 0.1 for the velocity. The performance of the vehicle platoon control system under the noise in the measurement signals is shown in *Figure 8* which represents the string stability state.

7. CONCLUSION

The paper addresses the coordination problem of a multi-vehicle system with a leader. The analysis of

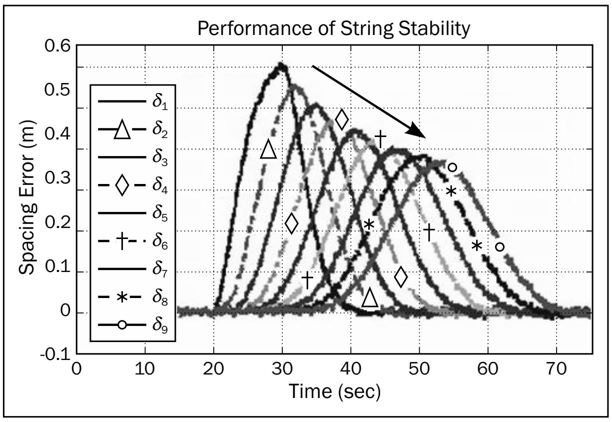

Figure 8 - Performance of string stability of ten vehicles with the noise measurement

error accumulation was also performed. The paper has extended the results related to the string stability analysis of a platoon of vehicles based on the more practical scenarios such as the parasitic time delays and lags. Also, the restriction which needs to be imposed on the control parameters is considered, in order to be well-suited with bounds on the dynamic variables. The results have shown that when the condition $K_c > K_v(\tau+\Delta)/(\tau\Delta)$ holds, the string stability is obtained. At the end, the robustness of the proposed algorithm is validated through computer simulations under noise in the sensor measurement signal.

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خلاصه

کنترل یک دسته از خودروها با تضمین راحتی با درنظرگرفتن تاخیرهاي زماني و وقفه هاي پارازيتي

کنترل طولی یک دسته از خودروها در نظر گرفته شده است به نحوی که خودروها یک خودرو راهنما را با یک فاصله بین خودروئی ثابت دنبال می کنند. هدف از این مقاله در نظر گرفتن اثرات تاخيرهاي زماني و وقفه هاي پارازيتي با تضمین راحتی می باشد. این مقاله دارای دو لایه است. اول، از آنجا که تاخيرهاي زماني و وقفه هاي پارازيتي پارازيتي از خصوصيات طبيعي و اجتناب ناپذير عملگرها و سنسورها در سیستم هاي فیزیکی اند، اثرات منفی آن در پایداری رشته ای بررسی شده است. دوم، برای تضمین محدودیت هاي عملي در متغيرهاي ديناميكي، محدوديت هائي بر روي پارامترهاي كنترلی با تضمین پایدارای رشته ای در نظر گرفته شده است. در انتها، با انجام شبیه سازي برای یک دسته از خودروها، صحت مرزهاي بدست آمده براي پارامترهاي كنترلي، ارزيابي شده است.

کلمات کلیدی

دسته خودروها، پایداری رشته ای، تاخيرهاي زماني و وقفه هاي پارازيتي، معیار راحتی

REFERENCES

[1] **Kesting, A.**, **Treiber, M.**, **Schäonhof, M.** and **Helbing, D.**: *Adaptive cruise control design for active congestion avoidance*, Transportation Research C 16, 668, 2008

[2] **Yadlapalli, S.K.**, **Darbha, S.** and **Rajagopal, K.R.**: *Information flow and its relation to stability of the motion of vehicles in a rigid formation*, IEEE Transactions on Automatic Control, Vol. 51, No. 8, pp. 1315–1319, 2006

[3] **Fax, J.A.** and **Murray, R.M.**: *Information Flow and Cooperative Control of Vehicle Formations*, IEEE Transactions on Automatic Control, Vol. 49, No. 9, pp. 1465–1476, 2004

[4] **Gattami, A.** and **Murray, R.M.**: *A Frequency Domain Condition for Stability of Interconnected MIMO Systems*, Proceedings of the American Control Conference, pp. 3723–3728, 2004

[5] **Chu, K.C.**: *Optimal Decentralized Regulation for a String of Coupled Systems*, IEEE Trans. Automat. Contr., Vol. 19, No. 3, pp. 243-246, 1974

[6] **Swaroop, D.** and **Hedrick, J.K.**: *String stability of interconnected systems*, IEEE Transactions on Automatic Control, Vol. 41, No. 3, pp. 349–357, 1996

