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De Aurora Boreali (1737) – Contemporary insight to the young Bošković's treatise

Vladis Vujnović and Inga Lisac

University of Zagreb, Faculty of Science, Department of Geophysics, Zagreb, Croatia

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An analysis of Bošković's early paper concerned with the polar light (aurora borealis) gives an insight into the validity of the mathematical method which the young Bošković applied to significant geophysical phenomena in order to comprehend their nature. Based on the given data, we have examined numerical results about the height of atmosphere and the heights of the auroras which were observed over Europe in 1726 and in 1737. In his text there is not a single mathematical formula. Following his instructions we have derived a formula which may be used to determine the distance of an aurora by observing it from one station. For the assumed aurora model, his method is applicable. We find that described ideas on the physical cause of aurora, conceived in a fluid coming from the sun, have elements of modern knowledge.

Keywords: Ruđer Bošković, aurora borealis, atmosphere dimensions and density

1. Introduction

Ruđer Josip Bošković (1711–1787), Croatian scientist and philosopher, one of the last universal scholars, was born in Dubrovnik. At the age of 14 he entered the Collegium Romanum where studied rhetoric, philosophy, theology, physics, mathematics and astronomy. There he became a priest without pastoral duties, had heavy teaching duties and conducted different research – purely scientific or applied. After 1760 Bošković worked and resided in several European places, became a member of scientific societes, and used to serve in diplomatic missions. Belonging to Jezuits, distinguished promotors of science and scientific institutions (Udias, 2003), he had easier approach to high society of the time. Having a thorough knowledge of mathematical physics, he was engaged in different scientific and practical tasks, in statistics, astronomy (improved instruments, developed a method for finding orbit of celestical object from the three close positions), geodesy and cartography (measured meridian between Rome and Rimini, introduced a sort of geodetic stand), geophysics, meteorology, optics, civil engineering and archeology. Under his guidance the Brera astronomical observatory was built.

Bošković is mainly recognized as a founder of a theory which unifies all forces in nature. His fundamental work *Theory of natural philosophy reduced to the single law of forces existing in nature*, printed 1758 in Vienna, influenced works of many famous physicists and philosophers (Whyte, 1961).

Our comments refer to Bošković's treatise which the 27-year old Bošković submitted to the academic public on two occasions, first in August 1738 at the Roman Seminary (*Seminario Romano*), and then in September of the same year at the Roman College (*Collegio Romano*) (Bošković, 1738). In the education of the Jesuits, scientific discussion were stimulated, in which prominent students got a chance to prepare and lead discussion and shape its conclusions in a literary form. A leader of discussions was named „academician". Title of treatise does not bear his name, but it is acknowledged as his work and is listed in his references (Kutleša, 2011).

The treatise has ten pages of text and eight figures. Original text in Latin was translated to Croatian by Martinović (2012) from the specimen found in the Archivium Ragusa, Dubrovnik. As far as we know, this is the first translation to Croatian. The treatise we analyse formally consists of propositions and corollaries, and its subject belongs to the physics of the atmosphere. The first two propositions serve to introduce measuring units used to describe the Earth's size and distances on its curved surface. In the first proposition, Roman miles are used to express the Earth's radius. In the second proposition an improved method to find the "height of the atmosphere" was described. In this way the reader is ready to grasp the planetary dimensions of the natural phenomenon as aurora borealis is.

In the original text some circumstances are not explicitly stated. There is no mention of the shape of the Earth, probably presuming that it is spherical. May we speculate that the Earth was considered still, non-rotating, although Newton already explained the Earth's oblateness by its rotation. In the case of a spherical Earth, Bošković's notion of the "largest circle" could be applied to meridians as well as to the equator. Anyway, flattening of the Earth's body should not influence accuracy of the physical dimensions of the natural phenomena treated in this paper. In the time we witness, Jesuits officially adhered to Ptolemaic cosmology. Notion of the Earth's rotation axis was suppressed. In the fifth proposition Bošković describes the geometry of the aurora and writes: "… aurora borealis is a bright circle equally far from the Earth, with the center on the equatorial axis (in Latin, axis Æquatoris), therefore it is parallel to the equator." Although the term "axes of a circle" in geometry is not usual, centre of aurora's circle is conveniently placed on the Earth's rotation axis. (Quotation marks we will use when citing the original text, translated to English.) However, rotation of the Sun was acknowledged, since proved by motion of the sunspots, and it was the crucial proof in sixth proposition, in the explanation of the aurora as a natural phenomenon.

Generally, text discloses Bošković as a young imaginative natural philosopher who adopts new ideas of others, elaborates his own attitudes and mathematical procedures.

2. Analysis of propositions one to four

The **first proposition** deals with the relation between different length units and the Earth's size. In order to recall past times, let us mention the Roman mile and the Paris mile with corresponding smaller units: double step, feet, palm and line. There are relations:

1 Roman mile = 1 000 double steps = 5 000 feet = 20 000 palms.

According to different sources, the Roman mile is about 1480 m. Jakobović (1981) gives 1 Roman mile = 1478.5 m. For further calculation we will use the round number 1480 m.

For the ratio between the Paris and the Roman measures Bošković uses

$$432 : 392 = 54 : 49 = 1.1020$$

Bošković quoted that one degree of the "largest Earth's circle" accommodates

343 752 Paris feet = 378 828.734 Roman feet.

It follows: 68.7504 Paris miles = 75.7657 Roman miles.

For the planetary radius Bošković quotes 4341.0545 Roman miles, which is equal to 6,425 km. He remarks than one degree of parallel going through Athens equals 60 roman miles, while it has about 59.22 roman miles. This shows the precision of the contemporary measurements; mean Earth's radius is equal to 6371 km. Bošković realized his interest in measurements of our planet's size later on (1750–1752), when measuring the arc of the meridian through Rome and Rimini together with Ch. Le Maire.

