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# A STUDY OF THE ROLE OF GOVERNMENT IN INCOME AND WEALTH DISTRIBUTION BY INTEGRATING THE WALRASIAN GENERAL EQUILIBRIUM AND NEOCLASSICAL GROWTH THEORIES

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## ABSTRACT

This paper proposes a growth model of heterogeneous households with economic structure, wealth accumulation, endogenous labour supply, and tax rates. The paper is focused on effects of redistribution policies on income and wealth distribution, economic structure and economic growth. The paper integrates the Walrasian general equilibrium theory and neoclassical economic growth within a comprehensive framework. We overcome the controversial features in the two traditional theories by applying an alternative approach to households. We build an analytical framework for a disaggregated and microfounded general theory of economic growth with endogenous wealth accumulation. We simulate the model to identify equilibrium, stability and to plot the motion of the dynamic system with three groups. We also carry out comparative dynamic analysis with regard to the lump tax, human capital and propensity to use leisure time.

## KEY WORDS

lump tax, tax rates, Walrasian general equilibrium theory, neoclassical growth theory, income and wealth distribution

## CLASSIFICATION

JEL: C53, C63, E47

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## INTRODUCTION

It is well known that Walras' theory of pure exchange and production economies have provided the underpinning of contemporary general equilibrium theory. The Walrasian general equilibrium theory was initially proposed by Walras and in the 1950s further formalized by Arrow, Debreu and others [1-9]. According to Arrow [10], "From the time of Adam Smith's Wealth of Nations in 1776, one recurrent theme of economic analysis has been the remarkable degree of coherence among the vast numbers of individuals and seemingly separate decisions about the buying and selling of commodities. In everyday, normal experience, there is something of a balance between the amounts of goods and services that some individuals want to supply and the amount that other different individuals want to sell. Would be buyers ordinarily count correctly on being able to carry out their intensions, and would-be sellers do not ordinarily find themselves producing great amounts of goods that they cannot sell. This experience of balance is indeed so widespread that it raises no intellectual disquiet among laymen; they take it so much for granted that they are not disposed to understand the mechanism by which it occurs." The general equilibrium theory is important to understand economic mechanisms of production, consumption, and exchanges with heterogeneous industries and households. Nevertheless, this theory has not been successfully generalized and extended to growth theory of heterogeneous households with endogenous wealth. The purpose of this study is to introduce economic mechanisms of endogenous wealth accumulation with redistribution policy.

Walras introduced saving and capital accumulation in his general equilibrium theory. Nevertheless, his treatments of capital accumulation have many shortcomings, particularly, in the light of modern neoclassical growth model. As pointed out by Impicciatore et al. [11], "because of the absence of an explicit temporal indexation of the variables, the timeframe of Walras' theory is left to the reader's interpretation. In particular, it remains an open question whether the model is static (that is, a single-period model) or dynamic, and, in the latter case, if it pertains to the short run or long run." In fact, there is no profound microeconomic mechanism for wealth accumulation in Walras' original theory. Over years there are different attempts to further develop Walras' capital accumulation within Walras' framework of heterogeneous households (e.g., [12-17]). The common problem for these approaches is the lack of proper microeconomic foundation for wealth accumulation. To overcome this problem, Impicciatore et al. [11] propose a model in which it is assumed that consumers store capital goods in order to supply their services to the production sector in the next period under the condition that capital goods exiting in one period totally depreciate at the end of the period. The approach still relies on the strict assumption on household saving behaviour. This study introduces an alternative approach for modeling wealth accumulation with the traditional Walrasian general equilibrium framework of heterogeneous households.

There are some other studies in the literature of economic growth which introduce neoclassical growth theory into the general equilibrium analysis (e.g., [18]). As reviewed by Shoven and Whalley [19], "Most contemporary applied general models are numerical analogs of traditional two-sector general equilibrium models popularized by James Meade, Harry Johnson, Arnold Harberger, and others in the 1950s and 1960s. Earlier analytical work with these models has examined the distortionary effects of taxes, tariffs, and other policies, along with functional incidence questions." The history of analytical economics shows that it is not easy to properly model economic growth with wealth and income distribution. In fact, there are only a few formal dynamic models which explicitly deal with distribution issues among heterogeneous households in the neoclassical growth theory [22]. On the other hand, the

Arrow-Debreu general economic theory deals with economic equilibrium issues with heterogeneous households and firms. It is desirable to integrate the economic mechanisms of the two main approaches in economics into a single analytical framework. This study builds a model of integrating the two theories with an alternative approach to households behaviour by Zhang [23]. We develop a model to deal with interdependence between wealth and income distribution among heterogeneous households within the Uzawa two-sector growth modeling framework. This study synthesizes the ideas in the two-sector model with endogenous labour by Zhang [24] and the growth model with heterogeneous groups by Zhang [25]. In Zhang's two papers, no government distribution is introduced. This study introduces lump taxes (and subsidies) and taxes on production, consumption, wealth income and wages. The paper is organized as follows. Section 2 introduces the basic model with wealth and income distribution with distribution policy. Section 3 examines dynamic properties of the model and simulates the model with three types of households. Section 4 carries out comparative dynamic analysis with regard to redistribution policies, propensities to save, and propensities to use leisure time. Section 5 concludes the study.

## THE BASIC MODEL

The economy consists of two sectors, like in the two-sector model by Uzawa [26]. Most aspects of the production sectors are neoclassical [21, 22, 27]. Different from the Solow one-sector growth model, the Uzawa two-sector growth model treats consumption and capital goods as different commodities, which are produced in two distinct sectors. The population is constant and homogeneous. There is only one malleable capital good. In the Uzawa model, capital goods can be used as an input in both sectors in the economy. Capital depreciates at a constant exponential rate $\delta_k$, which is independent of the manner of use. Households own assets of the economy and distribute their incomes to consume and save. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. Each group has a fixed population, $\overline{N}_j$, ($j = 1, ..., J$). It should be noted that in the Walrasian general equilibrium theory, $\overline{N}_j = 1$. Let prices be measured in terms of capital goods and the price of the commodity be unity. We denote the wage rate of worker of type $j$ and rate of interest by $w_j(t)$ and $r(t)$, respectively.