[7] **Swaroop, D.** and **Rajagopal, K.R.**: *A review of constant time headway policy for automatic vehicle following*, in Proceedings of the 2001 IEEE Intelligent Transportation Systems Conference, Oakland, CA, 2001

[8] **Yadlapalli, S.K.**, **Darbha, S.** and **Rajagopal, K.R.**: *Information Flow and Its Relation to Stability of the Motion of Vehicles in a Rigid Formation*, IEEE Transactions on Automatic Control, Vol. 51, No. 8, pp. 1315–1319, 2006

[9] **Santhanakrishnan, K.** and **Rajamani, R.**: *On spacing policies for highway vehicle automation*, IEEE Transactions on intelligent transportation systems, Vol. 4, No. 4, pp. 198–204, Dec. 2003

[10] **Zhou, J.** and **Peng, H.**: *Range policy of adaptive cruise control vehicle for improved flow stability and string stability*, IEEE Transactions on intelligent transportation systems., Vol. 6, No. 2, pp. 229–237, June 2005

[11] **Swaroop, D.** and **Hedrick, J.K.**: *Constant spacing strategies for platooning in automated highway systems*, Transactions of the ASME, Vol. 121, pp. 462–470, Sep. 1999

[12] **Ghasemi, A**, **Kazemi, R.** and **Azadi, S.**: *Stable Decentralized Control of Platoon of Vehicles with Heterogeneous Information Feedback*" IEEE Transactions on Vehicular Technology, Vol. 62 , Issue 7, 20 March, 2013

[13] **Xiao, L.** and **Gao, F.**: *Practical String Stability of Platoon of Adaptive Cruise Control Vehicles*, IEEE Transactions on intelligent transportation systems, Vol. 12, No. 4, Dec. 2011

[14] **Xiao, L**, **Darbha, S.** and **Gao, F.**: *Stability of String of Adaptive Cruise Control Vehicles with Parasitic Delays and Lags*", 11th International IEEE Conference on Intelligent Transportation Systems, 10/2008

[15] **Yi, K.** and **J. Chung, J.**: *Nonlinear brake control for vehicle CW/CA system*, IEEE/ASME Trans. Mechatron., Vol. 6, No. 1, pp. 17–25, March 2001

[16] **Hoberock, L.L.**: *A survey of longitudinal acceleration comfort studies in ground transportation vehicles*, J. Dyn. Syst., Meas. Contr., pp.76–84, June 1977

[17] **Cook, P.**: *Stable Control of Vehicle Convoys for Safety and Comfort*, IEEE Transactions on Automatic Control, Vol. 52, No. 3, March 2007

[18] **Rajamani, R.**: *Vehicles Dynamics and Control*, New York, USA: Springer, 2006

[19] **Moskwa, J.** and **Hedrick, J.**: *Modeling and Validation of Automotive Engines for Control Algorithm Development*", Journal of Dynamic Systems, Measurement, and Control, Vol. 114, pp. 278-285, June 1992

[20] **Sheikholeslam, Sh.** and **Desoer, Ch.A.**: *Longitudinal Control of a Platoon of Vehicles*, IEEE American Control Conference, 1990

[21] **Huang, S.** and **Ren, W.**: *Autonomous intelligent cruise control with actuator delays*, J. Intell. Robot. Syst., Vol. 23, No. 1, pp. 27–43, Sep. 1998

[22] **Yanakiev, D.** and **Kanellakopoulos, I.**: *Longitudinal control of automated CHVs with significant actuator delays*, IEEE Trans. Veh. Technol, Vol. 50, No. 5, pp. 1289–1297, Sep. 2001

[23] **Lu, X.** and **Hedrick, K.**: *Practical string stability for longitudinal control of automated vehicles*, in Proc. IAVSD Conf., Kanagawa, Japan, Aug. 2003

[24] **Yanakiev, D.** and **Kanellakopoulos, I**: *A Simplified Framework for String Stability Analysis in AHS*, Proc. of the 13th IFAC World Congress, Volume Q, pp.177-182,1996

[25] **Cook, P.**: *Conditions for String Stability*, System and Control Letters, Vol. 54, pp. 991-998, 2005

[26] **Seiler, P.**, **Pant, A.** and **Hedrick, K.**: *Disturbance propagation in vehicle strings*, IEEE Trans. Autom. Control, Vol. 49, No. 10, pp. 1835–1841, Oct. 2004

[27] **Guo, G.** and **Yue, W.**: *Hierarchical platoon control with heterogeneous information feedback*, IET Control Theory Appl., Vol. 5, Iss. 15, pp. 1766–1781, 2011