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# Mass Appraisal – International Background, Polish Solutions and Proposal of new Methods Application

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*ABSTRACT.* The aim of the paper was to investigate in general the mass appraisal solutions worked out throughout years in chosen countries and compare them to methods adopted in Poland. Moreover, authors tried to propose the use of new methods for estimating values of properties: geographically weighted regression, spatial autoregressive models, regression-kriging, underlining their advantages both on theoretical and practical background. The case study research conducted on the example of Olsztyn City (Warmia and Mazury Province, Poland) has shown particular advantages of proposed methods. First of all the combination of them enables not only assessment of different attributes of property on its value but also presentation of the analysis results on different maps. Statistical models including spatial relationship can be successfully used in mass appraisal. One ought to remember that these are better tools than the classical methods only when one can notice spatial autocorrelation in transaction prices. This condition is satisfied in most local markets, although there may be exceptional circumstances when the location of the property does not affect its price. In that case, the models would be equivalent to the classical methods.

*Keywords:* mass appraisal models, valuation, real estate, geographically weighted regression, spatial autoregressive, regression-kriging.

## 1. Introduction

The paper gives a substantial introduction to the cycle of original scientific papers on broad aspects of mass appraisal in international prospective taken on by academic staff of two Polish Universities – the University of Warmia and Mazury in Olsztyn and the Warsaw University of Technology in cooperation with foreign experts and scientists. The substantial purpose for taking up that subject was the

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growing importance of mass appraisal systems through last decade and the need of their further development. The scientific literature gives a number of reasons for that need:

- improvement in computing technology
- improving data registers and their availability
- launch of end-user friendly software (Kauko and d’Amato 2008).

Because of the nowadays economic globalization and the international need of homogenous property valuation which is significantly emphasized in for example International Accounting Standards, International Financing Reporting Standards or International Valuation Standards (Żróbek and Grzesik 2013), the authors tried to see if Polish Mass appraisal solutions follow the same patterns as methods applied abroad.

## 2. Mass appraisal systems on international background

Countries all over the world throughout the years have developed different mass appraisal solutions, but most of them follow the similar pattern. For example in European Union countries the predominating system is a value system of real estate taxation (ad valorem). What underlines A. Baranska (Baranska 2013) “within the valuation categories of the market systems, the proportions as for the number of union countries using particular constructions of real estate taxation are similar”. The models are generally using hedonic equations, which are based on multiple regression analysis, cooperating to develop computer-assisted mass appraisal systems (Aurelio et al. 2006). The latest methods adopted or proposed in chosen countries are presented in Table 1.

Table 1. *The latest methods adopted or proposed in chosen countries [source: authors own study on the basis of Brankovic (2013), Kuburic and Cirovic (2012), McCluskey and Trinh (2013), Davis et al. (2012), Aurelio et al. (2006)].*

Country	Method
Brasil	Genetic fuzzy rule-based systems
USA	Expert systems
UK	Simplified valuation approaches
Serbia	Case based reasoning, logical aggregation
Germany	Regression analysis by means of least squares collocation method
Ireland	Multiplicative models

One can easily notice significant number of different solutions proposed or adopted in different countries. The methods worked out at the beginning of mass appraisal in Poland had much in common with them.

### 3. Mass appraisal in Poland – historical background

The history of mass appraisal in Poland starts in late nineties of the last century which is quite late in comparison to other European countries, especially the eastern ones. The reason for that had socio-political background – Poland in 1989 changed the centrally planned economy into the market one. From that time both political and scientific effort had been taken in order to prepare legal act on property management dealing with particular issues connected with property markets. In terms of mass appraisal several research units prepared their proposals of mass appraisal models.