The total capital stock $K(t)$ is allocated between the two sectors. We use subscript index $i$ and $s$ to stand for capital goods and consumer goods sector, respectively. We use $N_j(t)$ and $K_j(t)$ to stand for the labor force and capital stocks employed by sector $j$. We use $T_j(t)$ and $\overline{T}_j(t)$ to stand for, respectively, the work time and leisure time of a typical worker in group $j$ The total qualified labor supply $N(t)$ is defined by

$$N(t) = \sum_{j=1}^{J} h_j T_j(t) \overline{N}_j . \tag{1}$$

We introduce

$$k_j(t) \equiv \frac{K_j(t)}{N_j(t)}, \ \ n_j(t) \equiv \frac{N_j(t)}{N(t)}, \ \ k(t) \equiv \frac{K(t)}{N(t)}, \ \ j = i, s .$$

The assumption of labour force being fully employed implies

$$N_i(t) + N_s(t) = N(t). \tag{2}$$

## THE CAPITAL GOODS SECTOR

It is well known that in modern literature of economic growth the Cobb-Douglas production function has been widely applied to different issues (see for instance [28-30]). We assume that production is to combine the labor force $N_i(t)$ and physical capital $K_i(t)$. We use $\tau_j$ to stand for the tax rate on sector $j's$ output, $j = i, s$. Let $\tau_i(t)$ represent the tax rate on the capital goods sector. The function $F_i(t)$ is specified as

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \ \ A_i, \alpha_i, \beta_i > 0, \ \ \alpha_i + \beta_i = 1 \tag{3}$$

where $A_i$, $\alpha_i$ and $\beta_i$ are parameters. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. The rate of interest and wage rate are determined by markets. For any individual firm $r(t)$ and $w_j(t)$ are given at each point of time. The production sector chooses the two variables $K_i(t)$ and $N_i(t)$ to maximize its profit. The marginal conditions are given by

$$r(t) + \delta_k = \alpha_i \bar{\tau}_i(t) A_i K_i^{-\beta_i}(t) N_i^{\beta_i}(t), \ \ w_j(t) = \bar{\tau}_i(t) h_j w(t), \tag{4}$$

where

$$\bar{\tau}_i(t) \equiv 1 - \tau_i(t), \ \ w(t) \equiv \beta_i A_i K_i^{\alpha_i}(t) N_i^{-\alpha_i}(t).$$

## CONSUMER GOODS SECTOR

We specify the production function of the consumer goods sector as follows

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \ \ A_s, \ \alpha_s, \beta_s > 0, \ \ \alpha_s + \beta_s = 1. \tag{5}$$

The marginal conditions are

$$r(t) + \delta_k = \alpha_s \bar{\tau}_s(t) p(t) A_s K_s^{-\beta_s}(t) N_s^{\beta_s}(t), \ \ w_j(t) = \beta_s \bar{\tau}_s(t) h_j \, p(t) A_s K_s^{\alpha_s}(t) N_s^{-\alpha_s}(t). \tag{6}$$

## CONSUMER BEHAVIOURS AND WEALTH DYNAMICS

In this study, we use an alternative approach to modeling behaviour of households proposed by Zhang [23]. The preference over current and future consumption is reflected in the consumer's preference structure over leisure time, consumption and saving. Let $\bar{k}_j(t)$ stand for the per capita wealth of group $j$ We have $\bar{k}_j(t) = \bar{K}_j(t) / \bar{N}_j$, where $\bar{K}_j(t)$ is the total wealth held by group $j$ We use $\tau_j$ to stand for the lump sum transfer that group $j$'s representative household receives from the government. Per capita current disposable income from the interest payment $r(t)\bar{k}_j(t)$ and the wage payment $T_j(t) w_j(t)$ is given by

$$y_j(t) = (1 - \tau_{rj}) r(t) \bar{k}_j(t) + (1 - \tau_{wj}) T_j(t) w_j(t) + \tau_j,$$

where $\tau_{rj}$ and $\tau_{wi}$ are respectively the tax rates on the income from wealth and on the wage income. The total value of wealth that consumers can sell to purchase goods and to save is equal to $\bar{k}_j(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is the sum of the current disposable income and the value of wealth. That is

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{7}$$

The disposable income is used for saving and consumption. It should be noted that the value, $\bar{k}_j(t)$, (i.e., $p(t)\bar{k}_j(t)$ with $p(t) = 1$), in the above equation is a flow variable. Under the assumption that selling wealth can be conducted instantaneously without any transaction cost, we consider $\bar{k}_j(t)$ as the amount of the income that the consumer obtains at time $t$ by selling all of his wealth. Hence, at time $t$ the consumer has the total amount of income equaling $\hat{y}_j(t)$ to distribute among saving and consumption.

The representative household from group $j$ would distribute the total available budget between savings $s_j(t)$ and consumption of goods $c_j(t)$. Let the tax rate on group $j's$ consumption be denoted by $\tau_{cj}$. The budget constraint is given by

$$\left(1 + \tau_{cj}\right)p(t)c_j(t) + s_j(t) = \hat{y}_j(t). \tag{8}$$

Denote $\overline{T}_j(t)$ the leisure time at time $t$ and the (fixed) available time for work and leisure by $T_0$. The time constraint is expressed by

$$T_j(t) + \overline{T}_j(t) = T_0. \tag{9}$$

Substituting (11) into (10) implies

$$\left(1 - \tau_{wj}\right)w_j(t)\overline{T}_j(t) + \left(1 + \tau_{cj}\right)p(t)c_j(t) + s_j(t) = \bar{y}_j(t), \tag{10}$$

where

$$\bar{y}_j(t) \equiv \left(\left(1 - \tau_{rj}\right)r(t) + 1\right)\bar{k}_j(t) + \left(1 - \tau_{wj}\right)T_0\, w_j(t) + \tau_j.$$

In this model, at each point of time, consumers have three variables to decide. We assume that utility level $U_j(t)$ that the consumers obtain is dependent on the leisure time, $T_j(t)$, the consumption level of consumption goods $c_j(t)$ and savings $s_j(t)$ as follows

$$U_j(t) = \overline{T}_j^{\sigma_{0j}}(t)c_j^{\xi_{0j}}(t)s_j^{\lambda_{0j}}(t), \;\; \sigma_{0j}, \xi_{0j}, \lambda_{0j} > 0,$$

where $\sigma_{0j}$ is the propensity to use leisure time, $\xi_{0j}$ is the propensity to consume consumption goods, and $\lambda_{0j}$ propensity to own wealth. Some growth models with endogenous wealth accumulation consider heterogeneous households. Nevertheless, the heterogeneity in these studies is by the differences in the initial endowments of wealth among different types of households rather than in preferences (see for instance [31-35]). Different households are essentially homogeneous in the sense that all the households have the same preference utility function in the approach. In our approach we consider different types of households have different utility functions.

