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COMPARISON OF COMPLEXITY INDICATORS FOR ASSESSING GENERAL PROCESS STRUCTURES

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Original scientific paper
This paper focuses on the comparison of different complexity indicators for complexity assessment of selected general process structures. The main objective in this study is to test their ability to uncover assumed differences in structural complexity among observed general process structures. The obtained results of this theoretical study show that all proposed indicators can be effectively used for analysing structural complexity of general process structures.

Keywords: *complexity, graph theory, structure, vertex degree*

Usporedba pokazatelja složenosti za procjenu općih struktura procesa

Izvorni znanstveni članak
Rad se bavi usporedbom različitih pokazatelja pri procjeni složenosti odabranih općih struktura procesa. Glavni cilj istraživanja je ispitivanje sposobnosti tih pokazatelja u otkrivanju očekivanih razlika u strukturnoj složenosti među promatranim općim strukturama procesa. Rezultati ovog teorijskog istraživanja pokazuju da svi predloženi pokazatelji mogu biti učinkovito upotrijebljeni pri analizi strukturne složenosti općih struktura procesa.

Ključne riječi: *složenost, struktura, stupanj čvora, teorija grafova*

1 Introduction

Nowadays, it is useful and important to manage process structures and to measure their complexity. This is one of the reasons for an increasing number of research works on complexity of manufacturing process structures. On the other hand a unified procedure for complexity assessment of process structures is still missing. Moreover, the tendencies and relations in the development of organizational, management and production structures increase the requirements concerning the characteristics of the process structures. In order to obtain the relationship, a set of complexity indicators is tested.

2 Literature review

Current research on process structures' complexity is characterized by different ways of complexity exploration. It is possible to identify the following research areas:

(i) Theoretical discussions of production systems complexity. They are primarily based on the systems theory or mathematical modelling of a structure. Simulation and non-linear dynamics are used to gain data for the structure's analysis [1]. Another relevant approach to the complexity measurement is based on graph theory (see for example [2]).

(ii) Measurements of system complexity and lowering the complexity of a system. There is a link between the complexity and the performance of the same system [3]. Mostly used key words in the context of measuring methods are: number of elements of the structure, manageability. Case studies have mostly been based on frameworks. It is necessary to decrease the complexities of production structures especially for planning and work distribution problem in the high variety systems. None of these studies solved the problem adequately [4].

(iii) Relation between process structure complexity measurement and process performance measurement. It is clear that the complexity is an important factor determining the production system's quality and therefore process complexity is connected with its performance. Different aspects of business performance measurements and control were discussed, for example, in [5] or [6]. Keeping in mind the need for process structure simplification, an Average Shortest Path (ASP) and a Modified Flow Complexity (MFC) have been extended and presented [7, 8]. We will use the indicator in this study in comparison with other known approaches. Obviously there are other literature sources discussing the issue of process complexity from different points of view (see for example [9, 10, 11]).

3 Methodology

One of the useful methods for evaluation of structural properties of manufacturing processes is a graph theory. The fundamental concept of a graph theory is the graph $G=(V; E)$ that conceptually consists of a set of vertices $V(G)$ and edges $E(G)$. The two points connected by a line are said to be *adjacent*. Two lines that share an endpoint of the graph are *incident*.

In the proposed approach the structural properties of a manufacturing process will be examined in terms of the primary elements of the process structure: vertices (or nodes denoted by N) and edges (of links denoted by L) are considered. In this research all initial parameters of the examined process graphs (see Fig. 1) are known with certainty, so that the problem is deterministic. The primary research methodology has been based on the application of general axioms of graph theory for selected problem area.

To gain some insight into the relation between the properties of the individual parameters under the given conditions, statistical analysis was performed. In classical statistical tests, such as the use of a single correlation coefficient r_c between two random variables, it is required that the random variables are normally distributed. Under the assumption of Gaussian violation (as in our case), nonparametric Spearman's rank order correlation (corrected) is calculated. The obtained values of the correlation coefficients are summarized in Tab. 2. Corrected Spearman correlation coefficient is formalized:

$$r_c = \frac{C_x + C_y - \sum_{i=1}^{n} d_i^2}{\sqrt{C_x \cdot C_y}}, \quad (1)$$

$$C_x = \frac{n^3 - n}{12} - T_x, \quad (2)$$

$$C_y = \frac{n^3 - n}{12} - T_y, \quad (3)$$

$$\sum_{i=1}^{n} d_i^2 = \sum_{i=1}^{n} \left[R(x_i) - R(y_i)\right]^2, \quad (4)$$

$$T_x = \sum_t \frac{t^3 - t}{12}, \quad (5)$$

$$T_y = \sum_t \frac{t^3 - t}{12}. \quad (6)$$

4 Description of compared complexity indicators for manufacturing systems

4.1 Restrictiveness estimator *RT*

RT is practically the same measure as Order Strength defined by Mastor [12]. Formally *RT* is expressed by the formula:

$$RT = \frac{2 \cdot \sum r_{ij} - 6 \cdot (N-1)}{(N-2) \cdot (N-3)}. \quad (7)$$

