

SYNCHRONIZED PHASOR MEASUREMENTS IN A DUAL LAYER HYBRID STATE ESTIMATOR

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The architecture of the proposed Dual Layer Hybrid State Estimator starts with the linear state estimator that only takes synchrophasors as inputs and calculates states of the buses with the deployed Phasor Measurement Units (PMUs) and their adjacent buses. These states are used to obtain the pseudo-measurements that are processed together with conventional SCADA measurements. As the relationship between the conventional measurements and the state variables is nonlinear, in this layer the iterative weighted least squares method is applied to estimate state of the complete power system. The developed formulation was tested on the IEEE systems as well as on the Croatian transmission power system model. The obtained results are compared with the classical solution that uses the conventional measurements and with another hybrid model. The hybrid state estimator based on the proposed architecture offers more accurate state estimation with enhanced filtering of measurements errors and with improved convergence capabilities.

Keywords: *hybrid state estimator, Phasor Measurement Unit (PMU), power system state estimation, synchronized measurement technology*

Sinkronizirana mjerenja fazora u dvoslojnom hibridnom procjenitelju stanja

Izvorni znanstveni članak

Arhitektura predloženog dvoslojnog hibridnog procjenitelja stanja polazi od linearnog procjenitelja stanja koji kao ulazne veličine koristi sinkronizirana mjerenja fazora te računa stanja sabirnica s ugrađenim jedinicama za sinkronizirano mjerenje fazora i stanja njima susjednih sabirnica. Izračunata se stanja koriste za dobivanje pseudo-mjerenja koja se zatim u drugom sloju obrađuju zajedno s konvencionalnim mjerenjima iz SCADA sustava. S obzirom da je veza konvencionalnih mjerenja i varijabli stanja nelinearna, u drugom se sloju koristi iterativna metoda najmanjih kvadrata s težinskim faktorima kako bi se odredilo stanje cjelokupnog sustava. Razvijena je formulacija ispitana na IEEE sustavima i na modelu hrvatskog prijenosnog elektroenergetskog sustava. Dobiveni su rezultati uspoređeni s procjeniteljem stanja zasnovanom na klasičnoj teoriji koji koristi konvencionalna mjerenja, kao i s još jednim hibridnim modelom. Hibridni procjenitelj stanja zasnovan na predloženoj arhitekturi omogućuje točniju procjenu stanja s poboljšanim filtriranjem pogrešaka u skupu mjerenja te s poboljšanom konvergencijom proračuna.

Ključne riječi: *procjena stanja elektroenergetskog sustava, hibridni procjenitelj stanja, jedinica za sinkronizirano mjerenja fazora, tehnologija sinkroniziranih mjerenja fazora*

1 Introduction

In recent years power utilities started to populate their transmission networks with Phasor Measurement Units (PMUs) that provide synchronized phasor measurements (synchrophasors) of voltage and current. The availability of the synchrophasors presents a basis for development of Wide Area Monitoring, Protection and Control (WAMPAC) systems that are based on the Synchronized Measurement Technology (SMT) [1 ÷ 4].

The state estimator is a cornerstone application in the power system control centre as it provides an optimal solution for the power system state, which consists of voltage phasors at all the buses. As the voltage phasors measured by PMUs are directly related to system state variables, it is expected that the state estimation would benefit from application of the SMT. The state estimator result is taken as an input for other applications that constitute the Energy Management System. Consequently, the application of the SMT in the state estimation would positively affect the power system operator's capability to operate the power system more economically and with a higher confidence in real-time.

Availability to obtain the voltage magnitudes and angles processed by the PMUs opens the possibility of establishing a linear state estimator, in which the power system state is not estimated but directly measured [5]. In this case, the power system should be completely observable by the PMUs, which is still a challenging task for larger power systems. On the other hand, the conventional SCADA measurements should not be

discarded, as their high levels of redundancy could be useful for the detection and identification of gross errors in a set of measurements. Rather than focusing on the linear state estimator, a trend in the implementation of the SMT in the area of state estimation is the development of the hybrid state estimators, which are intended to combine the conventional measurements with the synchrophasors.

There are numerous methodologies for finding optimal PMU locations, which is an extensively investigated topic and it is out of the scope of this paper. An interested reader is suggested to refer to [6], in which a thorough description of the state of the art of the optimization methods for optimal PMU placement problem is provided. Further literature overview reveals that a key goal when developing the hybrid state estimators is improvement of their performance. In [7] a measurement vector is augmented with the phasors of current that are transformed from polar to rectangular coordinates. In this way the possible convergence issues of the iterative procedure are avoided. To avoid the propagation of measurement uncertainties connected with transformation of measurements in [8] the synchrophasors are introduced through a set of constraints to relate the phasors of current with the bus voltages. Another solution is to derive voltage phasors at buses that are adjacent to PMU buses. These pseudo-voltages are obtained from the measured voltage and current phasors and the known branch parameters, as given in [9] and [10].

