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COMPARATIVE ANALYSIS OF LIMIT BEARING CAPACITY OF FRAMES DEPENDING ON THE CHARACTER OF THE LOAD

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Original scientific paper

Determination of the bearing capacity of a structure is very valuable, not only as a simple control of structural bearing capacity, but also as a significant basis and factor in designing of structures. Limit load of structures determined by application of the limit analysis is one of the indicators of bearing capacity of structure exposed to the action of proportional load. When the structure is exposed to the action of variable repeated load, the limit theorems do not yield the adequate solutions, thus the adaptation theorems which made safe limit load determination possible were developed simultaneously. By applying the limit and shakedown analysis in this paper structural analysis of frame structures with different degrees of static indeterminacy was carried out. Also displayed is the difference between the values of failure forces depending on the character of load and ratio of height and width of frame in order to assess justification for application of the shakedown method in the analysis of the limit bearing capacity of the frame structures.

Keywords: *alternative failure force, frame structure, incremental failure force, limit failure force*

Komparativna analiza granične nosivosti okvira u ovisnosti o karakteru opterećenja

Izvorni znanstveni članak

Određivanje nosivosti konstrukcije u zavisnosti od karaktera opterećenja je ne samo dragocjeno kao jednostavna kontrola nosivosti konstrukcije, već je i značajna baza i faktor pri projektiranju konstrukcija. Kada je konstrukcija izložena djelovanju opterećenja koje proporcionalno raste primjenom granične analize moguće je odrediti granično opterećenje loma koje je jedan od pokazatelja moći nošenja. U slučaju djelovanja promjenjivo ponovljenog opterećenja granični teoremi ne daju adekvatna rješenja, tako da su se paralelno s njima razvijali i teoremi adaptacije koji su omogućili određivanje sigurnog graničnog opterećenja. Primjenom granične analize i metode adaptacije u radu je sprovedena analiza nosivosti ramovskih nosača različitog stupnja statičke neodređenosti. Prikazana je procentualna razlika između veličina sila loma u zavisnosti od karaktera opterećenja i odnosa visine i širine rama kako bi se došlo do zaključka o opravdanosti primjene teorema adaptacije u analizi granične nosivosti ramovskih nosača.

Ključne reči: *alternativna sila loma, granična sila loma, inkrementalna sila loma, okvirni nosač*

1 Introduction

When the load acting on the structure is proportionally increasing, at some point it reaches a certain critical value at which the plastic failure of the structure occurs (i. e. unlimited increase of deformation at a constant load) after which the structure is unable to receive the further accrual of the load. This critical state is called the limit state of the structure, and the load causing it is called the limit load. Determination of the structural bearing capacity (limit load) is an important factor in designing of structures.

Limit structural analysis is the alternative analytical procedure determining the maximum safe load parameter, or the load increase parameter which can be endured by an ideal elasto-plastic structure. In comparison to incremental analysis (step-by-step method), the efficiency in the limit analysis is achieved by observing the ultimate state, the failure state, irrespective of what happened to the structure and the load since the moment of formation of the first plastic joint right until the failure. The limit analysis method is based on the theorems of plastic failure for ideal elasto-plastic bodies. These theorems are known as lower (static) and upper (kinematic) theorem of the limit structural analysis.

It should be mentioned, that apart from the limit bearing state there are other limit states, which may occur prior to the limit equilibrium state which can be limiting in terms of external load bearing capacity, such as the limit usability state or even limit crack state in the structures made of reinforced or pre-stressed concrete [1]. In order to determine the limit bearing capacity of a structure applying limit analysis, previously it must be proved that the limit state relevant for it will occur by formation of the failure mechanism, that is, any other limit state occurrence should be eliminated, and any effects which could lead to the structural failure should be ruled out prior to formation of a sufficient number of plastic joints or plastic members.

If a structure, made of an elasto-plastic material, is exposed to variable loads, then, the following situations are possible [1]:

- If the load intensities remain sufficiently low, the structural response is perfectly elastic;
- If the load intensities become sufficiently high, the instantaneous load-carrying capacity of the structure becomes exhausted, plastic, unconstrained flow mechanism develops and the structure collapses. Obviously, plastic deformations can develop also for loads below the collapse load;
- If the plastic strain increments in each load cycle are of the same sign then, after a sufficient number of cycles, the total strains (and therefore displacements) become so large that the structure departs from its original form and becomes unserviceable. Such behaviour can be observed in experimental investigations. For sufficiently high load amplitude (although below the load-carrying capacity) the deflection grows in each cycle. This phenomenon is called ***incremental collapse***;
- If the strain increments change sign in every cycle, they tend to cancel each other out and the total deformation remains small (***alternating plasticity***). In this case, however, after a sufficient number of cycles, material at the most stressed points begins to break due to low-cycle fatigue;

- It may also happen that, after some plastic deformation in the initial load cycles, the structural behaviour becomes eventually elastic, for lower load amplitudes. Such stabilization of plastic deformations is called ***shakedown*** or ***adaptation.***

The fact that the collapse loads calculated according to limit analysis may fail to provide a proper measure of structural safety in the case of variable repeated loads, was pointed out for the first time by Grüning and later by Bleich, who proved the static shakedown theorem for a system of beams of ideal I-cross sections. In 1936 Melan presented a more general theorem and later extended it to the general case of a continuum [2]. In 1957 Prager and Rozenblum further extended the Melan theorem to account for thermal stress. The temperature dependence of elastic moduli was accounted later by König.

