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Designing Gaussian Membership Functions for Fuzzy Classifier Generated by Heuristic Possibilistic Clustering

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Abstract

The paper deals with the problem of constructing Gaussian membership functions of fuzzy sets for fuzzy rules derived from the data by using heuristic algorithms of possibilistic clustering. Basic concepts of the heuristic approach to possibilistic clustering are reminded and the extended technique of constructing membership functions of fuzzy sets is proposed. An illustrative example is given and preliminary conclusions are made.
Keywords: fuzzy cluster, fuzzy rule, antecedent, consequent, Gaussian membership function

1. Introduction

Fuzzy rule-based classifiers are one of the most famous applications of fuzzy logic and fuzzy sets theory. They can be helpful to achieve classification tasks, process simulation and diagnosis, online decision support tools and process control. Fuzzy classifiers are based on fuzzy inference systems. One key feature of fuzzy inference systems is their comprehensibility because each fuzzy classification rule is clearly interpretable. So, the problem of generation of fuzzy rules is one of more than important problems in the development of fuzzy inference systems.

There are a number of approaches to learning fuzzy rules from data based on techniques of fuzzy or possibilistic clustering [2]. The idea of deriving fuzzy classification rules from data can be formulated as follows: the training data set is divided into homogeneous group and a fuzzy rule is associated to each group.

Let assume that the training set contains n data pairs. Each pair is made of a m_1-dimensional input-vector and a c-dimensional output-vector. We assume that the number of rules in the fuzzy inference system rule base is c. So, Mamdani and Assilian's [4] fuzzy rule l within the fuzzy inference system is written as follows:

$$\textit{If } \hat{x}^1 \textit{ is } B_l^1 \textit{ and } \ldots \textit{and } \hat{x}^{m_1} \textit{ is } B_l^{m_1} \textit{ then } y_1 \textit{ is } C_1^l \textit{ and} \ldots \textit{and } y_c \textit{ is } C_c^l , \quad (1)$$

where $B_l^{t_1}$, $t_1 \in \{1,\ldots,m_1\}$ and C_l^l, $l \in \{1,\ldots,c\}$ are fuzzy sets that define an input and output space partitioning. A fuzzy inference system which is described by a set of fuzzy classification rules with the form (1) is the multiple inputs, multiple outputs system. Membership functions of fuzzy sets $B_l^{t_1}$, $t_1 \in \{1,\ldots,m_1\}$ and C_l^l, $l \in \{1,\ldots,c\}$ can be trapezoidal, triangular or Gaussian.

A heuristic approach to possibilistic clustering was outlined in [7] and the approach was developed in other publications. The essence of the proposed heuristic approach to possibilistic clustering is that the sought clustering structure of the set of objects is formed based directly on the formal definition of fuzzy cluster and possibilistic memberships are determined also directly from the values of the pairwise similarity of objects.

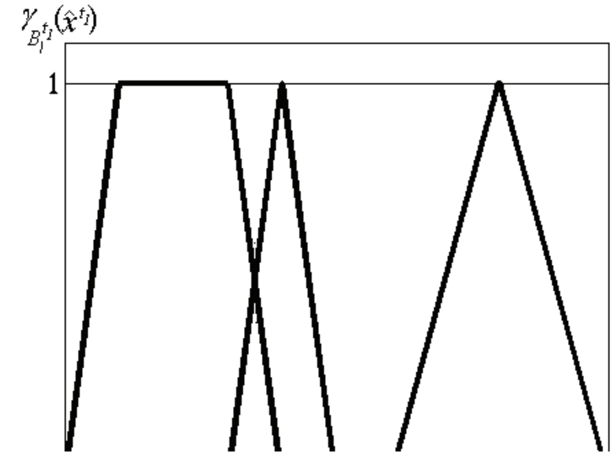
Heuristic clustering algorithms which are based on a definition of the cluster concept are called algorithms of direct classification or direct clustering algorithms. Direct heuristic algorithms of possibilistic clustering can be divided into two types: relational versus prototype-based. A fuzzy tolerance relation matrix is a matrix of the initial data for the direct heuristic relational algorithms of possibilistic clustering and a matrix of attributes is a matrix of the initial data for the prototype-based algorithms. In particular, the group of direct relational heuristic algorithms of possibilistic clustering includes

- D-AFC(c)-algorithm: using the construction of the allotment among given number c of partially separate fuzzy clusters;
- D-PAFC-algorithm: using the construction of the principal allotment among an unknown minimal number of at least c fully separate fuzzy clusters;
- D-AFC-PS(c)-algorithm: using the partially supervised construction of the allotment among given number c of partially separate fuzzy clusters.

On the other hand, the family of direct prototype-based heuristic algorithms of possibilistic clustering includes

- D-AFC-TC-algorithm: using the construction of the allotment among an unknown number c of fully separate fuzzy clusters;
- D-PAFC-TC-algorithm: using the construction of the principal allotment among an unknown minimal number of at least c fully separate fuzzy clusters;
- D-AFC-TC(α)-algorithm: using the construction of the allotment among an unknown number c of fully separate fuzzy clusters with respect to the minimal value α of the tolerance threshold.