Where r_{ij} is an element of the reachability matrix, such that $r_{ij} = 1$ if there is a path from the vertex v_i to v_j, otherwise $r_{ij} = 0$, and N is a number of nodes in a graph. *RT* ranges from 0 to 1, where the zero is for parallel directed graphs and 1 for series directed graphs.

4.2 Aggregate complexity indicator *AC*

In order to measure structural complexity of supply chains it seems to be useful to apply *AC* indicator constructed by Modrak [13]. A concept of this indicator is based on the aggregation of three sub-indicators: binding of structure B, structure diversity SD and diameter of network D. The following expression for an aggregate complexity indicator is formulated:

$$AC = \lg\left(B + SD + \frac{D}{3}\right), \quad (8)$$

where:

$$B = \frac{L}{N-1} - 1, \quad (9)$$

$$SD = \frac{1}{N_1 N_2} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} c_{ij} - 1, \quad (10)$$

$$D = \max_{ij}(D_{ij}), \quad (11)$$

in which:
- N_1, N_2 are numbers of initial and final nodes,
- c_{ij} represents number of heterogeneous paths of the i^{th} input node to the j^{th} output node of the graph (without any possibility to pass twice through the same node within one route),
- D_{ij} is the shortest path between i and j.

4.3 Average shortest length *ASP*

The *ASP* is a network indicator which is applicable for determination distance of network between every pair of nodes. Alex and Efstathiou [7] used it for interpretation of robustness complex networks as fragmentation of network. Formally it can be described as follows:

$$ASP = \frac{1}{N \cdot (N-1)} \cdot \sum\sum d_{ij}, \quad (12)$$

where:
d_{ij} – is the shortest path in the network for all nodes from i till j.

4.4 Number of trees *T*

Temperley [14] introduced the classification of graphs by the number of trees they contain. It is calculated using the tree-generating determinant D_i which is defined by the number of outputs of every node of the structure.

To count the total number of distinct trees, we need to multiply every tree-generating determinant with each other for certain structure. It can be expressed by the formula:

$$T = \sum D_i. \quad (13)$$

4.5 Complexity degree *κ*

Maksimovic and Petrovic [15] described a complexity degree κ indicator and its extended definition based on two fundamental constituents of each structure. The κ indicator takes beside the number of elements also the interrelation between elements within the structure. It is mainly focused on the flows in a system. Formally κ is expressed by the formula:

$$\kappa = \frac{\sum_{i=1}^{m} m_i}{m}. \quad (14)$$

4.6 Flow complexity *FC*

The *FC* is proposed by Crippa [16]. It can be expressed by (15) and it counts all Tiers (including Tier 0), Nodes and Links and adds all these counts, weighted with arbitrary chosen α, β and γ coefficients. Nodes are counted only once, even if they are repeated in Tiers. Presence of repetition is included in Links count.

$$FC = \alpha \cdot \sum_{i=1}^{n} T_i + \beta \cdot \sum_{s=1}^{m} N_s + \gamma \cdot \sum_{i=1}^{n} \sum_{j=1}^{k} LK_{ij}, \tag{15}$$

where: T_i - i^{th} Tier, N_s – s^{th} Node, LK - i^{th} and j^{th} Link.

4.7 Modified flow complexity *MFC*

Modified flow complexity indicator [8] combines *FC* together with Multi-Tier ratio *MTR* and index *MTI*, and Multi-Link ratio *MLR*. Using *MTI, MTR* and *MLR* we can determine α, β and γ coefficients. *MFC* basically counts all Tiers (including Tier 0), Nodes and Links and adds all these counts, weighted with determined α, β and γ coefficients. In *MFC* indicator, Nodes and Links are counted only once, even if they are repeated in graph. Presence of Nodes and Links repetition is included in coefficients. In mathematical term, the *MFC* indicator can be expressed as follows:

$$MFC = \alpha \cdot T + \beta \cdot N + \gamma \cdot L, \tag{16}$$

$$\alpha = MTI = \frac{TN - N}{(T-1) \cdot N}, \tag{17}$$

$$\beta = MTR = \frac{TN}{N}, \tag{18}$$

$$\gamma = MLR = \frac{LK}{L}, \tag{19}$$

where: N - Number of Nodes, TN - Number of Nodes per the i^{th} Tier Level, L - Number of Links, LK - Number of Links per the i^{th} Tier Level, T - Number of Tiers.