The first one was the so called “Brzeski unit” that prepared mass appraisal model called the “Krakow model” at the turn of 1992-1993. Mathematical function of mass valuation in that model was based on market data and the dependence of properties attributes describing it. The basic statistics of the collected market data involved the following attributes: the distance from the center, azimuth distance, the current usage, destination in land use plan and equipment in the technical infrastructure. The formula for the model was as follows:

$$C = C_0 + P_1 \cdot ODL CEN + P_2 \cdot ACN 12 + P_3 \cdot ACN 3 + P_4 \cdot ACN 8 + P_5 \cdot OEN 5 + P_6 \cdot PEN 12 + P_7 \cdot SIECO \quad (1)$$

The second unit research team headed by Jozef Czaja in 1993 prepared another mass appraisal model. It was based on the multiple regression and the correlation coefficient. The model was described by eleven independent variable which were estimated on the basis of mathematical models. This model gave the ability of precise determination of the cadastral value of the property for the full and stable property market. Its disadvantage was that determination of the unit price of land was done on the basis of the regression model, in which none of the variables were expressed in monetary units. The formula for the model was as follows:

$$W = S_0 \cdot f(X_1) \cdot \dots \cdot f(X_5) \cdot [1 + f(X_6 X_7) + f(X_8) + \dots + f(X_{11})] \quad (2)$$

The third model called the “Lodz model” was developed by a team led by Przewłocki and involved determining and verifying taxation units areas for the mass appraisal. It described the dependence of the initial value of the field strength valuables. For each area separate individual average valuable comparative unit was set. The results were used to prepare three types of taxation maps areas: buildings, land and rents. These has been demonstrated in a lines of equal field strength valuables. Fixed ranges of areas resulted from geographical, but eventually progress has been corrected by a competent team of experts which included both the technical and social aspects (Kuryj 2001) (Fig. 1).

All the proposals of mass appraisal methods presented above led to creation of the technical procedure of mass appraisal in Poland.



Fig. 1. Particular steps of “Lodz model” application [source: own study on the basis of Kuryj (2001)].

#### 4. Present solutions worked out in Polish mass appraisal

In 1997 the legal Act on Property Management dated 21 August 1997 had been accepted. From that time Mass appraisal system in Poland is defined as legal and technical procedure of real estate valuation in order to establish cadastral value of the property. It is conducted for properties which are the objects of taxation according to legal acts concerning property tax. It involves a number of public authorities which are responsible for particular things (Act on Property Management 1997, Walacik et al. 2012).

The technical procedure of mass appraisal in Poland is initiated with a monitoring of property market. The monitoring enables taxation zones formation and description of the representative properties. The next stage of technical procedure involves determination of the unit value of the representative property and the unit value of the taxation unit which are used for determination of the cadastral value of the taxation unit in zone. Very important elements of technical procedure of mass appraisal are taxation maps and tables. Taxation maps and tables are prepared separately for each municipality.

The taxation map is drawn up on the cadastral map. It includes boundaries for each zone, its identifier, the value of 1 m<sup>2</sup> of land and features of the representative property. An integral part of the map is a list of the taxation zones. The list of taxation zones includes:

- taxation zone identifier
- identification of land parcels according to the real estate cadastre
- the kind of the soil
- the value of 1 m<sup>2</sup> of land of representative real estate
- characteristics of a representative property
- the weightings differences between the characteristics of a real estate representative features of other properties in the taxation zone (determined with the use of the methods of statistical analysis)
- the unit value of each parcel of land cadastre.

The starting point of preparation of the taxation table are lists of the same taxation values of 1 m<sup>2</sup> of land components (units), identified on the basis of an assessment of representative properties. In each zone, each taxation group of components of land is prepared. The representative properties are chosen with the consideration of the main features of properties in particular taxation zone. The value of real estate in the mass appraisal is determined with taking into account the characteristics of the properties that influence their cadastral value. For the purpose of the mass appraisal two kinds of land are distinguished:

- built-up land or designated for development, and the land used for purposes other than agricultural and forestry
- agricultural and forest land.

The characteristic features of land, buildings and premises, you can also classify other features, if they are typical of the taxation zone.

## 5. Proposal of new methods application – justification and theoretical background

Classical regression models used to analyze the real estate market in general do not directly include potential interaction (spatial autocorrelation), which may occur between the level of phenomena in space. They assume the stability of the process associated with the formation of prices in the geographic space (Kulczycki and Ligas 2007). The significance of the parameters of classical regression models is in this case dependent on the spatial structure of the studied phenomenon, which can lead to misinterpretation of the results (Charlton and Fotheringham 2009), particularly from the spatial heterogeneity of the real estate market.