It should be noted, that these prototype-based heuristic algorithms of possibilistic clustering are based on the transitive closure of initial fuzzy tolerance. New direct prototype-based heuristic algorithms of possibilistic clustering were proposed in [9] and the family of algorithms is based on the transitive approximation of fuzzy tolerance [1]. A method of the rapid prototyping fuzzy inference systems based on deriving fuzzy classification rules from the data was also proposed in [8]. However, the method of constructing membership functions of fuzzy sets was developed for cases of trapezoidal and triangular membership functions. Moreover, obtained fuzzy model cannot be transparent in all cases because bad coverage can be constructed [5]. The situation is illustrated by Figure 1.



The purpose of the paper is a consideration of the technique of constructing Gaussian membership functions of fuzzy sets for antecedents and consequents of fuzzy rules (1). So, the contents of the paper is as follows: in the second section basic concepts of the heuristic method of possibilistic clustering are considered, in the third section the extended methodology for constructing Gaussian membership functions of fuzzy sets for antecedents and consequents of extracted fuzzy rules is presented, in the fourth section an illustrative example is given and in the fifth section some final remarks are stated.

2. Basic Definitions of the Heuristic Approach to Possibilistic Clustering

Let us remind the basic concepts of the heuristic method of possibilistic clustering [8]. Let $X = \{x_1,...,x_n\}$ be the initial set of elements and $T : X \times X \to [0,1]$ some fuzzy tolerance on X with $\mu_T(x_i,x_j) \in [0,1]$, $\forall x_i, x_j \in X$ being its membership function. Let α be the α-level value of the fuzzy tolerance T, $\alpha \in (0,1]$. Columns or rows of the fuzzy tolerance matrix are fuzzy sets $\{A^1,..., A^n\}$ on X. Let A^l, $l \in \{1,\ldots,n\}$ be a fuzzy set on X with $\mu_{A^l}(x_i) \in [0,1]$, $\forall x_i \in X$ being its membership function. The α-level fuzzy set $A^l_{(\alpha)} = \{(x_i, \mu_{A^l}(x_i)) \mid \mu_{A^l}(x_i) \geq \alpha, x_i \in X\}$ is fuzzy α-cluster. So, $A^l_{(\alpha)} \subseteq A^l$, $\alpha \in (0,1]$, $A^l \in \{A^1,\ldots,A^n\}$ and $\mu_{A^l}(x_i)$ is the membership degree of the element $x_i \in X$ for some fuzzy α-cluster $A^l_{(\alpha)}$, $\alpha \in (0,1]$, $l \in \{1,\ldots,n\}$. The membership degree will be denoted μ_{li} in further considerations. The membership degree of the element $x_i \in X$ for some fuzzy α-cluster $A^l_{(\alpha)}$, $\alpha \in (0,1]$, $l \in \{1,\ldots,n\}$ can be defined as a

$$\mu_{li} = \begin{cases} \mu_{A^l}(x_i), & x_i \in A^l_\alpha \\ 0, & otherwise \end{cases}, \tag{2}$$

where the α-level $A^l_\alpha = \{x_i \in X \mid \mu_{A^l}(x_i) \geq \alpha\}$, $\alpha \in (0,1]$ of a fuzzy set A^l is the support of the fuzzy α-cluster $A^l_{(\alpha)}$ and value of α is the tolerance threshold of fuzzy α-cluster elements.

Let $\{A^1_{(\alpha)},..., A^n_{(\alpha)}\}$ be the family of fuzzy α-clusters for some α. The point $\tau^l_e \in A^l_\alpha$, for which

$$\tau^l_e = \arg\max_{x_i} \mu_{li}, \ \forall x_i \in A^l_\alpha, \tag{3}$$

is called a typical point of the fuzzy α-cluster $A^l_{(\alpha)}$, $\alpha \in (0,1]$, $l \in [1,n]$. A set $K(A^l_{(\alpha)}) = \{\tau^l_1,\ldots,\tau^l_{|l|}\}$ of typical points of the fuzzy cluster $A^l_{(\alpha)}$ is a kernel of the fuzzy cluster and $card\left(K(A^l_{(\alpha)})\right) = |l|$ is a cardinality of the kernel. If the fuzzy cluster have an unique typical point, then $|l| = 1$.

Let $R^\alpha_z(X) = \{A^l_{(\alpha)} \mid l = \overline{1,c}, 2 \leq c \leq n\}$ be a family of fuzzy α-clusters for some value of tolerance threshold α, which are generated by a fuzzy tolerance T on the initial set of elements $X = \{x_1,..., x_n\}$. If condition

$$\sum_{l=1}^{c} \mu_{li} > 0, \ \forall x_i \in X, \tag{4}$$

is met for all $A^l_{(\alpha)}$, $l = \overline{1,c}$, $c \leq n$, then the family is the allotment of elements of the set $X = \{x_1,..., x_n\}$ among fuzzy α-clusters $\{A^l_{(\alpha)}, l = \overline{1,c}, 2 \leq c \leq n\}$ for some value of the tolerance threshold α. It should be noted that several allotments $R^\alpha_z(X)$ can exist for some tolerance threshold α. That is why symbol z is the index of an allotment.

Obviously, the definition of the allotment among fuzzy clusters (4) is similar to the definition of the possibilistic partition [3]. So, the allotment among fuzzy clusters can be considered as the possibilistic partition and fuzzy clusters in the sense of (2) are elements of the possibilistic partition.

Thus, the problem of cluster analysis can be defined as the problem of discovering the unique allotment $R_c^*(X)$, resulting from the classification process and detection of fixed or unknown number c of fuzzy α-clusters can be considered as the aim of classification.