In 1950 Neal presented a method of shakedown analysis for frames by analysing possible mechanisms of plastic flow. It was Koiter who formulated a general kinematical shakedown theorem. Rozenblum and De Donato extended it to allow for thermal loadings. Gokhfeld and Sawczuk derived from Koiter's theorem a criterion of incremental collapse and showed that, in the case of piecewise linear conditions, the inequality in this theorem can be effectively integrated with respect to time. Many new solutions have been obtained by using this approach. Quite later a separate criterion of alternating plasticity has been derived by König. The notion of shakedown is applicable also in the case of strain-hardening. An appropriate static theorem was worked out by Melan, holding true for the generalized Bauschinger effect. A particular case of that theorem was proved independently by Neal.

In the recent years, the shakedown analysis of elasto-plastic structures has become increasingly applied in the analysis of engineering problems due to the increased demands of modern technologies. It is thus successfully applied in many engineering problems, such as designing of nuclear reactors, railways, civil engineering designing and safety assessment of some building structures.

The goal of this paper is to implement limit and shakedown theorems and thus analyse bearing capacity of frame structures depending on the degree of static indeterminacy, as well as the character of the load. Applying the limit theorems, bear capacity analysis was performed in the case when frame is exposed to the proportionally increasing load, while in the case of the variable repeated load, the analysis was performed applying the shakedown theorems. Change of the limit and incremental failure force is presented as dependent on the change of coefficient α, which defines the ratio of width to height of the frame, while in case when the frame is exposed to the action of alternative loading, the change of limit and alternative failure force is presented as dependent on α, as well as on the coefficient of the cross-section shape α_{form} On the basis of conducted analysis, the conclusion of justification of application of shakedown theorem in analysis of frame structure bearing capacity is drawn.

2 Basic postulates and theorems of limit and shakedown analysis

The analysis of the static plastic collapse of a structure is usually accomplished by proportionally increasing the loads acting on it until a sufficient number of plastic hinges appears to cause the structure to become a mechanism.

In the area of elastic behaviour the stresses and strains are proportionally dependant. Due to the increase of load, there is a gradual build-up of stress, until the value of the stress in the most loaded fibre reaches the value of yield stress. Further increase of load causes plasticization of the entire cross section, and thus formation of plastic hinge [3].

It is known, that in statically determinate structures, the complete plastification of one cross-section of a beam (formation of a plastic hinge on the location of maximum bending moment) and transition of the structure into the failure mechanism means the loss of load bearing capacity. In statically indeterminate structure, formation of one plastic joint does not lead to formation of failure mechanism, and the bearing capacity of one n times statically indeterminate structure is fully exhausted when in the beam an n+1 plastic joint is formed.

It can be stated that a structure is in the state of limit equilibrium when the bearing capacity of the structure has been fully exhausted, and when the structure behaves fully plastic in a sufficient number of cross sections [4]. On this basis it can be concluded that at the moment of formation of a sufficient number of plastic hinges, the deformations are progressive, and the structure transits into the failure mechanism. The moment immediately preceding the formation of failure mechanism represents the moment of limit equilibrium of the structure.

If the structure is unloaded prior to formation of failure mechanism, certain residual strain occurs, which causes occurrence of retained bending moments. By applying the limit analysis it is not possible to include the retained bending moments in the calculations, in the case of repeated loading of the beam. This is possible by applying the shakedown analysis. In the shakedown analysis all the assumptions of the limit analysis are also valid, whereby this method makes possible the analysis of the behaviour of the structure exposed to repeated load.

The theorems of limit analysis are known and presented in detail in the paper [5].

Shakedown theorems have a role to set the main conditions under which the plastic yield in the structure finally ceases, regardless of how frequently and in what sequence the load was applied [6]. As well as in the limit analysis, in the shakedown analysis there are static and kinematic theorems, on whose basis it is possible to determine the safe limit load depending on the type of variable repeated load.

The bending moment of the observed cross section j can be presented as:

$$M_{\mathrm{j}} = m_{\mathrm{j}} + (M_{\mathrm{e}})_{\mathrm{j}}, \tag{1}$$

where:
M_{j} – is the actual bending moment of the cross section j,
$(M_{\mathrm{e}})_{\mathrm{j}}$ – is the elastic bending moment of the cross section,
m_{j} – is the residual bending moment of the cross section.

Any distribution of residual bending moments, defined in this way must be statically possible in case when the structure is unloaded, because the moment M_j and $(M_e)_j$ must be in equilibrium with the external load [3]. Thus it can be said that the structure has adapted under the action of variable repeated load, if at some point the condition (1) has been satisfied, and all the following loads cause only elastic change of bending moments.

On the basis of conditions (1) the static shakedown theorem can be expressed in the following form: if there exists any distribution of residual bending moment m_j throughout structure, which is statically admissible in the case with zero external loading and which also satisfies at every cross section j, it is necessary to meet one of the conditions:

$$m_j + \lambda (M_j)_{max} \le (M_p)_j, \tag{2}$$

$$m_j + \lambda (M_j)_{min} \ge -(M_p)_j, \tag{3}$$

$$\lambda \big((M_j)_{max} - (M_j)_{min}\big) \le 2(M_e)_j, \tag{4}$$

the value λ will be less than or equal to the shakedown load factor λ_S. Herein M_p is moment of full plastification of cross section (plastic moment), which depends only on cross section geometry.

Each girder strives to adapt to the action of variable repeated load in the best possible way. Thus, if λ exceeds the value λ_S, the unlimited plastic yield occurs, and in this case no distribution of residual moments is possible, which is a necessary condition for determination of safe limit load. Similarly, under the action of proportional load, the structure will fail when the load factor λ reaches the value λ_C, above which the structure is not safe, and simultaneously there is a statically possible distribution of bending moments. Depending on the calculated load factor λ it is possible to determine the safe limit load which depends on the type of variable repeated load, on the basis of meeting some of the requirements of the equations (2) and (3), as incremental conditions of plasticity and equation (4), as alternating plasticity conditions.

