

# Privatization and environmental pollution in a mixed duopoly\*

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## Abstract

*This paper establishes mixed duopoly game-theoretical models to investigate the economic impacts exerted by privatization in the presence of the environmental pollution. When the residents' environmental preference is introduced to the public firm's objective function, we mainly find that privatization may increase the public firm's output, decrease the private firms' outputs, and exert no impacts on social welfare. These findings run contrast to the common findings of the studies on privatization. Moreover, Cournot competition and Stackelberg competition are separately analyzed to show that our findings are robust and irrelevant with the firms' moves. This paper highlights the role the environmental pollution and residents' environmental preference play in determining the economic impacts exerted by privatization.*

**Key words:** privatization, mixed duopoly, environmental pollution, residents' environmental preference, Cournot competition, Stackelberg competition

**JEL classification:** H21, L33, Q58

## 1. Introduction

Mixed markets where publicly owned firms and privately owned firms coexist and compete with each other are prevalent in both developed and developing countries. One can easily find such sorts of markets appear in the telecommunications, airline, rail, natural gas and postal industries. Since the 1980s, privatization waves

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spread worldwide (Vickers and Yarrow, 1988; Ribnikar and Košak, 2011). For example, some European countries privatized their public firms. Brazil and other Latin American countries employed the privatization policy to fight against high inflation, currency and debt crises, and activate the economy. China also carried out the privatization policy to reform the state-owned enterprises, which exerted a salient impact on the China's economic development.

Along with privatization waves, discussions of privatization has become one of the hottest topics in economic studies. The basic motivation for privatization is to make the public firm operate efficiently. The objective of the public firm is to maximize social welfare, which leads to a higher output quantity than that obtained by the competitive markets. Besides, privatization can be a useful policy tool for the government to influence the economic efficiency and social welfare level (see De Fraja and Delbono, 1990). However, it is commonly known that the establishment of the public firm is to correct market failure. The environmental pollution is a typically negative externality. In reality, environmental problems capture more and more attentions of governmental officials and local residents. If the government does not care about the residents' environmental preference, the residents as voters will vote for a benevolent government who takes care of the environmental quality (see Roelfsema, 2007). Therefore, when the government carries out the privatization policy, the residents' environmental preference and relevant environmental problems must be seriously taken into account. Such arguments are also supported by the academic studies like Beladi and Chao (2006), Wang and Wang (2009), Naito and Ogawa (2009), and Prašnikar et al. (2012). For example, as suggested by Wang and Wang (2009), in Taiwan in steel industries one public firm competes with one private firm and their production processes generate the environmental pollution. After privatization, the environmental problem may be even worsened and is always criticized by the general public. Therefore, as stated above, it is reasonable for us to integrate the environmental pollution into the public firm's objective function and investigate the economic impacts of the privatization policy when the environmental pollution is considered by the public firm.

This paper tries to address such an issue by employing mixed duopoly game-theoretical models. When we introduce the environmental pollution to the public firm's objective function, privatization may increase the public firm's output, decrease the private firms' outputs, and exert no impacts on social welfare, which runs contrast to the conventional wisdom of privatization. What's more, we also show that our findings are robust and irrelevant with the firms' moves.

The rest parts of this paper are organized as follows. In Section 2, we briefly review the related studies of mixed oligopoly (duopoly). In Section 3, we separately investigate the impacts exerted by the privatization policy in the presence of the environmental pollution under Cournot (1838) competition and Stackelberg (1934) competition. In Section 4, we compare the outcomes under Cournot competition

and Stackelberg competition before and after privatization. In Section 5, some concluding remarks are provided.

## 2. Literature review

The decision concerning whether the government should privatize the public firm arouses the great interest of many theoretical economists. They try to address such an issue from various perspectives. The representative literature can be referred to De Fraja and Delbono (1989), White (1996), Matsumura (1998), Fjell and Heywood (2004), Matsumura et al. (2009), Wang et al. (2009), Wang and Wang (2009), Kato (2011), Barcena-Ruiz (2012), and Kato (2012). For example, De Fraja and Delbono (1989) contend that the number of private firms existing in the industry and the order of the firms' moves (Cournot or Stackelberg competition) influence the government's decision-making in privatization. Matsumura (1998) investigates the optimal partial privatization level when the government carries out privatization. White (1996) and Fjell and Heywood (2004) explore how the government's subsidy influences privatization, and they find that the firms' moves matter for the results. Wang et al. (2009) argue that the firms' moves are important for the privatization decision-making and the choices of government's strategic trade policies. Wang and Wang (2009) contend that when the government lowers the environmental tax and the product is less substitutable, the environment is damaged with the implementation of the privatization policy. Matsumura et al. (2009) discuss how the entry of foreign enterprises affects privatization. Kato (2011) argues that a differentiated emission quota can do more to improve welfare than an emission tax in a mixed duopoly. Barcena-Ruiz (2012) analyzes the government's choice of privatization in the situation where the public firm works as efficiently as the private firms. Kato (2012) finds that the optimal degrees of privatization and environmental damage are irrelevant with each other when the private firm is owned by domestic private investors, and that full privatization or full nationalization can be optimal when it is owned by foreign private investors.

However, although some aforementioned studies consider the situation where the firms' production process generates the environmental pollution, they largely neglect to explore how privatization influences the whole society when the environmental pollution and the residents' environmental preference are introduced to the public firm's objective function. Besides, although much attention has been paid to the employment and investment effects exerted by the privatization policy, the current studies greatly ignore the crucial role that the residents' environmental preference plays in determining the government's privatization policy making.<sup>4</sup>

<sup>4</sup> Although Barcena-Ruiz and Garzon (2006), Beladi and Chao (2006), Naito and Ogawa (2009), Chen and Wang (2010), and Wang and Wang (2009) consider the issue related to privatization in the presence of environmental pollution, they do not consider the situation discussed by this paper.

In order to fill the current research gap, we introduce the environmental pollution and the residents' environmental preference to the social welfare function and the public firm's objective function. Our approach to introduce the environmental pollution and the residents' environmental preference to the social welfare function (i.e., the objective of public firm) follows Copeland and Taylor (1994) and Roelfsema (2007), and is also similar to that adopted in Naito and Ogawa (2009) and Wang and Wang (2009). We address the issue concerning the government's privatization decision from a greatly different perspective. Our main findings are that the residents' environmental preference plays a crucial role in determining socioeconomic impacts of privatization. When the residents' environmental preference is introduced to the objective function of the public firm, privatization may even raise the output of the public firm, reduce the profit of the public firm, increase the profit of a private firm, enhance the consumer surplus, and decrease the producer's surplus. In some situations, the public firm can operate as efficiently as the private firm, and there is no need for the government to carry out the privatization policy. Cournot competition and Stackelberg competition are separately analyzed to show that our findings are robust and irrelevant with the firms' moves. When the residents' environmental preference is included in the public firm's objective function, privatization exerts significantly different economic impacts, which runs contrast to the common findings of the studies on privatization.

### 3. The theoretical analysis

Consider a single market where a public firm and a private firm coexist. These two firms produce the homogenous product and compete with each other.<sup>5</sup> In reality, there exist such firms and their production processes generate the environmental pollution in some industries (see, e.g., Wang and Wang, 2009).

It is worth noting that the situation discussed in this paper is how the government's privatization policy influences the whole economy in the presence of the environmental pollution. Thus, the environmental regulation policies are ignored by this paper. Besides, the firms' pollution abatement costs can be treated exogenously, and are also neglected.

<sup>5</sup> Different from the situation considered by the current literature where one public firm and  $n$  private firms operate in the same market producing the homogenous product, here we only take into account a market with one public firm and one private firm. This paper focuses on how the change of the residents' environmental preference, other than the number of private firms in the market, influences the government's privatization decision. Our assumption can keep our model as simple as possible, and the investigation of a market with one public firm and  $n$  private firms brings little additional insight to the present analysis. It is notable that the assumption described by this paper also fits well with the practical situation.

The inverse market demand curve is assumed to be linear, which is given by:

$$p = a - q_0 - q_1, \quad (1)$$

where  $p$  is the price of the homogenous good produced by the public and private firms.  $a$  describes the market scale.  $q_0$  and  $q_1$  are the output quantities of the public and private firms respectively.