Maximizing the utility subject to (10) yields

$$w_j(t)\overline{T}_j(t) = \sigma_j\, \bar{y}_j(t), \;\; p(t)c_j(t) = \xi_j\, \bar{y}_j(t), \;\; s_j(t) = \lambda_j\, \bar{y}_j(t), \tag{11}$$

where

$$\sigma_j \equiv \frac{\rho_j\, \sigma_{0j}}{1 - \tau_{wj}}, \;\; \xi_j \equiv \frac{\rho_j\, \xi_{0j}}{1 + \tau_{cj}}, \;\; \lambda_j \equiv \rho_j\, \lambda_{0j}, \;\; \rho_j \equiv \frac{1}{\sigma_{0j} + \xi_{0j} + \lambda_{0j}}.$$

We now find dynamics of capital accumulation. According to the definition of $s_j(t)$, the change in the household's wealth is given by

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t) = \lambda_j\, \bar{y}_j(t) - \bar{k}_j(t). \tag{12}$$

This equation simply states that the change in wealth is equal to the saving minus dissaving.

## DEMAND AND SUPPLY

The output of the consumer goods sector is consumed by the households. That is

$$\sum_{j=1}^{J} c_j(t)\overline{N}_j = F_s(t) \tag{13}$$

As output of the capital goods sector is equal to the depreciation of capital stock and the net savings, we have

$$S(t) - K(t) + \delta_k K(t) = F_i(t), \tag{14}$$

where

$$S(t) \equiv \sum_{j=1}^{J} s_j(t) \bar{N}_j, \quad K(t) = \sum_{j=1}^{J} \bar{k}_j(t) \bar{N}_j .$$

### CAPITAL BEING FULLY UTILIZED

Total capital stock $K(t)$ is allocated to the two sectors and households. As full employment of labor and capital is assumed, we have

$$K_i(t) + K_s(t) = K(t). \tag{15}$$

### THE GOVERNMENT'S BUDGET

The government spends all the tax income on redistribution. We have

$$\tau_i(t) F_i(t) + \tau_s(t) p(t) F_s(t) + \Gamma_r(t) + \Gamma_w(t) + \Gamma_c(t) = \sum_{j=1}^{J} \tau_j \bar{N}_j , \tag{16}$$

where

$$\Gamma_r(t) \equiv \sum_{j=1}^{J} \tau_{rj}\, r(t) \bar{k}_j(t) \bar{N}_j , \quad \Gamma_w(t) \equiv \sum_{j=1}^{J} \tau_{wj}\, T_j(t) w_j(t) \bar{N}_j , \quad \Gamma_c(t) \equiv \sum_{j=1}^{J} \tau_{cj}\, p(t) c_j(t) \bar{N}_j .$$

### THE BEHAVIOUR OF THE GOVERNMENT

The government chooses the following tax and subsidy rates $\tau_i(t)$, $\tau_s(t)$, $\tau_{rj}(t)$, $\tau_{wj}(t)$, $\tau_{cj}(t)$, $\tau_j(t)$, $j = 1, ..., J$. There is only one budget constraint. For simplicity of discussion, we assume that the tax rates on the two sectors are interrelated as follows

$$\tau_s(t) = \tau_0\, \tau_i(t), \tag{17}$$

where $\tau_0$ is a constant. The two sectors' tax rates are proportional. We further assume that the tax rate on the capital sector is determined by (16). From (16) and (17)we have

$$\tau_i(t) = \frac{\Gamma(t)}{F_i(t) + \tau_0\, p(t) F_s(t)}, \tag{18}$$

where

$$\Gamma(t) \equiv \sum_{j=1}^{J} \tau_j \bar{N}_j - \Gamma_r(t) - \Gamma_w(t) - \Gamma_c(t).$$

We complete the model. As far as economic structure and growth theory with endogenous capital are concerned, our model is general in the sense that the model is built on the basis of economic mechanisms of the Walras-Arrow-Debreu general economic theory, the Solow growth model and the Uzawa two sector model. For instance, if the economic system has only two sectors, then the Arrow-Debreu equilibrium theory (which treats capital exogenous) can be considered as a special case of our model with heterogeneous households with endogenous leisure time and wealth. It is straightforward to see that the Solow-one sector and the Uzawa two sector model are special cases of our model. As our model also includes labor supply and tax policies, it is closely related with some other growth models in the literature of, for instance, public economics. We now examine behaviour of the economic system.

## THE DYNAMICS AND ITS PROPERTIES

The dynamic system consists of any (finite) number of households. As behavioural patterns vary among different types, the dynamic system is of high dimension. The following lemma shows that the dimension of the dynamical system is equal to the number of types of households. We also provide a computational procedure for calculating all the variables at any point of time. Before stating the lemma, we introduce a new variable $z(t)$ by

$$z(t) \equiv \frac{r(t) + \delta_k}{w_j(t)/h_j}.$$

### LEMMA

The motion of the economic system is determined by $J$ differential equations with $z(t)$, $\tau_i(t)$ and $\{\bar{k}_j(t)\}$, where $\{\bar{k}_j(t)\} \equiv (\bar{k}_3(t), \cdots, \bar{k}_J(t))$, as the variables

$$\dot{z}(t) = \Lambda_1\left(z(t),\, \tau_i(t),\, \{\bar{k}_j(t)\}\right),$$
$$\dot{\tau}_i(t) = \Lambda_2\left(z(t),\, \tau_i(t),\, \{\bar{k}_j(t)\}\right),$$
$$\dot{\bar{k}}_j(t) = \Lambda_j\left(z(t),\, \tau_i(t),\, \{\bar{k}_j(t)\}\right),\ \ j = 3, \ldots, J, \tag{19}$$

in which $\Lambda_j(t)$ are unique functions of $z(t)$, $\tau_i(t)$, and $\{\bar{k}_j(t)\}$ defined in the appendix. At any point of time the other variables are unique functions of $z(t)$, $\tau_i(t)$, and $\{\bar{k}_j(t)\}$ determined by the following procedure: $\bar{k}_1(t)$ and $\bar{k}_2(t)$ by (A21) $\rightarrow$ $r(t)$ and $w_j(t)$ by (A3) $\rightarrow$ $\bar{y}_j(t)$ by (A4) $\rightarrow$ $N(t)$ by (A13) $\rightarrow$ $K_i(t)$ and $K_s(t)$ by (A15) $\rightarrow$ $N_i(t)$ and $N_s(t)$ by (A1) $\rightarrow$ $F_i(t)$ by (3) $\rightarrow$ $F_s(t)$ by (5) $\rightarrow$ $p(t)$ by (A8) $\rightarrow$ $\bar{T}(t)_j$, $c_j(t)$, and $s_j(t)$ by (11) $\rightarrow$ $T_j(t) = T_0(t) - \bar{T}_j(t) \rightarrow K(t) = K_i(t) + K_s(t)$.