The above stated hybrid state estimators process the conventional measurements and the synchrophasors at the same time, and therefore require significant modifications

of the existing EMS software, as the classical state estimators use the conventional measurements only. The electric power industry is usually conservative towards new technologies and power utilities might be reluctant to substantial changes of solutions that have been operating for decades. In the light of this, the evolutionary approach is proposed, in which the majority of the calculations with synchrophasors are conducted separately from the conventional measurements in a pre-processing layer. The proposed Dual Layer Hybrid State Estimator (DLHSE) starts with a linear state estimator, which uses only the synchrophasors and prepares the pseudo-measurements for the state estimator based on the weighted least squares method (WLS). Here, the conventional measurements are combined with the pseudo-measurements to obtain the state estimate for the complete power system.

Section 2 gives an explanation of a classical power system state estimation approach. In Section 3 an overview of the Croatian Wide Area Monitoring System (WAMS) is provided. In Section 4 the proposed new estimation methodology is presented, while in Section 5 case studies are described. In Section 6 the results are presented, whereas Section 7 gives discussion and Section 8 concludes the paper.

2 Classical approach to state estimation

The classical state estimation theory is applied for the majority of the state estimators in the control rooms all over the world. A brief overview of the classical state estimation approach is given, while a comprehensive description of numerous topics regarding the power system state estimation is available in [11] and [12].

The classical state estimators use the conventional SCADA measurements that include voltage magnitudes, power flows and power injections, which are all grouped into the measurement vector \mathbf{z} . As the measurements collected from the power system are imperfect, the error vector \mathbf{e} presents measurement errors that are assumed to be uncorrelated and with the Gaussian distribution. The state vector \mathbf{x} , which comprises a set of voltage phasors at all the buses in the power system, is then related to the measurement vector \mathbf{z} using the set of nonlinear equations $\mathbf{h}(\mathbf{x})$:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e}. \quad (1)$$

According to the WLS method the state estimate closest to the true power system state is obtained when the objective function $J(\mathbf{x})$ is minimized:

$$J(\mathbf{x}) = \frac{1}{2} [\mathbf{z} - \mathbf{h}(\mathbf{x})]^T \mathbf{R}^{-1} [\mathbf{z} - \mathbf{h}(\mathbf{x})], \quad (2)$$

where matrix \mathbf{R} is the measurement covariance matrix. If all the errors are mutually independent, \mathbf{R} is a diagonal matrix filled with σ^2 , while $\mathbf{W} = \mathbf{R}^{-1}$ is the weight matrix with the weight factors on its diagonal.

The minimum of the objective function $J(\mathbf{x})$ is obtained by satisfying the first-order optimality condition $\mathbf{g}(\mathbf{x}) = \partial J(\mathbf{x}) / \partial \mathbf{x} = 0$, where the Jacobian matrix $\mathbf{H}(\mathbf{x}) =$

$\partial \mathbf{h}(\mathbf{x}) / \partial \mathbf{x}$ contains partial derivatives of $\mathbf{h}(\mathbf{x})$ with respect to the state variables.

Since the relationship between the conventional measurements and the state vector elements is nonlinear an iterative procedure is required. The change of the state vector in the k^{th} iteration is:

$$\mathbf{G}(\mathbf{x}^k) \Delta \mathbf{x}^k = \mathbf{H}^T(\mathbf{x}^k) \cdot \mathbf{R}^{-1} \cdot [\mathbf{z} - \mathbf{h}(\mathbf{x}^k)], \quad (3)$$

$$\Delta \mathbf{x}^k = \mathbf{x}^{k+1} - \mathbf{x}^k, \quad (4)$$

where $\mathbf{G}(\mathbf{x}^k) = \partial \mathbf{g}(\mathbf{x}^k) / \partial \mathbf{x}$ is the gain matrix. The iterative procedure finishes when a maximal change in the state vector $\Delta \mathbf{x}^k$ becomes smaller than a tolerance.

Literature [11] reports ill-conditioning of the Gain matrix in case of using highly accurate measurements such as zero-injections that have small values of standard deviations σ . Instead of using high weight factors for these measurements, a set of constraints is introduced in the WLS based estimation:

$$L(\mathbf{x}, \boldsymbol{\lambda}) = \frac{1}{2} [\mathbf{z} - \mathbf{h}(\mathbf{x})]^T \mathbf{R}^{-1} [\mathbf{z} - \mathbf{h}(\mathbf{x})] - \boldsymbol{\lambda}^T \mathbf{c}(\mathbf{x}), \text{ s.t. } \mathbf{c}(\mathbf{x}) = 0, \quad (5)$$

where $\boldsymbol{\lambda}$ is a vector of the Lagrange multipliers. The following conditions should be fulfilled to minimize $L(\mathbf{x}, \boldsymbol{\lambda})$:

$$\frac{\partial L(\mathbf{x}, \boldsymbol{\lambda})}{\partial \mathbf{x}} = 0, \quad \frac{\partial L(\mathbf{x}, \boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = 0. \quad (6)$$

The following system is solved for $\Delta \mathbf{x}$ and $\boldsymbol{\lambda}$:

$$\begin{bmatrix} \alpha \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} & \mathbf{C}^T \\ \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{x} \\ -\boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \alpha \mathbf{H}^T \mathbf{R}^{-1} [\mathbf{z} - \mathbf{h}(\mathbf{x}^k)] \\ -\mathbf{c}(\mathbf{x}^k) \end{bmatrix}, \quad (7)$$

where $\mathbf{C}(\mathbf{x}) = \partial \mathbf{c}(\mathbf{x}) / \partial \mathbf{x}$ and $\Delta \mathbf{x} = \mathbf{x}^{k+1} - \mathbf{x}^k$.