We assume that the public and private firms share the same quadratic cost functions, which are described by:

$$C_i = \frac{1}{2}q_i^2 + F, \quad (2)$$

where  $i = 0, 1$ ,  $C_i$  is the total cost of firm  $i$  ( $i = 0$ , the public firm;  $i = 1$ , the private firm).  $F$  is the entry cost. The above cost function shares the property of the increasing marginal cost. The increasing marginal cost function used by this paper with specific forms (e.g., the variable cost being equal to  $1/2 q_i^2$ ) is also used by Han and Ogawa (2008), Mendaz-Naya (2008), Wang et al. (2009), and Naito and Ogawa (2009). This paper does not consider the entry problem of the private firms. Reasonably,  $F$  is set to be zero.

We further assume that the production processes of both public and private firms will produce the environmental pollutants. The higher the output quantities, the more environmental pollutants the production processes of public and private firms produce. For the sake of simplicity, following Taylor and Copeland (1994), we also use a linear function to describe the relation between the output and the environmental pollution. It is hypothesized that one unit of output generates one unit of environmental pollutant.

Thus, we have:

$$E = q_0 + q_1, \quad (3)$$

where  $E$  is the environmental pollution discharged by the production procedures of two firms.

The profits of the public and private firms can be stated as:

$$\pi_0 = (a - q_0 - q_1)q_0 - \frac{1}{2}q_0^2, \quad (4)$$

$$\pi_1 = (a - q_0 - q_1)q_1 - \frac{1}{2}q_1^2, \quad (5)$$

where  $\pi_0$  and  $\pi_1$  is the profits of public and private firms, respectively.

The goal of the private firm is to maximize its profit. As mentioned before, for a benevolent government, the environmental pollution should be also included in the social welfare function. Otherwise, if the environmental pollution in the economy is serious, residents may vote for a new government who cares about their environmental preference. Generally speaking, the goal of the public firm is in accord with that of the government. That is, the goal of the public firm is to maximize social welfare. In this sense, the environmental pollution is introduced to the social welfare function, and hence treated as the objective function of the public firm. In other words, the goal of the public firm is to correct market failure and maximize social welfare. It is reasonable to see that when the environmental pollution is introduced to the social welfare function, it will be also included in the public firm's objective function. The social welfare function (i.e., the objective function of the public firm) is stated as:

$$W = CS + PS - \lambda E, \quad (6)$$

where  $CS$  and  $PS$  represent the consumer surplus and the producer surplus, respectively.  $CS$  (namely consumer surplus) represents the consumers' behooves in the social welfare function and is measured by calculating the difference between what consumers are willing to pay for the good relative to its market price.  $PS$  (namely, producer surplus) stands for the interests of producers in the social welfare and is reasonably measured by the sum of profits earned by both public and private firms. In this paper,  $CS = \frac{1}{2}(q_0 + q_1)^2$  and  $PS = \pi_0 + \pi_1$ . The parameter  $\lambda$  describes the environmental preference of the residents.  $\lambda$  can also be interpreted as the weight of the environmental pollution in the social welfare function. The higher the value of  $\lambda$  is, the more the residents take care about the environmental pollution. The commonly used social welfare function employed by the existent literature is a special case of Equation (6) and can be achieved if we let  $\lambda = 0$ .

Here, we would like to point out why we introduce the environmental pollution to the public firm's objective function. Because this paper does not consider the political manipulation, we treat both the consumer surplus and the producer surplus of equal weight. In some literature on the mixed oligopoly (e.g., Naito and Ogawa, 2009; Wang and Wang, 2009), the environmental pollution is introduced to the social welfare function in a quadratic form. However, we find that if the environmental pollution is introduced to the social welfare function with a quadratic form, it only makes the calculation more complex and adds little insight. In order to keep the model as simple as possible, we use a linear form to accommodate the environmental pollution to our social welfare function. The social welfare function used by the present paper with a linear form of the environmental pollution is also utilized by Copeland and Taylor (1994) and Roelfsema (2007).

Now, we will analyze the socioeconomic consequences of privatization in the presence of the environmental pollution. We try to address the issue by considering the following two cases, namely Cournot competition and Stackelberg competition. The significant difference between these two cases is the order of the firm's moves. The current studies concerning the mixed duopoly argue that the change of the order of the firm's moves will exert salient impacts or even opposite impacts (see White, 1996; Pal, 1998; Fjell and Heywood, 2004, Wang et al 2009). Moreover, Cournot competition and Stackelberg competition are commonly used by the current literature to examine how the firm's moves influence the original findings (see De Fraja and Delbono, 1989; Pal, 1998; Fjell and Heywood, 2004).

#### *a. Cournot competition*

In this situation, the public firm and the private firm will simultaneously choose  $q_0$  and  $q_1$  to maximize their objective functions. For simplicity, this paper only considers the inner solutions of our established theoretical model. Specifically, we know that positive solutions for variables under Cournot competition can be obtained when  $\lambda$  is restricted within the interval  $[0, \frac{5}{8}a)$ , and positive solutions for variables under Stackelberg competition can be founded when  $\lambda$  is located in the interval  $[0, \frac{20}{33}a)$ . But for the sake of conducting the comparisons between Cournot competition and Stackelberg competition, we assume that  $\lambda \in [0, \frac{20}{33}a)$  in view of  $\frac{20}{33}a < \frac{5}{8}a$ . Thus, throughout this paper, we analyze the issues under the condition that  $\lambda \in [0, \frac{20}{33}a)$ . Such an assumption implies that the environmental preference of the residents is not too high.

The equilibrium in mixed duopoly under Cournot competition can be stated as:

$$\begin{aligned}
 q_{c0}^{ex} &= \frac{2a-3\lambda}{5}, q_{c1}^{ex} = \frac{a+\lambda}{5}, \\
 \pi_{c0}^{ex} &= \frac{(2a+7\lambda)(2a-3\lambda)}{50}, \pi_{c1}^{ex} = \frac{3(a+\lambda)^2}{50}, \\
 CS_c^{ex} &= \frac{(3a-2\lambda)^2}{50}, PS_c^{ex} = \frac{7}{50}a^2 + \frac{7}{25}a\lambda - \frac{9}{25}\lambda^2, \\
 W_c^{ex} &= \frac{8}{25}a^2 - \frac{14}{25}a\lambda + \frac{3}{25}\lambda^2.
 \end{aligned} \tag{7}$$

Here,  $q_{c0}^{ex}$  and  $q_{c1}^{ex}$  are the output quantities of the public and private firms before privatization, respectively.  $\pi_{c0}^{ex}$  and  $\pi_{c1}^{ex}$  are the profits of the public and private firms before privatization, respectively.  $CS_c^{ex}$  and  $PS_c^{ex}$  are consumer surplus and producer surplus before privatization, respectively.  $W_c^{ex}$  is the social welfare level before privatization.

Now we will establish Lemma 1 to summarize the economic impacts exerted by the residents' environmental preference in the mixed duopoly under Cournot competition.

Lemma 1: In the mixed duopoly under Cournot competition, an increase in the residents' environmental preference will exert the following economic impacts:

- (i) The output of the public firm will decrease and that of the private firm will increase. (ii) If  $0 \leq \lambda < \frac{4}{21}a$ , the profit of the public firm will rise; and if  $\frac{4}{21}a < \lambda < \frac{20}{33}a$ , the profit of the public firm will fall. The profit of the private firm will increase. (iii) The consumer surplus will be reduced. If  $0 \leq \lambda < \frac{7}{18}a$ , the producer surplus will rise; and if  $\frac{7}{18}a < \lambda < \frac{20}{33}a$ , the producer surplus will fall.
- (iv) Social welfare will decrease.

Proof: See Appendix A.

An increase in the residents' environmental preference will decrease the output of the public firm. The higher the residents' environmental preference is, the larger the negative impact the environmental pollution exerts on social welfare. At this time, the public firm has an incentive to reduce its output to smooth the increased negative impact of the environmental pollution on social welfare. Meanwhile the



private firm behaves more aggressively and increases its output due to the reduction of the public firm's output. From Equation (7) we know that an increase in the residents' environmental preference will decrease the total output of both public and private firms, resulting in a reduction of consumer surplus. On the other hand, a decrease in the total output quantity leads to a higher product price. If the residents take less care about the environmental pollution (i.e.,  $0 \leq \lambda < \frac{4}{21}a$ ), the public

firm can benefit more from the higher product price, which offsets the impact exerted by the output reduction. Hence, the profit of the public firm will increase. However, if the residents take more care about the environmental pollution (i.e.,  $\frac{4}{21}a < \lambda < \frac{20}{33}a$ ), the public firm will suffer from a reduction of its output, which

offsets the impact exerted by the increase in the product price. The profit of the public firm will be reduced. But for the private firm, it will take both advantages of the increased output and the decreased product price, and thus its profit will rise. It is reasonable to see that if the residents take less care of the environmental pollution (i.e.,  $0 \leq \lambda < \frac{7}{18}a$ ), the decrease of the public firm's profit will exceed

the increase of the private's profit and the producer surplus will be reduced. If the residents take more care about the environmental pollution (i.e.,  $\frac{7}{18}a < \lambda < \frac{20}{33}a$ ),

the opposite will occur and the producer surplus will increase. Besides, due to an increase in the residents' environmental preference, the residents will suffer more from the environmental pollution and social welfare will decrease.