The lemma gives a computational procedure for plotting the motion of the economic system with any number of types of households. It is well known that calibration of general equilibrium involves solving high-dimensional nonlinear equations. With regard to the Arrow-Debreu concept of general equilibrium the final stage of analysis is to find a price vector at which excess demand is zero [36]. There are numerical approaches for calculating equilibria (e.g., [37-38]). We can apply these traditional methods to find how the prices and other variables are related to the variables in the differential equations. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model with the following parameters:

$$A_i = 1.3,\ A_s = 1,\ \alpha_i = 0.29,\ \alpha_s = 0.32,\ T_0 = 1,\ \tau_0 = 0.8,\ \delta_k = 0.05,$$
$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 50 \\ 300 \\ 200 \end{pmatrix},\ \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0.6 \end{pmatrix},\ \begin{pmatrix} \xi_{10} \\ \xi_{20} \\ \xi_{30} \end{pmatrix} = \begin{pmatrix} 0.12 \\ 0.16 \\ 0.18 \end{pmatrix},\ \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.78 \\ 0.75 \\ 0.7 \end{pmatrix},\ \begin{pmatrix} \sigma_{10} \\ \sigma_{20} \\ \sigma_{30} \end{pmatrix} = \begin{pmatrix} 0.25 \\ 0.18 \\ 0.15 \end{pmatrix},$$
$$\begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} -0.01 \\ 0.03 \\ 0.15 \end{pmatrix},\ \begin{pmatrix} \tau_{r1} \\ \tau_{r2} \\ \tau_{r3} \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.03 \\ 0.03 \end{pmatrix},\ \begin{pmatrix} \tau_{w1} \\ \tau_{w2} \\ \tau_{w3} \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.03 \\ 0.03 \end{pmatrix},\ \begin{pmatrix} \tau_{c1} \\ \tau_{c2} \\ \tau_{c3} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.01 \\ 0.01 \end{pmatrix}. \tag{20}$$

The population of group $2$ is largest, while the population of group $3$ is the next. The human capital level of group $1$ is highest, while the human capital level of group $3$ is lowest. The capital goods sector and consumer goods sector's total productivities are respectively 1,3 and 1.

We specify the values of the parameters, $\alpha_j$ in the Cobb-Douglas productions approximately equal to 0,3 (for instance [39, 40]). The depreciation rate of physical capital is specified at 0,05. Group 1 propensity to save is 0,78 and group 3 propensity to save is 0,7 The value of group 2 propensity is between the two groups. The tax rates on different groups are mild. The rich group pays lump tax, $\tau_1 = -0{,}01$. Groups 2 and 3 receive subsidies, respectively, $\tau_3 = 0{,}02$. We specify the initial conditions as follows

$$z(0) = 0.048, \quad \tau_i(0) = 0.047, \quad \bar{k}_3(0) = 2.3.$$

The motion of the variables is plotted in Figure 1. The output level of the capital goods sector is enhanced and the output level of the consumer goods sector is slightly lowered over time. The tax rates on the capital and consumer goods sectors fall slightly. The rate of interest rises slightly. The price of consumer goods and the wage rates of the three groups vary slightly. The total supply and labor force employed by the consumer goods sector are reduced, and the labor force employed by the capital goods sector is augmented slightly. The total capital and the capital input of the consumer goods sector are reduce slightly, and the capital input of the consumer goods sector is increased. The national output is lowered. Group 1 and group 3 wealth levels are increased, group 2 wealth is diminished. Group 1 and group 3 reduce work hours, and group 2 increases work hours. Group 1 and group 3 consumption levels are increased, group 2 consumption level is diminished. It should be noted that there are empirical studies which find negative relationships between wealth and labor supply (for instance [41-43]). In our model with the specified parameter values, the negative relationship is obvious for the groups.

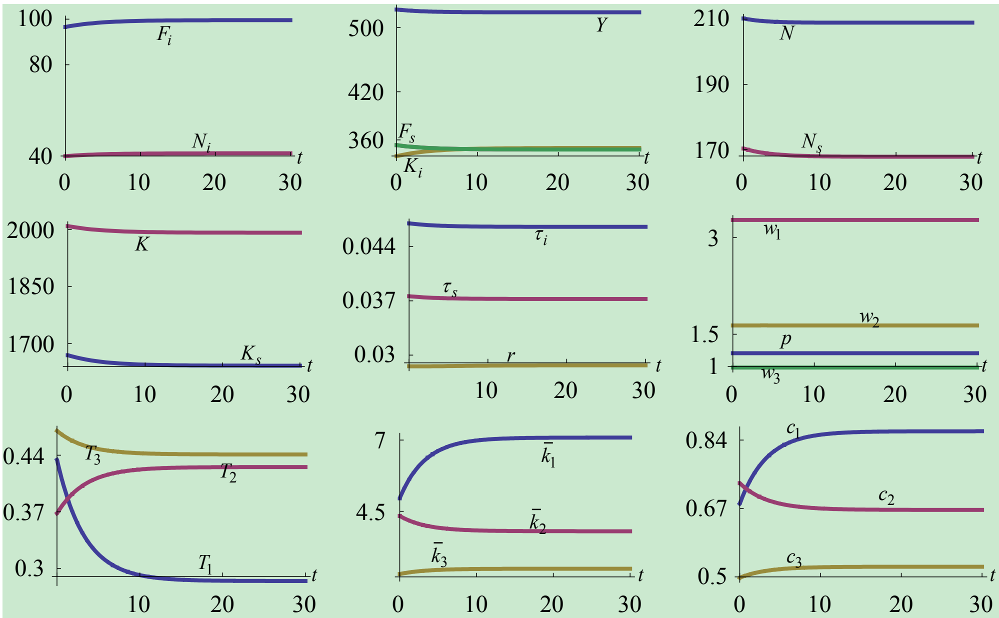

**Figure 1.** The motion of the economic system.

It is straightforward to confirm that the variables become stationary. The simulation confirms that the system has a unique equilibrium. We list the equilibrium values in (21).

