A NOVEL LOAD FLOW ALGORITHM FOR RADIAL DISTRIBUTION NETWORKS WITH DISPERSED GENERATION

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During the planning and designing stage of power distribution networks development, a large number of load flow calculations have to be made. The purpose of these calculations is to check if the thermal constraints and voltage limits of network elements are breached in order to comply with the EN 50160 quality standard. With the integration of distributed generation, the once passive distribution networks are rapidly changing to active networks. As a consequence, the planners have to take into consideration the bidirectional power flow in distribution networks. Also, the existing power flow algorithms for unidirectional radial distribution feeders need to adapt to new circumstances. This paper considers a new iterative algorithm for load flow calculation of a radial distribution feeder with distribution lines represented by a detailed π equivalent model including dispersed generation. In so doing, the influence on voltage profile and technical power losses is analysed in cases with and without the connection of dispersed generation. The algorithm is implemented in Matlab programming language.

Keywords: active distribution networks, bidirectional power flow, distribution line π equivalent model and dispersed generation

Proračun tokova snaga za distribucijske mreže s uključenim distribuiranim izvorima električne energije

Izvorni znanstveni članak

Prilikom planiranja i projektiranja distribucijskih elektroenergetskih mreža potrebno je izvršiti veći broj proračuna tokova snaga i padova napona. Svrha proračuna je provjera termičkih opterećenja elemenata mreže i mogućih naponskih prekoračenja, a sve sa zadatkom kvalitetne isporuke električne energije krajnjim kupcima. Distribucijske mreže su donedavno imale pasivnu ulogu, koja se očitovala kroz činjenicu da su napajane iz jednog čvora (napojne trafostanice) i projektirane za jednosmjerni tok snaga od napojnih trafostanica prema potrošačima. Priključkom novih potrošača i proizvođača električne energije značajno se mijenjaju prilike u okolnoj distribucijskoj mreži, koja mijenja svoju ulogu iz pasivne u aktivnu. U ovom se radu razmatra točan iterativni proračun tokova snaga za pogonski radijalnu distribucijsku mrežu proizvoljne konfiguracije, s vodovima modeliranim π ekvivalentnim modelom i s uključenim distribuiranim izvorom električne energije. Pritom se analizira utjecaj distribuiranog izvora na naponske prilike, tokove snaga i tehničke gubitke u mreži. Program je napisan i realiziran u programskom paketu Matlab.

Ključne riječi: aktivne distribucijske mreže, dvosmjerni tokovi snaga, π model distribucijskog voda i distribuirani izvori električne energije

1 Introduction

Distribution networks are a part of an electrical power system and they represent the final stage in the delivery of electricity to end users. A distribution network transfers power from the transmission system and is responsible for its delivery to consumers on medium and/or low voltage. The distribution network is owned and operated by Distribution System Operator (DSO). Typically, the Croatian distribution networks include voltage levels of 35, 20, 10 and 0,4 kV. The tendency is to replace 35 and 10 kV voltage with a 20 kV voltage level.

However, due to a recent unbundling and restructuring of the power sector, and with the introduction of dispersed generation (DG), consumers can now produce electricity and deliver it to the distribution grid. Thus, the term consumer is nowadays replaced by a term prosumer. Also, because of a strong development of dispersed generation production technology, further reduction in capital and plant operation & maintenance costs is expected and higher government subsidies are being introduced all over the world.

Some of the advantages of dispersed generation are:
- Electricity is generated near the place of its consumption, which results in lower power and energy losses,
- Improvement in network reliability and security of supply,
- Improved voltage profile of the feeders,
- Improved voltage stability,
- etc.

However, the main disadvantage of dispersed generation includes the mass introduction of intermittent power sources to distribution networks. The only remedy to this problem is to develop the Smart Grids concept in parallel with the introduction of DG units.

Distribution networks' operation is radial (although, their topology is often meshed in urban areas), and its main difference compared to transmission networks is a much higher resistance to reactance (R/X) ratio. Because of higher R/X ratio, radial operation and higher voltage drop, the usual load flow algorithms developed for high voltage (HV) analysis (like "Gauss-Seidel" or "Newton-Raphson" method), perform unsatisfactory in distribution networks. For a large number of nodes (MV/LV substations), the convergence of the aforementioned methods is slow, and often not possible.

As a result, new methods are proposed which specialize in load flow calculation for radial distribution networks with a large number of nodes. Most of them are based on backward/forward sweep algorithm, as in [1 ÷ 6]. This paper will address and modify the backward/forward sweep method for a detailed π equivalent model of a distribution line, with the inclusion of dispersed active and reactive power generation.