The basis for the application of geographically weighted regression is the assumption that the model parameters can be estimated separately for each point in space, for which the value is unknown dependent variable and the explanatory variables. Interactions occurring between the analyzed objects in space characterized in many cases that the means which close similarities are usually more than objects that are far apart (Tobler 1970). Using this principle one can estimate the model parameters at a given location on the assumption that the observations made at points closer to the test point will have a correspondingly higher weight than observations that are further (Charlton and Fotheringham 2009). Typical Equation of GWR model have the following form:

$$Y = \beta_0(x_i, y_i) + \sum_{i=1}^k \beta_j(x_i, y_i) \cdot X_i + \varepsilon_i, \text{ for } i = 1, 2, \dots, n. \quad (3)$$

The size of model parameters is associated with a location, in this case, expressed by the coordinates  $(x_i, y_i)$ . GWR model parameter estimation is carried out in a similar way to classical models, but takes into account the weights depend ant on the location of observation:

$$\hat{\beta}(x_i, y_i) = (X^T W_{(i)} X)^{-1} X^T W_{(i)} Y, \quad (4)$$

where  $W_{(i)}$  is a matrix representing the function of the distance between the location specified coordinates  $(x_i, y_i)$ , and the location of each point at which the observations were made. In order to determine the spatial scales nuclear functions are used, which set the value of the weights in such a way that they are decreased with distance from the point at which the estimation is made of the weighted regression model coordinates.

As a result of the application of GWR model one obtain a number of areas designated by the estimated parameters. The diversity of values of these parameters indicate the impact of local variation in the dependent variables on explanatory variable, and thus the spatial heterogeneity of the studied phenomenon (Charlton and Fotheringham 2009).

When real estate market reveals spatial interactions between transaction prices one can notice the phenomenon of spatial autoregression. This means that the value of variable from other locations affect the evolution of the value of the variable in the location analyzed. If interactions are random component, the phenom-

enon of spatial autocorrelation of the same character takes place. Depending on the type of spatial interaction two basic spatial regression models are usually used: spatial lag model and spatial error model (Anselin 1988, Wilhelmsson 2002, Páez and Scott 2004, Arbia 2006). The general form of models, taking into account the spatial lag one, also called spatial autoregressive models (SAR – spatial autoregressive models), is as follows (Anselin 1988, Arbia 2006):

$$y = \rho W y + X \beta + \varepsilon, \quad (5)$$

where  $X$  is a matrix of explanatory variables,  $\beta$  the vector of coefficients (model parameters), and  $\varepsilon \sim N(0, \sigma^2 I)$  is the error vector model.  $W y$  refers to as delayed spatially dependent variable.  $\rho$  factor is the spatial autocorrelation coefficient.

If the rest of the regression models are spatially correlated, improvement of parameter estimation accuracy can improve the spatial models use. Then some of the variables may be included in the model, and the impact of other variables that cannot be taken into account in the model will be expressed in the form of residuals (Osland 2010). This means therefore that the global autocorrelation dependency will be included as an error model. The general form of the spatial error model is as follows (Anselin 2003):

$$y = X \beta + \varepsilon \quad (6)$$

$$\varepsilon = \lambda W \varepsilon + \xi, \quad (7)$$

where  $\lambda$  is the spatial autocorrelation coefficient.  $W \varepsilon$  is the lag in space error, which should be interpreted as the average error of the neighboring location, and  $\xi$  is an independent error model.

To determine the type of spatial autocorrelation two Lagrange Multiplier tests are used. They allow to determine which case of autocorrelation takes place in the model (Anselin 1988). Using the method of least squares to estimate the spatial regression models will get inconsistent estimators (Anselin 1999), so the most frequently used method for these models is the method of maximum likelihood. Methods of estimation of SAR models are quite detailed in the extensive literature (Anselin 1999, Arbia 2006, LeSage and Pace 2009).