For the Ludolph's number π, the ratio between half circumference and radius, Bošković uses 355 : 113 = 3.141 592 92, which differs a little from the more exact value equal to 3.141 592 65.

The associated **Corollary** gives notion of the geographical foot. Bošković gives the ratio:

1 geographical foot = 100 000 : 79 191.51 Roman feet = 1.263 Roman feet.

This ratio of the Roman and the geographical feet follows from the ratio of numbers of Roman and geographical miles which are needed for one degree of the largest circle, which means, 60 geographical miles = 75.7657 Roman miles. (The number 60 was also mentioned when stating that one degree of the parallel through Athens, Greece, is equal to 60 Roman miles; nowadays we find it is 60.98 Roman miles.)

For the Roman miles we will hereinafter use the abbreviation r. m.

In the **second proposition** Bošković presents a geometrical model for calculating the "height of the Earth's atmosphere" – whatever it may mean. Also, he states that this height "is higher than the true one". This expression will be elaborate further.

The model starts with assumption that the first solar rays are seen when the Sun is 18° under the observer's horizon. This angle is obviously the result of astronomical practice. Today we also use the solar depression of 18° as the end of astronomical twilight, 12° as the end of nautical twilight, and 6° as the end of the civil twilight.

Twilight arises since the solar rays are scattered by air molecules. Bošković elaborates the procedure of his predecessors who used only one reflection. In order to get solar ray directly from the horizon, Bošković used a model with two reflections (Fig. 1). The first reflection is at *M*, and the second at *H*; the deflected ray reaches an observer at *A*. The height of the atmosphere is then defined as the height above the Earth where the second reflection is present. By dividing the angle of 18° into four angles equalling 4° 30', the both reflections should be at the same height above the Earth's surface. In this way one gets the same atmospheric density at the place of reflections. If the angle *ACF* was divided into three equal angles, the reflections would take place at different heights.

In the described method one has to calculate the distance $\overline{BH}$ which is a part of triangle *CAH*, by using the known Earth's radius and the angle of 4° 30'.

The phraseology for the 18th century is usuall one. Bošković writes that hypotenuse of triangle *CAH* is the „secans of the angle 4° 30'". This is understand-

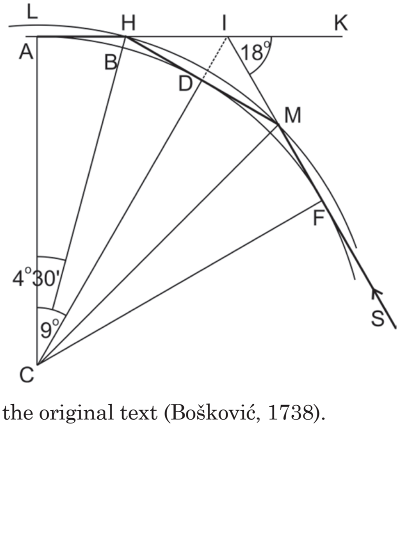

Figure 1. Equal to Fig. 1 in the original text (Bošković, 1738).

able if the other triangle side $\overline{AC}$ is a measuring unit. Then the author explains that the ratio of $\overline{AC}$ and $\overline{BH}$ is equal to the ratio of 100 000 versus 309.22, e.g. as „whole sinus 100 000 versus excess of secans 4° 30' above the radius", i.e.

$$\frac{\overline{BH}}{\overline{AC}} = \frac{\sec 4.5^\circ - 1}{1} \quad \text{where} \quad \sec 4.5° = 1.003\,092\,2.$$

The „whole sinus" means the sinus of the right angle; "excess of secans 4° 30' above the radius" is sec 4.5° – 1.

Let us calculate: $\overline{BH} = \overline{AC} \cdot 0.003\,092\,2 \approx 13.4$ r. m. ≈ 20 km. This height approximately corresponds to the height of troposphere, the atmospheric layer of highest density. When taking only one reflection at *I*, the height should be 54 r. m.

Now we can understand the phrase "height which is higher than the true one". If the refraction measured by Tycho Brahe was taken into account, then the ray *HA* (the refraction is continuous as the ray proceeds towards the Earth) would be bent from below, lowering the point *H* named as a place of the "last reflection". The model used by Bošković neglects the continuous refraction.

We have to acknowledge that scientists at the beginning of 18th century had a good insight into the extension of the atmosphere. Although Bošković's result is the best estimate of the atmoshpere height for his time, it is rather the result of happy chance than of a sound model, since the passage of light through the atmosphere is much more complex. Sky light is the multiply scattered light of all primary rays. Moreover, one makes a logical error by drawing a primary ray tangentially to the Earth's surface from *S* to *M*, and from *M* to *H*; touching the ground, ray should stop.

Contemporary knowledge of density variation with height, owing to Newton, satisfies our knowledge, at least to the height of 50 km, as shown in the table:

Height		Relative density	
r. m.	km	Time of Bošković	Today (Hedin, 1991)
0	0	1	1
7	10	1/4	1/3
14	21	1/16	1/14
21	31	1/64	1/63
28	41	1/256	1/280
35	52	1/1024	1/1180
70	104	10^{-6}	$3 \cdot 10^{-4}$
140	207	10^{-12}	$2.7 \cdot 10^{-10}$
210	311	10^{-18}	$2.5 \cdot 10^{-11}$

Bošković took the tabulated data from Newton's *Opticks*, first published in Latin in 1706. The cognition that the air is a real medium was proved by the famous experiment of Magdeburg's Lord Mayor Otto van Guericke in 1657, not much before Newton's analysis. Newton properly described gravitational sedimentation of the atmosphere and calculated variation of its density. Bošković's achievement was in understanding the atmospheric density structure. By his rational mind he adhered to Newtonists.

Bošković identifies the problem in the **third proposition** by saying: "To find the nearest distance of aurora borealis, which is less than the true one." This phrase prepares readers to grasp the approximate manner of calculations.