2 Basic concept of a distribution network load flow study

As it was mentioned, in case when distribution feeders are very long, with a large R/X ratio, the traditional load flow methods very often diverge and are not fit for calculations.

The new methods, based on "backward/forward sweep" algorithm, converge much faster for any radial network topology. Also, they are much simpler to implement in programming languages and require less memory usage and processing power [7 ÷ 14].

The algorithm described in this paper can be applied for load flow and voltage drop calculations for primary substation feeders and their laterals. The main presumption for a proposed load flow algorithm is that the three phase power distribution network is balanced; thus it can be replaced by an equivalent single phase model. A distribution network planner can use the proposed load flow algorithm for any radial configuration of a distribution network.
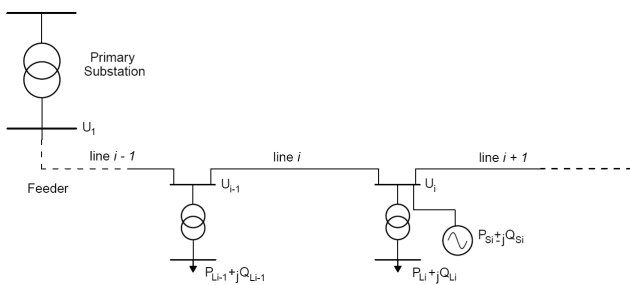

Figure 1 Model of a distribution network feeder with dispersed generation

Also, in this paper the distribution line is modelled with a detailed π equivalent model, which does not neglect parallel capacitance. Although most load flow algorithms very often use serial impedance as a substitute model for a distribution line, the fact is that parallel line capacitance injects substantial reactive power in the grid (for a 20 kV cable NA2XS(F)2Y 1×185RM/25 mm^2 the injection of reactive current is 3 A/km). In that way, the lines generate reactive power to a certain extent (rather small, but still significant) and compensate reactive (typically inductive) load, minimizing feeder total losses in the process. Finally, the proposed method for load flow covers the influence of dispersed generation which can be placed at any node on a primary HV/MV substation feeder. Although in this paper only one dispersed generation unit will be integrated in the network, the algorithm is designed to allow connection of any number of dispersed generation units.

The input presumption for the algorithm is that during the backward sweep all node voltages are equal to the base (nominal) network voltage. The backward sweep calculates all the line active and reactive power flows and the associated line losses, beginning from the end feeder node and finishing at the first feeder node. Then, during the forward sweep, from the first feeder node (HV/MV bus) to the end feeder node, the algorithm calculates the node voltages and voltage drops. The backward and forward sweep form one load flow iteration. The procedure keeps repeating until the node voltage difference between two consecutive iterations is less than specified in advance. After the desired accuracy is achieved, the algorithm has converged and automatically stops.

3   Defining input parameters of a load flow calculation

In order to correctly define the mathematical model of a load flow study, first it is necessary to numerate all the nodes and branches in a distribution network under consideration. A case study example of a distribution network is used, which consists of 30 nodes, as displayed in Fig. 2.
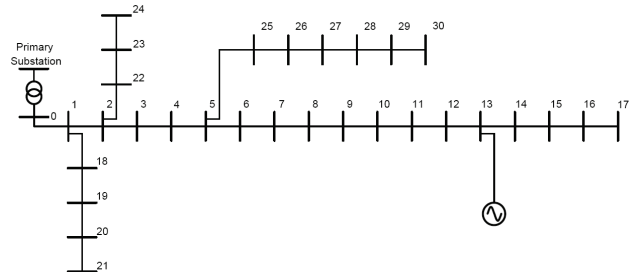

Figure 2 An example of a 30 node distribution network with a DG unit

Every node represents a MV bus of a MV/LV substation, except the first node, which represents a MV bus of a HV/MV primary substation. Each node is described by an amount of active and reactive power (P_L, Q_L) which it consumes from the grid. On the other hand, dispersed generation unit injects active power P_S in the network, and can either inject or consume reactive power Q_S in/from the network (as in case of a DG unit with induction/asynchronous generators, like wind farms). The DG unit is also shown in Fig. 2. In this example, node number 0 represents the MV bus of a HV/MV primary substation, and a DG unit is connected to the grid on a node (bus) no. 13.