Spatial regression models are used when real estate prices, or the rest of the regression models transaction prices are spatially correlated. Basu and Thibodeau (1998) point out two main causes of spatial autocorrelation in the property market: structural similarity of the local property and the impact of the same location factors within the same area. Although the location of the property is one of the most important factors determining transaction prices, the real estate market modeling using spatial regression models is still outside the mainstream of empirical studies (Kim 2003). Spatial effects in models of house prices (Anselin 1998, Can and Megbolugbe 1997, Pace and Gilley 1997). Based on the work of Kwak (1997), Beron (2003), Kim (2003) and Conway (2010) it can be concluded that the use of spatial regression models has specific reasons when the transaction prices are influenced by a continuous characterization the environment (air pollution, noise, saturation of urban greenery). Besner (2002) showed that the use of

autoregressive models significantly improves the accuracy of prediction. He stressed that the spatial structure based not only on the impact of the prices of neighboring properties but also other factors that are not always possible to identify are important. Gao (2002) and Bourassa (2007) suggest, however, that not all models taking into account the spatial relationship to the real estate market will reflect its status in a more accurate way, despite the fact that they generally have a better fit to the data. These models are mainly more complex as regards the estimation process as well as the interpretation of the results.

Recent proposals aim to combine statistical models of spatial relationships and geostatistical models, which allows for significant improvement in the accuracy of estimating the mass of property valuation (Bourassa 2010). Combining these methods obtain the regression-kriging model in which the classical form used linear multiple regression model, and parameter estimation is done using the generalized least squares method (GLS) using spatial relationship formulated as variogram (or semivariograms) and spatial interpolation by kriging.

According to Matheron (1969) the value of na analyzed variable in a given location can be modeled as a sum of a deterministic and stochastic component, defined by him as a universal model of spatial variability (universal model of spatial variation). Both deterministic component (trend) and stochastic (the rest of the model) can be modeled separately, then these components is added each other:

$$\hat{z}(s_0) = \hat{m}(s_0) + \hat{e}(s_0) \quad (8)$$

where  $\hat{m}(s_0)$  is the deterministic component and are considered interpolated rest kriging method.

This model can be represented as (Hengl 2007):

$$\hat{z}(s_0) = \sum_{k=0}^p \hat{\beta}_k X_k(s_0) + \sum_{i=1}^n w_i(s_0) e(s_i), \quad (9)$$

where:

$\hat{\beta}_k$  – the estimated coefficients of the regression model

$w_i$  – kriging weights

$e(s_i)$  – the rest of the model at the location  $s_i$ .

Model coefficients are estimated using GLS generalized least squares, where the weight matrix is covariance matrix of residuals (Cressie 1993):

$$\hat{\beta}_{GLS} = (X^T \cdot C^{-1} \cdot X)^{-1} \cdot X^T \cdot C^{-1} \cdot z. \quad (10)$$

Predictor in the regression-kriging method in matrix notation can be represented as follows (Christensen 2001, Schabenberger and Gotway 2005, Hengl 2007, Ligas 2009):

$$z(s_0) = X_0^T \cdot \hat{\beta}_{GLS} + w_0^T \cdot e(s_i) \quad (11)$$

or

$$z(s_0) = X_0^T \cdot \hat{\beta}_{GLS} + w_0^T \cdot [z - X \cdot \hat{\beta}_{GLS}], \quad (12)$$

where:

$z(s_0)$  – the estimated value of a location  $s_0$

$x_0$  – vector of explanatory variables in location  $s_0$

$w_0$  – vector of kriging weights

$X$  – matrix of explanatory variables

$z$  – vector of dependent variables.

A regression-kriging disadvantage stems from the fact that both the estimation of the regression coefficients and the semivariogram function should be carried out simultaneously. The estimation of the regression coefficients must know the covariance matrix of residuals, which can be determined only after the estimation of coefficients. Thus, it may be iteratively computing the residuals and their covariance. Iteration is the following (Schabenberger and Gotway 2005, Ligas 2009):

- 1) estimation of the initial values of the parameters  $\beta$  using ordinary least squares method
- 2) estimation of residues
- 3) determination of semivariogram and covariance matrix structure
- 4) estimation of the parameters  $\beta$  using GLS
- 5) returning to step 2) until the change in the parameter estimators will be relatively small.

According Kitanidisa (1994) there is no need for an excessive number of iterations. In practice, even one iteration can give satisfactory results.