3 Synchronized measurement technology in Croatian transmission power system

The Croatian Transmission Power System (TPS) represents an electric energy link from north-eastern to south-western parts of Europe, which became even more important when two synchronous zones of the European power system (ENTSO-E, former UCTE) were reconnected in 2004 [13]. In line with the objective of renewable energy sources integration, 1200 MW of wind power plants will be integrated into the Croatian TPS by the year 2020 according to the Croatian energy development strategy. Meanwhile, the regulating capacity of the Croatian power plants constraints the power of the wind power plants connected to the TPS to 400 ÷ 500 MW. New methods for the power system real-time monitoring should be deployed in order to cope with oncoming challenges, which will affect the power system operation.

The Croatian TSO uses the classical WLS based state estimator that takes voltage magnitudes, active and reactive power flows and power injections. The conventional measurements provide complete

observability of the Croatian 110, 220 and 400 kV TPS. The state estimator performance is limited as there are no measurements of voltage phase angles and the conventional measurements are unsynchronized and refreshed at a slow rate. Operational practice shows that the solution is sensitive to the loss of the part of measurements from neighbouring power systems and that topological errors sometimes cause convergence issues. Considering the given drawbacks of the classical state estimator, it is difficult to monitor the power system operation in real-time. A new hybrid model, which combines the more frequent and precise synchrophasors with the conventional measurements, would enhance the state estimation practice.

The PMUs in the Croatian TPS were deployed in several phases and their outputs are collected within two Phasor Data Concentrators (PDC). The Croatian WAMS uses the originally developed software support that is tailored to particularities of the Croatian TPS. The locations of the PMUs deployed in the Croatian TPS are available in [4]. It should be emphasized that in some nodes there are several PMUs accommodated, in order that measurements from different bays are not mixed. In this way each PMU collects only one voltage and one current phasor, although the deployed units have a sufficient number of measurement channels. Such placement does not result in the minimum number of PMUs, but it significantly improves redundancy levels. With 14 PMUs deployed at all 400 kV buses, the complete 400 kV network is observable and a linear state estimator for 400 kV network could be established. In the described configuration a number of PMUs is not minimized, but the given placement ensures observability even under a single PMU outage, as each 400 kV bus is observed by at least two PMUs. Nevertheless, the availability of the conventional measurements from 110, 220 and 400 kV networks should not be disregarded, as they ensure the observability of the complete Croatian TPS and increase measurement redundancy levels. In order to take advantage of both the conventional measurements and the synchrophasors, a hybrid state estimator is proposed.

4 Hybrid state estimators

4.1 Hybrid state estimator with pseudo-voltages

The existing state estimator with a structure similar to the model proposed in this paper is the hybrid state estimator with pseudo-voltages. In this model the current phasors are transformed into the voltage phasors on the buses adjacent to the PMU buses. In order to make the paper self-supporting, a brief overview of the Pseudo-Voltages State Estimator (PVSE) is given, as proposed in [9] and [10]. The PVSE starts with a pi-model of the network branch, shown in Fig. 1.

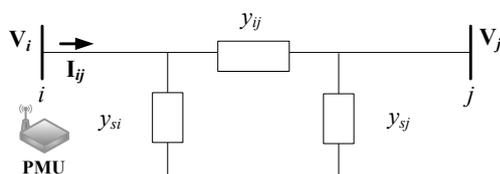


Figure 1 A pi-model of the network branch

where $V_i = V_i \angle \theta_i$ is the voltage phasor at the bus i measured by the PMU, $I_{ij} = I_{ij} \angle \theta_{ij}$ is the current phasor measured by the PMU, $V_j = V_j \angle \theta_j$ is the voltage phasor at the bus j adjacent to the PMU bus i , $y_{ij} = g_{ij} + jb_{ij}$ is the series admittance, while $y_{si} = g_{si} + jb_{si}$ and $y_{sj} = g_{sj} + jb_{sj}$ are the shunt admittances of the branch connecting the bus i and the bus j .

The voltage phasor V_j at any bus j adjacent to the PMU bus i can be obtained by using the measured voltage V_i and the current phasor I_{ij} with known branch parameters:

$$V_j = \frac{V_i(y_{ij} + y_{si}) - I_{ij}}{y_{ij}} \quad (8)$$

The relationship between the measurement vector z and the state vector x is given by:

$$z = [x_{conv}, V_{PMU}, \alpha_{PMU}, V_{pseV}, \alpha_{pseV}]^T = H \cdot \begin{bmatrix} V \\ \alpha \end{bmatrix} + e, \quad (9)$$

where z_{conv} is the vector of the conventional measurements:

$$z_{conv} = [V; P_{flow}; Q_{flow}; P_{inj}; Q_{inj}]^T, \quad (10)$$

while V_{PMU} and α_{PMU} are sets of the voltage magnitudes and angles directly measured at the PMU buses. The pseudo-voltages include sets of the voltage magnitudes V_{pseV} and angles α_{pseV} at the buses adjacent to the PMU buses, calculated by using Eq. (8). The state vector includes the voltage magnitudes V and angles α at all the buses in the system. Since the relationship between the conventional measurements and the system state variables is nonlinear, the iterative WLS method is applied to estimate the system state.