Table 1 Branch numeration in the case study distribution network

Branch no.	Sending node	Receiving node
1	0	1
2	1	2
3	2	3
4	3	4
5	4	5
6	5	6
7	6	7
8	7	8
9	8	9
10	9	10
11	10	11
12	11	12
13	12	13
14	13	14
15	14	15
16	15	16
17	16	17
18	1	18
19	18	19
20	19	20
21	20	21
22	2	22
23	22	23
24	23	24
25	5	25
26	25	26
27	26	27
28	27	28
29	28	29
30	29	30

The network consists of a main feeder (between nodes 0 and 17), and 3 laterals:

- The first lateral is situated between nodes 1 and 21
- The second lateral is situated between nodes 2 and 24
- The third lateral is situated between nodes 5 and 30.

Every branch will be designated with the same number as its receiving node. Tab. 1 presents the list of all the branches in the case study network.

In order to convert the network topology in Matlab programming language into a source code, first it is necessary to form an incidence matrix IM. The rows of the matrix represent the branches (lines) of the network under study, and the nodes are represented by matrix columns. Every matrix element $IM(i,j)$ has the following values:

- 1, if j is the receiving node of branch i
- −1, if j is the sending node of branch i
- 0, otherwise.

For a network example in Fig. 2, the incidence matrix IM is as follows:

$$IM = \begin{bmatrix} \cdots \end{bmatrix} \tag{1}$$

Using the mathematical operation of matrix inversion, the incidence matrix IM will be transformed to a new matrix INV. In a new formed matrix the rows represent the nodes, and the columns the branches of the network under study. For a network in Fig. 2, the matrix INV is as follows:

$$INV = \begin{bmatrix} \cdots \end{bmatrix} \tag{2}$$

Now it is necessary to define what each element of matrix $INV(i,j)$ represents. For each branch (column) the nodes which are fed from that branch are all the matrix elements in the aforementioned column equal to non-zero values. For a branch no. 1 (1st column), all the nodes that are fed by that branch are marked as elements:

$$INV(i,1) = 1, \ i = 1, 2, ..., 30 . \tag{3}$$

For a branch no. 5 (5th column), all the nodes that are fed by it, are marked as elements:

$$INV(i,5) = 1, \ i = \{5, 6, ..., 17\} \cup \{25, 26, ..., 30\}. \tag{4}$$

As seen from Fig. 2, all the nodes fed from the 5th branch (line) are nodes 5, 6, …, 17, and the nodes 25, 26, …, 30.

Now it is necessary to construct a new matrix BR, from the existing INV. The rows of the new matrix will represent the branches (lines) of the network under study, and the matrix elements $BR(i,j)$ will represent the ordinal number of nodes that are fed by each branch. For an example of the distribution network from Fig. 2 used in this paper, the matrix BR is:

$$BR = \begin{bmatrix} \cdots \end{bmatrix} \tag{5}$$

For an example a branch no. 22 can be taken into consideration (22nd row). From a corresponding row of matrix $BR(22,j)$ it can be observed that the branch 22 feeds nodes 22, 23 and 24 with electric power. The same applies to all branches in the distribution network example under study.

The total number of nodes fed by a branch i is specified by a one-dimensional vector $M(i)$. For the 1st branch the $M(1)$ is equal to 30. For the 8th branch $M(8)=10$, and for the 20th branch $M(20)=2$. The vector $M(i)$ will serve in the formulation of the load flow mathematical model, as described in the next section.

4 The mathematical model of a proposed load flow calculation

It is presumed that a distribution network under study is operationally and topologically radial. Also, the load is assumed to be balanced in all three phases, which practically means that the load flow calculation can be

observed only for one phase (for the other two phases the absolute values of electrical units are the same, only phase shifted by 120° and 240°).

The distribution line is represented by a detailed π equivalent model, and in this paper the capacitive currents generated by parallel admittance (shunt capacitors) will not be neglected. Only the shunt resistor (conductance) will be neglected, due to its very low value in practical examples. This is very important because, using this model the load flow results will be more exact, including the node voltages and active/reactive line losses. Most of load flow calculation algorithms use only serial impedance, but neglect the parallel admittance. The model of a distribution line used in developing a load flow algorithm in this paper is shown in Fig. 3.

Figure 3 A detailed π model of a distribution line

The load flow algorithm will be solved iteratively, through two sets of equations (two sweeps).

The first sweep begins from the last node of the feeder, and calculates all the branch currents (backward sweep). All the node voltages in the first sweep are assumed to be equal to the nominal network voltage. In this example a HV/MV feeding node is a 110/20 kV Substation which feeds a 20 kV MV feeder. The voltage on a MV side of a 110/20 kV Substation is regulated with an OLTC device (on-line tap changer) which alters the power transformer turns ratio in a number of predefined steps and in that way changes the secondary side voltage. Standard tap changers offer between ± 10 steps (i.e. 20 positions), with each step representing a change in 1,5 % nominal voltage. In practical cases, the voltage on a secondary side of 110/20 kV Substations is maintained at 21 kV to compensate a possible large voltage drop in 20 kV network.