Assuming that the observed failure mechanism is known, rotations of formed plastic hinges θ can be noticed in a certain number of characteristic cross sections [7]. If the rotation in any cross section is positive (θ^+), then it can be said that the total bending moment in this cross section aspires to reach the value $+M_p$, and if the rotation of formed plastic joint is negative (θ^-), the bending moment aspires to reach the value $-M_p$. On the basis of the introduced assumptions, the equations (2) and (3) can be written in the form:

$$m_j + \lambda (M_j)_{max} \le (M_p)_j, \text{ for } \theta_j^+, \tag{5}$$

$$m_j + \lambda (M_j)_{min} \le -(M_p)_j, \text{ for } \theta_j^-. \tag{6}$$

If the equations (5) and (6) are multiplied by the corresponding rotation of the formed plastic joint in the cross section j, then, they have the form:

$$m_j\theta_j + \lambda (M_j)_{max}\,\theta_j^+ = (M_p)_j\,|\theta_j|, \tag{7}$$

$$m_j\theta_j - \lambda (M_j)_{max}\,\theta_j^- = (M_p)_j\,|\theta_j|, \tag{8}$$

Adding up of equations (7) and (8), of all the plastic hinges which have been formed on the observed failure mechanism, give the following:

$$\sum_j m_j\theta_j + \lambda\left[\sum_j (M_j)_{max}\,\theta_j^+ + \sum_j (M_j)_{max}\,\theta_j^-\right] = \\ = \sum_j (M_p)_j\,|\theta_j|. \tag{9}$$

As the distribution of residual bending moments is in equilibrium when the structure is unloaded, and the θ is rotation of the cross section where the plastic joint has been formed, the equation of the principle of virtual work can be written in the following form:

$$\sum_j m_j\theta_j = 0,$$

thus (9) becomes:

$$\lambda\left[\sum_j (M_j)_{max}\,\theta_j^+ + \sum_j (M_j)_{max}\,\theta_j^-\right] = \sum_j (M_p)_j\,|\theta_j|, \tag{10}$$

which represents the basic equation of incremental failure.

On the basis of equation (10) it is possible to express the kinematic shakedown theorem in the following way: the value of parameter λ corresponding to any assumed failure mechanism of alternating plasticity λ_a or of incremental collapse λ_I must be either greater than or equal to the shakedown load factor λ_S.

The kinematic shakedown theorem in this form was first defined by Koiter (1956, 1960), though it can be said that he had done that on the basis of the work of P. S. Symods and B. G. Neal [8], which was published at the First National Congress of Applied Mechanics in Chicago in 1951. They started from the assumption that the work of all the residual moments on the possible failure mechanism is equal to zero. In this paper the incremental failure force will be calculated applying the Symonds and Neal method.

3 Analysis of the bearing capacity of frame structures depending on the load character and degree of static indeterminacy

There are two methods to calculate the failure load (limit and shakedown failure load), and those are incremental method (incremental elasto-plastic analysis) and direct method. The incremental method is based on an incremental, detailed calculation of failure load, which requires extensive calculation. The direct method is based on determination of failure load on the potential failure mechanism with no incremental monitoring of development and formation of plastic hinges, which is simpler in practical application.

When the structure is exposed to the action of variable repeated load, the failure may occur due to development of excessive plastic yield in some part of the structure, even if no individual applied load is sufficiently large to cause formation of failure mechanism. If the load has an alternate character, the cross section exhibits repetition of plastic deformations of the opposite sign (without accumulation of plastic strain) causing in this way the phenomenon of low cycle fatigue, and the load causing the failure is called alternative limit load. Another kind of failure can occur if during action of variable load, several critical combinations of load occur, and they succeed each other in certain cycles. The failure mechanism formation occurs due to accumulation of plastic deformations at every load cycle (progressive deformation), causing reduction of structural service life. In this case it is the incremental limit load.

Applying the adequate method depending on the character of the load, the analysis of limit bearing capacity of the frames displayed in Fig. 1 was performed. The procedure of failure force calculation depending on the change of the coefficient representing the ratio of height and width of the frame $\alpha = h/l$ was performed. Depending on the coefficient α the distribution of internal forces changes, which leads to the change of relevant failure condition, that is, to the change of failure force value while there are three possible failure mechanisms for the frame in Fig. 1b, shown in Fig. 3.

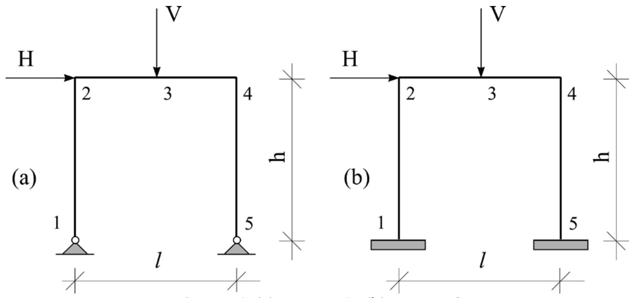

Figure 1 (a) Frame 1, (b) Frame 2

There are two possible failure mechanisms for the frame in Fig. 1a, which are displayed in Fig. 2.