4.2 Dual layer hybrid state estimator

The idea of the proposed DLHSE is to obtain estimates of the complex voltages at the subset of the PMU observable buses by processing the linear model of the synchronized voltage and current measurements. These estimates are used as pseudo-measurements and combined with the conventional measurements in the standard WLS state estimator. The DLHSE starts with the linear state estimator that uses the synchrophasors only. With the PMUs (a number of the PMUs is n_{PMU}) deployed in the system we can denote x_{meas} as the subset of the voltage phasors that are directly measured at the PMU buses (a number of such buses is n_{meas}). Also, x_{calc} is the subset of the voltage phasors at the buses adjacent to the PMU buses that can be calculated (a number of such buses is n_{calc}). The state vector is combined of the two subsets in rectangular coordinates:

$$x_{LIN} = [x_{meas}, x_{calc}]^T = [V_R, V_I]^T, \quad (11)$$

where V_R and V_I are real and imaginary parts of the state vector, respectively. With a reference to Fig. 1, the

voltage and current phasors measured by the PMUs are transformed from polar to rectangular coordinates and joined in the measurement vector \mathbf{z}_{LIN} :

$$\mathbf{z}_{\text{LIN}} = [\mathbf{V}_{iR}, \mathbf{V}_{iI}, \mathbf{I}_{ijR}, \mathbf{I}_{ijI}]^T. \quad (12)$$

To calculate the uncertainty propagation associated with the transformation from polar to rectangular coordinates the classical uncertainty propagation theory is used [14]. The measurement vector and the state vector are related using the linear measurement model [5]:

$$\mathbf{z}_{\text{LIN}} = \mathbf{H}_{\text{LIN}} \cdot \mathbf{x}_{\text{LIN}} + \mathbf{e}_{\text{LIN}}, \quad (13)$$

where $\mathbf{e}_{\text{LIN}} = [\mathbf{e}_{V_{iR}}, \mathbf{e}_{V_{iI}}, \mathbf{e}_{I_{ijR}}, \mathbf{e}_{I_{ijI}}]^T$ is the vector of measurement errors. The Jacobian matrix \mathbf{H}_{LIN} contains the partial derivatives of the measurements in respect to the real and imaginary parts of the state vector:

$$\mathbf{H}_{\text{LIN}} = \begin{bmatrix} \frac{\partial V_{iR}}{\partial V_R} & \frac{\partial V_{iI}}{\partial V_R} & \frac{\partial I_{ijR}}{\partial V_R} & \frac{\partial I_{ijI}}{\partial V_R} \\ \frac{\partial V_{iR}}{\partial V_I} & \frac{\partial V_{iI}}{\partial V_I} & \frac{\partial I_{ijR}}{\partial V_I} & \frac{\partial I_{ijI}}{\partial V_I} \end{bmatrix}^T, \quad (14)$$

where $\frac{\partial V_{iR}}{\partial V_R}$ and $\frac{\partial V_{iI}}{\partial V_I}$ are $n_{\text{PMU}} \times (n_{\text{meas}} + n_{\text{calc}})$, $n_{\text{PMU}} \times (n_{\text{meas}} + n_{\text{calc}})$ identity matrices, with zeros on the main diagonal where the voltage phasors have not been measured. The partial derivatives $\frac{\partial V_{iR}}{\partial V_I}$ and $\frac{\partial V_{iI}}{\partial V_R}$ are $n_{\text{PMU}} \times (n_{\text{meas}} + n_{\text{calc}})$ zero matrices. The partial derivatives for the current measurements in respect to the state variables are calculated using:

$$\begin{bmatrix} \mathbf{I}_{ijR} \\ \mathbf{I}_{ijI} \end{bmatrix} = \begin{bmatrix} (g_{ij} + g_{si}) & -g_{ij} & -(b_{ij} + b_{si}) & b_{ij} \\ (b_{ij} + b_{si}) & -b_{ij} & (g_{ij} + g_{si}) & -g_{ij} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{V}_{iR} \\ \mathbf{V}_{jR} \\ \mathbf{V}_{iI} \\ \mathbf{V}_{jI} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_{I_{ijR}} \\ \mathbf{e}_{I_{ijI}} \end{bmatrix}. \quad (15)$$

The estimation of the state vector elements for the linear state estimator is obtained by directly converting the matrices:

$$\hat{\mathbf{x}}_{\text{LIN}} = (\mathbf{H}_{\text{LIN}}^T \mathbf{R}_{\text{LIN}}^{-1} \mathbf{H}_{\text{LIN}})^{-1} \mathbf{H}_{\text{LIN}}^T \mathbf{R}_{\text{LIN}}^{-1} \mathbf{z}_{\text{LIN}}, \quad (16)$$

where \mathbf{R}_{LIN} is the covariance matrix.