The first two propositions were not only a sort of exercise before the main task, but made a didactic introduction to the extension of the atmosphere whose integral part aurora is.

In this proposition Bošković shows how to calculate the height of the aurora seen over Europe on October 19, 1726, whose elevations were measured in Tusculum (or Tuscul, a city close to Rome) and in Paris. The method – which together with the measured elevations uses corrections for the atmospheric refraction – is explained in Figs. 2. and 3.

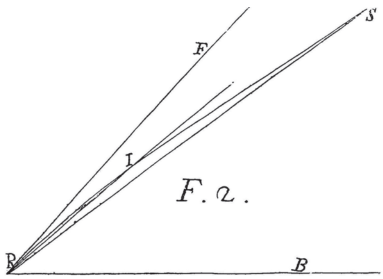

Figure 2. Original Fig. 2., Bošković (1738).

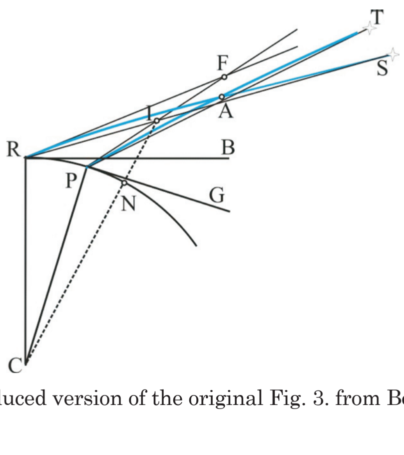

Figure 3. Reduced version of the original Fig. 3. from Bošković (1738).

First, in Fig. 2 (the original figure reproduced) Bošković explains the influence of the atmospheric refraction, refering to the astronomical observation of Tycho Brahe (1546–1601). The line *RF* is a tangent on the ray path *SIR*. True star height is given by the straight line *SR*. Apparent star height is given by the tangent *RF*. Astronomical refraction, the angle which should be subtracted from the apparent star height, is the angle *SRF*. However, the aurora is situated inside the atmosphere on the curved ray path *SIR* at *I*, not at infinity. When one subtracts the astronomical refraction from the observed aurora height *BRF*, the correction will be greater than necessary. The error is negligible only if point *I* was very far.

The phenomenon was observed by observers, in Paris, by Jacques Philippe Maraldi (Italo-French astronomer and mathematician, 1665–1729) and in Tusculum (Rome) by Francesco Bianchini (Italian philosopher and naturalist). The observers described the phenomenon in the form of an arc open towards the Earth's pole. They measured its highest position. It is not likely that Bošković witnessed the aurora in year 1726 when he was still a teenager, and we may only speculate about that. The elevation measured in Paris (*P*) was 37° 20', and in Rome (*R*) it was 20°.

Aurora is situated at *A* where the light paths of two stars, one at *T* and another at *S*, have their crossing. It is clear that point *A* where the aurora is situated, is not very distant; it should be nearer than stars since the difference of elevations measured in Paris and Rome is substantially larger than the difference of the observers' geographical latitudes; in other words, the tangents *RF* and *PF* are not parallel. Using the tabulated values for the astronomical refraction, Bošković diminishes the elevation measured in Rome by 3', obtaining 19° 57'. Certainly, instead of the exact refraction of the light coming from aurora *A* (angle *FRA*) he uses the refraction angle *FRS*. In this way there appears a new point *I* on the intersection of lines *SR* and *PF*, line of sight from Paris. (Bošković leaves refraction of the curved line *PA* uncorrected, probably because the refraction is small, about 1'.). Selecting the point *I* whose height from the Earth $|NI|$ should be determined, Bošković justifies the task of determining the distance of aurora borealis which is "less than a true one": the true height should be the height of the aurora spot *A*. Position of *I* is obviously lower than the position of *A*.

Figure 3. places the position of observers at *R* and *P* on the same meridian, a procedure borrowed from Mairan (reduction to the same meridian), whose important book on aurora borealis (Mairan, 1733) Bošković was obviously knew. The reason for this procedure was the following: "...since in both cases the auroral phenomenon appeared as a circular arc, he cleverly concluded: the elevation on which the phenomenon was observed at Tusculum should be observed on the whole parallel going through Tusculum."

This is an important intervention to the observational circumstances. The elevation measured at different parallels could be set at the same meridian only in the case if the aurora is *A* circular phenomenon parallel to the equator, and

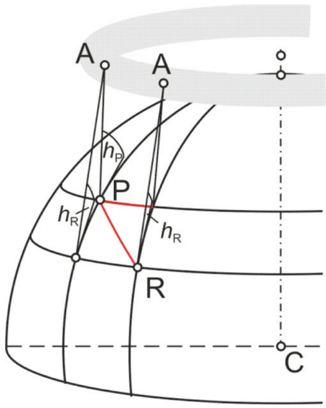

Figure 4. Explanation of the aurora model used by Bošković (1738).

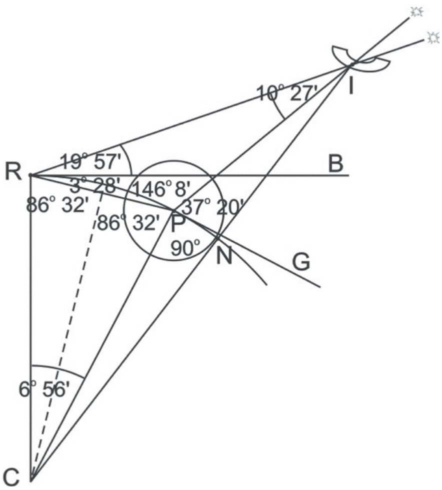

Figure 5. Angles in the geometrical analysis of aurora (Bošković, 1738).

centered on the equatorial axis, as the Fig. 4 shows. Point A is therefore at any point on the auroral circle closest to the observer, either in R or in P.