4.8 Vertex degree index I_{vd}

The information entropy of a graph with a total weight W and vertex weights w_i can be expressed in the form of the equation:

$$H(W) = W \cdot \text{lb}\, W - \sum_{i=1}^{V} w_i \cdot \text{lb}\, w_i. \tag{20}$$

Since the maximum entropy is when all $w_i = 1$, then

$$H_{\max} = W \cdot \text{lb}\, W. \tag{21}$$

By substituting $W = \sum \deg(v)_i$ and $w_i = \deg(v)_i$, the information content of the vertex degree distribution of a network called as Vertex degree index I_{vd} is derived by Bonchev and Buck [17] that is expressed as follows:

$$I_{vd} = \sum_{i=1}^{V} \deg(v)_i \,\text{lb} \deg(v)_i. \tag{22}$$

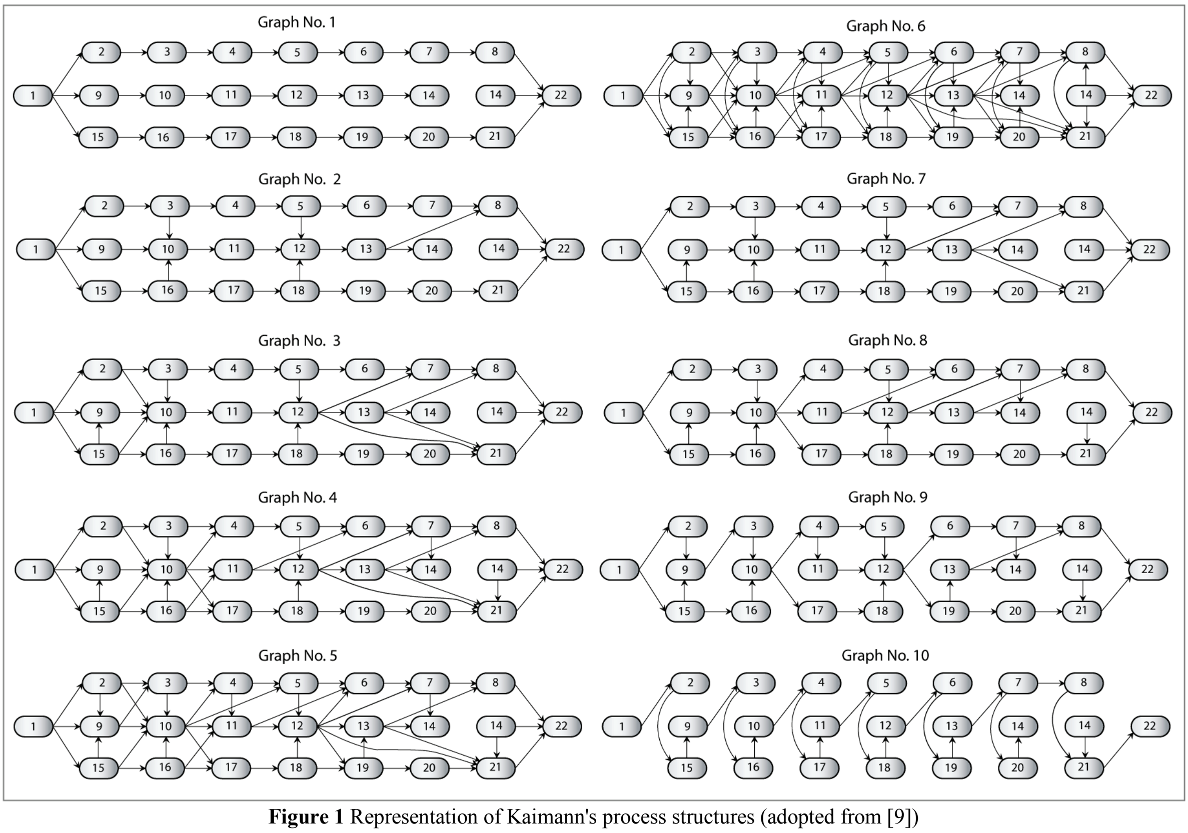

Figure 1 Representation of Kaimann's process structures (adopted from [9])