In the first layer of the DLHSE each PMU observes the PMU bus and its adjacent buses. The synchrophasors from disjointed parts of the system refer to the slack bus phase angle reference that is equal to zero and they are mutually connected by the difference between PMU angle measurements.

The estimated state of the PMU buses and their adjacent buses obtained by the linear state estimator is used to derive the pseudo-measurements. The transformation from rectangular to polar coordinates is applied to obtain the voltage magnitudes and angles of the pseudo-measurements at each bus observable by the PMU:

$$V_{\text{PSEUDO}} = \sqrt{\hat{\mathbf{x}}_{\text{LINR}}^2 + \hat{\mathbf{x}}_{\text{LINI}}^2}, \quad (17)$$

$$\alpha_{\text{PSEUDO}} = \tan^{-1} \left(\frac{\hat{\mathbf{x}}_{\text{LINI}}}{\hat{\mathbf{x}}_{\text{LINR}}} \right). \quad (18)$$

The standard deviations of the pseudo-measurements, needed to determine the weight factors, are calculated by using the classical uncertainty propagation theory.

The sets of the pseudo-measurements of the voltage magnitudes V_{PSEUDO} and angles α_{PSEUDO} are together with the conventional measurements used as inputs for the state estimator that is based on the WLS method:

$$\mathbf{z}_{\text{WLS}} = [\mathbf{z}_{\text{conv}}, V_{\text{PSEUDO}}, \alpha_{\text{PSEUDO}}]^T = \mathbf{H}_{\text{WLS}} \cdot \mathbf{x}_{\text{WLS}} + \mathbf{e}_{\text{WLS}}, \quad (19)$$

where \mathbf{z}_{conv} is the set of the voltage magnitudes, active and reactive power flows and injections. For the zero-injection buses the equality constraints are set. The state vector \mathbf{x}_{WLS} includes the voltage phasors at all the buses in the system in the polar form so the final result is the estimated state of the complete power system.

The measurement vector \mathbf{z}_{WLS} of the DLHSE has a structure similar to the measurement vector of the PVSE, given in Eq. (9). For the DLHSE the pseudo-measurements include the voltage phasors of both the PMU buses and their adjacent buses, where all these phasors are pre-processed by the linear state estimator. On the other hand, in the PVSE the pseudo-voltages at the buses adjacent to PMU buses are calculated, while for the voltage phasors at the PMU buses the PMU measurements are taken directly without any pre-processing.

5 Case studies

The developed DLHSE was tested on the standard IEEE test systems with 14, 30, 57 and 118 buses [15]. The hybrid model was compared with the PVSE described in Section 4 and with the classical state estimator that uses the conventional measurements only. The conventional measurements were placed to ensure complete observability of the test systems. The placement of the PMUs was chosen to improve local redundancy levels. The PMUs are assumed to have enough measurement channels for observing all the branches emanating from the PMUs buses. The locations of the PMUs for the IEEE test systems are given in Tab. 1.

As an example of a real power system, a model of the complete Croatian TPS, comprising 110, 220 and 400 kV levels was used. For the Croatian TPS the DLHSE was compared with the classical state estimator based on the WLS method that runs in the Croatian National Dispatching Centre. The locations of the conventional measurements and the PMUs were selected in line with the real placement of the measurement units in the Croatian TPS. The PMUs in the Croatian TPS are deployed to observe only one branch each. Therefore, after simulating the basic scenario with such a configuration, additional tests were carried out with the PMUs observing all the branches emanating from each PMU bus.

Table 1 PMU locations for the IEEE test systems

Test System	Buses with PMUs
IEEE 14	1 and 6
IEEE 30	1, 5, 10, 12, 15 and 27
IEEE 57	1, 6, 9, 18, 19, 30 and 55
IEEE 118	24, 40, 59, 69, 75, 80, 100, 103, 113 and 114

The set of noisy measurements was used as input into the tested state estimators. These measurements were created from the power flow results, which were considered to be true measurements, to which the random noise was added. Tab. 2 provides standard deviations for the conventional measurements and synchrophasors, expressed in percentage of the actual values. These standard deviations were used to generate the random Gaussian noise with a zero mean and given uncertainties.

Table 2 Standard deviations of measurements

Conventional measurements		
Power flow	Power injection	Voltage magnitude
2 %	2 %	0,2 %
Synchrophasors		
Current	Voltage	Voltage and current phase
0,03 %	0,02 %	0,01°

The state estimator performance was compared by using several criteria. The first criterion shows the capability of the state estimator to filter measurement errors:

$$\xi(\hat{x}) = \frac{\sum_{i=1}^m (\hat{z}_i - z_i^t)^2}{\sum_{i=1}^m (z_i - z_i^t)^2}, \quad (20)$$

where m is the number of measurements, while \hat{z} , z^t and z are the estimated, true and noisy measurements, respectively.