$$\begin{gathered}\tau_i = 0.047, \ \tau_s = 0.037, \ F_i = 99.6, \ F_s = 348.09, \ w_1 = 3.27, \ w_2 = 1.64, \ w_3 = 0.98, \\ r = 0.029, \ p = 1.21, \ N = 208.85, \ N_i = 41.20, \ N_s = 167.75, \ K = 1991.98, \ K_i = 349.98, \\ K_s = 1642, \ \bar{k}_1 = 7.09, \ \bar{k}_2 = 3.80, \ \bar{k}_3 = 2.49, \ T_1 = 0.29, \ T_2 = 0.43, \ T_3 = 0.44, \ c_1 = 0.86, \\ c_2 = 0.67, \ c_3 = 0.53. \end{gathered} \tag{21}$$

It is straightforward to calculate the three eigenvalues as follows

$$\{-0.30,\ -0.30,\ -0.24\}.$$

The eigenvalues are real and negative. The unique equilibrium is locally stable.

## COMPARATIVE DYNAMIC ANALYSIS

We already simulated the motion of the national economy under (20). We are now concerned with how the economic system reactions to some exogenous change. As the lemma gives the computational procedure to calibrate the motion of all the variables, it is straightforward to examine effects of change in any parameter on transitory processes as well stationary states of all the variables. We introduce a variable $\overline{\Delta} x_j(t)$ which stands for the change rate of the variable, $x_j(t)$ in percentage due to changes in the parameter value.

### GROUP 1 PAYS MORE LUMP TAX

First, we examine the case that group 1 pays more lump tax to the government in the following way: $\tau_1$: $-0{,}01 \rightarrow 0{,}015$. The simulation results are given in Figure 2. The immediate effects on group 1 are that the group reduces the consumption and wealth levels and works less hours, even though the change in the lump tax has little effects on these variables. Group 2 increases the consumption and wealth levels and works longer hours, even though these variables are slightly affected in the long term. Group 3 consumption and wealth levels and work hours are slightly affected. The tax rates on the capital and consumer goods are increased in association with the lessened lump tax on the rich group. The wage rates are slightly augmented. The rate of interest falls initially and rises in the long term. The total labor supply, total capital and national output are increased initially but become to the original stationary values in the long term. The economic structure shifts initially but maintains unaffected in the long term. The price is slightly affected. We see that the long-term effect of increasing group 1 lump tax is to reduce the tax rates on the production sectors and has almost no impact on the other variables (except the small change the rate of interest). It should be noted that from Figure 1 we see that group 1 consumption and wealth experience large changes during very short period, even though it does not take long for the variables to approach their stationary values. This character makes the group's wealth and consumption levels have strong reactions to exogenous changes as illustrated in Figure 2.

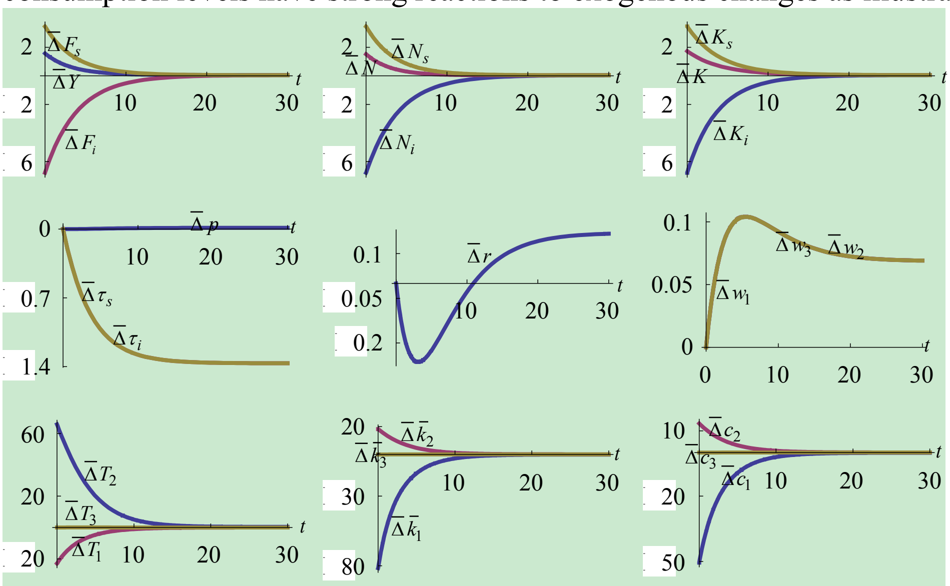


**Figure 2.** A rise in Group 1 lump tax.

## GROUP 3 HUMAN CAPITAL BEING ENHANCED

Group 3 human capital is changed as follows: $h_3$: 0,6 $\rightarrow$ 0,7. We plot the simulation results in Figure 3. Group 3 wage rate is increased, while the other two groups' wage rates are only slightly affected. Group 3 work time is increased, as the opportunity cost of staying at home is increased. The other two groups' work hours are slightly reduced. As group 3 works more effectively, the national output, the output levels and two input factors of the two sectors are all increased. Hence, an improvement in the group's human capital enhances the national and sectorial economic performance. The tax rates on the two sectors are slightly increased. Group 3 wealth and consumption levels are increased. The other two groups' wealth and consumption levels are slightly affected in the long term. It should be noted that relations between wealth and income distribution and growth have caused attention of economists long time ago. For instance, Kaldor [44] argues that as income inequality is enlarged, growth should be encouraged as savings are promoted. This positive relation between income inequality and growth is also observed in studies, [45-47]. There are other studies which find negative relations between income inequality and economic growth. Solow [48] makes a hypothesis on a negative relationship between income inequality and growth. Some formal models which predicate negative relations are referred to, for instance [49-51]. Some empirical studies by, for instance, Persson and Tabellini [52] also confirm negative relations. From our simulation, we see that relations between inequality and economic growth are complicated in the sense that these relations are determined by many factors. For instance, as group increases the level of human capital, the income and wealth gaps between group 1 and group 3 are reduced in association with positive economic growth. On the other hand, if group 1 reduces the level of human capital, the income and wealth gaps between group 1 and group 3 are reduced in association with negative economic growth. It can be seen that different empirical studies expectably may give different answers.