In the privatized duopoly under Cournot competition, the equilibrium is described by:

$$\begin{aligned}
 q_{c0}^{pri} &= q_{c1}^{pri} = \frac{1}{4}a, \\
 \pi_{c0}^{pri} &= \pi_{c1}^{pri} = \frac{3}{32}a^2, \\
 CS_c^{pri} &= \frac{1}{8}a^2, PS_c^{pri} = \frac{3}{16}a^2, \\
 W_c^{pri} &= \frac{5}{16}a^2 - \frac{a}{2}\lambda.
 \end{aligned} \tag{8}$$

Here,  $q_{c0}^{pri}$  and  $q_{c1}^{pri}$  are the output quantities of the public and private firms after privatization, respectively.  $\pi_{c0}^{pri}$  and  $\pi_{c1}^{pri}$  are the profits of the public and private firms after privatization, respectively.  $CS_c^{pri}$  and  $PS_c^{pri}$  are consumer surplus and

producer surplus after privatization, respectively.  $W_c^{pri}$  is the social welfare level after privatization.

After privatization, the objectives of two firms are both profit maximization. Hence, their outputs and profits, as well as the consumer surplus and producer surplus, are irrelevant to the residents' environmental preference. However, because the residents' environmental preference is introduced to the social welfare function, the consumers will suffer more from the environmental pollution due to an increase in the residents' environmental preference. This leads to a reduction of social welfare.

Proposition 1 is given to investigate the socioeconomic impacts exerted by privatization.

Proposition 1: In the mixed duopoly under Cournot competition, if  $\lambda = \frac{1}{4}a$ , privatization will exert no economic impacts. Otherwise, privatization will exert the following economic impacts: (i) If  $0 \leq \lambda < \frac{1}{4}a$ , privatization will decrease the output of the public firm and raise that of the private firm; and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , privatization will increase the output of the public firm and reduce that of the private firm. (ii) If  $0 \leq \lambda < \frac{11}{84}a$  and  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , privatization will increase the public firm's profit; and if  $\frac{11}{84}a < \lambda < \frac{1}{4}a$ , privatization will reduce the public firm's profit. If  $0 \leq \lambda < \frac{1}{4}a$ , privatization will decrease the profit of the private firm; and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , privatization will increase the profit of the private firm. (iii) If  $0 \leq \lambda < \frac{1}{4}a$ , privatization will decrease the consumer surplus; and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , privatization will increase the consumer surplus. If  $0 \leq \lambda < \frac{1}{4}a$  and  $\frac{19}{36}a < \lambda < \frac{20}{33}a$ , privatization will increase the producer surplus; and if  $\frac{1}{4}a < \lambda < \frac{19}{36}a$ , privatization will reduce the producer surplus. (iv) if  $0 \leq \lambda < \frac{1}{4}a$  and  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , privatization will decrease social welfare.

Proof: See Appendix B.

The key point to understand Proposition 1 is to know how privatization influences the output quantities of public and private firms. In the situation that the environmental pollution is introduced to the objective function of the public firm, the public firm will face a trade-off between increasing its output to raise the consumer surplus and decreasing its output to reduce the environmental pollution. If the residents care less about the environmental pollution, that is, the residents' environmental preference is relatively small (i.e.,  $0 \leq \lambda < \frac{1}{4}a$ ), the public firm will take more care about the residents' surplus. Therefore, the public firm's output will be higher than the socially efficient level and privatization will decrease the output of the public firm, thus making the public firm work efficiently after privatization. The decrease of the public firm's output will inspire the private firm to raise its output after privatization. This is in accord with the motivation of privatization and the conventional wisdom of privatization. However, if the residents care more about the environmental pollution, that is, the residents' environmental preference is large enough (i.e.,  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ ), the public firm will take more care about the environmental pollution. Therefore, the public firm's output will be lower than the socially efficient level and privatization will increase the output of the public firm. Therefore, when the environmental pollution is taken into account by the objective function of the public firm, it is possible for the public firm (the private firm) to raise (decrease) its output after privatization. It is not hard to see the establishment of the second, third and fourth parts of Proposition 1.

It is notable that if we set  $\lambda = 0$ , from Equation (6) we know that in this situation, the environmental pollution will exert no impact on social welfare. This is the simplest case to discuss the socioeconomic impacts exerted by privatization. From Proposition 1 we know that if the residents take less care about the environmental pollution, the results due to privatization will be the same as those obtained by the simplest case where the environmental pollution is not introduced to the social welfare function. However, if the residents take more care about the environmental pollution, the opposite findings will be obtained. From the first part of Proposition 1 we can get that privatization can even increase the output of the public firm. From the second part of Proposition 1, it is possible that the profit of the public firm will decrease and that of the private firm will increase after privatization. From the third part of Proposition 1, it is also shown that privatization will increase the consumer surplus and decrease the producer surplus. Therefore, we can conclude that if the environmental pollution is embedded in the public firm's objective function, privatization will exert significantly different socioeconomic impacts, compared with the situation that the residents' environmental preference is not taken into account. Here we would like to highlight that for some value of the residents' preference (e.g.,  $\lambda = \frac{1}{4}a$ ), the public firm can operate as efficiently as the private firm and there is no need for the government to carry out the privatization policy.

*b. Stackelberg competition*

In this case, we will consider the following two-stage game. In the first stage, the public firm chooses  $q_0$  to maximize the social welfare function. In the second stage, given the value of  $q_0$ , the private firm chooses  $q_1$  to maximize its profit function. In this two-stage game, the public firm is a leader, and its leadership can be gained due to the historical, technological or certain legal reasons, hence gaining the competitive advantage over the private firm (see Fjell and Heywood, 2004). In reality, large public firms can usually be the leaders in their industries, and even after privatization, they still are the private leaders in the same industries (see Fjell and Heywood, 2004). In order to get the inner solutions of our established model under Stackelberg competition, we assume that  $\lambda \in [0, \frac{20}{33}a)$ .

The equilibrium in mixed duopoly under Stackelberg competition is given by:

$$\begin{aligned} q_{s0}^{ex} &= \frac{5}{14}a - \frac{3}{7}\lambda, q_{s1}^{ex} = \frac{3}{14}a + \frac{1}{7}\lambda, \\ \pi_{s0}^{ex} &= (\frac{1}{4}a + \frac{1}{2}\lambda)(\frac{5}{14}a - \frac{3}{7}\lambda), \pi_{s1}^{ex} = \frac{3}{98}(\frac{3}{2}a + \lambda)^2, \\ CS_s^{ex} &= \frac{2(2a - \lambda)^2}{49}, PS_s^{ex} = \frac{31}{196}a^2 + \frac{8}{49}a\lambda - \frac{9}{49}\lambda^2, \\ W_s^{ex} &= \frac{9}{28}a^2 - \frac{4}{7}a\lambda + \frac{1}{7}\lambda^2. \end{aligned} \tag{9}$$

Here,  $q_{s0}^{ex}$  and  $q_{s1}^{ex}$  are the output quantities of the public and private firms before privatization, respectively.  $\pi_{s0}^{ex}$  and  $\pi_{s1}^{ex}$  are the profits of the public and private firms before privatization, respectively.  $CS_s^{ex}$  and  $PS_s^{ex}$  are consumer surplus and producer surplus before privatization, respectively.  $W_s^{ex}$  is the social welfare level before privatization.

Now we use Lemma 2 to summarize the economic impacts exerted by the residents' environmental preference in the mixed duopoly under Stackelberg competition.