4.9 Link tiers index *LTI*

When comparing two or more structures with the same number of *tiers* "*t*" and *nodes* "*n*" but with different number of links "*l*" (see Fig. 1), the following argument can be constructed:

The structure with the smallest number of links is topologically less complex than other one(s). Then, it is proposed to measure structural complexity by formula Links/Tiers Index [18]:

$$LTI = \sum_{j=1}^{p} \sum_{l=1}^{m} l_j \cdot t_l. \tag{23}$$

5 Comparison of complexity indicators
5.1 Representing of manufacturing process structures

In order to assess the relevance of the compared complexity indicators for the selected complexities of manufacturing process structures they have been assessed for a set of graphs. For this purpose we selected 10 models that are shown in Fig. 1.

5.2 Results of computational experiments

Tab. 1 shows the results of the implementation of compared complexity indicators that were described above. All selected models of manufacturing processes which are presented in Tab. 1 are listed in ascending order based on the indicator I_{vd}.

Table 1 Results of compared indicators

No.	*RT*	*AC*	*ASP*	lg *T*	*κ*	*FC*	*MFC*	I_{vd}	*LTI*
Graph 10	0,99	0,82	3,60	0,30	0,96	53	45,05	45,51	19,8
Graph 1	0,30	0,48	0,67	0,48	1,00	54	46,05	49,51	20,7
Graph 2	0,52	0,73	0,95	1,98	1,22	59	51,05	71,28	15,2
Graph 9	0,88	1,26	1,87	2,41	1,26	60	52,05	84,40	26,1
Graph 8	0,78	1,13	1,39	2,89	1,35	62	54,05	89,91	27,9
Graph 7	0,63	0,99	1,10	2,58	1,30	61	53,05	90,40	27,0
Graph 3	0,61	1,21	0,96	3,41	1,48	65	57,05	113,81	30,6
Graph 4	0,79	1,83	1,15	4,97	1,74	71	64,05	158,67	36,0
Graph 5	0,88	2,59	1,22	6,40	2,04	78	70,05	201,13	42,3
Graph 6	0,99	2,88	1,27	9,30	2,74	94	86,05	318,75	56,7

The graphs in Fig. 2 and Fig. 3 show that in spite of different concept of the paired indices, they generate comparable results.

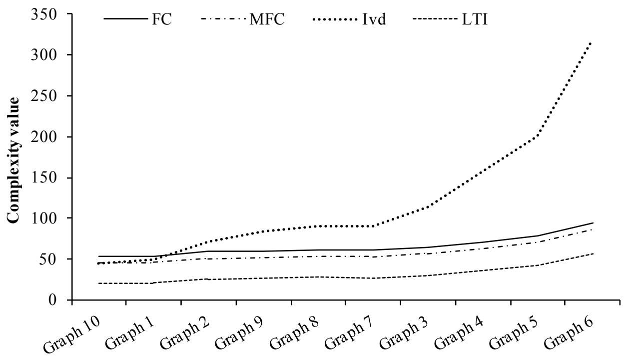

Figure 2 Comparison of *FC*, *MFC*, I_{vd} and *LTI*

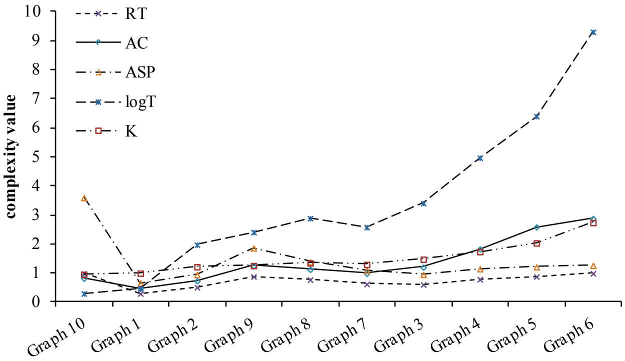

Figure 3 Comparison of *RT*, *AC*, *ASP*, lg*T* and *κ*

The obtained values of correlation coefficients are summarized in Tab. 2. Statistically, a significant positive correlation was found between variables I_{vd} and *MFC*. However, sometimes the correlation coefficient may not necessarily express the true causal relationship between two variables.