For a distribution network under study it will be taken that the secondary bus voltage of a 110/20 kV Substation is equal to the nominal MV network voltage:

$$U_n = 20\angle 0° \, kV. \tag{6}$$

The second sweep begins from the first feeder node (secondary side of a 110/20 kV Substation) and moves towards the last feeder node. During the sweep all the node voltages are calculated, both absolute values and phase values (forward sweep). It is very important to emphasize that the load flow algorithm includes the integration of dispersed generation at any distribution network feeder node.

4.1 The calculation of branch active/reactive power and currents (backward sweep)

From Fig. 3 the following set of equations, which specify the branch active/reactive power flow, can be made:

$$P_i = \sum_{k=BR(i,M(i))}^{BR(i,1)} P_{Lk} - \sum_{k=BR(i,M(i))}^{BR(i,1)} P_{Sk} + \sum_{k=BR(i,M(i))}^{BR(i,2)} ploss_k, \tag{7}$$

$$Q_i = \sum_{k=BR(i,M(i))}^{BR(i,1)} Q_{Lk} - \sum_{k=BR(i,M(i))}^{BR(i,1)} Q_{Sk} + \sum_{k=BR(i,M(i))}^{BR(i,2)} qloss_k - \sum_{k=BR(i,M(i))}^{BR(i,2)} (qc_{kend} + qc_{kbeg}), \tag{8}$$

where:
$M(i)$ – the total number of nodes fed by branch i
$BR(i,j) - 1 \leq j \leq M(i)$: each element in a BR matrix row denotes an ordinal number of the node j fed by a branch i,
P_{Lk} and Q_{Lk} – active and reactive load of node k
P_{Sk} – active power of a dispersed generation unit connected at node k, taken with a negative algebraic sign because it is injected at node k
Q_{Sk} – reactive power of a dispersed generation unit connected at node k, taken with a negative algebraic sign because it is injected at node k, (however, in case of a asynchronous generator the reactive power will be consumed from the node k and will be taken with a positive algebraic sign in the above equation)
P_i – active power that is injected in node i and is equal to the sum of all the active loads fed by branch i. From that sum we deduct the active injected power of all the dispersed generation units between node i and the end feeder node. To the above sum we add the active loss of all the branches between branch i and the feeder end branch (branch i is not included)
Q_i – reactive power that is injected in node i and is equal to the sum of all the reactive loads fed by branch i. From that sum we deduct the reactive injected power of all the dispersed generation units between node i and the end feeder node (except in the case of asynchronous generator, which consumes reactive power). To the above sum we add the active loss of serial inductance of all the branches between branch i and the feeder end branch (branch i is not included). Also, from the above sum we need to deduct the reactive power generated by parallel capacitance of the branches between branch i and the feeder end branch (branch i is not included)
$ploss_k$: active power loss of the kth branch (on a serial resistance of a detailed π equivalent distribution line model)
$qloss_k$ – reactive power loss of the kth branch (on a serial inductance of a detailed π equivalent distribution line model)
qc_{kbeg} – reactive power generated by the parallel capacitance in the beginning of the kth branch of a detailed π equivalent distribution line model
qc_{kend} – reactive power generated by the parallel capacitance in the end of the kth branch of a detailed π equivalent distribution line model

The current flowing through the ith branch of the distribution network is given by the following equation:

$$\overline{I}_i = \frac{\overline{S_i^*}}{\sqrt{3}U_i^*} = I_{ai} + jI_{ri} = \frac{P_i - jQ_i}{\sqrt{3}U_i^*}. \qquad (9)$$

If a complex value of a line voltage (the term voltage in this paper will refer to line voltage, not phase voltage) is expressed in polar coordinate system through its real and imaginary component, the following expression can be constituted:

$$\overline{U}_i = U_i \cdot (\cos\varphi_i + j\sin\varphi_i). \qquad (10)$$

The current flowing through the ith branch is then:

$$\overline{I}_i = \frac{\overline{S_i^*}}{\sqrt{3}U_i^*} = I_{ai} + jI_{ri} =$$
$$= \frac{P_i\cos\varphi_i + Q_i\sin\varphi_i}{\sqrt{3}U_i} + j\left(-\frac{Q_i\cos\varphi_i - P_i\sin\varphi_i}{\sqrt{3}U_i}\right). \qquad (11)$$