3. Constructing Gaussian Membership Functions for Fuzzy Rules

An extended technique of fuzzy rules antecedents learning is presented in the first subsection. A method of constructing Gaussian membership functions for fuzzy rules consequents is given in the second subsection of the section.

3.1. Antecedents learning

In the following, we will consider that the Mamdani-type fuzzy inference system is a multiple inputs, multiple outputs system. A method of constructing trapezoidal membership functions for fuzzy rules [8] should be reminded in the first place.

Let us consider a fuzzy rule (1) where $B_l^{t_1}$, $t_1 = 1,\ldots,m_1$, $l \in \{1,\ldots,c\}$ is a fuzzy set associate with the attribute $\hat{x}^{t_1}$. The antecedent of a fuzzy rule in a fuzzy inference system defines a decision region in the m_1-dimensional feature space. Let $B_l^{t_1} = (\underline{a}_{(l)}^{t_1}, \underline{m}_{(l)}^{t_1}, \overline{m}_{(l)}^{t_1}, \overline{a}_{(l)}^{t_1})$ be characterized by the trapezoidal membership function $\gamma_{B_l^{t_1}}(\hat{x}^{t_1})$ which is presented in Figure 2.

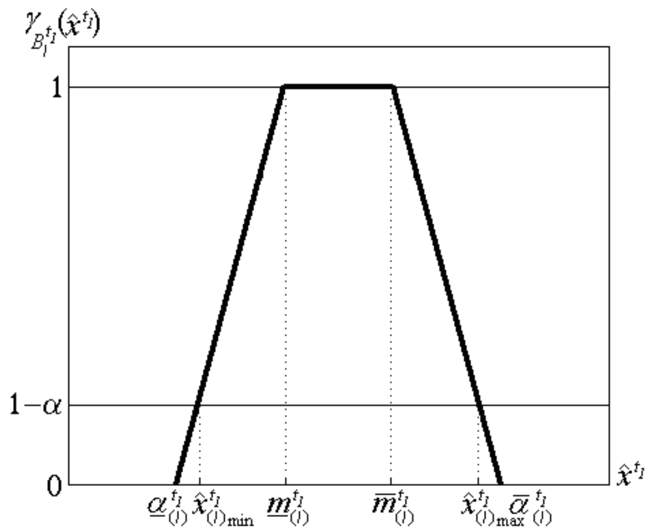


Figure 2. Trapezoidal membership function of an antecedent fuzzy set.

So, a triangular fuzzy set $B_l^t = (\underline{a}_{(l)}^t, m_{(l)}^t, \overline{a}_{(l)}^t)$ can be considered as a particular case of the trapezoidal fuzzy set where $\underline{m}_{(l)}^t = \overline{m}_{(l)}^t$. A technique for extracting fuzzy rules and calculating parameters of fuzzy set $B_l^{t_1}$, $t_1 = 1,\ldots,m_1$, $l = 1,\ldots,c$ is considered in detail in [8].

Let us consider a technique of constructing Gaussian membership function of fuzzy set $B_l^{t_1}$. In general, asymmetric Gaussian membership function $\gamma_{B_l^{t_1}}(\hat{x}^{t_1})$ of fuzzy set $B_l^{t_1}$ can be defined as

$$\gamma_{B_l^{t_1}}(\hat{x}^{t_1}) = \begin{cases} \exp\left[-\dfrac{1}{2}\left(\dfrac{\hat{x}^{t_1} - m_{(l)}^{t_1}}{\sigma_{L(l)}^{t_1}}\right)^2\right], & \hat{x}^{t_1} < m_{(l)}^{t_1} \\ \exp\left[-\dfrac{1}{2}\left(\dfrac{\hat{x}^{t_1} - m_{(l)}^{t_1}}{\ldots}\right)^2\right], & \hat{x}^{t_1} \geq m_{(l)}^{t_1} \end{cases}, \quad (5)$$

where $m^{t_1}_{(l)} = (\underline{m}^{t_1}_{(l)} + \overline{m}^{t_1}_{(l)})/2$ denotes the membership function center and $\sigma^{t_1}_{L(l)}$ and $\sigma^{t_1}_{R(l)}$ represent the left and right spreads. So, values of $\sigma^{t_1}_{L(l)}$ and $\sigma^{t_1}_{R(l)}$ can be defined as

$$\sigma^{t_1}_{L(l)} = \frac{\hat{x}^{t_1}_{(l)\min} - m^{t_1}_{(l)}}{\sqrt{-2\ln(1-\alpha)}}, \tag{6}$$

and

$$\sigma^{t_1}_{R(l)} = \frac{\hat{x}^{t_1}_{(l)\max} - m^{t_1}_{(l)}}{\sqrt{-2\ln(1-\alpha)}}. \tag{7}$$

That is why symmetric Gaussian membership function $\gamma_{B^{t_1}_l}(\hat{x}^{t_1})$ of fuzzy set $B^{t_1}_l = (m^{t_1}_{(l)}, \sigma^{t_1}_{(l)})$ can be defined as

$$\gamma_{B^{t_1}_l}(\hat{x}^{t_1}) = \exp\left[-\frac{1}{2}\left(\frac{\hat{x}^{t_1} - m^{t_1}_{(l)}}{\sigma^{t_1}_{(l)}}\right)^2\right], \; -\infty < \hat{x}^{t_1} < \infty, \tag{8}$$

where $\sigma^{t_1}_{(l)} = \max\{\sigma^{t_1}_{L(l)}, \sigma^{t_1}_{R(l)}\}$. The symmetric Gaussian membership function $\gamma_{B^{t_1}_l}(\hat{x}^{t_1})$ of fuzzy set $B^{t_1}_l = (m^{t_1}_{(l)}, \sigma^{t_1}_{(l)})$ is presented in Figure 3.