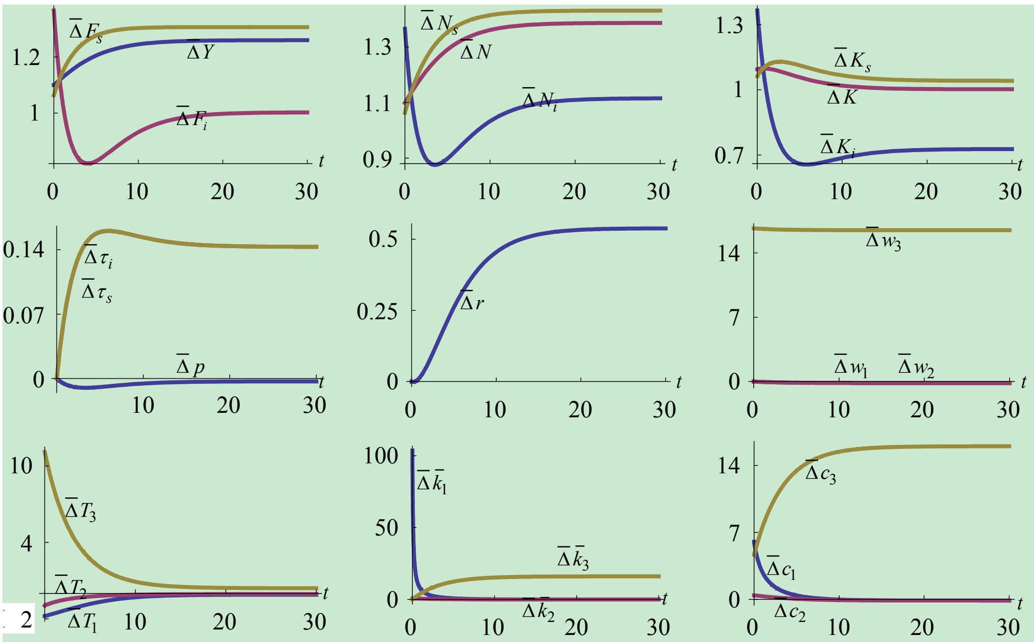


**Figure 3.** Group 3 human capital being enhanced.

## GROUP 3 PROPENSITY TO USE LEISURE TIME BEING AUGMENTED

We now study what will happen to the economic system if group 3 propensity to use leisure time is increased as follows: $\sigma_{03}$: 0,15 $\rightarrow$ 0,18. The simulation results are plotted in Figure 4. As group 3 propensity to use leisure time is increased, the group's work hours are reduced. The households from the group stay at home longer. The total labor supply is reduced. The reduction in the total labor supply partly explains the rise in the wage rates of the three groups. Each of the two sectors employs less labor. The price of consumer goods is increased slightly. Group 3 wealth and consumption are reduced as the household stays longer at home. In the long term the other two groups' time distribution, and wealth and consumption levels. The tax rates on the two sectors are reduced. The total capital and capital stocks employed by each sector are reduced. The national output level and output levels of the two sectors are all reduced. The rate of interest rises initially and subsequently falls.

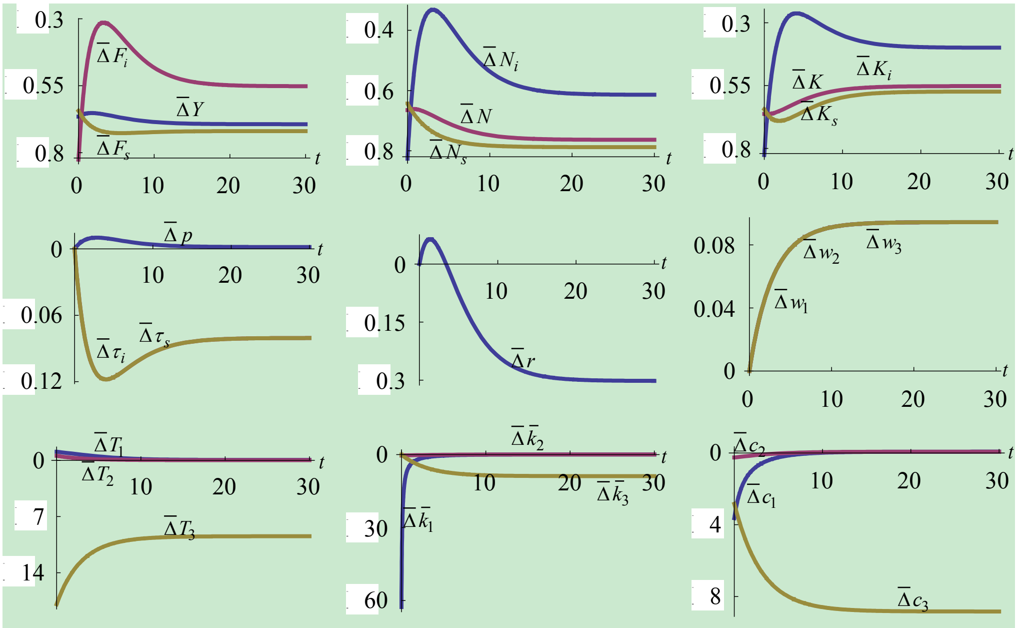

**Figure 4.** A rise in Group 3 propensity to save.

## CONCLUDING REMARKS

This paper proposed a growth model of heterogeneous households with economic structure. The framework is influenced by the Walrasian general equilibrium and neoclassical growth theories. We were mainly concerned with the role of government in income and wealth distribution in an economy with endogenous wealth accumulation. The economic system consists of one capital goods sector, one consumer goods sector, and any number of households. Different from the traditional Uzawa model where the population is homogeneous, the population is classified into different groups. The model shows how wealth accumulation, income and wealth distribution, time distribution and division of labor interact under perfect competition and government intervention over time. The motion is described by a set of differential equations. For illustration, we simulated the motion of the economic system with three groups. We identified the existence of a unique stable equilibrium point. We also carried out comparative dynamic analysis. We discussed implications of our simulation results for empirical studies in the literature of relations between work time and wealth and the literature of relations among wealth and income distribution and economic growth. Because our model is structurally general, it may be generalized and extended.