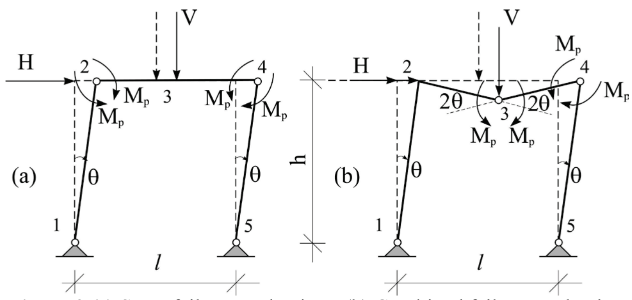
Figure 2 (a) Sway failure mechanisms (b) Combined failure mechanism

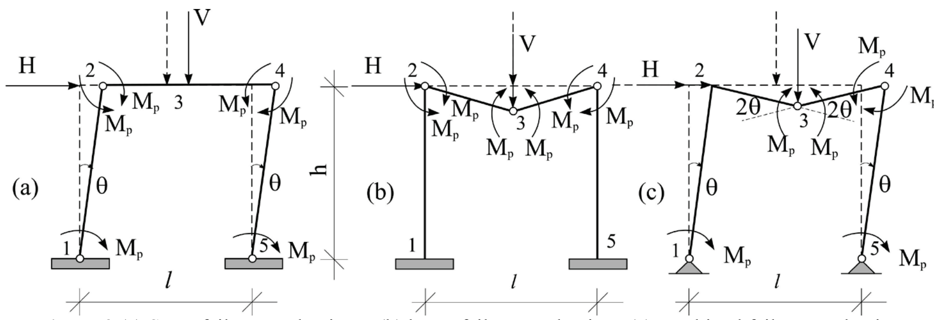
Figure 3 (a) Sway failure mechanisms, (b) beam failure mechanism, (c) combined failure mechanism

3.1 Frame 1 limit bearing capacity analysis

In case the Frame 1 is exposed to the proportionally increasing load, the failure force can be determined by some of the limit analysis theorems. For each of possible failure mechanisms, applying the kinematic theorem, the following expressions are obtained:

$$M_{\mathrm{p}}\theta + M_{\mathrm{p}}\theta = hH\theta, \tag{10}$$

$$M_{\mathrm{p}}2\theta + M_{\mathrm{p}}2\theta = hH\theta + V\frac{l}{2}\theta, \tag{11}$$

On the basis of expression (10) failure force is obtained, that corresponds to the sway failure mechanism (Fig. 2a)

$$H = \frac{2M_{\mathrm{p}}}{h}, \tag{12}$$

and on the basis of (11) the following expression is obtained

$$2hH + Vl = 8M_{\mathrm{p}}, \tag{13}$$

which corresponds to the combined failure mechanism (Fig. 2b).

If $H = V = F$, based on (12) and (13) the failure force values are

$$F_{\mathrm{crt}} = \frac{2M_{\mathrm{p}}}{\alpha l}, \tag{14}$$

$$F_{\mathrm{crt}} = \frac{8M_{\mathrm{p}}}{l(2\alpha+1)}. \tag{15}$$

Change of the limit failure force depends on the change of the ratio of height to width of the frame, as displayed in Fig. 4, while change of the limit failure force depending on the change of height h and width l of the frame displayed in Fig. 5.

In Fig. 4 it can be observed that for $\alpha < 0{,}50$ the combined failure mechanism forms, while in case of $\alpha > 0{,}50$ the sway failure mechanism forms.

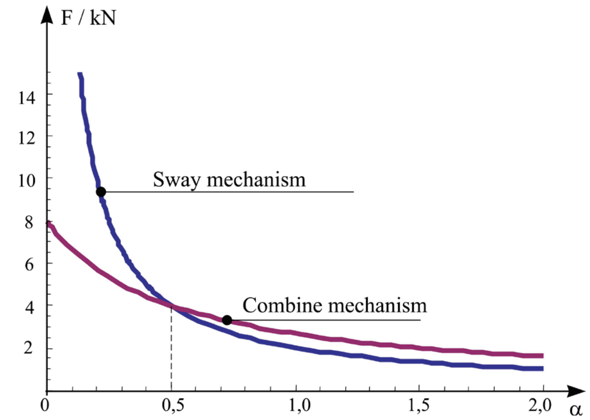

Figure 4 Limit failure force and change of relevant failure mechanism depending on α

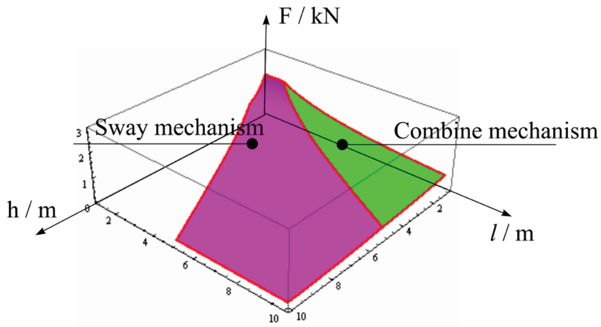

Figure 5 Limit failure force depending on h and l

If Frame 1 is exposed to the action of variable repeated load acting in the range: (0, H), (0, V) failure load (incremental failure force) is determined applying the kinematic shakedown theorem.

Elastic distribution of bending moments if only V force acts on the frame, that is, only the force H, is presented in Fig. 6.

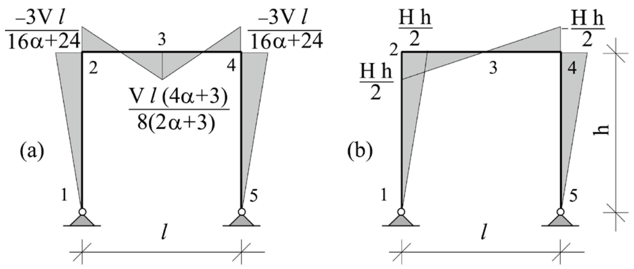

Figure 6 (a) Elastic distribution of bending moments due to V, (b) elastic distribution of bending moments due to H