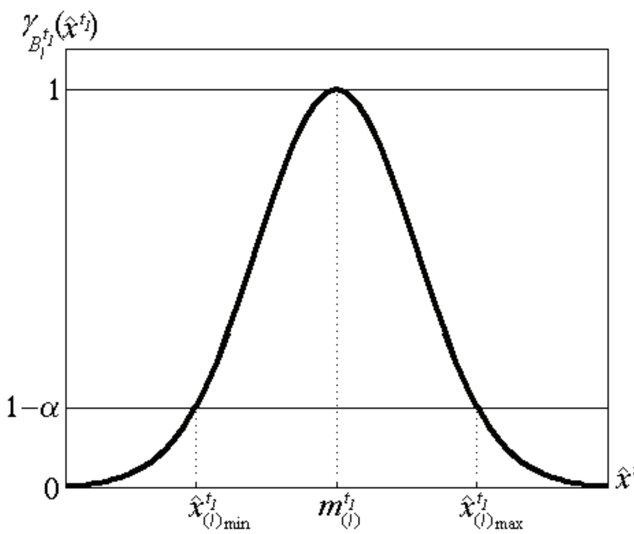


Figure 3. Symmetric Gaussian membership function of an antecedent fuzzy set.

Thus, symmetric Gaussian membership function $\gamma_{B^{t_1}_l}(\hat{x}^{t_1})$ of an antecedent fuzzy set $B^{t_1}_l$ can be constructed very simply. The proposed method is based on calculating parameters of a trapezoidal fuzzy set $B^{t_1}_l = (\underline{a}^{t_1}_{(l)}, \underline{m}^{t_1}_{(l)}, \overline{m}^{t_1}_{(l)}, \overline{a}^{t_1}_{(l)})$ and these parameters should be used for calculating parameters of each symmetric Gaussian fuzzy set $B^{t_1}_l = (m^{t_1}_{(l)}, \sigma^{t_1}_{(l)})$.

3.2. Consequents learning

Symmetric Gaussian membership function $\gamma_{C^l_l}(y_l)$ of consequents fuzzy sets $C^l_l = (\mu_l, \sigma_{(l)})$, $l = 1,\ldots,c$ can be constructed in similar manner. However, the method of constructing trapezoidal membership function $\gamma_{C^l_l}(y_l)$ of consequents fuzzy sets C^l_l, $l = 1,\ldots,c$ must be can be defined on the interval of memberships $[0,1]$ and these fuzzy sets can be presented as follows: $C_l^l = (\alpha, \underline{\mu}_l, \overline{\mu}_l, 1)$, where α is the tolerance threshold, $\underline{\mu}_l = \min_{x_i \in A_\alpha^l} \mu_{li}$ and $\overline{\mu}_l = \max_{x_i \in A_\alpha^l} \mu_{li}$.

The membership function can be interpreted as a high membership. The case is presented in Figure 4.

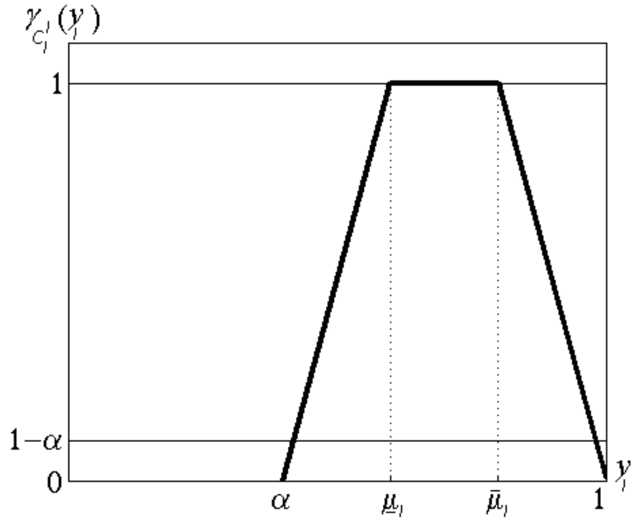


Figure 4. Trapezoidal membership function of a consequent fuzzy set in the case of a high degree of belongingness.

From other hand, if $A_{(\alpha)}^l$ and $A_{(\alpha)}^m$, $l \neq m$ are two particularly separated fuzzy clusters then a fuzzy set $C_m^l = (0, 1-\overline{\mu}_m, 1-\underline{\mu}_m, 1-\alpha)$ is the consequent for the variable y_m of the l-th fuzzy rule for a case of low membership. The case is presented in Figure 5.

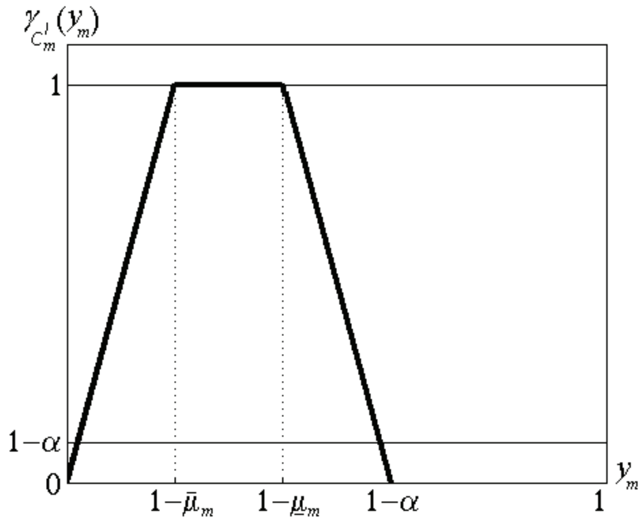


Figure 5. Trapezoidal membership function of a consequent fuzzy set in the case of a low degree of belongingness.