The formula used to calculate the variance of the prediction is the same as the universal kriging variance and has the following form (Hengl 2007):

$$\sigma_{RK}^2 = (C_0 + C_1) - c_0^T \cdot C^{-1} \cdot c_0 + (X_0 - X^T \cdot C^{-1} \cdot X_0)^T \cdot (X^T \cdot C^{-1} \cdot X)^{-1} \cdot (X_0 - X^T \cdot C^{-1} \cdot c_0) \quad (13)$$

where:

$(C_0 + C_1)$  – the threshold variation

$c_0$  – vector covariance residuals.

If the rest do not show spatial autocorrelation regression-kriging is equivalent to the classical model of multiple regression. Similarly, if a variable is not correlated with additional variables, regression-kriging model is reduced to an ordinary kriging model (Hengl 2007).



The regression – kriging model seems to be a very good solution for spatial modeling of economic phenomena, resulting in mass valuation of the property, however there are some limitations of which one should be aware. First, the data must be of sufficient quality. Even one outlier observation may distort the regression model, hence may appear large prediction errors over the analyzed area. Another issue is the appropriate size of the data. Webster and Oliver (2001) suggests that the estimation of variogram should take at least 50 observations. Neter et al. (1996) argue further that the observations should be at least 10 times more than the predictors (explanatory variables). An important role is also played by the spatial distribution points where the observations were made. If the observation points do not represent sufficiently well studied area, or if they represent only the central part of this leads to a large error model and at the same time large prediction errors. This is particularly important in the case of a linear model, where the variance of the prediction increases exponentially as it approaches the limits of the study area. Hence it is important to point where the observations were made were evenly distributed and possibly represent an area close to the borders of the.

## 6. Case study

The study was conducted in the city of Olsztyn, located in the north-eastern part of Poland. The study used 277 transactions undeveloped plots of land allocated for housing (low-intensity). The following explanatory variables were used: technical infrastructure area (equipped with water, electricity, sewer and gas), the area and the geometric features of the plot. The evaluation of variables are chosen in such a way that the zero values related to common property occurring in the course (fully armed land area of about 800 m<sup>2</sup>, having a favorable geometric properties). The scale of this greatly simplifies the interpretation of the constant in the different models. It is the average value of the properties of a typical. Relatively simplest solution that uses statistical models for mass measurement is the use of classical linear multiple regression model. This model enables the analysis of linear dependence between the values of the explanatory variables (characteristic properties) and transaction prices of multiple regression analysis are presented in Table 2.

Table 2. *Results of the estimation of linear multiple regression model (source: own study).*

	<b>B</b>	<b>standard error. <math>\beta</math></b>	<b>t</b>	<b>P</b>
constant	273,281	6,234	43,838	0,000
water	-44,024	16,861	-2,611	0,009
energy	12,786	15,462	0,827	0,409
sewerage	-14,673	15,056	-0,974	0,331
gas	-58,934	17,174	-3,431	0,001
surface	-93,841	30,531	-3,074	0,002
geometry	-6,177	11,423	-0,541	0,589

The coefficient of determination for this model was 0.39, and the standard error of the estimate 59.79. Linear multiple regression models do not always fully reflect the conditions of the real estate market because of the assumptions about the proportionality of, or lack of interrelated variables. Take into account the fact that in this type of location models can be taken into account only in an indirect way by means of appropriately chosen interval scale.

GWR models, although it does not take into account the location directly as an explanatory variable, however, location information is used in the estimation process as a broadcast observations weights depending on the distance from the point where the model parameters are determined. In this case, you should expect slightly better fit of the model to the realities of the real estate market. The estimation of geographically weighted regression models, the studied area, adopted the calculated weight based on the inverse distance (Charlton and Fotheringham 2009). Delay parameter search facilities adopted the model was set on the basis of AICC (Akaike 1973, Hurvich et al. 1998). Overall the results of GWR model are shown in Table 3.

Table 3. *The overall results of geographically weighted regression analysis (source: own study).*

	<b>Min</b>	<b>Max</b>	<b>Average</b>	<b>Standard deviation</b>
constant	200,467	338,518	287,522	32,002
water	-115,879	75,104	-18,465	23,562
energy	-73,399	140,181	-1,132	20,927
sewerage	-188,229	24,419	-65,364	60,844
Gas	-161,495	94,751	-33,409	50,514
Area	-539,037	107,608	-160,717	95,585
geometry	-75,993	53,287	-19,408	24,331
residuals	-200,707	159,648	-0,774	49,690
Local R2	0,241	0,730	0,469	0,121

The global coefficient of determination R2 was 0.57, indicating a slightly better fit than the linear multiple regression model. Delay parameter (the range from which the models were built) was adopted by AICC at 1908 m. Estimated values are characterized by a relatively large spread, but they are clearly correlated with the location. The advantage of this type of model compared to traditional linear models to take into account the spatial heterogeneity manifested in the uneven impact of dependent variables on transaction prices in different locations.