On the basis of the condition that residual bending moments on the potential failure mechanisms are in equilibrium (Fig. 2) the following equations can be written:

$$m_2(\theta) + m_4(-\theta) = 0, \tag{16}$$

$$m_3(2\theta) + m_4(-2\theta) = 0, \tag{17}$$

By solving the equations (16) and (17) the following expressions are obtained

$$3Vl + 24Hl\alpha + 16Hl\alpha^2 = 16M_\text{p}(3+2\alpha), \tag{18}$$

$$Vl + 2Hl\alpha = 8M_\text{p}, \tag{19}$$

on whose basis, in case when $H = V = F$ the incremental failure forces are obtained in the form:

$$F_\text{inc} = \frac{16M_\text{p}(3+2\alpha)}{l(3+24\alpha+16\alpha^2)}, \tag{20}$$

$$F_\text{inc} = \frac{8M_\text{p}}{l(1+2\alpha)}. \tag{21}$$

On the basis of the expression (20) and (21) the diagrams are constructed (Fig. 7) where change of incremental failure force and change of failure mechanism depending on α is observed. Thus for $\alpha <$ 0,29 combined failure mechanism is formed and for $\alpha >$ 0,29 sway mechanism is formed.

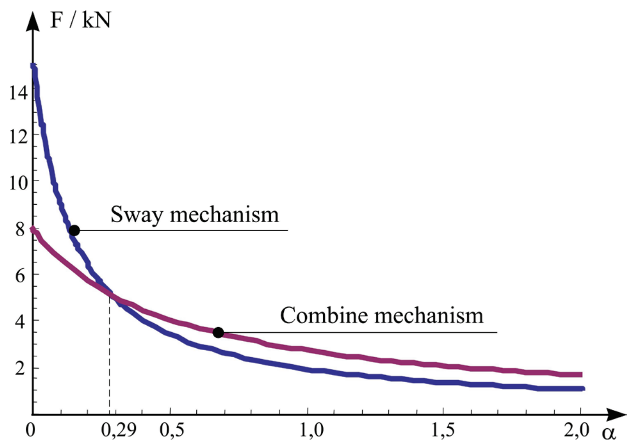

Figure 7 Incremental force and change of relevant failure mechanism depending on coefficient α

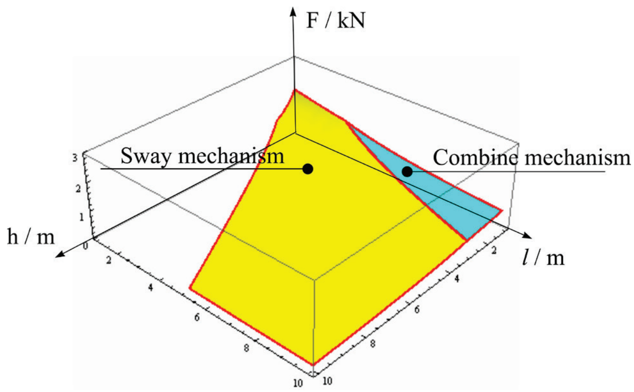

Figure 8 Incremental failure force depending on h and l

Table 1 Maximum and minimum values of bending moment in elastic area

Moment	M^+	M^-
1	/	/
2	$\frac{Hh}{2}$	$-\frac{3Vl}{16\alpha+24}-\frac{Hh}{2}$
3	$\frac{Vl(4\alpha+3)}{8(2\alpha+3)}$	/
4	$\frac{Hh}{2}$	$-\frac{3Vl}{16\alpha+24}-\frac{Hh}{2}$
5	/	/

Change of incremental failure force and the possible failure mechanism in dependence of the change of height and width of the frame is displayed in Fig. 8.

In further analysis of the limit analysis of Frame 1, it is assumed that the horizontal force has alternate character ($-H$, H), while the vertical force is in the range (0, V). When the frame is exposed to the action of alternate load, the alternative failure force is determined applying conditions (4) of the static shakedown theorem.

For the cross-sections 2 and 4, as well as the cross-section 3, on the basis of alternative plasticity condition (4) of the static shakedown theorem the following equations can be written:

$$\left(\frac{Hh}{2}+\frac{3Vl}{16\alpha+24}+\frac{Hh}{2}\right)\leq\frac{2M_\text{p}}{\alpha_\text{form}}, \tag{22}$$

$$\frac{Vl(4\alpha+3)}{8(2\alpha+3)}\leq\frac{2M_\text{p}}{\alpha_\text{form}}. \tag{23}$$

By solving the equations (22) and (23) the following expressions are obtained:

$$\frac{3Vl+Hl(24\alpha+16\alpha^2)}{3+2\alpha}=\frac{16M_\text{p}}{\alpha_\text{form}}, \tag{24}$$

$$\frac{Vl(3+4\alpha)}{3+2\alpha}=\frac{16M_\text{p}}{\alpha_\text{form}}, \tag{25}$$

on whose basis are obtained alternative failure forces for characteristic cross section when $H = V = F$:

$$F_\text{alt}^{2,4}=\frac{16M_\text{p}(3+2\alpha)}{\alpha_\text{form}l(3+24\alpha+16\alpha^2)}, \tag{26}$$

$$F_\text{alt}^{3}=\frac{16M_\text{p}(3+2\alpha)}{\alpha_\text{form}l(3+4\alpha)}. \tag{27}$$

Since the value of alternative failure force depends on the coefficient of cross section form α_form, in further analysis this impact will be also considered. On the basis of expressions (26) and (27) change of alternative failure force depending on the change of coefficient α, as well as coefficient α_form is presented in Fig. 9.

In Fig. 9, it can be observed that irrespective of the change of coefficients α and α_form alternative failure force is determined on the basis of failure mechanism occurring by formation of plastic hinges in cross-sections 2 and 4. The force corresponding to such failure mechanism increases with the change of coefficient of cross section shape, at low value of the coefficient α, and decreases as this ratio approaches $\alpha = 2$. In Fig. 10 is presented the change of alternative failure force depending on h and l for the rectangular cross section frame.