Fuzzy clusters can be subnormal fuzzy sets. The fact is shown in [8]. So, cases which are presented in Figures 4 and 5 are general cases. If the allotment $R_c^*(X)$ among fully separate

Let us consider a technique of constructing Gaussian membership function of fuzzy sets C_l^l, $l=1,\dots,c$. Asymmetric Gaussian membership function $\gamma_{C_l^l}(y_l)$ for a case of high membership can be defined as

$$\gamma_{C_l^l}(y_l)=\begin{cases}\exp\left[-\frac{1}{2}\left(\frac{y_l-\mu_l}{\sigma_{L(l)}^c}\right)^2\right], & y_l<\mu_l\\ \exp\left[-\frac{1}{2}\left(\frac{y_l-\mu_l}{\sigma_{R(l)}^c}\right)^2\right], & y_l\geq\mu_l\end{cases}, \tag{9}$$

where the membership function $\gamma_{C_l^l}(y_l)$ center can be calculated according to a formula

$$\mu_l=(\underline{\mu}_l+\overline{\mu}_l)/2, \tag{10}$$

and values of $\sigma_{L(l)}^c$ and $\sigma_{R(l)}^c$ can be calculated as

$$\sigma_{L(l)}^c=\frac{\hat{\underline{\mu}}_l-\mu_l}{\sqrt{-2\ln(1-\alpha)}}, \tag{11}$$

and

$$\sigma_{R(l)}^c=\frac{\hat{\mu}_l-\mu_l}{\sqrt{-2\ln(1-\alpha)}}. \tag{12}$$

where values of coefficients $\hat{\underline{\mu}}_l$ and $\hat{\mu}_l$ can be obtained according to formulas

$$\hat{\underline{\mu}}_l=\underline{\mu}_l-\alpha(\underline{\mu}_l+\alpha), \tag{13}$$

$$\hat{\mu}_l=1-\overline{\mu}_l-\alpha(1-\overline{\mu}_l), \tag{14}$$

and the value of $\sigma_{(l)}^c=\max\{\sigma_{L(l)}^c,\sigma_{R(l)}^c\}$ can be defined as $\sigma_{(l)}^c=\max\{\sigma_{L(l)}^c,\sigma_{R(l)}^c\}$. So, the corresponding membership function can be defined according to a formula

$$\gamma_{C_l^l}(y_l)=\exp\left[-\frac{1}{2}\left(\frac{y_l-\mu_l}{\sigma_{(l)}^c}\right)^2\right], -\infty<y_l<\infty, \tag{15}$$

and the corresponding case is presented in Figure 6.

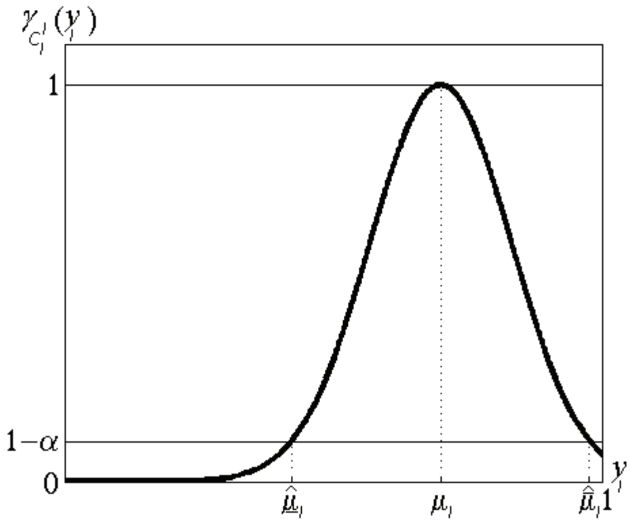


Figure 6. Symmetric Gaussian membership function of a consequent fuzzy set in the case of a high degree of belongingness.

$$\gamma_{C_m^l}(y_m)=\begin{cases}\exp\left[-\frac{1}{2}\left(\frac{y_m-\tilde{\mu}_m}{\sigma_{L(m)}^c}\right)^2\right], & y_m<\tilde{\mu}_m\\ \exp\left[-\frac{1}{2}\left(\frac{y_m-\tilde{\mu}_m}{\sigma_{R(m)}^c}\right)^2\right], & y_m\geq\tilde{\mu}_m\end{cases}, \tag{16}$$

where the membership function $\gamma_{C_m^l}(y_m)$ center can be calculated according to a formula

$$\tilde{\mu}_m=((1-\underline{\mu}_m)+(1-\overline{\mu}_m))/2, \tag{17}$$

and values of $\sigma_{L(m)}^c$ and $\sigma_{R(l)}^c$ can be calculated as

$$\sigma_{L(m)}^c=\frac{\underline{\tilde{\mu}}_m-\tilde{\mu}_m}{\sqrt{-2\ln(1-\alpha)}}, \tag{18}$$

and

$$\sigma_{R(m)}^c=\frac{\tilde{\tilde{\mu}}_m-\tilde{\mu}_m}{\sqrt{-2\ln(1-\alpha)}}. \tag{19}$$