The accuracy of the obtained state estimate is given in the variance of the estimated states:

$$\sigma_{\Sigma}^2 = \sum_{i=1}^{2N} (\hat{x}_i - x_i)^2, \quad (21)$$

where N is the number of buses in the system, while \hat{x} and x are the estimated and true state values, respectively.

The convergence of the state estimator is given by the total number of iterations that is needed to reach the tolerance 10^{-6} .

In order to provide a reliable comparison of the state estimators operation, when comparing the proposed DLHSE with the PVSE and the classical state estimator, 100 Monte Carlo simulations were carried out for all the test systems. For each Monte Carlo simulation a different set of random noise was used to generate the noisy measurements. The results provided in Section 6 are average values for 100 Monte Carlo simulations.

6 Results

6.1 The IEEE test systems

For the IEEE test systems with 14, 30, 57 and 118 buses the developed DLHSE was compared with the hybrid PVSE and with the classical state estimator.

The graphical results are given for the IEEE 14 test system only, as for the IEEE 30, 57 and 118 test systems the numbers of buses and branches are limiting factors for the graphical representation. For the IEEE 14 test system Fig. 2 gives errors of the estimated voltage angles and magnitudes together with errors of the calculated active and reactive power injections at the buses, while Fig. 3 gives errors of the calculated active and reactive power flows on the branches.

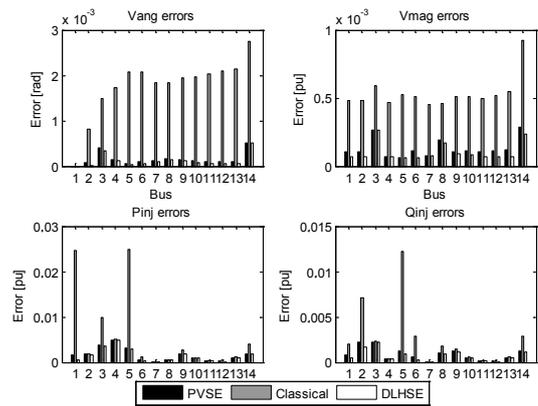


Figure 2 Voltage and power injection errors for the IEEE 14 test system

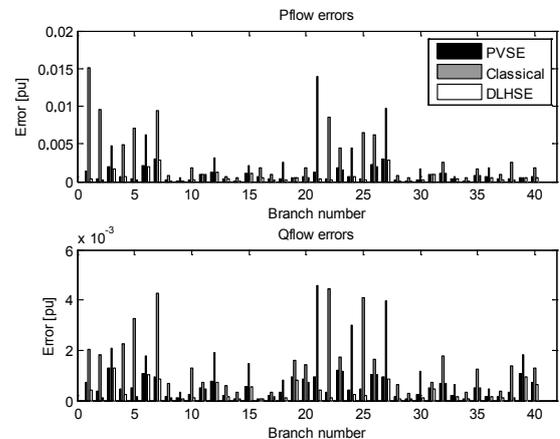


Figure 3 Power flow errors for the IEEE 14 test system

Tab. 3 presents the results of the state estimators for the IEEE test systems with 14, 30, 57 and 118 buses.

When comparing the classical state estimator with the two hybrid state estimators, it can be concluded that the estimation practice is enhanced with the inclusion of the synchrophasors into the set of measurements. The comparison with the PVSE shows that the DLHSE additionally improves accuracy, providing smaller variances σ_{Σ}^2 of the estimated states, as well as better filtering ξ of measurement errors for all the test systems. The reason for such improvement, in comparison with the pseudo-voltages used in the PVSE method, is that the linear state estimator provides the more accurate pseudo-measurements by pre-processing the voltage and current

phasors. The total number of iterations for the DLHSE was calculated by taking the number of the WLS state estimator iterations to which a single iteration is added to take into account the non-iterative conversion of matrices in the linear state estimator. The total number of iterations is smaller when compared with the other estimators.

Table 3 Results for the IEEE test systems

System	State estimator	ξ	σ_{Σ}^2	Iteration
IEEE 14	PVSE	0,0355	$1,3293 \times 10^{-6}$	4,00
	Classical	0,4501	$7,9950 \times 10^{-5}$	4,96
	DLHSE	0,0282	$9,7456 \times 10^{-7}$	3,05
IEEE 30	PVSE	0,0319	$5,3171 \times 10^{-7}$	4,00
	Classical	0,3760	$2,5582 \times 10^{-5}$	4,58
	DLHSE	0,0208	$4,5506 \times 10^{-7}$	3,00
IEEE 57	PVSE	0,0389	$8,6362 \times 10^{-4}$	5,00
	Classical	0,3080	$1,3508 \times 10^{-3}$	5,00
	DLHSE	0,0252	$8,5835 \times 10^{-4}$	4,20
IEEE 118	PVSE	0,4334	$5,5797 \times 10^{-5}$	4,37
	Classical	0,6809	$5,7932 \times 10^{-4}$	4,96
	DLHSE	0,4253	$5,3195 \times 10^{-5}$	4,00

6.2 The Croatian transmission power system

Once that the proposed DLHSE was compared with the hybrid PVSE and the classical state estimator using the IEEE test systems, it was shown that it provides a more accurate state estimation with the enhanced capability of filtering the measurement errors and with the lowered number of the iterations necessary to reach the desired tolerance. For the Croatian TPS the DLHSE was also compared with the hybrid model PVSE and the classical state estimator, which uses only the conventional measurements and is running in the Croatian control centre. The DLHSE was tested on the complete model of the Croatian TPS that comprises 200 buses and 287 branches. Therefore, the graphical representation of the results is given only for the buses with the deployed PMUs.