Lemma 2: In the mixed duopoly under Stackelberg competition, an increase in the residents' environmental preference will exert the following economic impacts:

- (i) The output of the public firm will decrease and that of the private firm will increase; (ii) If  $0 \leq \lambda < \frac{1}{6}a$ , the profit of the public firm will rise; and if  $\frac{1}{6}a < \lambda < \frac{20}{33}a$ , the profit of the public firm will fall. The profit of the private firm will increase; (iii) The consumer surplus will decrease. If  $0 \leq \lambda < \frac{4}{9}a$ , the producer

surplus will rise; and if  $\frac{4}{9}a < \lambda < \frac{20}{33}a$ , the producer surplus will fall. (iv) The social welfare will decrease.

Proof: See Appendix C.

The economic explanations for Lemma 2 are similar to those for Lemma 1. In order to save space, here we do not give detailed analysis.

In the private duopoly under Stackelberg competition, the equilibrium is stated as:

$$\begin{aligned} q_{s0}^{pri} &= \frac{2}{7}a, q_{s1}^{pri} = \frac{5}{21}a, \\ \pi_{s0}^{pri} &= \frac{84}{882}a^2, \pi_{s1}^{pri} = \frac{75}{882}a^2, \\ CS_s^{pri} &= \frac{121}{882}a^2, PS_s^{pri} = \frac{53}{294}a^2, \\ W_s^{pri} &= \frac{20}{63}a^2 - \frac{11}{21}a\lambda. \end{aligned} \tag{10}$$

Here,  $q_{s0}^{pri}$  and  $q_{s1}^{pri}$  are the output quantities of the public and private firms after privatization, respectively.  $\pi_{s0}^{pri}$  and  $\pi_{s1}^{pri}$  are the profits of the public and private firms after privatization, respectively.  $CS_s^{pri}$  and  $PS_s^{pri}$  are consumer surplus and producer surplus after privatization, respectively.  $W_s^{pri}$  is the social welfare level after privatization.

The analysis of Equation (10) is similar to that of Equation (8). After privatization, the output quantities and profits of the two firms, as well as the consumer surplus and producer surplus, are irrelevant to the residents' environmental preference. The change of the residents' environmental preference only influences social welfare. Thus, an increase in the residents' environmental preference means that residents will pay more attention to the environmental pollution, and ceteris paribus, social welfare will decrease.

We will establish Proposition 2 to investigate the socioeconomic impacts exerted by privatization.

Proposition 2: In the mixed duopoly under Stackelberg competition, if  $\lambda = \frac{1}{6}a$ , then privatization will exert no economic impacts. Otherwise, privatization will exert the following economic impacts: (i) If  $0 \leq \lambda < \frac{1}{6}a$ , privatization will reduce the output of the public firm and raise that of the private firm; and if

$\frac{1}{6}a < \lambda < \frac{20}{33}a$ , privatization will increase the output of the public firm and decrease that of the private firm; (ii) If  $0 \leq \lambda < \frac{20}{33}a$ , privatization will decrease the profit of the public firm. If  $0 \leq \lambda < \frac{1}{6}a$ , privatization will increase the profit of the private firm; and if  $\frac{1}{6}a < \lambda < \frac{20}{33}a$ , privatization will decrease the profit of the private firm. (iii) If  $0 \leq \lambda < \frac{1}{6}a$ , privatization will decrease the consumer surplus; and if  $\frac{1}{6}a < \lambda < \frac{20}{33}a$ , privatization will increase the consumer surplus. If  $0 \leq \lambda < \frac{1}{6}a$ , privatization will increase the producer surplus, and if  $\frac{1}{6}a < \lambda < \frac{20}{33}a$ , privatization will reduce the producer surplus. (iv) If  $0 \leq \lambda < \frac{1}{6}a$  and  $\frac{1}{6}a < \lambda < \frac{20}{33}a$ , privatization will decrease social welfare.

Proof: See Appendix D.

The economic intuitions behind Proposition 2 are almost the same as those depicted by Proposition 1. Even under Stackelberg competition, privatization may also increase the output of the public firm, decrease the profit of the public firm, raise the profit the private firm, increase the residents' surplus, and reduce the producer surplus. These findings are quite different from those obtained by the situation that  $\lambda = 0$  under Stackelberg competition. Obviously, for some value of the residents' preference (e.g.,  $\lambda = \frac{1}{6}a$ ), the public firm can work as efficiently as the private firm, and there is no need for the government to carry out the privatization policy.

It is worth noting that under Stackelberg competition, the public firm will have the first mover advantage, and compared with the situation under Cournot competition, the public firm's behavior will generate larger impact on the private firm. It is more likely for the public firm to raise its output after privatization under Stackelberg competition (recall that the critical value of  $\lambda$  is  $\frac{1}{6}a$ ) than that under Cournot competition (recall that the critical value of  $\lambda$  is  $\frac{1}{4}a$ ). Under Stackelberg competition, the public firm is more sensitive to the environmental pollution and the residents' environmental preference.

Now we use Table 1 to summarize the main findings by Proposition 1 and Proposition 2.<sup>6</sup>

Table 1: Impacts of privatization under Cournot competition and Stackelberg competition

Competition modes	Cournot Competition		Stackelberg Competition	
	Increase	Decrease	Increase	Decrease
Public firm's output	$\frac{1}{4}a < \lambda < \frac{20}{33}a$	$0 \leq \lambda < \frac{1}{4}a$	$\frac{1}{6}a < \lambda < \frac{20}{33}a$	$0 \leq \lambda < \frac{1}{6}a$
Private firm's output	$0 \leq \lambda < \frac{1}{4}a$	$\frac{1}{4}a < \lambda < \frac{20}{33}a$	$0 \leq \lambda < \frac{1}{6}a$	$\frac{1}{6}a < \lambda < \frac{20}{33}a$
Public firm's profit	$0 \leq \lambda < \frac{11}{84}a,$ $\frac{1}{4}a < \lambda < \frac{20}{33}a$	$\frac{11}{84}a < \lambda < \frac{1}{4}a$	/	$0 \leq \lambda < \frac{20}{33}a$
Private firm's profit	$\frac{1}{4}a < \lambda < \frac{20}{33}a$	$0 \leq \lambda < \frac{1}{4}a$	$0 \leq \lambda < \frac{1}{6}a$	$\frac{1}{6}a < \lambda < \frac{20}{33}a$
Consumer surplus	$\frac{1}{4}a < \lambda < \frac{20}{33}a$	$0 \leq \lambda < \frac{1}{4}a$	$\frac{1}{6}a < \lambda < \frac{20}{33}a$	$0 \leq \lambda < \frac{1}{6}a$
Producer surplus	$0 \leq \lambda < \frac{1}{4}a,$ $\frac{19}{36}a < \lambda < \frac{20}{33}a$	$\frac{1}{4}a < \lambda < \frac{19}{36}a$	$0 \leq \lambda < \frac{1}{6}a$	$\frac{1}{6}a < \lambda < \frac{20}{33}a$
Social welfare	/	$0 \leq \lambda < \frac{1}{4}a,$ $\frac{1}{4}a < \lambda < \frac{20}{33}a$	/	$0 \leq \lambda < \frac{1}{6}a,$ $\frac{1}{6}a < \lambda < \frac{20}{33}a$

Source: Authors' calculation

From Table 1, we can conclude that when the residents' environmental preference is introduced to the objective function of the public firm, privatization may exert the significantly different economic impacts, which runs contrast to the conventional wisdom of privatization. Examined by Cournot competition and Stackelberg competition, our findings are proved to be irrelevant with firms' moves.

<sup>6</sup> We ignore the situation that there is no need for the government to conduct the privatization policy (i.e.,  $\lambda = \frac{1}{4}a$  in Cournot competition case and  $\lambda = \frac{1}{6}a$  in Stackelberg competition case) in Table 1.

#### 4. Results and discussion

In this section, we will conduct the comparisons of the firms' output quantities and profits, residents' surplus, producer surplus, and social welfare under Cournot competition and Stackelberg competition before and after privatization.