where coefficients $\underline{\tilde{\mu}}_m$ and $\tilde{\tilde{\mu}}_m$ can be defined in the following way

$$\underline{\tilde{\mu}}_m=(1-\alpha)(1-\overline{\mu}_m), \tag{20}$$

$$\tilde{\tilde{\mu}}_m=(1-\alpha)(-1-\underline{\mu}_m-\alpha), \tag{21}$$

and $\sigma_{(m)}^c=\max\{\sigma_{L(m)}^c,\sigma_{R(m)}^c\}$. So, the membership function can be defined as follows:

$$\gamma_{C_m^l}(y_m)=\exp\left[-\frac{1}{2}\left(\frac{y_m-\tilde{\mu}_m}{\sigma_{(m)}^c}\right)^2\right], -\infty<y_m<\infty, \tag{22}$$

and the membership function is presented in Figure 7.

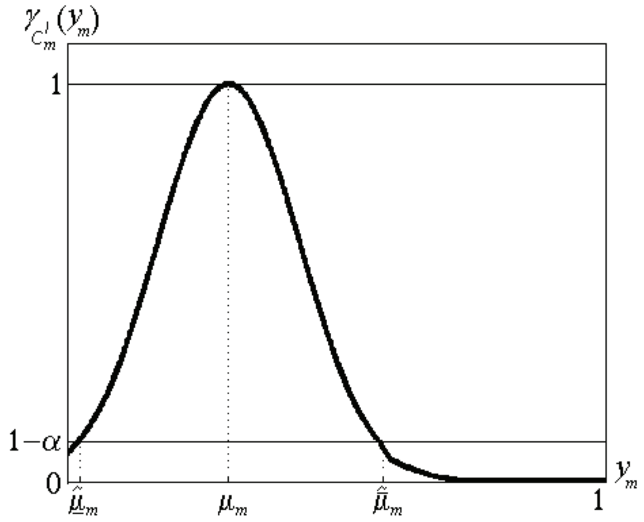


Figure 7. Symmetric Gaussian membership function of a consequent fuzzy set in the case of a low degree of belongingness.

So, symmetric Gaussian membership function $\gamma_{B_l^{t_1}}(\hat{x}^{t_1})$ of an antecedent fuzzy set $B_l^{t_1}$ and symmetric Gaussian membership functions $\gamma_{C_l^l}(y_l)$ and $\gamma_{C_m^l}(y_m)$ of consequents fuzzy sets C_l^l and C_m^l can be obtained immediately by using parameters of corresponding trapezoidal membership functions. The proposed technique should be explained by a simple illustrative example.

4. An Illustrative Example

The first subsection of the section includes the results of the two-dimensional Sneath and Sokal [6] dataset clustering by the D-AFC(c)-algorithm. The designed fuzzy inference system with Gaussian membership functions of fuzzy sets for antecedents and consequents is presented in the second subsection and the results are compared with the results of fuzzy inference system with triangular membership functions of corresponding fuzzy sets.

4.1. Results of clustering of Sneath and Sokal dataset

Let us consider results of clustering which were obtained from the heuristic D-AFC(c)-algorithm of possibilistic clustering algorithm which was applied to two-dimensional artificial Sneath and Sokal [6] dataset. The original data set is presented in Figure 8.

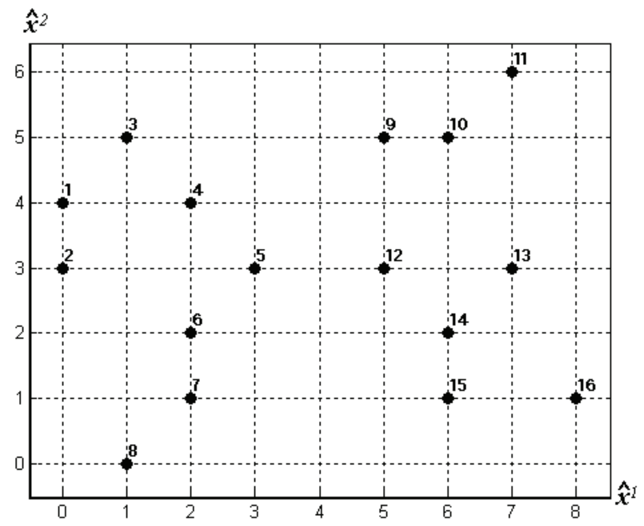


Figure 8. Sneath and Sokal dataset.

The D-AFC(c)-algorithm was applied to the matrix of fuzzy tolerance for $c = 2,\ldots,5$ using a validity measure [8]. The actual number of fuzzy clusters is equal 2 and this number corresponds to the first minimum of the measure of separation and compactness of the allotment [8].

By executing the D-AFC(c)-algorithm for two classes we obtain the following: the first class is formed by 8 elements and the second class is composed of 9 elements. The fifth element belongs to both classes. The allotment $R_c^*(X)$, which corresponds to the result, was obtained for the tolerance threshold $\alpha = 0.81944$. The value of the membership function of the fuzzy cluster which corresponds to the first class is maximal for the second object and is equal one. So, the second object is the typical point of the first fuzzy cluster. The membership value of the thirteenth object is equal one for the second fuzzy cluster. Thus, the thirteenth object is the typical point of the second fuzzy cluster. Membership functions of two classes of the allotment are presented in Figure 9 and values which equal zero are not shown in the presented figure.