The difference of the geographic latitudes between Paris and Rome amounts to 6° 56'. Following the geometrical analysis, as described by Bošković, we present the angles in Fig. 5, from where we find the aurora's height $|NI|$ = 779 r. m. = 1152 km. The original gives 720 r. m. = 1054 km.

In the investigation described hereinafter, Bošković neglected the atmospheric refraction. The omission of the refraction and its consequences are discussed in the first and second corollary. As written in the second corollary, Bošković was aware of observational difficulties and concluded that the correction of the refraction has a little sense. This is quite reasonable since the maximum normal refraction of the stellar light amounts to only half degree – that is, when the rays come parallel to the horizon. The aurora as a phenomenon usually covers a very large part of the sky and is not clearly outlined.

A question remains, do observers at different locations observe the same auroral spot? The auroral height is a pretty strong function of the measured elevation. By substituting the elevation at Paris by 40º, instead of 38º 20', for the height of the aurora we obtain 708 r. m. instead of 779 r. m.

The **fourth proposition** is concerned with the aurora of December 16, 1737. The task was to find its distance which is „less than a true one". We do not know if Bošković observed this phenomenon. However, this 1737 event was observed in Croatia (Lisac and Marki, 1998, 2003; Penzar and Penzar, 1997). Fr. Nikola Gojak, annalist in Franciscan monastery in Makarska (note extracted from the Franciscan monastery annals in Makarska), writes:

„16/10/1737 Around 02 h Makarska – On the clear night and W-ward the air was illuminated fiery-red; the light was distributed into three parts: towards Split, towards the Island of Hvar and towards the mountains."

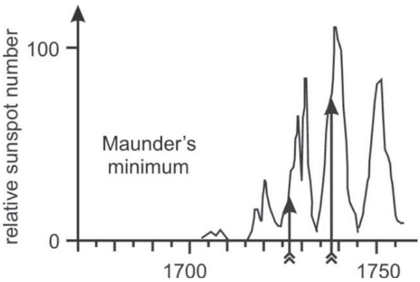

Figure 6. The onset of auroras treated by Bošković (1738).

At the beginning of the 18th century the aurora was given extreme attention since the phenomenon had not been observed for sixty years (Maunder's minimum of solar activity).

During 1726, as well as in 1737, solar activity was on its rising slope. It should be noted that the solar cycles to which the auroras of 1726 and 1737 belong were quite prominent and they should have strong impression on population. The frequency and intensity of auroras depend on the solar activity. Auroras usually appear almost daily at south and north latitudes above 70°, but during the maximum of the solar activity, their range may extend towards middle and lower latitudes; so they can be observed in the whole central Europe, including Paris and Rome .

In the year 1737, Giovanni Poleni in Padua measured the elevation of 20° of some prominent aurora details with a quadrant; the quadrant was at that time in

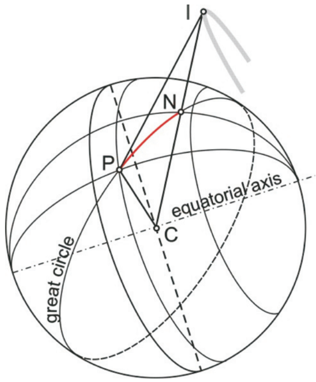

Figure 7. Explanation of the Bošković's procedure for calculating properties of aurora observed in 1737.

general use in astronomical observations. According to original Fig. 4, aurora was seen in zenith at the "ultimate borders of Britain" (the term indistinctly defines the geographical position). This time Bošković did not place both observations on the same meridian but used the vertical circle through the "borders of Britain" and Padua. We therefore sketched a great circle through Padua (*P*) and Britain (*N*), Fig. 7. Such a situation is much better founded than in the previous case since only two data are needed, the arc $\widehat{PN}$ (or its central angle), and the elevation at Padua. Bošković obtained the arc $PN = 18°$ from geographical tables; the choice depends on the notion "ultimate borders". In the triangle *CPI* all angles are then known and so is the Earth's radius as its one side.

Bošković quoted the height of the aurora in 1737 as being 836 r. m. = 1220 km; our calculation using the sine rule gave the same result.

From the physical point of view, aside from the purely observational indefiniteness, it is rather difficult to explain the heights determined in years 1726 and 1737. They can be understood as overestimated values.

The aurora may start at the level of 80–150 km, broaden and grow to 600 km, and on some occasions to 1100 km. When observing from the Earth's surface, observers from higher latitudes notice the most brilliant parts, in the majority of cases, close to 100 km. However, observations from satellites, or from the International space station, reveal the vertical structure of aurora in the form of a curtain, extending vertically over several hundred kilometers. In their lower parts colors are bright, green, and in higher parts hazy and red. The colors depend on the molecular atmospheric constitution and on excitation conditions.

The auroral base is visually dominant due to a higher density of the lower atmosphere; in these layers blue and magenta-red light is emitted by nitrogen molecules. In the upper auroral parts, formed inside the less dense atmosphere, prevailes green light of the monoatomic oxygen, and in still higher and more rarefied parts, prevailes the red light of the monoatomic oxygen. Given oxygen spectral lines originate at metastable levels having vastly different lifetimes; a level having longer lifetime deexcites radiatively when atoms are found in medium of lower density, otherwise it is deexcited collisionally.

Due to the Earth's curvature, observers from central or southern Europe look at higher parts, red and foggy. This leads to the observation of red paint aurora seen from moderate latitudes. In Croatia the last auroras were seen around midnight over several places, in April 2000 and in October 2003, all in diffuse red colour.

3. Analysis of the fifth proposition

The **fifth proposition** deals with a very interesting mathematical problem. Bošković conducted this task following F. C. Mayer, whose treatises on aurora were published in St. Petersburg 1728 and 1735. Bošković corroborated it only in principle, giving advice.

The problem consists in determining the height of the auroral oval (circular aurora) which is centered on the geographical pole and whose plane is orthogonal to the Earth's „equatorial axis" (Fig. 8a). (It is instructive to present original Bošković's drawing, Fig. 8b.)