## APPENDIX: PROVING THE LEMMA

By (4) and (6), we obtain

$$z \equiv \frac{r + \delta_k}{w_j / h_j} = \frac{N_i}{\bar{\beta}_i K_i} = \frac{N_s}{\bar{\beta}_s K_s}, \tag{A1}$$

where $\bar{\beta}_j \equiv \beta_j / \alpha_j$. From (A1) and (2), we obtain

$$\bar{\beta}_i K_i + \bar{\beta}_s K_s = \frac{N}{z}. \tag{A2}$$

Insert (A1) in (4)

$$r = \alpha_r \bar{\tau}_i z^{\beta_i} - \delta_k, \;\; w_j = \alpha_j \bar{\tau}_i z^{-\alpha_i}, \tag{A3}$$

where

$$\alpha_r = \alpha_i A_i \bar{\beta}_i^{\beta_i}, \;\; \alpha_j = h_j \beta_i A_i \bar{\beta}_i^{-\alpha_i}.$$

Hence, we determine the rate of interest and the wage rates as functions of $\tau_i$ and z From (A3) and the definitions of $\bar{y}_j$, we have

$$\bar{y}_j = g_j \bar{k}_j + \bar{g}_j, \tag{A4}$$

where

$$g_j(z, \tau_i) \equiv \alpha_r \bar{\tau}_i \bar{\tau}_{rj} z^{\beta_i} + \delta_j, \;\; \bar{g}_j(z, \tau_i) \equiv T_0 \alpha_j \bar{\tau}_{wj} \bar{\tau}_i z^{-\alpha_i} + \tau_j, \;\; \delta_j \equiv 1 - \bar{\tau}_{rj} \delta_k,$$
$$\bar{\tau}_{rj} \equiv 1 - \tau_{rj}, \;\; \bar{\tau}_{wj} \equiv 1 - \tau_{wj}.$$

Insert $p c_j = \xi_j \bar{y}_j$ in (13)

$$\sum_{j=1}^{J} \xi_j \bar{N}_j \bar{y}_j = p F_s. \tag{A5}$$

Substituting (A4) in (A5) yields

$$\sum_{j=1}^{J} \tilde{g}_j \bar{k}_j = p F_s - \tau, \tag{A6}$$

where

$$\tilde{g}_j(z, \tau_i) \equiv \xi_j \bar{N}_j g_j, \;\; \tau(z, \tau_i) \equiv \sum_{j=1}^{J} \xi_j \bar{N}_j \bar{g}_j.$$

From (4) and (6), we solve

$$r + \delta_k = \alpha_i \bar{\tau}_i A_i K_i^{-\beta_i} N_i^{\beta_i} = \alpha_s \bar{\tau}_s p A_s K_s^{-\beta_s} N_s^{\beta_s}. \tag{A7}$$

Inserting (A1) in (A7), we have

$$p = \frac{\alpha_i A_i \bar{\beta}_i^{\beta_i} \bar{\tau}_i z^{\beta_i - \beta_s}}{\alpha_s A_s \bar{\tau}_s \bar{\beta}_s^{\beta_s}}. \tag{A8}$$

From (6), we have

$$p F_s = \frac{w_1 N_s}{h_1 \beta_s \bar{\tau}_s}. \tag{A9}$$

From (A9) and (A1), we have

$$p F_s = \frac{\bar{\beta}_s w_1 z K_s}{h_1 \beta_s \bar{\tau}_s}. \tag{A10}$$

Insert (A10) in (A6)

$$\sum_{j=1}^{J} \tilde{g}_j \bar{k}_j = g_0 K_s - \tau, \tag{A11}$$

where

$$g_0(z, \tau_i) \equiv \frac{\bar{\beta}_s w_1 z}{h_1 \beta_s \bar{\tau}_s}.$$

Using (1) and (9), we get

$$N = T_0 \sum_{j=1}^{J} h_j \bar{N}_j - \sum_{j=1}^{J} \frac{h_j \sigma_j \bar{y}_j \bar{N}_j}{w_j}, \tag{A12}$$

in which we also use $w_j \bar{T}_j = \sigma_j \bar{y}_j$. Substitute (A4) into (A12)

$$N = \tilde{\varphi}_0 - \sum_{j=1}^{J} \tilde{\varphi}_j \bar{k}_j, \tag{A13}$$

where

$$\tilde{\varphi}_0(\tau_i, z) \equiv \sum_{j=1}^{J} \left( T_0 - \frac{\sigma_j \bar{g}_j}{w_j} \right) h_j \bar{N}_j, \ \ \tilde{\varphi}_j(\tau_i, z) \equiv \frac{h_j \sigma_j \bar{N}_j g_j}{w_j}.$$

From (15), we have

$$K_i + K_s = K = \sum_{j=1}^{J} \bar{k}_j \bar{N}_j. \tag{A14}$$

Solve (A2) and (A14) with $K_i$ and $K_s$ as the variables

$$K_i = \beta \bar{\beta}_s \sum_{j=1}^{J} \bar{k}_j \bar{N}_j - \frac{\beta N}{z}, \ \ K_s = \frac{\beta N}{z} - \beta \bar{\beta}_i \sum_{j=1}^{J} \bar{k}_j \bar{N}_j, \tag{A15}$$

where $\beta \equiv 1/(\bar{\beta}_s - \bar{\beta}_i)$. From (A3), we determine $r$ and $w_j$ as functions of $\tau_i$ and $z$. Insert $K_s$ from (A15) in (A11)

$$\sum_{j=1}^{J} \left( \tilde{g}_j + \beta \bar{\beta}_i g_0 \bar{N}_j \right) \bar{k}_j = \frac{\beta g_0 N}{z} - \tau. \tag{A16}$$