On the basis of conducted bearing capacity analysis depending on the load character, it is possible to perform the comparative analysis of Frame 1. When two independent load systems H and V simultaneously act on the frame, which are in the random relationship, it is possible to perform the analysis of limit bearing capacity and define the domain within the frame is safe to occurrence of failure by use of interaction diagram on whose basis connectedness of failure mechanisms and relations of load is best identified. For Frame 1, in case when $h = l$, ($\alpha = 1$), the interaction diagram is presented in Fig. 11. It can be concluded that for the load ratio which is defined on the basis of **ab** segment the sway failure mechanism is formed when load ratio is $(V/H) \leq 1$, while for the segment **bcdef** the load ratio is $(V/H) \geq 1$ and the combined failure mechanism is formed.

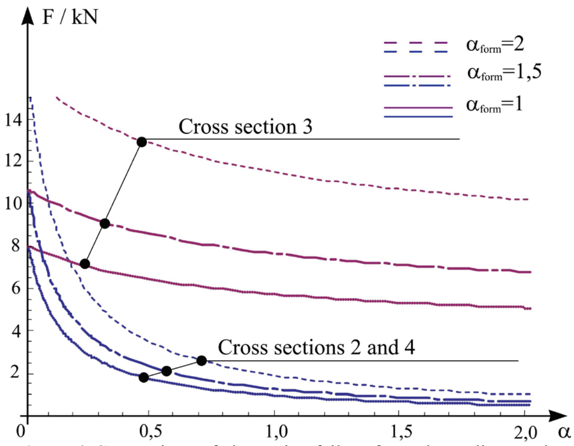

Figure 9 Comparison of alternative failure force depending on the coefficient α for various α_form

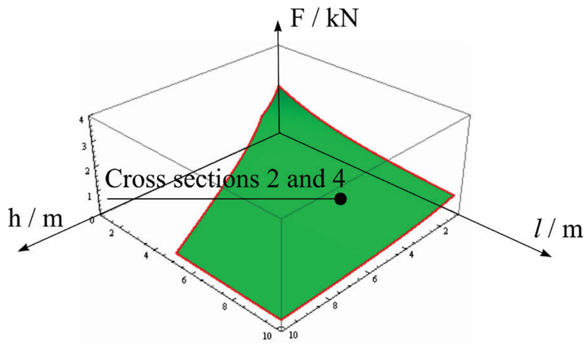

Figure 10 Alternative failure force depending on h and l for the rectangular cross-section frame

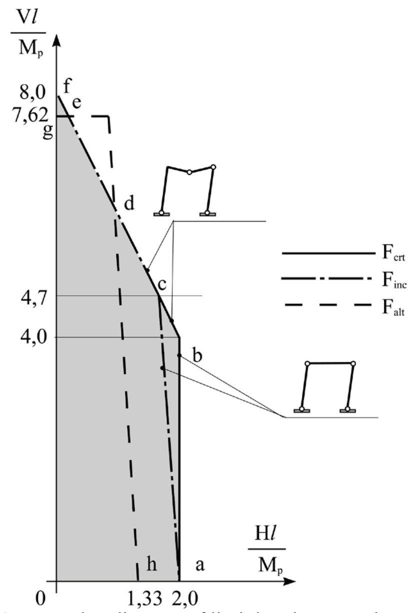

Figure 11 Interactive diagram of limit bearing capacity of Frame 1

For any ratio of load inside **0abcdef0** area there will be no failure mechanism, therefore, no frame failure will occur when the load gradually increases. If the ratio of load is defined by some of the segments, then the failure mechanism defined by the segment is formed. The frame is safe to the occurrence of failure when it is exposed to the action of variable repeated load, when the ratio of H and V load is within **0acdef0** area. When the ratio of H and V loads is defined on the basis of the **cdef** segment, combined failure mechanism occurs at the same value of limit and incremental failure force. Area **0hdeg0** is defined on the basis of the alternative failure conditions, whereby on the basis of the segments **hd** and **fg** formation of plastic hinges in the cross sections 1 and 5 is defined, while on the basis of the segment **ed** the ratio of forces bringing about combined failure mechanism is defined.

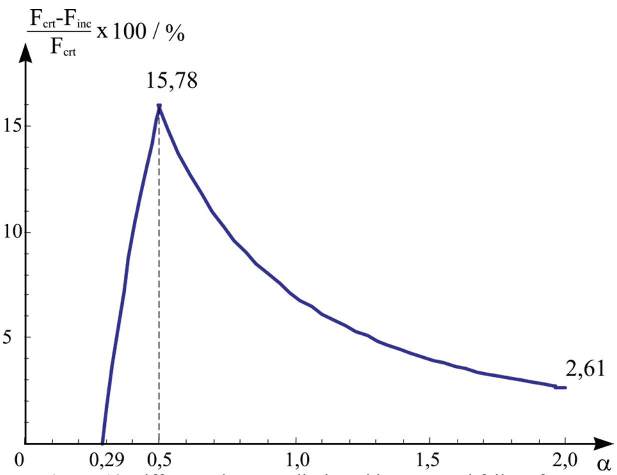

Figure 12 Difference between limit and incremental failure force depending on α in percentage

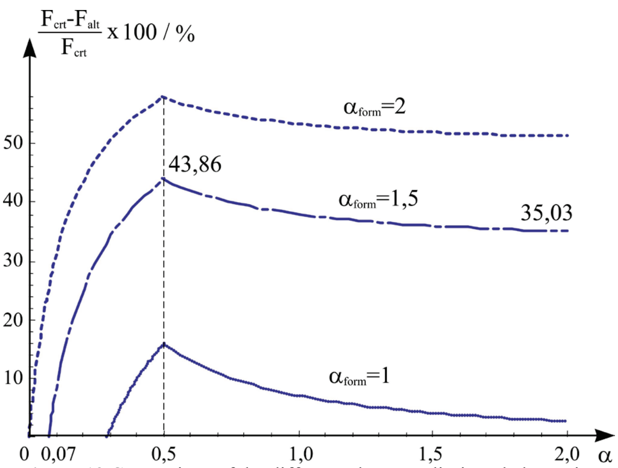

Figure 13 Comparison of the difference between limit and alternative failure force depending on the coefficient α for various α_{form}

If comparison of percentage of limit (Fig. 5) and incremental failure forces (Fig. 8) is performed, as well as of limit (Fig. 5) and alternate failure force (Fig. 10), this change could be displayed in the diagrams in Fig. 12 and Fig. 13.