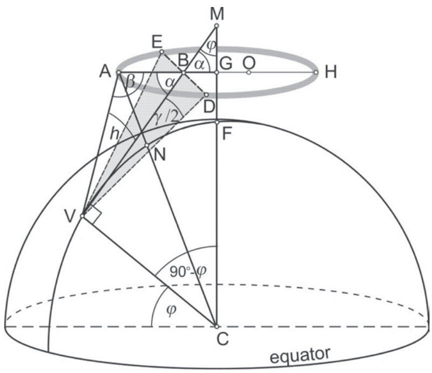

Figure 8a. Observation of polar light in a special case, from only one position.

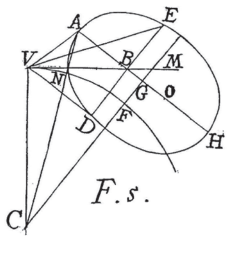

Figure 8b. Original drawing.

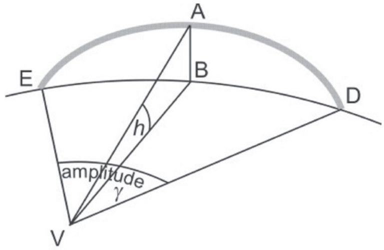

Figure 9. Situation at the observation position.

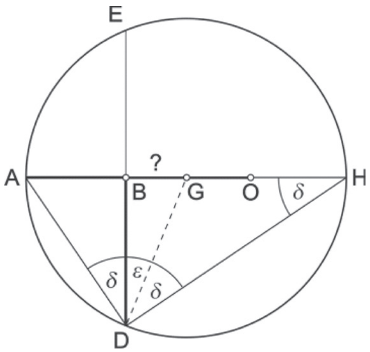

Figure 10. Geometry of polar light. The problem is solved when one finds the relation between $\overline{AB}$ and $\overline{BG}$.

From a single position *V*, the observer should measure the elevation *h* of the closest aurora point and the angular width γ of the auroral arc $\widehat{EAD}$ seen above the horizon (Fig. 9). The situation within the auroral circle is presented in Fig. 10 (Figs. 8, 9 and 10 were derived from the original Fig. 5 in order to explain the original text in Corollary 1).

Let us enumerate what is known or measured: $\overline{CV}$ = 4341 r. m., *h*, the elevation of the point *A*, and the amplitude γ. One has to determine the distance $\overline{AN}$, the height of the prominent auroral point *A* above the Earth's surface. We followed the original text as a prescription (written in italics).

(A) For a given place V, arc $\widehat{VF}$, *the complement of the geographic latitude, and therefore the angle VCM is also given.*

The right-angled triangle VCM is given, and since the Earth's radius $\overline{CV}$ *is known,* $\overline{VM}$ *is also given.*

(B) The right-angled triangle BGM will also be given, since the angle in M is common with the given right-angled triangle CVM.

Hence angle GBM and angles at tips of triangle ABV will be given.

(C) As observation gives the elevation AVB, triangle VBA will be separately given. And lastly, as semiamplitude DVB is given, the right-angled triangle DBV is also given.

(D) Thus, ratios $\overline{AB}:\overline{BV}$ *and* $\overline{BV}:\overline{BD}$, *are given, hence the ratio composed of them,* $\overline{AB}:\overline{BD}$.

On the circle it holds: $\overline{AB}:\overline{BD}=\overline{BD}:\overline{BH}$, *hence the latter ratio, as a ratio* $\overline{AB}:\overline{BH}$.

(E) Or, if one makes $\overline{HO}$ *equal to* $\overline{AB}$, *ratio* $\overline{AB}$ *against the difference* $\overline{BO}$, *and against semidifference* $\overline{BG}$ *will be given.*

(F) The ratio $\overline{BG}:\overline{BM}$ *is already known, and hence the ratio* $\overline{AB}:\overline{BM}$ *will be known, and since the ratio* $\overline{VB}:\overline{BA}$ *was given, it follows that the ratio composed of them:* $\overline{VB}:\overline{BM}$ *or* $\overline{VM}:\overline{BV}$ *will also be given.*

(G) Therefore, since $\overline{VM}$ *is given,* $\overline{VB}$ *and the second side* $\overline{VA}$ *of the triangle AVB will be given.*

(H) Finally, in the triangle CVA with given sides $\overline{CV}$, $\overline{VA}$, *and the angle CVA between them, one will know* $\overline{CA}$, *and, when the radius* $\overline{CN}$ *is subtracted, one will obtain the sought distance* $\overline{NA}$. *Q.E.F. (Quod Errat Faciendum.)*

The mathematical translation of Bošković's prescription:

(A)
$$\measuredangle VF = 90° - \varphi = \measuredangle VCM$$
$$\overline{CV} = 4341 \text{ r. m.}$$
$$\overline{VM} = \overline{CV}\tan\left(90° - \varphi\right) = \overline{CV}\cot\varphi$$

(B)
$$\measuredangle M = \measuredangle BMG = \measuredangle VMC = \varphi$$
$$\measuredangle GBM = 90° - \varphi = \measuredangle VBA = \alpha, \quad \measuredangle VAB = \beta = 180° - \alpha - h,$$

(C)
$$\alpha = 90° - \varphi$$
$\beta = 180° - \alpha - h = 90° + \varphi - h$, h is the measured elevation

$\measuredangle DBV = \dfrac{\gamma}{2}$ is the measured semiamplitude.

$$\measuredangle VBD = 90°, \quad \measuredangle VBD = 90° - \frac{\gamma}{2}$$

(D)
$$\frac{\overline{AB}}{\overline{BV}} = \frac{\sin h}{\sin\beta}, \quad \frac{\overline{BV}}{\overline{BD}} = \cot\frac{\gamma}{2}, \quad \frac{\overline{AB}}{\overline{BD}} = \frac{\sin h}{\sin\beta}\cot\frac{\gamma}{2} = \tan\delta$$