It is important to note that the current I_{lossi}, which flows through the serial impedance of the detailed π equivalent distribution line model (and generates real and reactive line loss), is equal to the sum of the current I_i and the amount of reactive (capacitive) current which is generated by the parallel ith branch capacitance $C_{iend}/2$, as displayed in Fig. 3:

$$\overline{I}_{Ciend/2} = \frac{\overline{U}_i}{\sqrt{3}X_{Ci/2}} = \frac{U_i \cdot (\cos\varphi_i + j\sin\varphi_i)}{\sqrt{3} \cdot \dfrac{1}{j\omega C_i/2}}. \qquad (12)$$

$$\overline{I}_{Ciend/2} = \frac{-\omega C_i \cdot U_i \cdot \sin\varphi_i + j\omega C_i \cdot U_i \cdot \cos\varphi_i}{2\sqrt{3}}. \qquad (13)$$

$$\overline{I}_{lossi} = \overline{I}_i + \overline{I}_{Ciend/2} = I_{lossai} + jI_{lossri} =$$
$$\left(\frac{P_i\cos\varphi_i + Q_i\sin\varphi_i}{\sqrt{3}U_i} - \frac{\omega C_i \cdot U_i \cdot \sin\varphi_i}{2\sqrt{3}}\right) +$$
$$+ j\left[-\left(\frac{Q_i\cos\varphi_i - P_i\sin\varphi_i}{\sqrt{3}U_i} - \frac{\omega C_i \cdot U_i \cdot \cos\varphi_i}{2\sqrt{3}}\right)\right]. \qquad (14)$$

The equations that specify the three phase distribution network real and reactive line losses (the index k will be used instead of i, because of the losses' affiliation to equations (7) and (8)) are:

$$ploss_k = 3 \cdot |I_{lossk}|^2 \cdot R_k =$$
$$3 \cdot \left| \begin{array}{c} \left(\dfrac{P_k\cos\varphi_k + Q_k\sin\varphi_k}{\sqrt{3}U_k} - \dfrac{\omega C_k \cdot U_k \cdot \sin\varphi_k}{2\sqrt{3}}\right)^2 + \\[4mm] + \left[-\left(\dfrac{Q_k\cos\varphi_k - P_k\sin\varphi_k}{\sqrt{3}U_k} - \dfrac{\omega C_k \cdot U_k \cdot \cos\varphi_k}{2\sqrt{3}}\right)\right]^2 \end{array} \right| \cdot R_k. \qquad (15)$$

$$qloss_k = 3 \cdot |I_{lossk}|^2 \cdot X_k =$$
$$3 \cdot \left| \begin{array}{c} \left(\dfrac{P_k\cos\varphi_k + Q_k\sin\varphi_k}{\sqrt{3}U_k} - \dfrac{\omega C_k \cdot U_k \cdot \sin\varphi_k}{2\sqrt{3}}\right)^2 + \\[4mm] + \left[-\left(\dfrac{Q_k\cos\varphi_k - P_k\sin\varphi_k}{\sqrt{3}U_k} - \dfrac{\omega C_k \cdot U_k \cdot \cos\varphi_k}{2\sqrt{3}}\right)\right]^2 \end{array} \right| \cdot X_k. \qquad (16)$$

The reactive (capacitive) power generated by parallel capacitance is:

$$qc_{kbeg} = 3 \cdot \left(\frac{U_{SE(k)}}{\sqrt{3}}\right)^2 \cdot \frac{1}{X_{Ck/2}} = \omega\frac{C_k}{2}U_{SE(k)}^2, \qquad (17)$$

$$qc_{kend} = 3 \cdot \left(\frac{U_{RE(k)}}{\sqrt{3}}\right)^2 \cdot \frac{1}{X_{Ck/2}} = \omega\frac{C_k}{2}U_{RE(k)}^2, \qquad (18)$$

where:

R_k – series resistance of branch (line) k, equal to the product of branch per unit resistance and line length
X_k – series inductive reactance of branch (line) k, equal to the product of branch per unit inductive reactance and line length
C_k – parallel capacitance of branch (line) k, equal to the product of branch per unit capacitance and line length
$B_k = \omega \cdot C_k$ – parallel susceptance of branch (line) k, equal to the product of branch per unit capacitance and angular network frequency
$SE(k)$ – abbreviation for sending end of node i
$RE(k)$ – abbreviation for receiving end of node i.