When α < 0,29 is relevant limit failure force, while for α = 0,50, when the relevant failure mechanism change occurs, difference of limit and incremental failure force is maximum and amounts to 15,78 %. For α > 0,50 this difference decreases from 15,78 % to 2,61 % (α = 2,0).

In Fig. 13 is presented change of limit and alternative failure force, depending on the change of coefficients α and α_{form}. Along with the increase of the cross section shape coefficient, this difference increases and exceeds 50 % for α_{form} = 2,0 and α = 0,50. For the rectangular cross section frame, (α_{form} = 1,50) the greatest difference is 43,86 % for α = 0,50, and for α = 2,0 it is 35,03 %, so it can be said that beam span has no significant impact on the difference between the values of limit and alternate failure force.

3.2 Analysis of Frame 2 limit bearing capacity

Depending on the load character, Frame 2 limit bearing capacity was analysed (Fig. 1b), by the analogous procedure as for Frame 1. The following limit failure forces are obtained for the possible failure mechanism displayed in Fig. 3

$$F_{crt} = \frac{4M_p}{\alpha l}, \tag{28}$$

$$F_{crt} = \frac{8M_p}{l}, \tag{29}$$

$$F_{crt} = \frac{12M_p}{l(2\alpha+1)}. \tag{30}$$

In Fig. 14 based on the expressions (28), (29) and (30) the change of limit failure force is presented in dependence on the coefficient α. It can be observed that for α < 0,25 the frame failure mechanism is relevant, for α > 1,0 sway failure mechanism, and for 0,25 < α < 1,0 combined failure mechanism.

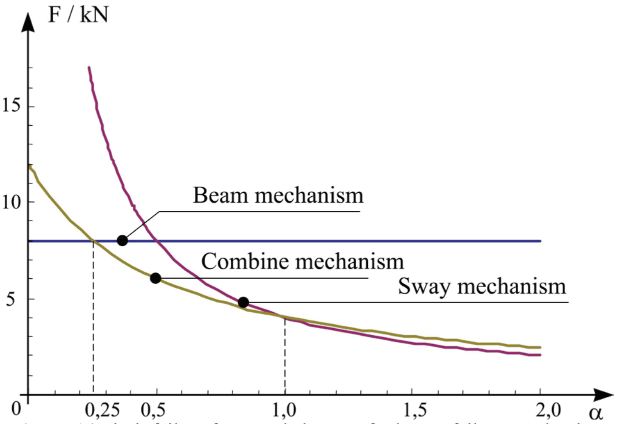

Figure 14 Limit failure force and change of relevant failure mechanism depending on coefficient α

When the frame is exposed to the variable repeated load (0, H), (0, V), for potential failure mechanisms presented in Fig. 3, the expressions for corresponding failure forces are obtained:

$$F_{inc} = \frac{32M_p(2+\alpha)}{l(3+16\alpha+8\alpha^2)}, \tag{31}$$

$$F_{inc} = \frac{8M_p(1+6\alpha)}{l(1+6\alpha+3\alpha^2)}, \tag{32}$$

$$F_{\text{inc}} = \frac{48M_{\text{p}}(2+\alpha)}{l(9+20\alpha+8\alpha^2)}, \tag{33}$$

on whose basis is presented the change of incremental failure force and possible failure mechanism in Fig. 15.

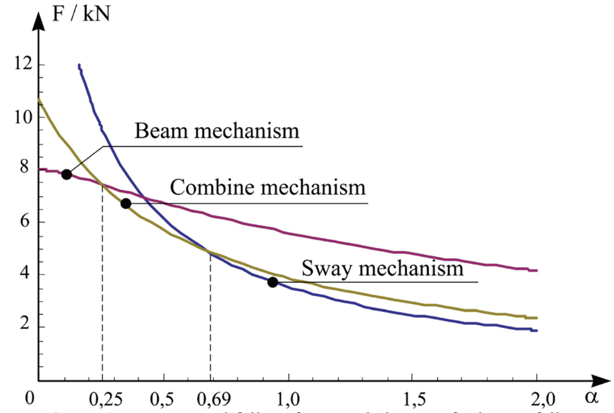

Figure 15 Incremental failure force and change of relevant failure mechanism depending on coefficient α

When $\alpha < 0,25$ the beam failure mechanism is relevant, for $\alpha > 0,69$ sway failure mechanism, and for $0,25 < \alpha < 0,69$ combined failure mechanism.