Insert (A13) in (A16)

$$\phi_1 \bar{k}_1 + \phi_2 \bar{k}_2 = \phi_0, \tag{A17}$$

where

$$\phi_0\left(z, \tau_i, \{\bar{k}_j\}\right) \equiv \frac{\beta g_0}{z} \tilde{\varphi}_0 - \tau - \sum_{j=3}^{J} \phi_j \bar{k}_j, \ \ \phi_j(z, \tau_i) \equiv \tilde{g}_j + \beta \bar{\beta}_i g_0 \bar{N}_j + \frac{\tilde{\varphi}_j \beta g_0}{z}.$$

in which $\{\bar{k}_j\} \equiv (\bar{k}_3, \ldots \bar{k}_J)$. From (14) we have

$$\sum_{j=1}^{J} s_j \bar{N}_j - (1 - \delta_k) \sum_{j=1}^{J} \bar{k}_j \bar{N}_j = \frac{w N_i}{\beta_i}, \tag{A18}$$

where we also use $F_i = w N_i / \beta_i$. Insert $s_j = \lambda_j \bar{y}_j$ and $N_i = \bar{\beta}_i K_i z$ from (A1) in (A18)

$$\sum_{j=1}^{J} \lambda_j \bar{N}_j \bar{y}_j - (1 - \delta_k) \sum_{j=1}^{J} \bar{k}_j \bar{N}_j = \frac{\beta \bar{\beta}_s \bar{\beta}_i w z}{\beta_i} \sum_{j=1}^{J} \bar{k}_j \bar{N}_j - \frac{w \bar{\beta}_i \beta N}{\beta_i}. \tag{A19}$$

Insert (A4) and (A13) in (A19)

$$\varphi_1 \bar{k}_1 + \varphi_2 \bar{k}_2 = \varphi_0, \tag{A20}$$

where

$$\varphi_j(z, \tau_i) \equiv \left( \frac{w \bar{\beta}_i \beta \tilde{\varphi}_j}{\bar{N}_j \beta_i} - \lambda_j g_j + 1 - \delta_k + \frac{\beta \bar{\beta}_s \bar{\beta}_i w z}{\beta_i} \right) \bar{N}_j,$$

$$\varphi_0\left(z, \tau_i, \{\bar{k}_j\}\right) \equiv - \sum_{j=3}^{J} \varphi_j \bar{k}_j + \sum_{j=1}^{J} \lambda_j \bar{N}_j \bar{g}_j + \frac{w \bar{\beta}_i \beta \tilde{\varphi}_0}{\beta_i}.$$

Solving the linear equations (A17) and (A20) with $\bar{k}_1$ and $\bar{k}_2$ as the variables, we have

$$\bar{k}_j = \Omega_j\left(z, \tau_i, \{\bar{k}_j\}\right), \ \ j = 1, 2. \tag{A21}$$

Here, we do not give the expressions of the functions in (A21) as it is straightforward and the expressions are tedious. It is straightforward to confirm that all the variables can be expressed as functions of $z$, $\tau_i$, and $\{\bar{k}_j\}$ by the following procedure: $\bar{k}_1$ and $\bar{k}_2$ by (A21) $\rightarrow$ $r$ and $w_j$ by (A3) $\rightarrow$ $\bar{y}_j$ by (A4) $\rightarrow$ $N$ by (A13) $\rightarrow$ $K_i$ and $K_s$ by (A15) $\rightarrow$ $N_i$ and $N_s$ by (A1) $\rightarrow$ $F_i$ by (3) $\rightarrow$ $F_s$ by (5) $\rightarrow$ $p$ by (A8) $\rightarrow$ $\bar{T}_j$, $c_j$, and $s_j$ by (11) $\rightarrow$ $T_j = T_0 - \bar{T}_j$ $\rightarrow$ $K = K_i + K_s$ by (15). From this procedure, (A21), and (12), we have

$$\dot{\bar{k}}_j = \bar{\Omega}_j\left(z, \tau_i, \{\bar{k}_j\}\right) \equiv \lambda_j \bar{y}_j - \Omega_j, \ \ j = 1, 2, \tag{A22}$$

$$\dot{\bar{k}}_j = \Lambda_j\left(z, \tau_i, \{\bar{k}_j\}\right) \equiv \lambda_j \bar{y}_j - \bar{k}_j, \ \ j = 3, ..., J. \tag{A23}$$

Taking derivatives of equation (A21) with respect to $t$ and combining with (A23) implies

$$\dot{\bar{k}}_j = \frac{\partial \Omega_j}{\partial z}\dot{z} + \frac{\partial \Omega_j}{\partial \tau_i}\dot{\tau}_i + \sum_{j=3}^{J} \Lambda_j \frac{\partial \Omega_j}{\partial \bar{k}_j}, \ \ j = 1, 2. \tag{A24}$$

Equaling the right-hand sizes of equations (A24) and (A22), we get

$$\frac{\partial \Omega_j}{\partial z}\dot{z} + \frac{\partial \Omega_j}{\partial \tau_i}\dot{\tau}_i = \bar{\Omega}_j - \sum_{j=3}^{J} \Lambda_j \frac{\partial \Omega_j}{\partial \bar{k}_j}, \ \ j = 1, 2. \tag{A25}$$

Solving the linear equations (A25) with $\dot{z}$ and $\dot{\tau}_i$ as the variables, we have

$$\begin{aligned} \dot{z} &= \Lambda_1\left(z, \tau_i, \{\bar{k}_j\}\right), \\ \dot{\tau}_i &= \Lambda_2\left(z, \tau_i, \{\bar{k}_j\}\right). \end{aligned} \tag{A26}$$

Here, we do not give the expressions of the functions in (A26) as it is straightforward and the expressions are tedious. In summary, we proved the lemma.

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# A STUDY OF THE ROLE OF GOVERNMENT IN INCOME AND WEALTH DISTRIBUTION BY INTEGRATING THE WALRASIAN GENERAL EQUILIBRIUM AND NEOCLASSICAL GROWTH THEORIES

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## SAŽETAK

U radu se postavlja model rasta heterogenih kućanstava koji uključuje ekonomsku strukturu, akumulaciju bogatstva, endogenu ponudu radne snage i porezne stope. Fokus rada je na učincima mjera redistribucije na distribuciju prihoda i bogatstva, na ekonomsku strukturu i na ekonomski rast. Cjelovito su objedinjene Walrasova teorija opće ravnoteže i neoklasični ekonomski rast. Nadiđene su kontroverzne karakteristike dviju tradicionalnih teorija primjenom alternativnog pristupa kućanstvima. Postavljen je analitički okvir za disagregiranu i mikroutemeljenu opću teoriju ekonomskog rasta s endogenom akumulacijom bogatstva. Simulacijom modela identificirani su ravnoteža i stabilnost te iscrtana gibanja dinamičkog sustava s tri grupacije. Također je provedena komparativna dinamička analiza s osvrtom na paušalni porez, ljudski kapital i sklonost korištenju slobodnog vremena.

## KLJUČNE RIJEČI

paušalni porez, porezne stope, Walrasianska teorija opće ravnoteže, neoklasične teorije rasta, dohodak i raspodjela bogatstva