We have added angle δ which was not mentioned by Bošković. Furthermore:

$$\frac{\overline{AB}}{\overline{BD}} = \frac{\overline{BD}}{\overline{BH}}$$

$$\frac{\overline{AB}}{\overline{BH}} = \left(\frac{\overline{BD}}{\overline{BH}}\right)^2 = \tan^2\delta$$

(E) $$\overline{BO} = 2\overline{BG}, \quad \overline{OH} = \overline{AB}$$

$$\frac{\overline{AB}}{\overline{BH}} = \frac{\overline{AB}}{2\overline{BG} + \overline{AB}} = \frac{1}{2\frac{\overline{BG}}{\overline{AB}} + 1}$$

(F) $$\overline{BG} = \overline{BM}\sin\varphi, \quad \frac{\overline{AB}}{\overline{BH}}\tan\delta^2$$

$$\tan^2\delta = \frac{1}{2\frac{\overline{BM}}{\overline{AB}}\sin\varphi + 1}, \quad \frac{\overline{BM}}{\overline{AB}} = \frac{\cot^2\delta - 1}{2\sin\varphi}$$

$$\frac{\overline{AB}}{\overline{VB}} = \frac{\sin h}{\sin\beta}, \quad \overline{AB} = \overline{VB}\frac{\sin h}{\sin\beta}$$

$$\frac{\overline{BM}}{\overline{VB}} = \frac{\sin h}{\sin\beta}\frac{\cot^2\delta - 1}{2\sin\varphi}$$

(G) $$\overline{VB} + \overline{BM} = \overline{VM} = \overline{CV}\cot\varphi$$

$$\overline{VB} = \overline{CV}\cot\varphi - \overline{BM} = \overline{CV}\cot\varphi - \overline{VB}\frac{\sin h}{\sin\beta}\frac{\cot^2\delta - 1}{2\sin\varphi}$$

$$\boxed{\begin{gathered}\overline{VB} = \frac{\overline{CV}\cot\varphi}{1 + \frac{\sin h}{\sin\beta}\frac{\cot^2\delta - 1}{2\sin\varphi}} \\[2em] \overline{VA} = \overline{VB}\frac{\cos\varphi}{\sin\beta}\end{gathered}}$$

(H) The distance $\overline{CA}$ is obtained by using the cosine rule for the triangle CVA, knowing sides $\overline{CV}$ and $\overline{VA}$.

Let us substitute all from the above into the cosine law:

$$\overline{CA}^2 = \overline{CV}^2 + \overline{VA}^2 - 2\overline{CV}\,\overline{VA}\cos(90^\circ + h) \qquad \cos(90^\circ + h) = -\sin h$$

$$\frac{\overline{CA}^2}{\overline{CV}^2} = 1 + \left(\frac{\overline{VA}}{\overline{CV}}\right)^2 + 2\sin h\frac{\overline{VA}}{\overline{CV}}$$

$$\frac{\overline{CA}^2}{\overline{CV}^2} = 1 + \left(\frac{\cos^2\varphi}{\sin\varphi}\right)\left[\left(\frac{\cos^2\varphi}{\sin\varphi}\right)\left(\sin\beta + \sin h\frac{\cot^2\delta - 1}{2\sin\varphi}\right)^{-2} + 2\sin h\left(\sin\beta + \sin h\frac{\cot^2\delta - 1}{2\sin\varphi}\right)^{-1}\right]$$

$$\overline{CA} = \overline{CV}\left(\frac{\overline{CA}^2}{\overline{CV}^2}\right)^{\frac{1}{2}}, \quad \overline{AN} = \overline{CA} - 4341 \text{ r. m.}$$

$\frac{\gamma}{2}$ is the measured semiamplitude

h is the measured elevation

$$\beta = 90^\circ + \varphi - h$$

$$\cot\delta = \frac{\sin\beta}{\sin h}\tan\frac{\gamma}{2}$$

An example. By taking $\varphi = 56.5^\circ$, $\gamma = 58^\circ$, $h = 21.5^\circ$, we get the height $\overline{AN}$ of 865 r. m. or 1280 km.

What is the precision of an observation from only one position on the Earth's globe? The exact expression of relative errors, depending on the error of the measured elevation and of the arc amplitude, would be cumbersome. Therefore, by using the Excel spreadsheet we used the range of elevations between 10° and 25° for two amplitudes, $\gamma = 58^\circ$ and 90°, and $\varphi = 56.5^\circ$ for the latitude. The evaluated height is approximately linearly dependent on the elevation h. For amplitude $\gamma = 58^\circ$, the obtained heights $\overline{AN}$ are above 1000 km, and the error of 1° in h generates an error of about 50 km. For amplitude $\gamma = 90^\circ$ the heights are in the range of 500 km, and the error of 1° generates an error of about 30 km. This proves that the method is quite sound and can be used, only if the aurora has been modeled according to the given assumptions. In reality, the auroras inside the auroral oval are centered on the magnetic pole instead on the geographical pole. Therefore the geographical latitude should be replaced by the geomagnetic latitude. So much about the measurements from the Earth's surface.

One can find another expression in order to solve the crucial step in the calculation, e.g. to find ratio $\overline{VB}/\overline{BM}$. Therefore we have introduced an auxiliary angle ε (Fig. 10), as presented in the Appendix.

In the **first corollary,** the method is proposed of aurora observation from two positions. From both positions the direction to the Earth's pole should be known. The **second corollary** mentions observations of the phenomenon from many positions, since Bošković was, obviously, aware of spatially and temporally inhomogeneous aurora pattern. The best way were, as he suggested, precise measurements of elevations from two positions in the same "vertical" – what means the vertical plane passing through the observation positions.