When Frame 2 is exposed to the action of horizontal force H of alternate character ($-H$, H), and vertical force V in the range (0, V), the failure force is determined by application of conditions (4) of shakedown theorem. For characteristic cross sections 1 and 5, as well as for 2 and 4, and 3, the following expressions for failure forces are obtained:

$$F_{\text{alt}}^{1,5} = \frac{16M_{\text{p}}(2+13\alpha+6\alpha^2)}{\alpha_{\text{form}} l(1+22\alpha+56\alpha^2+24\alpha^3)}, \tag{34}$$

$$F_{\text{alt}}^{2,4} = \frac{8M_{\text{p}}(2+\alpha)(1+6\alpha)}{\alpha_{\text{form}} l(1+6\alpha+24\alpha^2+12\alpha^3)}, \tag{35}$$

$$F_{\text{alt}}^{3} = \frac{8M_{\text{p}}(2+\alpha)}{\alpha_{\text{form}} l(1+\alpha)}. \tag{36}$$

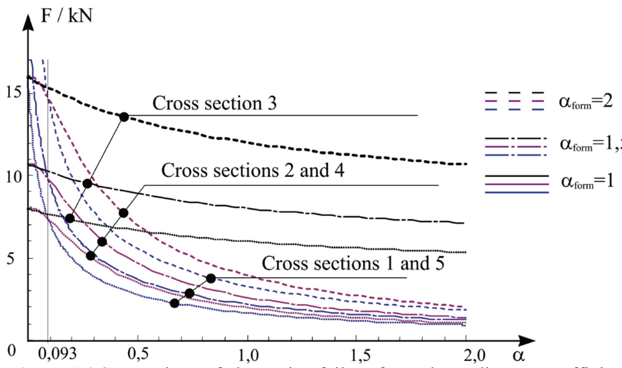

Figure 16 Comparison of alternative failure force depending on coefficient α for various α_{form}

Since the value of alternate failure force depends on the cross section shape coefficient, in Fig. 16 is displayed change of failure force and failure conditions for various values of cross section shape coefficient. For $\alpha \geq 0,093$ the alternate failure force is determined on the basis of formation of plastic hinges in the cross sections 1 and 5, regardless of the vile of cross section shape coefficient, while for $\alpha < 0,093$ the alternate failure force corresponds to formation of plastic hinges in cross sections 2 and 4.

In Fig. 17 is displayed a percentage difference between the values of limit and incremental failure force depending on the changes of α coefficient, while in Fig. 18 is displayed percentage change of difference between limit and alternate failure force depending on the change of coefficients α and α_{form}.

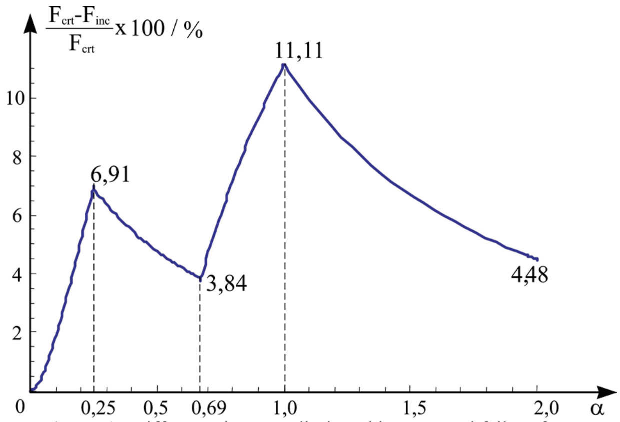

Figure 17 Difference between limit and incremental failure force depending on coefficient α for various α_{form} in percentage

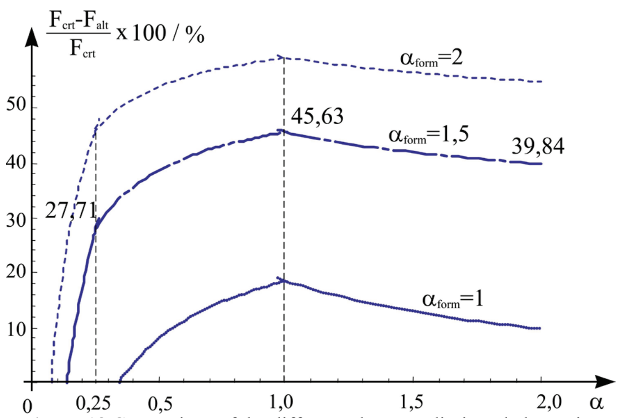

Figure 18 Comparison of the difference between limit and alternative failure force depending on coefficient α for various α_{form}

The greatest difference between the values of limit and incremental failure force is 11,11 % for $\alpha = 1,0$. For other values of α this difference is smaller, and the minimum of 3,84 % is reached for $\alpha = 0,69$.

Depending on the change of cross section shape coefficient, in Fig. 18 is presented percentage difference between limit and alternate failure force. The greatest difference for all the coefficients of cross section is reached for $\alpha = 1,0$, while with the increase of α it slightly increases.

4 Conclusion

It is possible to perform the analysis of limit bearing capacity and bearing capacity of linear structures depending on the character of the load, applying limit and shakedown theorems, whose basic postulates are presented in the introduction of this paper. Depending on the character of the load, static indeterminacy, as well as of the ratio between height and width of the frame, a bearing capacity analysis was performed, and limit, incremental and alternate failure load are determined.

Through the comparative analysis of the values of limit and incremental failure force, it is concluded that for frame, when the ratio of the height to width of the frame is $\alpha \leq 0{,}29$, the limit failure force is relevant, while for $\alpha = 0{,}50$ the difference of forces is the highest and amounts to 15,78 %. For a certain ratio of forces H and V the limit load is equal to the incremental failure load, as displayed in the interaction diagram in Figure 11. When the Frame 2 is exposed to the action of repeated load, the incremental failure force is relevant, and the maximum difference of limit and incremental force is 11,11 % for $\alpha = 1{,}0$.

Difference between limit and alternative force for Frame 1 and Frame 2 is considered depending on the change of coefficient α and cross section shape coefficient α_{form}. For both frames, the difference between values of failure forces is the highest for shape coefficient $\alpha_{\text{form}} = 2{,}0$ and for certain ratios of coefficient α it exceeds 50 %. On the basis of the conducted analysis, it is concluded that the application of shakedown theorems is justified for all considered frame tapes, static indeterminacy and ratios height to width of the frame.

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