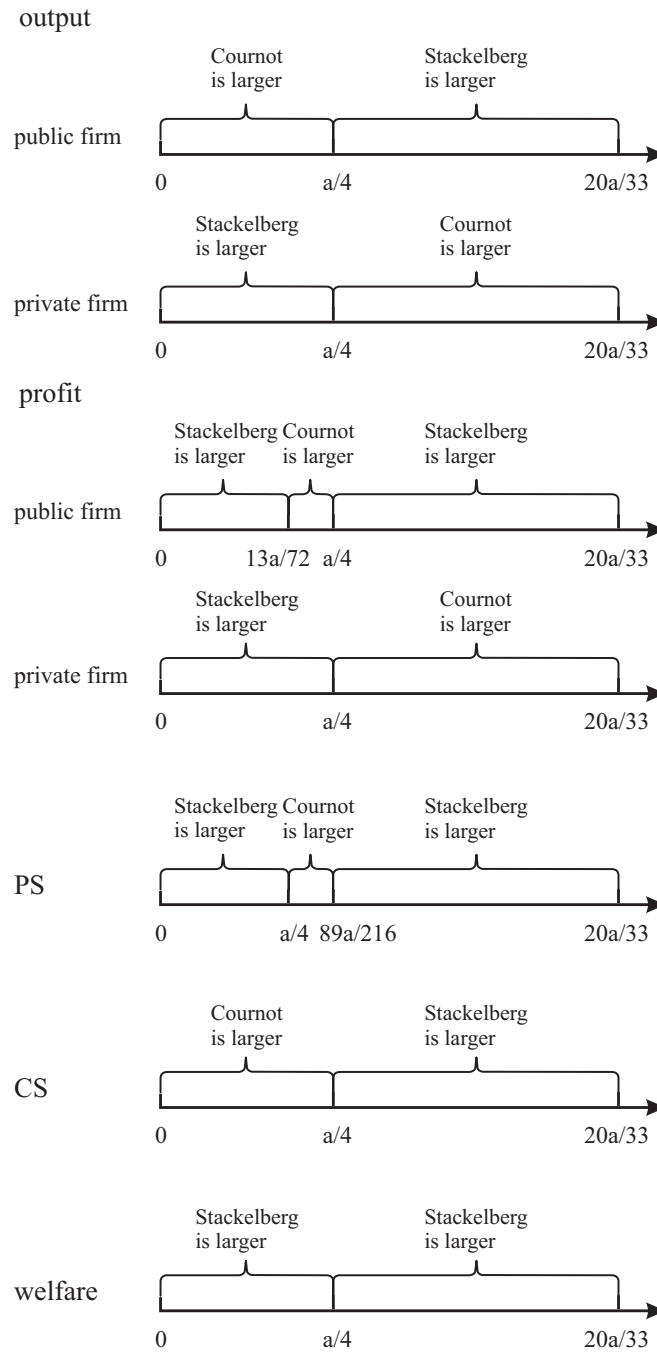
First, we will compare the firms' output quantities and profits, residents' surplus, producer surplus and social welfare under Cournot competition with those under Stackelberg competition, which are summarized by Proposition 3.

Proposition 3: In the mixed duopoly under Stackelberg competition and Cournot competition: (i) If  $0 \leq \lambda < \frac{1}{4}a$ , the public (private) firm's output under Stackelberg competition will be less (larger) than that under Cournot competition; and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , the public (private) firm's output under Stackelberg competition will be larger (less) than that under Cournot competition. (ii) If  $0 \leq \lambda < \frac{13}{72}a$  and  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , the public firm will gain more profit under Stackelberg competition; and if  $\frac{13}{72}a < \lambda < \frac{1}{4}a$ , the public firm will gain more profit under Cournot competition. If  $0 \leq \lambda < \frac{1}{4}a$ , the private firm will gain more profit under Stackelberg competition; and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , the private firm will gain more profit under Cournot competition. (iii) If  $0 \leq \lambda < \frac{1}{4}a$ , Stackelberg competition will lead to a lower consumer surplus; and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , Cournot competition will result in a lower consumer surplus. If  $0 \leq \lambda < \frac{1}{4}a$  and  $\frac{89}{216}a < \lambda < \frac{20}{33}a$ , Stackelberg competition will enjoy a higher producer surplus; and if  $\frac{1}{4}a < \lambda < \frac{89}{216}a$ , Cournot competition will have a higher producer surplus. (iv) If  $0 \leq \lambda < \frac{1}{4}a$  and  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , the social welfare level under Stackelberg competition will be larger than that under Cournot competition.

Proof: See Appendix E.



Figure 1: Comparisons of Cournot competition and Stackelberg competition before privatization



Source: Authors' calculation

From Proposition 3, we see that it is hard to verify that Stackelberg competition is better than Cournot competition, although the social welfare level under Stackelberg competition is larger than (or at least equal to) that under Cournot competition. The comparisons among the firms' output quantities and profits, residents' surplus and producer surplus between Cournot competition and Stackelberg competition before privatization are largely determined by the residents' environmental preference. We use Figure 1 to summarize the main findings of Proposition 3.

After privatization, both the public firm and the private firm are profit maximizers. The firms' output quantities and profits, residents' surplus and producer surplus are all irrelevant to the environmental pollution. The comparisons among firms' output quantities and profits, residents' surplus and producer surplus between Cournot competition and Stackelberg competition can be easily found in the Microeconomic textbook. The comparisons between the social welfare levels under Cournot competition and Stackelberg competition will be summarized by Proposition 4.

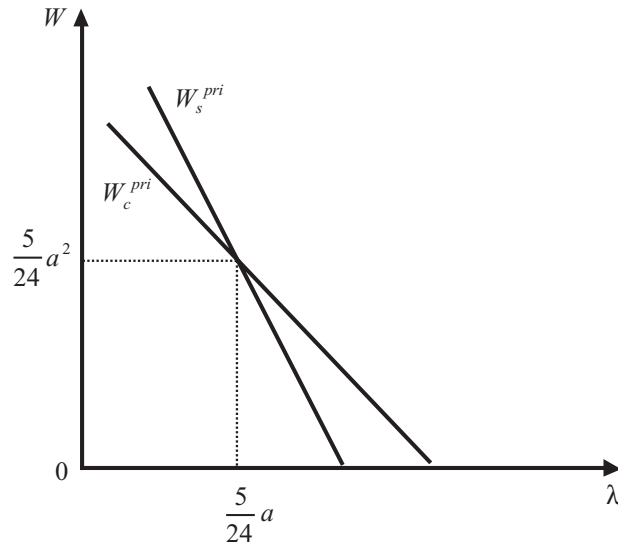
Proposition 4: In the mixed duopoly, if  $0 \leq \lambda < \frac{5}{24}a$ , then the Stackelberg competition will have a higher social welfare level after privatization. If  $\frac{5}{24}a < \lambda < \frac{20}{33}a$ , then the Cournot competition will have a higher social welfare level after privatization.

Proof: See Appendix F.

If we set  $\lambda = 0$ , from Equation (6) we know that in this situation, the environmental pollution will exert no impact on social welfare. This is the common case discussed by the Microeconomic textbook, which contends that the social welfare level under the Stackelberg competition is higher than that under Cournot competition. It is also common to know that the total output quantity under Stackelberg competition is larger than that under Cournot competition.

However, when we introduce the environmental pollution to the social welfare function, the society under Stackelberg competition will suffer more from the environmental pollution, and the welfare impact exerted by privatization will be ambiguous. The main findings of Proposition 4 can be illustrated by Figure 2.

Figure 2: Welfare comparisons of Cournot competition and Stackelberg competition after privatization



Source: According to equations (8) and (10)

From Figure 2 we know that if the residents care less about the environment and the residents' environmental preference is relatively low (i.e.,  $0 \leq \lambda < \frac{5}{24}a$ ), the environmental pollution will have a trivially negative impact on social welfare, and the finding obtained by the textbook can also be established. However, if they also care more about the environment, and the residents' environmental preference is sufficiently high (i.e.,  $\frac{5}{24}a < \lambda < \frac{20}{33}a$ ), the environmental pollution will exert a non-negligible negative impact on social welfare, and the textbook finding cannot be achieved. In this situation, the social welfare level under the Cournot competition will be higher than that under Stackelberg competition.

Therefore, we can conclude that if the residents care less about the environment, after privatization Stackelberg competition will be better than Cournot competition from the perspective of social welfare. However, if the residents care more about the environment, the opposite will occur.

## 5. Conclusion

On the one hand, along with more and more serious environmental problem prevailing in the real world, the environmental pollution attracts more and more attentions of both government officials and local residents. On the other hand, privatization of public firms, working as a policy instrument aimed at raising the economic efficiency, prevails in both developing and developed countries. Our hypothesis is that when we introduce the environmental pollution and the residents' environmental preference to the public firm's objective function, some results running contrast to the common findings of the studies on privatization can be achieved. This paper establishes game-theoretical models in the framework of Cournot and Stackelberg competitions to confirm the validity of our hypothesis.