4. Physical cause of the aurora

The physical cause of the aurora is the subject of the **sixth proposition**, and Bošković discusses the viewpoints which existed at the time. According to one viewpoint, the polar light causes solar rays, refracted in increased density (an explanation for such a claim, is missing in the original text) of the atmosphere around the pole. Bošković opposes this opinion since reflecting body is not seen. Another viewpoint seeks the polar light in the so-called atmospheric *combustion* connected with evaporated water in the atmosphere. The difficulty in this opinion is the height to which the evaporations, when ignited, rise to be seen from lower geographical latitudes.

Bošković adheres to the third theory, the theory of Mairan which explains the auroral phenomenon by invoking an extension of the solar atmosphere. The solar rotation, documented by the regular motion of sunspots over the solar disk, deforms the solar atmosphere which therefore may obtain a convex shape and extend far into the ecliptic plane. In his words, the solar substance flows to the Earth, into its atmosphere, and even further from the Earth. This is, in Bošković's opinion, proved by the appearance of the zodiacal light; it is found in the sky opposite to the sun. The conical shape of the zodiacal light is pointing along the ecliptic plane and this corresponds to the convex shape of the rotating solar atmosphere. The another effect he envisaged, is the occasional excitation of polar lights. "The refined solar substance", Bošković writes, is a light source. It enters into the high atmosphere and flows towards polar areas, heating and igniting the Earth's atmosphere. He does not explain the reason of attracting the solar substance (light) above the polar areas.

We may compare this scenario with the modern scientific picture about the solar wind – a flow of rarefied and extremely hot gas composed of atomic particles, ions and electrons, which penetrates through the geomagnetic field, forces the magnetosphere to react and direct very rarefied content of the magnetosphere together with the solar wind particles into the auroral ionosphere. So the very strong electric current is formed which excites the atmospheric constituents to emit light.

Bošković's convinction of solar-terrestrial interaction and his results regarding auroral structure and dimension was acknowledged by contemporaries. De Mairan himself comments his findings (Marković, 1950): „He understood very well that property of my view of the aurora borealis which is the most probable. Even more, he substantiated my view and proved it in a way honourably to him ... Bošković enlarges value of my hypothesis with conclusions derived from the phenomenon seen in 1737, and particulary with his calculations applied to the distance to which the matter of the phenomenon was far from the Earth.“

Somehow vague, but sufficiently general visions about the genesis of the phenomena like the zodiacal and polar light, have close resemblance with the modern view. The source of both phenomena is found in the sun. However, the truth is closer in the case of a glowing aurora than in the case of a feeble zodiacal light. The zodiacal light is seen because the solar light is scattered by tiny solid particles, grains, in the space where most planetary bodies are moving – around the plane of ecliptic. The polar light is more specifically caused and induced by the genuine solar medium. The interaction between the solar wind and the Earth's high, geomagnetically active atmosphere, manifests itself in dynamics, shape details and colors of the polar light.

The solar activity which was fully renewed after Maunder's minimum and during Bošković's life, three hundred years ago, shows ambiguously less activity during the beginning of the present 24th solar cycle period 2010–2012, Fig. 11. Since the solar cycle in 1755 has the number 1, the cycles of the auroras in 1726 and in 1737 appeared immediately before it.

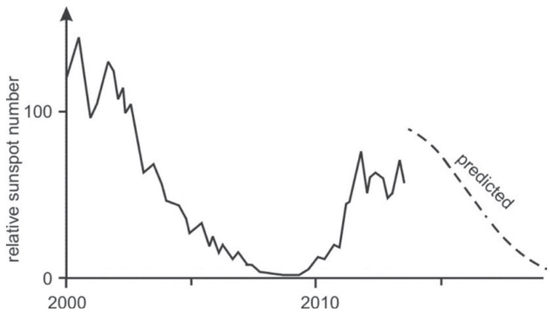

Figure 11. End of 23rd and part of the present 24th solar cycle period.

In the near future there is a probability that we shall see the polar light from our middle latitudes, more or less expressively. The phenomenon should be expected during the spring and autumn equinoxes, as supported by the observational statistic and explained by the convenient orientation of active regions on the Sun towards the Earth.

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References

Bošković, R. J. (1738): De Aurore Borealis Dissertatio, Rome, 1738, 1–12.

(„De Aurora Boreali, Dissertatio habita in Seminario Romano ab Augustino Fanucci Academico redivivo, Jo. Bapt. Amalfitano Equite Hier. Ac. Rediv. …“, 12 pages, 8 figures, in Latin, published twice: in August 1738 printed by Komarek and in September 1738 by Antonio de Rubeis).

Hedin, A. E. (1991): Extension of the MSIS thermospheric model into the middle and lower atmosphere, *J. Geophys. Res.,* **96**, 1159–1172.

Jakobović, Z. (1981): *Leksikon mjernih jedinica* (Lexicon on measure units). Školska knjiga, Zagreb, 120 pp, in Croatian.

Kutleša, S. (2011): *Ruđer Josip Bošković*. Technical Museum, Zagreb, 174 pp, in Croatian and English.

Lisac, I. and Marki, A. (2003): Report on polar light, *Croat. Meteorol. J.*, **38**, 103–111, in Croatian.

Lisac, I. and Marki, A. (1998): The auroral events observed from Croatia and a part of surrrounding countries, *Geofizika*, **15**, 53–68.

Mairan, D. de (1733): Traité physique et historique de l'aurore boréale. Suite de Mémoires de l' Académie Royale de Sciences, Paris.

Marković, Ž. (1950): Ruđe Bošković, *Almanach Bošković*, 137–191.

Marković, Ž. (1968–69): *Ruđe Bošković*. JAZU, Zagreb, 1144 pp.

Martinović, I. (2012): Ruđer Josip Bošković, On the Aurora Borealis, Rome, 1738. (in Croatian, translation from Latin, part of a Sci. project sponsored by State Meteorological and Hydrological Service, Zagreb), private communication.

Penzar, B. and Penzar, I. (1997): Notes from the 17th and 18th century about the weather in Makarska, *Croatian Met. J.*, **32**, 59–68, in Croatian.