To build the SAR model assumes that the spatial structure of the matrix will be the inverse of the prices included the distance between the centroids of parcels, which were the subject of the transaction. Choosing the right model (delayed spatial or spatial error) was made using the LM test (Lagrange Multiplier) and PE (Robust Lagrange Multiplier) (Anselin 1988). The results are presented in Table 4.

Table 4. *Results of the LM and RLM (source: own study).*

		<b>LM</b>	<b>RLM</b>
lag	statistic	57,841	0,538
	p-value	< 0,001	0,463
error	statistic	81,510	24,207
	p-value	< 0,001	< 0,001

In the test conducted shows that a more appropriate model is the spatial delays, which means the rest of the model are strongly correlated spatially than transaction prices. It seems indeed justified. In this model the spatial interactions of spatial delay directly affected transaction prices. The spatial error model assumes that these interactions are residuals, understood in this case as prices, which have already been taken into account the influence of factors nonspatial (explanatory variables in the model). So we can accept the hypothesis that, regardless of the method of determining the neighborhood spatial weights matrix is just the spatial error model better reflects depending on the prevailing real estate market. Results SAR model parameter estimation are shown in Table 5.

Table 5. *SAR model estimation results (source: own study).*

	<b>B</b>	<b>standard error. <math>\beta</math></b>	<b>z value</b>	<b>Pr(&gt;  z )</b>
constant	257,759	28,271	9,117	0,000
water	-13,738	17,671	-0,777	0,437
energy	-21,759	18,097	-1,202	0,229
sewerage	-60,697	17,492	-3,470	0,000
gas	-18,181	17,696	-1,027	0,304
ares	-101,608	28,995	0,000	0,000
geometry	-20,827	11,662	0,074	0,074

In this model, as in the previously constructed models of some of the variables appear to be statistically insignificant. However, in this case, all the parameters that are the dependent variables are negative, which, given the choice of scales for the variables point to the correct structure substantially depending on the prevailing real estate market. The standard deviation of residues in this case was 51.54, which is slightly better fit to the data than in the case of the classical multiple regression model. Advantage SAR models compared to traditional regression models stems not only from a better fit to the data but also to the research, the rationale for having more parameters determining the values of attributes impact on transaction prices.

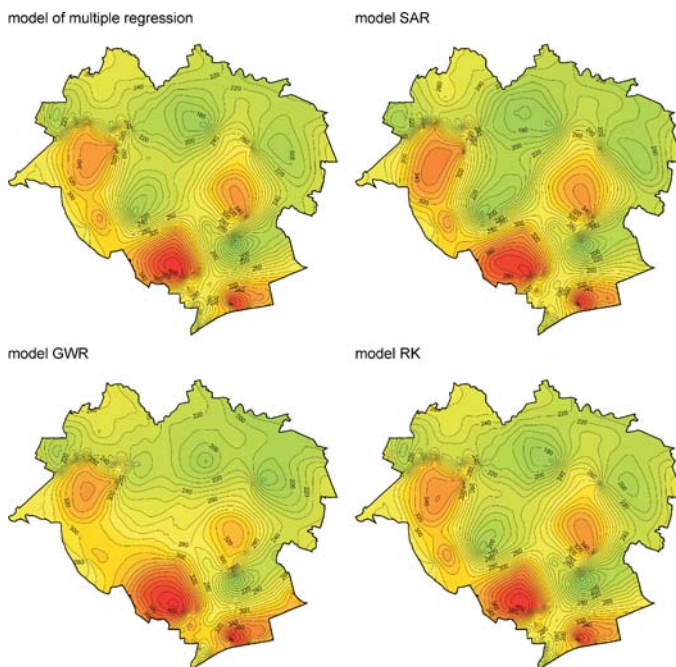
During the construction of the model regression – kriging, to construct variance – covariance matrix used spherical semivariograms models. In total, three iterations performed which led to the results shown in Table 6.