The issue addressed by this paper and the angle of view on the issue are largely ignored by the current studies of privatization in the framework of mixed oligopoly. Thus, we contribute to the works on privatization in the framework of mixed oligopoly by introducing the residents' environmental preference to the public firm's objective function. As expected, our analytical extensions challenge the common results obtained by the literature on privatization. In addition, although the existent studies have accommodated the environmental pollution and the residents' environmental preference to the social welfare function, but they seldom conduct the analysis in the framework of mixed duopoly. Our paper is a beneficial supplement to this strand of literature, and we highlight the role that the environmental pollution and the residents' environmental preference play in a mixed duopoly market. Much attention has rested on the issues related to the employment and investment effects during the privatization process, but little has been paid to the environmental problems. As shown by Beladi and Chao (2006), Naito and Ogawa (2009), Wang and Wang (2009) and Prašnikar et al. (2012), environmental issues are also of great importance when the government conducts its privatization policy. Moreover, our findings indicate that when residents pay much attention to the environmental problems, they can benefit from government's privatization policy.

Here we would like to point out that the establishment of our findings is based on the assumptions that the firms' cost functions are quadric, the firms' outputs are homogenous, only one public firm prevails, and the environmental pollution is linearly introduced to the social welfare function. If these assumptions are relaxed, will our results still be robust? Besides, due to non-availability of empirical data, we cannot conduct an empirical analysis to test our findings. It cannot deny that the above-mentioned points are our research limitations, but they also motivate us to relax these assumptions and find relevant data in the future studies.

This paper highlights that when the government carries out some development policies, their environmental impacts cannot be neglected. When the residents'

environmental preference is accommodated to the public firm's objective function, it is possible for the public firm to under-produce. Privatization is not always a good way to improve the production efficiency of the public firm. When residents pay less attention to the environmental problems, privatization is beneficial to the producers' interests. When the government carries out the privatization policy, they should take the resident's environmental preference into account, make a tradeoff between the benefits of residents and firms, and decide whose interests should be placed as the priority during the privatization process. Therefore, in the situation discussed by this paper, the government should take more care of its privatization decision.

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## Privatizacija i zagađenje okoliša u mješovitom duopolu

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### Sažetak

U radu se uspostavlja mješoviti duopol model teorijskih igara kako bi se istražili ekonomski učinci privatizacije u prisutnosti onečišćenja okoliša. Kad se preferiranje zaštite okoliša uključi u funkciju cilja javnog poduzeća, nalazimo da privatizacija povećava proizvodnju javnih poduzeća, smanjuje proizvodnju privatnih poduzeća i nema utjecaja na socijalnu skrb. Rezultati ovog istraživanja u suprotnosti su s uobičajenim rezultatima istraživanja o privatizaciji. Štoviše, Cournotova konkurencija i Stackelbergova konkurencija analiziraju se zasebno kako bi se pokazalo da su naši rezultati robusni i irelevantni u odnosu na poslovne poteze poduzeća. U radu se ističe uloga onečišćenja okoliša i preferiranje zaštite okoliša stanovništva u određivanju ekonomskog utjecaja prouzročenog privatizacijom.

**Ključne riječi:** privatizacija, mješoviti duopoly, zagađenje okoliša, preferiranje zaštite okoliša stanovništva, Cournotova konkurencija, Stackelbergova konkurencija

**JEL klasifikacija:** H21, L33, Q58

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### Appendix A

Differentiating Equation (7) with respect to  $\lambda$  yields:

$$\frac{\partial q_{c0}^{ex}}{\partial \lambda} = -\frac{3}{5} < 0, \quad \frac{\partial q_{c1}^{ex}}{\partial \lambda} = \frac{1}{5} > 0; \quad \frac{\partial \pi_{c0}^{ex}}{\partial \lambda} = \frac{8a - 42\lambda}{50}, \quad \text{if } 0 \leq \lambda < \frac{4}{21}a, \quad \text{then}$$

$$\frac{\partial \pi_{c0}^{ex}}{\partial \lambda} > 0, \quad \text{and if } \frac{4}{21}a < \lambda < \frac{20}{33}a, \quad \text{then } \frac{\partial \pi_{c0}^{ex}}{\partial \lambda} < 0; \quad \frac{\partial \pi_{c1}^{ex}}{\partial \lambda} = \frac{3(a + \lambda)}{25} > 0;$$

$$\frac{\partial CS_c^{ex}}{\partial \lambda} = -\frac{6a - 4\lambda}{25} < 0; \quad \frac{\partial PS_c^{ex}}{\partial \lambda} = \frac{7}{25}a - \frac{18}{25}\lambda, \quad \text{if } 0 \leq \lambda < \frac{7}{18}a, \quad \text{then } \frac{\partial PS_c^{ex}}{\partial \lambda} > 0,$$

and if  $\frac{7}{18}a < \lambda < \frac{20}{33}a$ , then  $\frac{\partial PS_c^{ex}}{\partial \lambda} < 0$ ;  $\frac{\partial W_c^{ex}}{\partial \lambda} = -\frac{14}{25}a + \frac{6}{25}\lambda < 0$ .

### Appendix B

From Equation (7) and Equation (8), we have:

$$q_{c0}^{ex} - q_{c0}^{pri} = \frac{3}{20}a - \frac{3}{5}\lambda, \quad \text{if } 0 \leq \lambda < \frac{1}{4}a, \quad \text{then } q_{c0}^{ex} > q_{c0}^{pri} \quad \text{and if } \frac{1}{4}a < \lambda < \frac{20}{33}a,$$

then  $q_{c0}^{ex} < q_{c0}^{pri}$ .

$$q_{c1}^{ex} - q_{c1}^{pri} = -\frac{1}{20}a + \frac{1}{5}\lambda, \quad \text{if } 0 \leq \lambda < \frac{1}{4}a, \quad \text{then } q_{c1}^{ex} < q_{c1}^{pri} \quad \text{and if } \frac{1}{4}a < \lambda < \frac{20}{33}a,$$

then  $q_{c1}^{ex} > q_{c1}^{pri}$ .

$$\pi_{c0}^{ex} - \pi_{c0}^{pri} = \frac{4a^2 + 8a\lambda - 21\lambda^2}{50} - \frac{3}{32}a^2. \quad \text{When } \lambda = \frac{11}{84}a \quad \text{and } \lambda = \frac{1}{4}a, \quad \text{we have}$$

$$\pi_{c0}^{ex} = \pi_{c0}^{pri}.$$

$$\frac{\partial(\pi_{c0}^{ex} - \pi_{c0}^{pri})}{\partial \lambda} = \frac{8a - 42\lambda}{50}. \quad \text{If } \lambda < \frac{4}{21}a, \quad \text{then } \frac{\partial(\pi_{c0}^{ex} - \pi_{c0}^{pri})}{\partial \lambda} > 0 \quad \text{and if } \lambda > \frac{4}{21}a,$$

then  $\frac{\partial(\pi_{c0}^{ex} - \pi_{c0}^{pri})}{\partial \lambda} < 0$ .

Thus, we have if  $0 \leq \lambda < \frac{11}{84}a$  and  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , then  $\pi_{c0}^{ex} < \pi_{c0}^{pri}$ , and if

$$\frac{11}{84}a < \lambda < \frac{1}{4}a, \quad \text{then } \pi_{c0}^{ex} > \pi_{c0}^{pri}.$$



$\pi_{c1}^{ex} - \pi_{c1}^{pri} = \frac{3(a+\lambda)^2}{50} - \frac{3}{32}a^2$ . When  $\lambda = \frac{1}{4}a$ , we have  $\pi_{c1}^{ex} = \pi_{c1}^{pri}$ . From the

proof of Lemma 1 we know,  $\frac{\partial \pi_{c1}^{ex}}{\partial \lambda} = \frac{3(a+\lambda)}{25} > 0$ . Thus, if  $0 \leq \lambda < \frac{1}{4}a$ , then

$\pi_{c1}^{ex} < \pi_{c1}^{pri}$ , and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , then  $\pi_{c1}^{ex} > \pi_{c1}^{pri}$ .

$CS_c^{ex} - CS_c^{pri} = \frac{(3a-2\lambda)^2}{50} - \frac{1}{8}a^2$ . When  $\lambda = \frac{1}{4}a$ , we have  $CS_c^{ex} = CS_c^{pri}$ . From

the proof of Lemma 1 we know,  $\frac{\partial CS_c^{ex}}{\partial \lambda} = -\frac{6a-4\lambda}{25} < 0$ . Thus, if  $0 \leq \lambda < \frac{1}{4}a$ ,

then  $CS_c^{ex} > CS_c^{pri}$ , and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , then  $CS_c^{ex} < CS_c^{pri}$ .