Udias, A. (2003): *Searching the heavens and the Earth: The history of Jesuit observatories*. Kluwer Academic Publishers, 373 pp, ISBN: 978-1-4020-1189-4.

Whyte, L. L., ed. (1961): *Roger Joseph Boscovich SJ FRS, 1711–1787: Studies of his life and work on the 250th anniversary of his birth*. George Allen and Unwin, London.

SAŽETAK

De Aurora Boreali (1737) – Suvremeni uvid u raspravu mladoga Boškovića

Vladis Vujnović i Inga Lisac

Analiza Boškovićeve rane rasprave o polarnoj svjetlosti pruža uvid u snagu matematičkog postupka koji je mladi Bošković primijenio na istaknute geofizičke pojave, s namjerom da shvati njihovu prirodu. Na temelju danih podataka provjerili smo numeričke rezultate o visini atmosfere i visini polarnih svjetlosti koje su u Europi viđene 1726. i 1737. godine. U njegovu tekstu nema niti jedne ispisane matematičke formule. Slijedeći pak njegove upute, izveli smo formulu koja se može koristiti da bi se visina polarne svjetlosti odredila promatranjem s jednog motrišta. Primjenjivost metode, uz pretpostavljeni model polarne svjetlosti, prikladna je. Nalazimo da izložena ideja o fizičkom uzroku polarne svjetlosti, koji se traži u zamišljenim emanacijama Sunca, sadrži elemente suvremenih shvaćanja.

Ključne riječi: Ruđer Bošković, polarna svjetlost, razmjeri i gustoća atmosfere

Authors' corresponding e-mail addresses: vvujnovi@irb.hr, lisac@irb.hr

Appendix

Instead of closely following Bošković's prescription regarding the relations within the auroral circle, we can find ratio $\overline{AB}/\overline{BG}$ by introducing an auxiliary angle ε (Fig. 10):

(E') $\varepsilon = 90° - 2\delta$

$$\overline{BG} = \overline{GD}\sin\varepsilon = \overline{AG}\sin\varepsilon = \left(\overline{AB} + \overline{BG}\right)\sin\varepsilon$$

since $\overline{GD}$ and $\overline{AG}$ are circle radii.

$$\frac{\overline{AB}}{\overline{BG}} = \frac{1-\sin\varepsilon}{\sin\varepsilon}$$

$$\frac{\overline{BG}}{\overline{BM}} = \sin\varphi, \quad \frac{\overline{AB}}{\overline{BM}} = \frac{\overline{AB}}{\overline{BG}}\frac{\overline{BG}}{\overline{BM}} = \frac{1-\sin\varepsilon}{\sin\varepsilon}\sin\varphi$$

(F') *...and since the ratio* $\overline{VB}:\overline{BA}$, *was given, the ratio composed of them:* $\overline{VB}:\overline{BM}$ *or* $\overline{VM}:\overline{BV}$ *follows*

$$\frac{\overline{VB}}{\overline{BM}} = \frac{\overline{VB}}{\overline{AB}}\frac{\overline{AB}}{\overline{BM}} = \frac{\sin\gamma}{\sin h}\frac{1-\sin\varepsilon}{\sin\varepsilon}\sin\varphi$$

$$\overline{VB} + \overline{BM} = \overline{VM} = \overline{CV}\tan\left(90° - \varphi\right) = \overline{CV}\cot\varphi$$

(G') *Therefore, since* $\overline{VM}$ *is given,* $\overline{VB}$ *and the second side* $\overline{VA}$ *of the triangle AVB will be given*:

$$\overline{VB} = \overline{CV}\,cot\varphi\left[1 + \left(\frac{\sin\beta}{\sin h}\frac{1-\sin\varepsilon}{\sin\varepsilon}\sin\varphi\right)^{-1}\right]^{-1}$$

$$\overline{VA} = \overline{VB}\frac{\cos\varphi}{\sin\beta} = \overline{CV}\frac{\cos\varphi}{\sin\beta}cot\varphi\left[1 + \left(\frac{\sin\beta}{\sin h}\frac{1-\sin\varepsilon}{\sin\varepsilon}\sin\varphi\right)^{-1}\right]^{-1}.$$

Comparing the expression for $\overline{VB}/\overline{BM}$ in (F) and (F'), we find an identity

$$\cot^2\delta - 1 = \frac{2\sin\varepsilon}{1-\sin\varepsilon}, \quad \varepsilon = 90° - 2\delta.$$

(H') *Finally, in the triangle CVA with given sides* $\overline{CV}$, $\overline{VA}$, *and angle between them CVA, one will know* $\overline{CA}$, *and, when the radius* $\overline{CN}$ *is subtracted, one will obtain sought distance* $\overline{NA}$:

$$\overline{CA}^2 = \overline{CV}^2 + \overline{VA}^2 - 2\overline{CV}\,\overline{VA}\cos(90° + h), \quad \cos(90° + h) = -\sin h,$$

$$\overline{CA} = \overline{AN} + \overline{CN}.$$

To sum up:

$$\left(\frac{\overline{AN}+\overline{CN}}{\overline{CN}}\right)^2 = 1 + \left(\frac{cos^2\varphi}{\sin\varphi\sin\beta}[\ \ ]^{-1}\right)^2 + 2\left(\frac{cos^2\varphi}{\sin\varphi\sin\beta}[\ \ ]^{-1}\right)\sin h, \quad (\overline{CV} = \overline{CN})$$

$$\text{where } [\ \ ]^{-1} = \left[1 + \left(\frac{\sin\beta}{\sin h}\frac{1-\sin\varepsilon}{\sin\varepsilon}\sin\varphi\right)^{-1}\right]^{-1}$$

$$\gamma = 90° + \varphi - h$$

$$\varepsilon = 90° - 2\delta = 90° - 2\arctan\left(\frac{\sin h}{\sin\beta}\cot\frac{\gamma}{2}\right).$$