In order to investigate how the measurements of the current phasors on all the branches emanating from the PMU buses would affect the state estimator performance, two tests were carried out. First the actual configuration was used, in which each PMU provides one voltage phasor and only one current phasor. Then another configuration was simulated, in which each PMU collects one voltage phasor and several current phasors, from all the branches emanating from the PMU buses. For both hybrid models these two configurations were denoted as "##" and "###", respectively.

Tab. 4 gives the results that present average values of 100 Monte Carlo simulations for the Croatian TPS, in configuration where each PMU provides only one current phasor ("##"). When comparing the PVSE and the DLHSE with the classical state estimator it has to be emphasized that the given results present variances of the estimated states for 200 buses, while only 14 buses are equipped with the PMUs. Also, the filtering of the measurement errors is given for 1362 measurements (out of which there are only 14 voltage phasors and 14 current phasors).

In Fig. 4 the estimation errors for the voltage magnitudes and angles together with errors of the calculated power injections at the PMU buses are given. In Fig. 5 errors of the calculated power flows on the

branches observable by the PMUs are presented.

Table 4 State estimation results for the Croatian TPS - configuration#

System	State estimator	ξ	σ_{Σ}^2	Iteration
CRO	PVSE	0,2502	$8,5356 \times 10^{-6}$	4,17
	Classical	0,2692	$1,4086 \times 10^{-5}$	4,00
	DLHSE	0,2471	$8,0563 \times 10^{-6}$	3,75

Each PMU measures one phasor of current

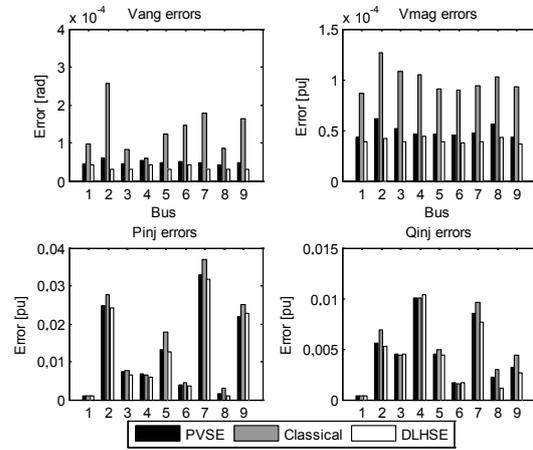


Figure 4 Voltage and power injection errors for the Croatian TPS – configuration#

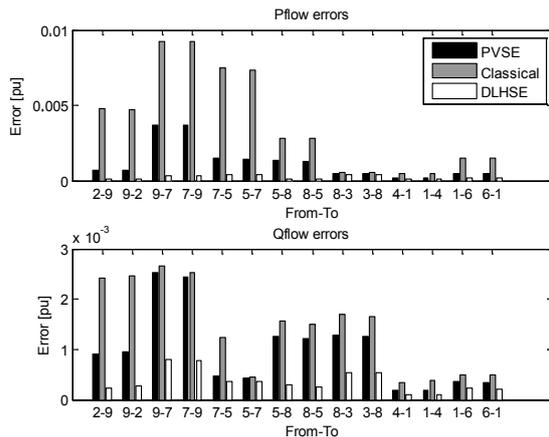


Figure 5 Power flow errors for the Croatian TPS – configuration#

It can be observed that the inclusion of the synchrophasors in the set of measurements brings a more accurate estimation for the PMU buses, which are actually important nodes in the Croatian TPS, as these are all 400 kV and a part of 220 kV nodes. Since the more accurate estimation of the voltage phasors is obtained, this allows us to calculate the power injections at the same buses more accurately. Finally, the more accurate calculation of the power flows helps the power system operator in operating the system.

When the current phasors on all the branches emanating from the PMU buses are taken into the set of measurements, the results for the DLHSE are additionally improved, as given in Tab. 5. In this case, there are 14 voltage phasors and 48 current phasors out of 1396 measurements. With the improved accuracy of the estimated states, visible through the lowered value of the variances of the estimated states, the filtering of the measurement errors is improved and the number of iterations is reduced. Although they use the same set of

measurements, it can be concluded that in comparison with the PVSE, the developed DLHSE benefits more from the inclusion of all the available current phasors.