$PS_c^{ex} - PS_c^{pri} = -\frac{3}{800}a^2 + \frac{7}{25}a\lambda - \frac{9}{25}\lambda^2$ . When  $\lambda = \frac{1}{4}a$  and  $\lambda = \frac{19}{36}a$ ,

$PS_c^{ex} = PS_c^{pri}$ .

$\frac{\partial (PS_c^{ex} - PS_c^{pri})}{\partial \lambda} = \frac{7}{25}a - \frac{18}{25}\lambda$ . If  $\lambda < \frac{7}{18}a$ , then  $\frac{\partial (PS_c^{ex} - PS_c^{pri})}{\partial \lambda} > 0$  and if

$\lambda > \frac{7}{18}a$ , then  $\frac{\partial (PS_c^{ex} - PS_c^{pri})}{\partial \lambda} < 0$ .

Thus, we have if  $0 \leq \lambda < \frac{1}{4}a$  and  $\frac{19}{36}a < \lambda < \frac{20}{33}a$ ,  $PS_c^{ex} < PS_c^{pri}$ , and if

$\frac{1}{4}a < \lambda < \frac{19}{36}a$ ,  $PS_c^{ex} > PS_c^{pri}$ .

$W_c^{ex} - W_c^{pri} = \frac{3}{400}a^2 - \frac{3}{50}a\lambda + \frac{3}{25}\lambda^2$ . When  $\lambda = \frac{1}{4}a$ , we have  $W_c^{ex} = W_c^{pri}$ .

It is not hard to know that  $\frac{\partial (W_c^{ex} - W_c^{pri})}{\partial \lambda} = -\frac{3}{50}a + \frac{6}{25}\lambda$ , if  $0 \leq \lambda < \frac{1}{4}a$ , then

$\frac{\partial (W_c^{ex} - W_c^{pri})}{\partial \lambda} < 0$ , and  $W_c^{ex} > W_c^{pri}$ ; and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , then

$\frac{\partial (W_c^{ex} - W_c^{pri})}{\partial \lambda} > 0$ , and  $W_c^{ex} > W_c^{pri}$ .

### Appendix C

Differentiating Equation (9) with respect to  $\lambda$  yields:

$$\frac{\partial q_{s0}^{ex}}{\partial \lambda} = -\frac{3}{7} < 0, \quad \frac{\partial q_{s1}^{ex}}{\partial \lambda} = \frac{1}{7} > 0;$$

$$\frac{\partial \pi_{s0}^{ex}}{\partial \lambda} = \frac{1}{14}a - \frac{3}{7}\lambda, \text{ if } 0 \leq \lambda < \frac{1}{6}a, \text{ then } \frac{\partial \pi_{s0}^{ex}}{\partial \lambda} > 0; \text{ and if } \frac{1}{6}a < \lambda < \frac{20}{33}a, \text{ then}$$

$$\frac{\partial \pi_{s0}^{ex}}{\partial \lambda} < 0.$$

$$\frac{\partial \pi_{s1}^{ex}}{\partial \lambda} = \frac{9}{98}a + \frac{3}{49}\lambda > 0; \quad \frac{\partial CS_s^{ex}}{\partial \lambda} = \frac{4}{49}\lambda - \frac{8}{49}a < 0;$$

$$\frac{\partial PS_s^{ex}}{\partial \lambda} = \frac{8}{49}a - \frac{18}{49}\lambda, \text{ if } 0 \leq \lambda < \frac{4}{9}a, \text{ then } \frac{\partial PS_s^{ex}}{\partial \lambda} > 0; \text{ and if } \frac{4}{9}a < \lambda < \frac{20}{33}a,$$

$$\text{then } \frac{\partial PS_s^{ex}}{\partial \lambda} < 0.$$

$$\frac{\partial W_s^{ex}}{\partial \lambda} = -\frac{4}{7}a + \frac{2}{7}\lambda < 0.$$

### Appendix D

From Equation (9) and Equation (10), we know:

$$q_{s0}^{ex} - q_{s0}^{pri} = \frac{1}{14}a - \frac{3}{7}\lambda, \text{ if } 0 \leq \lambda < \frac{1}{6}a, \text{ then } q_{s0}^{ex} > q_{s0}^{pri}, \text{ and if } \frac{1}{6}a < \lambda < \frac{20}{33}a,$$

$$\text{then } q_{c0}^{ex} < q_{c0}^{pri}.$$

$$q_{s1}^{ex} - q_{s1}^{pri} = -\frac{1}{42}a + \frac{1}{7}\lambda, \text{ if } 0 \leq \lambda < \frac{1}{6}a, \text{ then } q_{s1}^{ex} < q_{s1}^{pri}; \text{ and if } \frac{1}{6}a < \lambda < \frac{20}{33}a,$$

$$\text{then } q_{s1}^{ex} > q_{s1}^{pri}.$$

$$\pi_{s0}^{ex} - \pi_{s0}^{pri} = \frac{5}{56}a^2 + \frac{1}{14}a\lambda - \frac{3}{14}\lambda^2 - \frac{2}{21}a^2. \text{ Only when } \lambda = \frac{1}{6}a \text{ can we have}$$

$$\pi_{s0}^{ex} = \pi_{s0}^{pri}.$$

$\frac{\partial(\pi_{s0}^{ex} - \pi_{s0}^{pri})}{\partial\lambda} = \frac{a - 6\lambda}{14}$ . If  $\lambda < \frac{1}{6}a$ , then  $\frac{\partial(\pi_{c0}^{ex} - \pi_{c0}^{pri})}{\partial\lambda} > 0$ ; and if  $\lambda > \frac{1}{6}a$ , then  $\frac{\partial(\pi_{c0}^{ex} - \pi_{c0}^{pri})}{\partial\lambda} < 0$ . Thus, we have if  $0 \leq \lambda < \frac{20}{33}a$  and  $\lambda \neq \frac{1}{6}a$ , then

$\pi_{s0}^{ex} < \pi_{s0}^{pri}$ ; and if  $\lambda = \frac{1}{6}a$ , then  $\pi_{s0}^{ex} = \pi_{s0}^{pri}$ .

$\pi_{s1}^{ex} - \pi_{s1}^{pri} = \frac{3}{98}(\frac{3}{2}a + \lambda)^2 - \frac{25}{294}a^2$ . When  $\lambda = \frac{1}{6}a$ , we have  $\pi_{s1}^{ex} = \pi_{s1}^{pri}$ . From

the proof of Lemma 2 we know,  $\frac{\partial\pi_{s1}^{ex}}{\partial\lambda} = \frac{9}{98}a + \frac{3}{49}\lambda > 0$ . Thus, if  $0 \leq \lambda < \frac{1}{6}a$ ,

then  $\pi_{s1}^{ex} < \pi_{s1}^{pri}$ ; and if  $\frac{1}{6}a < \lambda < \frac{20}{33}a$ , then  $\pi_{s1}^{ex} > \pi_{s1}^{pri}$ .

$CS_s^{ex} - CS_s^{pri} = \frac{2(2a - \lambda)^2}{49} - \frac{121}{882}a^2$ . When  $\lambda = \frac{1}{6}a$ , we have  $CS_s^{ex} = CS_s^{pri}$ .

From the proof of Lemma 2 we know,  $\frac{\partial CS_s^{ex}}{\partial\lambda} = \frac{4}{49}\lambda - \frac{8}{49}a < 0$ . Thus, if

$0 \leq \lambda < \frac{1}{6}a$ , then  $CS_s^{ex} > CS_s^{pri}$ ; and if  $\frac{1}{6}a < \lambda < \frac{20}{33}a$ , then  $CS_s^{ex} < CS_s^{pri}$ .

$PS_s^{ex} - PS_s^{pri} = -\frac{13}{588}a^2 + \frac{8}{49}a\lambda - \frac{9}{49}\lambda^2$ . When  $\lambda = \frac{1}{6}a$ ,  $PS_s^{ex} = PS_s^{pri}$ .