Membership values of the first class are represented by ○, and membership values of the second class are represented by ■ in Figure 9. Values which equal zero are not shown in the figure. Obviously, that the both fuzzy α-clusters are particularly separated fuzzy α-clusters [8].

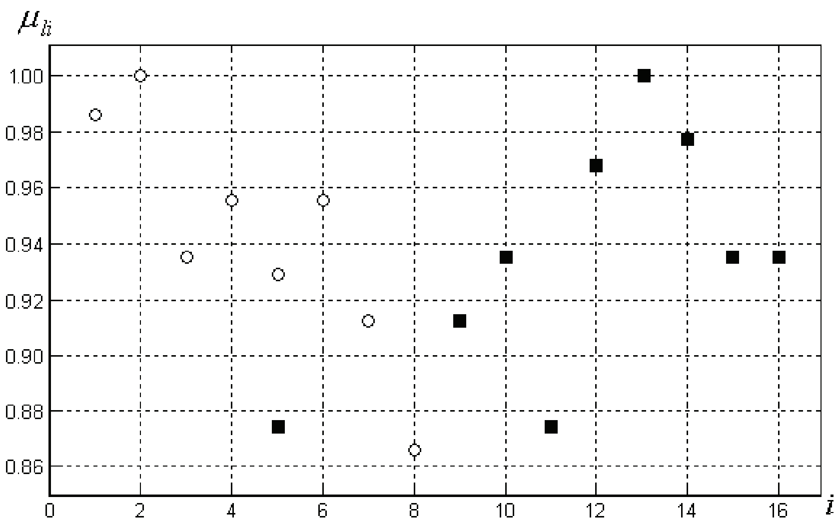


Figure 9. Membership functions of two fuzzy clusters which were obtained by using the D-AFC(c)-algorithm.

So, the results which were obtained by using the D-AFC(c)-algorithm seem to be satisfactory in comparison with fuzzy clustering algorithms [8].

4.2. Generated fuzzy inference systems

The technique for constructing fuzzy inference systems should be explained by examples. So, an example for the Sneath and Sokal two-dimensional dataset [6] can be considered for the purpose.

In the first place, let us consider the results obtained for the triangular membership functions. The corresponding technique for deriving fuzzy rules from fuzzy clusters was applied to the initial data. So, triangular membership functions $\gamma_{B_l^{t_1}}(\hat{x}^{t_1})$ for corresponding fuzzy sets $B_l^{t_1}$ and trapezoidal membership functions $\gamma_{C_l^l}(y_l)$ for fuzzy sets C_l^l, $t_1 = 1, 2$, $l = 1, 2$, were constructed immediately. The rule base induced by the D-AFC(c)-algorithm clustering result can be seen in Figure 10 [8] in which the performance of the designed fuzzy inference system is shown.

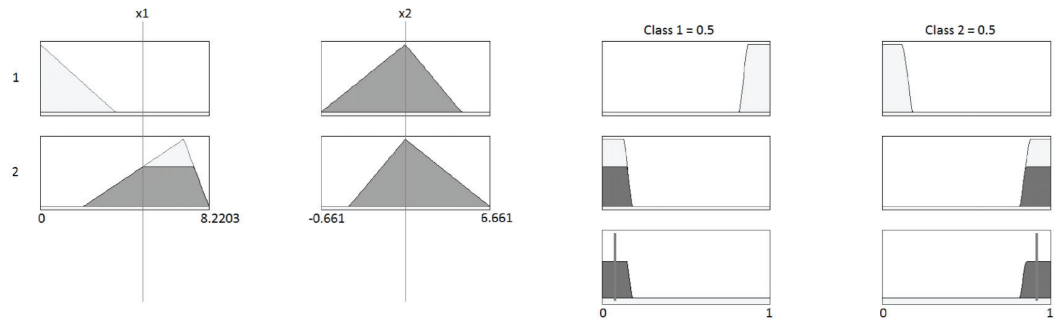


Figure 10. Performance of the fuzzy inference system which was generated from Sneath and Sokal dataset by using triangular membership functions.

Note that fuzzy sets C_2^1 and C_1^2 are non-empty because the allotment $R_c^*(X)$, which corresponds to the clustering result, is the allotment among two particularly separated fuzzy clusters. Figure 10 shows the example of classification of the fifth object which belongs to both classes. Labels $x1$ and $x2$ denote the first attribute and the second attribute in the data set, and $l = 1, 2$ is the number of rule in the Figure 10.

The value of the membership function of the fuzzy cluster which corresponds to the first class is equal 0.92969 for the fifth object. The value of membership function of the second fuzzy cluster is equal 0.875 for the fifth object. So, $\mu_{15} > \mu_{25}$ and the fact is shown in Figure 9. From other hand, Figure 10 shows that the fifth object is the element of both classes and corresponding values of an average of the range of output variables are equal 0.5. These values can be interpreted as average belonging of the object to both classes.

A plot of the output surface of the generated fuzzy inference system using both inputs and the second output is presented in Figure 11 [8].