Table 2 The results of Spearman correlation coefficients

No.	Correlation between	Corrected Spearman coefficient
1	*RT*, lg *T*	0,323
2	*AC*, lg T	0,891
3	*κ*, lg *T*	1,000
4	I_{vd}, lg *T*	0,988
5	I_{vd}, *MFC*	0,988
6	I_{vd}, *LTI*	0,988

To identify mutual correlation among respective values of testing indicators, Fig. 4 offers scatter plots and r_c - squared values. Based on these results we can state that both novel indicators, namely *AC* and *MFC* are comparable measures with the existing indicators and are usable to measure structural complexity of manufacturing processes.

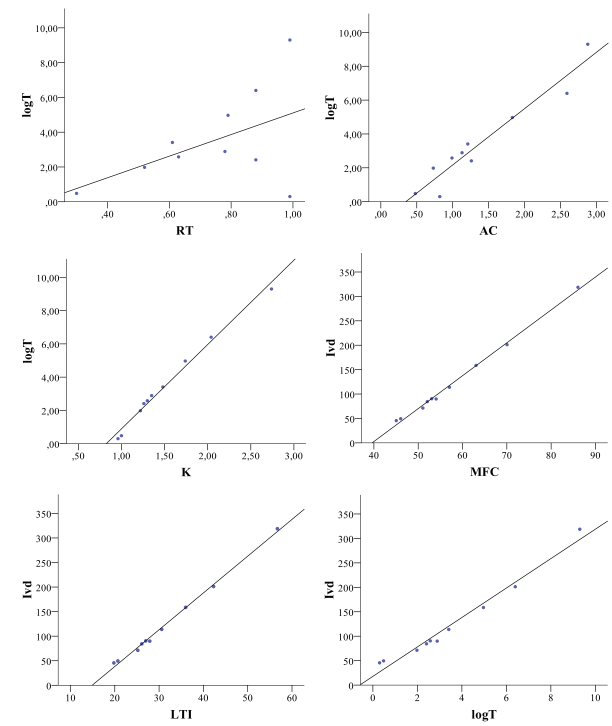

Figure 4 Mutual comparisons of selected indicators

5.3 Comprehensive comparison of the indicators

Based on the obtained results shown in Tab. 1, our experience with calculation of indices and mutual consideration, we assessed indices as shown in Tab. 4. According to Latva-Koivisto [9], the following criteria for comparison of complexity measures can be used:

(i) Validity (V) is ability of an indicator to measure what it is supposed to measure (1 - very low, 5 - high),

(ii) Usability (U) is an accuracy and completeness with which a certain metric can assess structural property (1 - not applicable, 5 - highly usable),

(iii) Ease of implementation (E) is a measure of difficulty of implementation of used indicators (1 - very difficult, 5 - very easy),

(iv) Time of computation (T) is a measure of time consumed by computation of a certain indicator (1 - long time, 5 - short time),

(v) Dependence on structure`s binding (D) is a structural property representing linkage of structural elements (1 - very low, 5 - high).

Quantified comparisons have been performed on the Kaimann's process structures (see Fig. 1).

Table 3 Result table of assessed indices

Criteria / Indicator	V	U	E	T	D	Σ Sum
I_{vd}	High (5)	Satisfactory usable (4)	Easy (4)	Long time (2)	High (5)	20
LTI	Rather high (4)	Medium (3)	Easy (4)	Very short (5)	Rather high (4)	20
lg T	Medium (3)	Usable (3)	Easy (4)	Very short (5)	Rather high (4)	19
κ	Medium (3)	Medium usable (2)	Easy (4)	Very short (5)	Rather high (4)	18
FC	Rather high (4)	Usable (3)	Rather difficult (3)	Short time (4)	Rather high (4)	18
MFC	Rather high (4)	Usable (3)	Rather difficult (3)	Medium long (3)	Rather high (4)	17
AC	Rather high (4)	Usable (3)	Difficult (3)	Long time (2)	High (5)	16
ASP	Medium (3)	Usable (3)	Difficult (3)	Medium long (3)	Low (2)	13
RT	Medium (3)	Medium usable (2)	Difficult (3)	Medium long (3)	Very low (1)	11

From Tab. 4 we consider that the most suitable indicators to measure structural complexity of process structures are indicators I_{vd} and *LTI*. Indicator lg T can be used as a supplementary indicator.

6 Summary and conclusion

This study shows that developed and existing indices are usable for assessing the given attribute in case we want to compare a static complexity of different process structures. It is well known that the Graph theory is useful for modelling and analysing a variety of empirical systems including general process structures, but it is necessary to say that the Graph theory does not have an answer for all the questions we have towards the overall process structural complexity. It is because the nodes and links consist of different entities that interact in a network. This paper also supports the need for parallel use of alternative indicators to be used as a basis for a development of objective evaluation of process structural properties.

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