$\frac{\partial(PS_s^{ex} - PS_s^{pri})}{\partial\lambda} = \frac{8}{49}a - \frac{18}{49}\lambda$ . If  $\lambda < \frac{4}{9}a$ , then  $\frac{\partial(PS_s^{ex} - PS_s^{pri})}{\partial\lambda} > 0$ ; and if

$\lambda > \frac{4}{9}a$ , then  $\frac{\partial(PS_s^{ex} - PS_s^{pri})}{\partial\lambda} < 0$ . Thus, if  $0 \leq \lambda < \frac{1}{6}a$ , then  $PS_s^{ex} < PS_s^{pri}$ ,

and if  $\frac{1}{6}a < \lambda < \frac{20}{33}a$ , then  $PS_s^{ex} > PS_s^{pri}$ .

$W_s^{ex} - W_s^{pri} = \frac{1}{252}a^2 - \frac{1}{21}a\lambda + \frac{1}{7}\lambda^2$ . When  $\lambda = \frac{1}{6}a$ , we have  $W_s^{ex} = W_s^{pri}$ . It is

not hard to know that  $\frac{\partial(W_s^{ex} - W_s^{pri})}{\partial\lambda} = -\frac{1}{21}a + \frac{2}{7}\lambda$ . If  $0 \leq \lambda < \frac{1}{6}a$ , then

$$\frac{\partial(W_s^{ex} - W_s^{pri})}{\partial \lambda} < 0 \quad \text{and} \quad W_s^{ex} > W_s^{pri}; \quad \text{and} \quad \text{if} \quad \frac{1}{6}a < \lambda < \frac{20}{33}a, \quad \text{then}$$

$$\frac{\partial(W_s^{ex} - W_s^{pri})}{\partial \lambda} > 0 \quad \text{and} \quad W_s^{ex} > W_s^{pri}.$$

### Appendix E

From Equation (7) and Equation (9), we know:

$$q_{c0}^{ex} - q_{s0}^{ex} = \frac{3}{70}a - \frac{6}{35}\lambda, \quad \frac{\partial(q_{c0}^{ex} - q_{s0}^{ex})}{\partial \lambda} = -\frac{6}{35} < 0, \quad \text{if} \quad 0 \leq \lambda < \frac{1}{4}a, \quad \text{then} \quad q_{c0}^{ex} > q_{s0}^{ex};$$

and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , then  $q_{c0}^{ex} < q_{s0}^{ex}$ .

$$q_{c1}^{ex} - q_{s1}^{ex} = -\frac{1}{70}a + \frac{2}{35}\lambda, \quad \frac{\partial(q_{c1}^{ex} - q_{s1}^{ex})}{\partial \lambda} = \frac{2}{35} > 0, \quad \text{if} \quad 0 \leq \lambda < \frac{1}{4}a, \quad \text{then} \quad q_{c1}^{ex} < q_{s1}^{ex};$$

and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , then  $q_{c1}^{ex} > q_{s1}^{ex}$ .

$$\pi_{c0}^{ex} - \pi_{s0}^{ex} = -\frac{13}{1400}a^2 + \frac{31}{350}a\lambda - \frac{36}{175}\lambda^2. \quad \text{When} \quad \lambda = \frac{1}{4}a \quad \text{and} \quad \lambda = \frac{13}{72}a, \quad \text{we have}$$

$$\pi_{c0}^{ex} = \pi_{s0}^{ex} \cdot \frac{\partial(\pi_{c0}^{ex} - \pi_{s0}^{ex})}{\partial \lambda} = \frac{31}{350}a - \frac{72}{175}\lambda. \quad \text{If} \quad \lambda < \frac{31}{144}a, \quad \text{then} \quad \frac{\partial(\pi_{c0}^{ex} - \pi_{s0}^{ex})}{\partial \lambda} > 0;$$

and if  $\lambda > \frac{31}{144}a$ , then  $\frac{\partial(\pi_{c0}^{ex} - \pi_{s0}^{ex})}{\partial \lambda} < 0$ . Thus, if  $0 \leq \lambda < \frac{13}{72}a$  and

$\frac{1}{4}a < \lambda < \frac{20}{33}a$ , then  $\pi_{c0}^{ex} < \pi_{s0}^{ex}$ ; and if  $\frac{13}{72}a < \lambda < \frac{1}{4}a$ , then  $\pi_{c0}^{ex} > \pi_{s0}^{ex}$ .

$$\pi_{c1}^{ex} - \pi_{s1}^{ex} = \frac{-87a^2 + 276a\lambda + 288\lambda^2}{9800}. \quad \text{When} \quad \lambda = \frac{1}{4}a, \quad \text{we have} \quad \pi_{c1}^{ex} = \pi_{s1}^{ex}.$$

$$\frac{\partial(\pi_{c1}^{ex} - \pi_{s1}^{ex})}{\partial \lambda} = \frac{276a + 576\lambda}{9800}, \quad \frac{\partial(\pi_{c0}^{ex} - \pi_{s0}^{ex})}{\partial \lambda} > 0. \quad \text{Thus, if} \quad 0 \leq \lambda < \frac{1}{4}a, \quad \text{then}$$

$\pi_{c1}^{ex} < \pi_{s1}^{ex}$ ; and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , then  $\pi_{c1}^{ex} > \pi_{s1}^{ex}$ .

$CS_c^{ex} - CS_s^{ex} = \frac{41a^2 - 188a\lambda + 96\lambda^2}{2450}$ . When  $\lambda = \frac{1}{4}a$ , we have  $CS_c^{ex} = CS_s^{ex}$ .

$\frac{\partial(CS_c^{ex} - CS_s^{ex})}{\partial\lambda} = -\frac{188}{2450}a + \frac{192}{2450}\lambda$ , if  $0 \leq \lambda < \frac{20}{33}a$ , then  $\frac{\partial(CS_c^{ex} - CS_s^{ex})}{\partial\lambda} < 0$ .

Thus, if  $0 \leq \lambda < \frac{1}{4}a$ , then  $CS_c^{ex} > CS_s^{ex}$ ; and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , then  $CS_c^{ex} < CS_s^{ex}$ .

$PS_c^{ex} - PS_s^{ex} = -\frac{89a^2 - 572a\lambda + 864\lambda^2}{4900}$ . When  $\lambda = \frac{1}{4}a$  and  $\lambda = \frac{89}{216}a$ ,

$PS_c^{ex} = PS_s^{ex}$ .  $\frac{\partial(PS_c^{ex} - PS_s^{ex})}{\partial\lambda} = \frac{572}{4900}a - \frac{864}{2450}\lambda$ . If  $\lambda < \frac{143}{432}a$ , then

$\frac{\partial(PS_c^{ex} - PS_s^{ex})}{\partial\lambda} > 0$ ; and if  $\lambda > \frac{143}{432}a$ , then  $\frac{\partial(PS_c^{ex} - PS_s^{ex})}{\partial\lambda} < 0$ . Thus, if

$0 \leq \lambda < \frac{1}{4}a$  and  $\frac{89}{216}a < \lambda < \frac{20}{33}a$ , then  $PS_c^{ex} < PS_s^{ex}$ ; and if  $\frac{1}{4}a < \lambda < \frac{89}{216}a$ ,

then  $PS_c^{ex} > PS_s^{ex}$ .

$W_c^{ex} - W_s^{ex} = -\frac{1}{700}a^2 + \frac{2}{175}a\lambda - \frac{4}{175}\lambda^2$ . When  $\lambda = \frac{1}{4}a$ , we have  $W_c^{ex} = W_s^{ex}$ .

It is easy to know that  $\frac{\partial(W_c^{ex} - W_s^{ex})}{\partial\lambda} = \frac{2}{175}a - \frac{8}{175}\lambda$ . If  $0 \leq \lambda < \frac{1}{4}a$ , then

$\frac{\partial(W_c^{ex} - W_s^{ex})}{\partial\lambda} > 0$  and  $W_c^{ex} < W_s^{ex}$ ; and if  $\frac{1}{4}a < \lambda < \frac{20}{33}a$ , then

$\frac{\partial(W_c^{ex} - W_s^{ex})}{\partial\lambda} < 0$  and  $W_c^{ex} < W_s^{ex}$ .

## Appendix F

From Equation (7) and Equation (9), we know:

$$W_c^{pri} - W_s^{pri} = \frac{1}{42}a\lambda - \frac{5}{1008}a^2. \quad \frac{\partial(W_c^{pri} - W_s^{pri})}{\partial\lambda} = \frac{1}{42}a > 0. \quad \text{If } 0 \leq \lambda < \frac{5}{24}a,$$

then  $W_c^{pri} < W_s^{pri}$ ; and if  $\frac{5}{24}a < \lambda < \frac{20}{33}a$ , then  $W_c^{pri} > W_s^{pri}$ .