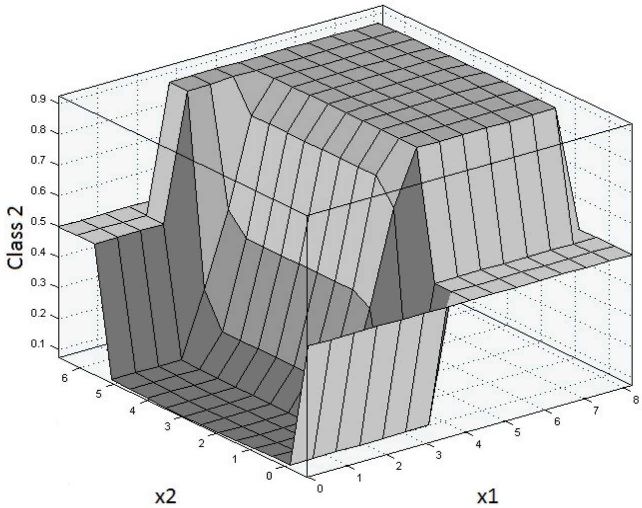


Figure 11. Output surface of the fuzzy inference system which was generated from Sneath and Sokal dataset by using triangular membership functions.

In the second place, the results obtained for the Gaussian membership functions should be considered. The proposed technique for deriving fuzzy rules from fuzzy clusters was applied to the initial data. That is why Gaussian membership functions $\gamma_{B_l^{t_1}}(\hat{x}^{t_1})$ and $\gamma_{C_l^l}(y_l)$ for fuzzy sets $B_l^{t_1}$ and C_l^l, $t_1 = 1, 2$, $l = 1, 2$, were constructed. The corresponding rule base induced by the D-AFC(c)-algorithm clustering result can be seen in Figure 12 where a graph of the performance of the designed fuzzy inference system is presented.

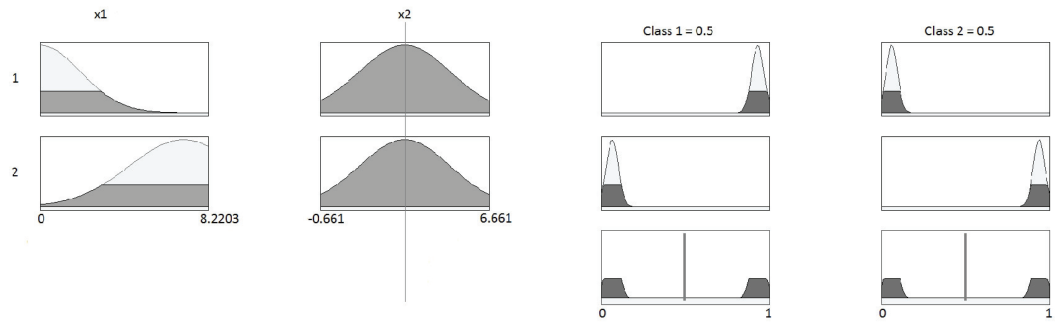


Figure 12. Performance of the fuzzy inference system which was generated from Sneath and Sokal dataset by using Gaussian membership functions.

Figure 12 shows that the fifth object is the element of both classes and corresponding values of an average of the range of output variables are also equal 0.5.

A plot of the output surface of the generated fuzzy inference system using both inputs and the second output is presented in Figure 13.

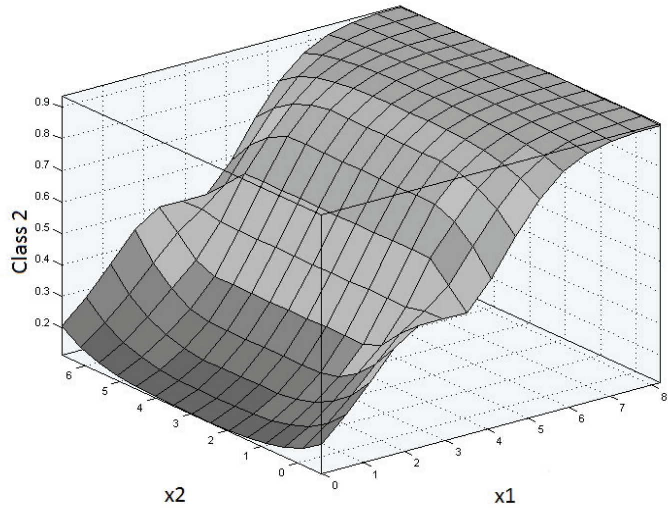


Figure 13. Output surface of the fuzzy inference system which was generated from Sneath and Sokal dataset by using Gaussian membership functions.

So, the results obtained from the fuzzy inference system which was generated by using Gaussian membership functions is not very different from the results obtained from the fuzzy inference system which was generated by using triangular membership functions. However, the constructed fuzzy inference system is transparent because the condition $-\infty < \hat{x}^{t_1} < \infty$ is met in the formula (8) and the condition $-\infty < y_l < \infty$ is met in formulas (15) and (22). Numerical experiments are shown that fuzzy inference system which was generated from the data by using Gaussian membership functions is precise and accurate.

5. Conclusions

We have presented an approach to construct Gaussian membership functions for fuzzy inference systems obtained from clustering results. Heuristic possibilistic clustering is used for the purpose. The proposed approach to construct Gaussian membership functions is simple, and it does not require any techniques such as genetic algorithms for tuning the Gaussian membership functions. Compact and transparent, yet accurate Mamdani-type fuzzy classifiers are the results.

It should be note, that the proposed approach can be generalized for a case of unstable clustering structure very simply [7]. On the other hand, the technique of rapid prototyping fuzzy inference systems using Gaussian membership functions can be extended for a case of unknown number of classes. The extended technique can be elaborated on a base of direct algorithms of possibilistic clustering which are based on the transitive approximation of fuzzy tolerance [9].

These perspectives for investigations are of great interest both from the theoretical point of view and from the practical one as well.

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