Table 6. *The estimates of regression-kriging model (source: own study).*

	<b>B</b>	<b>standard error. <math>\beta</math></b>	<b>t value</b>	<b>p value</b>
constant	269,669	112,499	2,397	0,017
water	-44,599	17,178	-2,596	0,009
energy	12,699	15,594	0,814	0,416
sewerage	-14,279	15,006	-0,951	0,342
gas	-60,167	17,133	-3,512	0,000
area	-86,230	30,602	2,818	0,005
geometry	-9,504	11,614	-0,818	0,414

The standard deviation of residuals was 59.16, which is comparable to the size of the linear multiple regression model. Also in this case not all the analyzed variables were statistically significant. RK advantage over the classical model regression because of the way the estimation and the inclusion of the spatial relationships in the form of spatial correlation in prices. The results of the study suggest, however, that the RK models are not always significantly better results than classical methods.

The models can be used for mass measurement using the additional information that carries spatial distribution of residues indicating the location of the impact.

Fig. 2. *Maps of land developed on the basis of the analyzed models (source: own study).*

Note that the scales assessing aspects of the property have been chosen in such a way that the adopted scale of zero value related to a typical property, usually appearing on the market. This makes it relatively easy opportunity to make the value of such property on a map, using spatial interpolation for example, by ordinary kriging (ordinary kriging). If we assume that the attribute values for a typical property is zero, then the value of the variable interpolation will be a fixed amount, and the rest of the model. In addition, SAR model should be considered a component resulting from spatial autocorrelation residuals. These maps are shown in Fig 2. The value of a particular property is the estimated amount was the result of spatial interpolation and appropriate adjustments due to the characteristics, resulting directly from the estimated model.

## 7. Conclusions

The problem of taking into account location as quantitative variable in mass appraisal models can be solved by the use of both spacial (GWR, SAR) and geostatistical spatial interpolation models. The combination of these two enables assessment of different attributes of property on its value but also presentation of the analysis results on different maps. Statistical models including spatial relationship can be successfully used in mass appraisal. One ought to remember that these are better tools than the classical methods only when one can notice spatial autocorrelation in transaction prices. This condition is satisfied in most local markets, although there may be exceptional circumstances when the location of the property does not affect its price. In that case the models would be equivalent to the classical methods. The current rapid development of both spatial data infrastructure and GIS tools support the conclusion that the presented methods are likely to widely used for mass valuation of real estate.

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## Masovna procjena – međunarodna pozadina, rješenja u Poljskoj i prijedlog novih metoda primjene

*SAŽETAK.* Cilj ovog rada je provesti općenito istraživanje rješenja masovne procjene izrađenih tijekom godina u izabranim zemljama te ih usporediti s metodama koje su usvojene u Poljskoj. Osim toga, autori su pokušali predložiti nove metode za procjenu vrijednosti nekretnina: geografski ponderirana regresija, prostorni autoregresivni modeli, regresijski kriging, naglašavajući njihove prednosti u teoretskom i u praktičnom smislu. Studija istraživanja provedena na primjeru grada Olsztyna (pokrajine Warmia i Mazury u Poljskoj) pokazala je određene prednosti predloženih metoda. Prije svega, njihovom kombinacijom omogućeno je ne samo procjenjivanje različitih atributa nekretnine prema njezinoj vrijednosti, već i prezentiranje rezultata analize na različitim kartama. Statistički modeli koji uključuju prostorne odnose mogu se uspješno koristiti u masovnoj procjeni. Treba se prisjetiti da su to bolji alati nego klasične metode samo kada se mogu ustanoviti prostorne autokorelacije u cijena transakcija. Ovaj uvjet je zadovoljen na većini lokalnih tržišta, iako ima i iznimnih okolnosti kada lokacija nekretnine ne utječe na njezinu cijenu. U tom slučaju, modeli bi bili jednaki klasičnim metodama.

*Ključne riječi:* modeli masovne procjene, vrednovanje, nekretnina, geografski ponderirana regresija, prostorno autoregresivan, regresijski kriging.

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