4.2 The calculation of node voltages, their absolute values and phase angles (forward sweep)

From Fig. 3 the following set of equations, which specify the node voltages, their absolute values and phase angles, can be made:

$$\overline{U}_{RE(i)} = \overline{U}_{SE(i)} - \sqrt{3} \cdot (R_i + jX_i) \cdot \overline{I}_{lossi}. \qquad (19)$$

If a complex value of a line voltage is expressed in polar coordinate system through its real and imaginary component, the following expressions can be constituted:

$$U_{aRE(i)} = U_{SE(i)} \cdot \cos\varphi_{SE(i)} - \sqrt{3} \cdot (R_i \cdot I_{lossai} - X_i \cdot I_{lossri}). \qquad (20)$$

$$U_{rRE(i)} = U_{SE(i)} \cdot \sin\varphi_{SE(i)} - \sqrt{3} \cdot (X_i \cdot I_{lossai} + R_i \cdot I_{lossri}). \qquad (21)$$

The voltage of the first node (secondary side of the primary 110/20 kV Substation) will be maintained and equal to 20 kV with a phase angle of 0°. The voltage and phase angle of all the other nodes in the distribution network under study is, according to the following equations, equal to:

$$U_{RE(i)} = \sqrt{U_{aRE(i)}^2 + U_{rRE(i)}^2}, \qquad (22)$$

$$\varphi_{RE(i)} = \arctan\frac{U_{rRE(i)}}{U_{aRE(i)}}. \tag{23}$$

If a desired accuracy is achieved, then the difference between two consequent iterations of every node voltage in the case study network must be as follows:

$$\Delta U_i = U_i - U_{i-1} \le \varepsilon. \tag{23}$$

In that case the algorithm has converged and is then stopped. In this paper the difference after two consequent iterations between the voltages of every node in the case study network will be taken as $\varepsilon = 10^{-15}$.

4.3 The flowchart of the load flow algorithm

A flowchart describing the steps of an algorithm developed for a load flow study is displayed in Fig. 4.
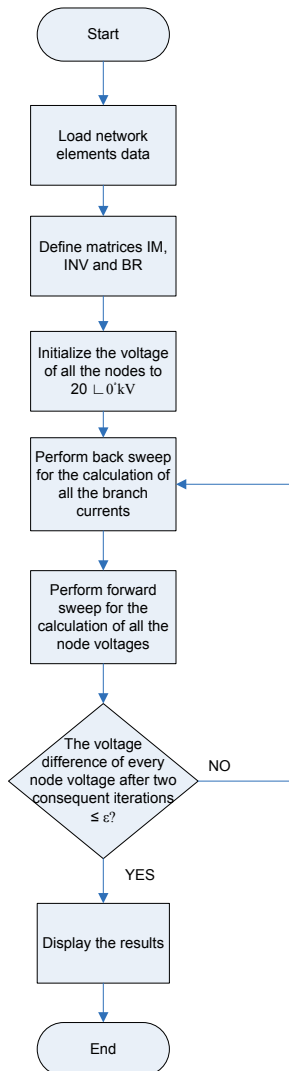

Figure 4 The flowchart of the load flow algorithm described in this paper

5 Results and discussion

The load flow algorithm described in this paper was implemented in Matlab programming language, version

7.11.0. The input data of the distribution network under study is presented in the following tables.

In Tab. 2 the electrical parameters of lines used in the network example are presented. In this paper one type of electric underground MV cables is used. Tab. 3 contains the electrical parameters of distribution network lines, considering their length and position.

Table 2 The data of the electrical cables used in the case study distribution network

U_n / kV	Type	R_1 / Ω/km	X_1 / Ω/km	B_1 / µS/km
20	XHE-49A 3x(1X150)	0,206	0,115	82

Table 3 The electrical parameters of the lines considering their position and length

Branch no.	Length / km	$R = R_1 \cdot l$ / Ω	$X = X_1 \cdot l$ / Ω	$B = B_1 \cdot l$ / µS
1	0,7	0,1442	0,0805	57,4
2	0,5	0,1030	0,0575	41
3	0,7	0,1442	0,0805	57,4
4	0,8	0,1648	0,0920	65,6
5	0,9	0,1854	0,1035	73,8
6	0,6	0,1236	0,0690	49,2
7	0,2	0,0412	0,0230	16,4
8	0,8	0,1648	0,0920	65,6
9	0,7	0,1442	0,0805	57,4
10	0,3	0,0618	0,0345	24,6
11	0,1	0,0206	0,0115	8,2
12	0,4	0,0824	0,0460	32,8
13	0,4	0,0824	0,0460	32,8
14	0,2	0,0412	0,0230	16,4
15	0,7	0,1442	0,0805	57,4
16	0,8	0,1648	0,0920	65,6
17	0,5	0,1030	0,0575	41
18	0,9	0,1854	0,1035	73,8
19	0,1	0,0206	0,0115	8,2
20	0,3	0,0618	0,0345	24,6
21	0,6	0,1236	0,0690	49,2
22	0,2	0,0412	0,0230	16,4
23	0,3	0,0618	0,0345	24,6
24	0,6	0,1236	0,0690	49,2
25	0,4	0,0824	0,0460	32,8
26	0,2	0,0412	0,0230	16,4
27	0,1	0,0206	0,0115	8,2
28	0,9	0,1854	0,1035	73,8
29	0,4	0,0824	0,0460	32,8
30	0,3	0,0618	0,0345	24,6