Table 5 State estimation results for the Croatian TPS - configuration##

System	State estimator	ξ	σ_{Σ}^2	Iteration
CRO	PVSE	0,0799	$5,8767 \times 10^{-6}$	4,00
	Classical	0,2692	$1,4086 \times 10^{-5}$	4,00
	DLHSE	0,0303	$3,6627 \times 10^{-6}$	3,63

Each PMU measures all available phasors of current

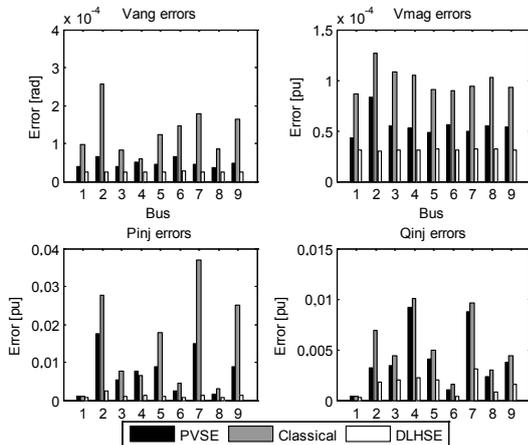


Figure 6 Voltage and power injection errors for the Croatian TPS - configuration##

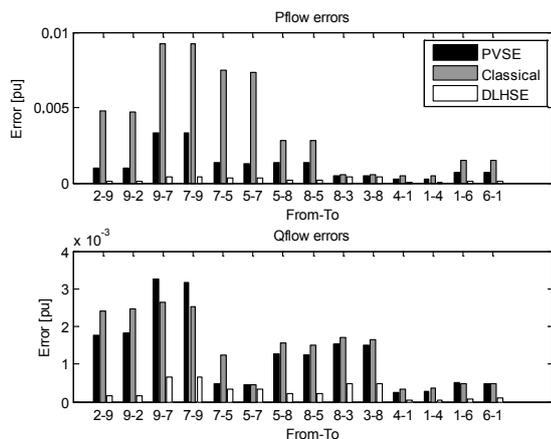


Figure 7 Power flow errors for the Croatian TPS - configuration##

In Fig. 6 the estimation errors at the PMUs buses are presented together with errors of the calculated power injections, while Fig. 7 gives errors of the calculated power flows. Due to limited space, the power flow errors are given for the same branches as in the previously tested configuration. The inclusion of the phasors of current from all the branches emanating from the PMU buses allows the DLHSE to provide the even more accurate estimation of the states at the PMU buses, which results in the more accurate calculation of the power injections and power flows.

7 Discussion

The SMT is expected to improve operation of several applications in the power system control centre, as the power utilities started to populate transmission grids with

the PMUs offering measurements of voltage and current phasors. One of the most promising applications for the SMT is in the area of state estimation, since the state estimator is a cornerstone application in the EMS and its outputs are used by other applications such as the power flow calculation and the contingency analysis.

The existing conventional measurements such as the voltage magnitudes, power flows and injections are used in the classical state estimators based on the WLS method, and therefore usually ensure the complete observability of the monitored power system. Although they are unsynchronized, their high levels of redundancy are useful for the gross errors detection and identification. On the other hand, the synchrophasors of voltage and current are provided by the PMUs in high sampling frequencies and are synchronized by the precise GPS signals. The optimal combination of the conventional measurements together with the synchrophasors leads us towards the hybrid state estimators that should overcome the drawbacks of the classical solution, and therefore ensure enhancement of the power system real-time monitoring.

In this paper the Dual Layer Hybrid State Estimator is proposed. Its architecture starts with the linear state estimator that uses the synchrophasors only. In the linear state estimator the state of the buses observed by the PMUs is obtained directly by converting the matrices. The states of these buses are propagated into the next layer as the pseudo-measurements, where they are processed together with the conventional measurements. Since for the conventional measurements the relationship with the state variables is nonlinear, the iterative WLS method is used.

The proposed hybrid state estimator was tested using several test systems of different sizes, topologies and configurations of measurements. The IEEE test systems with 14, 30, 57 and 118 buses were used as examples of the standard test systems. To investigate the state estimator functionality on an example of the real power system, the Croatian transmission power system model that comprises 110, 220 and 400 kV levels was used. For the Croatian transmission power system model the real placement of the conventional measurements and the synchrophasors was used. The provided results were obtained as average values of 100 Monte Carlo simulations to eliminate impact of the random errors in the set of noisy measurements on the state estimator performance. The results of the simulated case studies were compared with another hybrid model and with the classical solution that uses the conventional measurements only.

In the Croatian transmission power system the PMUs are configured to provide one voltage phasor and only one current phasor. The case study simulation in which the PMUs were reconfigured to observe the phasors of current on all the branches emanating from the PMU buses was carried out.

8 Conclusion

The proposed Dual Layer Hybrid State Estimator offers more accurate estimation of the power system state with the enhanced filtering of the measurement errors and

the improved convergence capability when compared with the classical state estimator.

In comparison with another hybrid state estimator, the Pseudo-Voltage State Estimator, which calculates the pseudo-voltages at the buses adjacent to the PMU buses, the developed model provides the more accurate pseudo-measurements for all the buses observable by the PMUs, i.e. both the PMU buses and their adjacent buses. The pseudo-measurements are the output of the linear state estimator, which takes into account random errors in the measured synchrophasors, thus resulting in the overall improvement of the estimator performance.

The results obtained for the Croatian transmission power system showed that it would be recommended to take advantage of all the potentially available current measurements and include them into the set of measurements, since this contributes to the developed state estimator performance.

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