Tab. 4 contains the data regarding the amount of active and reactive power consumed by every node in the network under study.

The load flow algorithm provides all the network electrical parameters output data, including node voltage RMS (root mean square) values, phase angle values, branches' currents and active/reactive power flows and associated power losses. In this example only the node voltage RMS (root mean square) values and active/reactive branch power losses will be considered. As the algorithm can calculate load flow of a distribution network with connected dispersed generation units, two cases will be considered:

- The load flow of the distribution network under study without a dispersed generation unit
- The load flow of the distribution network under study with a dispersed generation unit.

The dispersed generation unit will be connected to node 13, with the parameters shown in Tab. 5.

Table 4 The amount of active and reactive power consumed by each node in the distribution network under study

Node no.	P_L / kW	Q_L / kvar
1	100	60
2	90	40
3	120	80
4	60	30
5	60	20
6	200	100
7	200	100
8	60	20
9	60	20
10	45	30
11	60	35
12	60	35
13	120	80
14	60	10
15	60	20
16	60	20
17	90	40
18	90	40
19	90	40
20	90	40
21	90	40
22	90	50
23	420	200
24	420	200
25	60	25
26	60	25
27	60	20
28	120	70
29	200	600
30	150	70

Table 5 The amount of active and reactive power injected by dispersed generation unit

Node no.	P_S / kW	Q_S / kvar
13	700	300

5.1 The results of a load flow algorithm applied to distribution network under study without dispersed generation unit

The results of a load flow algorithm applied to distribution network under study without dispersed generation unit are displayed in Tab. 6.

Table 6 The results of a load flow algorithm applied to distribution network under study without dispersed generation unit

Node no.	Voltage RMS value / kV	Branch no.	Active power losses / kW
0	20	1	5,1199
1	19,969	2	2,9345
2	19,9491	3	1,7615
3	19,9309	4	1,7310
4	19,9116	5	1,7961
5	19,8908	6	0,3874
6	19,8831	7	0,0921
7	19,8809	8	0,1920
8	19,8750	9	0,1417
9	19,8702	10	0,0543
10	19,8681	11	0,0161
11	19,8674	12	0,0460
12	19,8652	13	0,0344
13	19,8634	14	0,0081
14	19,8627	15	0,0163
15	19,8611	16	0,0096
16	19,8598	17	0,0024
17	19,8592	18	0,0618
18	19,9654	19	0,0044
19	19,9651	20	0,0057
20	19,9644	21	0,0028
21	19,9637	22	0,1091
22	19,9467	23	0,1327
23	19,9435	24	0,0661
24	19,9402	25	0,2020
25	19,8864	26	0,0959
26	19,8843	27	0,0439
27	19,8833	28	0,3126
28	19,8755	29	0,1139
29	19,8725	30	0,0042
30	19,8719	Σ	15,4985

Despite the fact that the demanded accuracy of results was very high ($\varepsilon=10^{-15}$, unlike the usually taken $\varepsilon=10^{-4}$), the load flow algorithm has converged after 9 iterations. The time in which the CPU executed the algorithm is 0,003698 s.

5.2 The results of a load flow algorithm applied to distribution network under study with a dispersed generation unit

The results of a load flow algorithm applied to distribution network under study with a dispersed generation unit connected at node 13 are displayed in Tab. 7.

Table 7 The results of a load flow algorithm applied to distribution network under study with a dispersed generation unit connected at node 13

Node no.	Voltage RMS value / kV	Branch no.	Active power losses / kW
0	20	1	3,2420
1	19,9754	2	1,7522
2	19,9600	3	0,7509
3	19,9481	4	0,6762
4	19,9360	5	0,6668
5	19,9233	6	0,0439
6	19,9210	7	0,0034
7	19,9206	8	0,0236
8	19,9219	9	0,0177
9	19,9233	10	0,0049
10	19,9240	11	0,0022
11	19,9242	12	0,0187
12	19,9256	13	0,0273
13	19,9274	14	0,0080
14	19,9267	15	0,0162
15	19,9251	16	0,0095
16	19,9238	17	0,0024
17	19,9232	18	0,0618
18	19,9717	19	0,0044
19	19,9714	20	0,0057
20	19,9707	21	0,0028
21	19,9701	22	0,1090
22	19,9576	23	0,1326
23	19,9543	24	0,0660
24	19,9510	25	0,2012
25	19,9189	26	0,0956
26	19,9168	27	0,0437
27	19,9158	28	0,3115
28	19,9080	29	0,1135
29	19,9050	30	0,0042
30	19,9044	Σ	8,4179

The load flow algorithm has also converged after 9 iterations. The time in which the CPU executed the algorithm is 0,003654 s.

5.3 The comparison of the results of a load flow algorithm without and with a dispersed generation unit

Fig. 5 displays the comparison of all the network node voltages (RMS values) for a load flow calculation without a dispersed generation unit and with a dispersed generation unit connected at node 13.
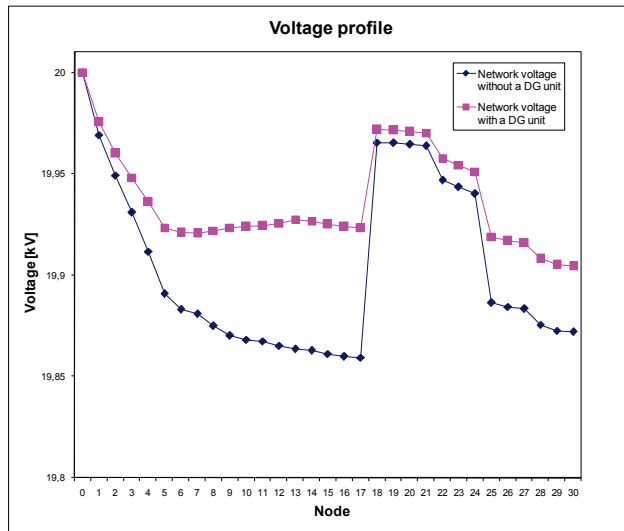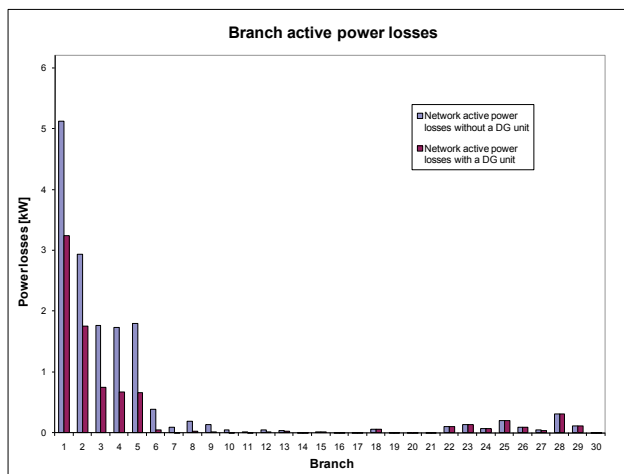

Figure 5 Voltage profile of a distribution network under study without and with a dispersed generation unit connected at node 13

Fig. 6 displays the comparison of all the network branch active power losses for a load flow calculation without a dispersed generation unit and with a dispersed generation unit connected at node 13.

Figure 6 Active power losses of a distribution network under study without and with a dispersed generation unit connected at node 13

The results demonstrate that after the connection of a single dispersed generation unit at the 13th node all the network node voltages were raised. The largest variation of voltage between the case without and with dispersed generation is at node no. 17 and amounts to 0,321234 %.

Also, after the connection of a dispersed generation unit at node 13, the active power losses of all the branches

between the primary 110/20 kV substation and a dispersed generation unit have fallen. The branch where the active power losses have decreased the most is the 7th branch, with a 96,30836 % decrease.

6 Conclusion

In this paper a novel load flow algorithm is described in which the distribution line is represented by a detailed π equivalent model (meaning that the capacitive currents generated by parallel admittance are not neglected) and with the inclusion of arbitrary number of dispersed generation units. The algorithm can be applied on any radial distribution network configuration with dispersed generation units connected simultaneously in any node of the network.

The algorithm is fast and accurate in calculating the electrical parameters of a distribution network, and for that reason it can later serve as a tool in calculating the optimal number, location and output power of any number of dispersed generation units with a goal of optimizing distribution network performance. The aforementioned problem belongs to multi-objective combinatorial optimization and in literature is termed "dispersed generation allocation". It will be the subject of the next paper.

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