TARGET TRACKING BASED ON A MULTI-SENSOR COVARIANCE INTERSECTION FUSION KALMAN FILTER

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Abstract:

In a multi-sensor target tracking system, the correlation of the sensors is unknown, and the cross-covariance between the local sensors can not be calculated. To solve the problem, the multi-sensor covariance intersection fusion steady-state Kalman filter is proposed. The advantage of the proposed method is that the identification and computation of cross-covariance is avoided, thus the computational burden is significantly reduced. The new algorithm gives an upper bound of the covariance intersection fused variance matrix based on the convex combination of local estimations, therefore, ensures the convergence of the fusion filter. The accuracy of the covariance intersection (CI) fusion filter is lower than and close to that of the optimal distributed fusion steady-state Kalman filter, and is far higher than that of each local estimator. A numerical example shows that the covariance intersection fusion Kalman filter has enough fused accuracy without computing the cross-covariance.

1 Introduction

In recent years, filtering algorithms have been widely used in maneuvering target tracking [1-3]. After the model of maneuvering target has been established, the goal of the target tracking system is to accurately estimate the parameters and states of the target by observed data in real time, which is called the state estimation. Compared to the traditional single-sensor system, a multi-sensor system can obtain higher estimation accuracy. A multi-sensor system analyses and combines the local state estimators or local measurements from each sensor to get the fused state estimation. It can eliminate redundancy and contradiction between sensor information, reduce uncertainty, and thus form a relatively complete and consistent understanding to the system. It has become a hot research topic that applies multi-sensor information fusion technology to resolve the target tracking problems.

The basic methods of information fusion include centralized and distributed fusion based on Kalman filtering. The centralized fusion Kalman method can give the globally optimal state estimation by directly combing local measurement equations to obtain an augmented measurement equation, but the computational burden is heavy. Compared with the centralized fuser, the distributed fusion Kalman method can reduce the computation burden and is more flexible and reliable.

Based on linear unbiased minimum variance rules,

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there are three kinds of distributed weighted fusion algorithm concepts: the matrix weights, scalar weights and diagonal matrix weights. They can give the globally suboptimal state estimators by weighting the local state estimators [4-7]. These weighting fusion estimators have the limitation that they need to compute the cross-covariance among the local estimation errors. However, in many theoretical and application problems, the cross-covariance is unknown, or the computation of the cross-covariance is very complicated [8, 9]. If the cross-covariance is assumed to be zero, it will cause not only an increase of the variance of the local filtering error, but even the divergence of the filtering. In order to overcome the drawback, the covariance intersection (CI) fusion algorithm is presented by Julier and Uhlmann in [10-12], and developed in references [13-15]. The advantage of the CI fusion Kalman filter is that it can avoid computing the local cross-covariance.
In this paper, based on the CI fusion algorithm, the CI fusion Kalman filter is proposed for multi-sensor target tracking systems. The accuracy comparison of the CI fusion Kalman filter, the local Kalman filters and the optimal distributed fusion Kalman filters is given. In order to verify the correctness of the theoretical accuracy relations, an experiment of a three sensor tracking system is given, and the simulation results validated by the proposed CI fusion Kalman filter has enough accuracy, good fusion performance and convergence.

2 Problem formulation

Consider a multi-sensor target tracking system

$$x(t+1)=\Phi x(t)+\Gamma w(t), \tag{1}$$

$$y_i(t)=H_i x(t)+v_i(t), i=1,\cdots,L, \tag{2}$$

where t is the discrete time, $x(t)=\left[x_1(t),x_2(t)\right]^{\mathrm{T}}$ is the state vector, the exponent T denotes the transpose. $x_1(t)$, $x_2(t)$ are the position and velocity of the target, $y_i(t)$ is the measurement signal of the *ith* sensor, L is the number of sensors, $w(t)$ is the input white noise, $v_i(t)$ is the measurement noise of the *ith* sensor, Φ, Γ and H_i are known constant matrices with compatible dimensions. Φ is the state transition matrix, H_i is the measurement matrix.

Assumption 1 $w(t)$ and $v_i(t)$ are uncorrelated white noises with zero mean and variances Q and R_i respectively.

$$\mathrm{E}\left\{\begin{bmatrix} w(t) \\ v_i(t)\end{bmatrix}\begin{bmatrix} w^{\mathrm{T}}(k) & v_j^{\mathrm{T}}(k)\end{bmatrix}\right\}=\begin{bmatrix} Q & 0 \\ 0 & R_{ij}\delta_{ij}\end{bmatrix}\delta_{tk} \tag{3}$$

where E denotes the mathematical expectation, δ_{ij} is the Kronecker δ function, $\delta_{ii}=1,\delta_{ij}=0(i\neq j)$.
Assumption 2 The initial state $x(0)$ is uncorrelated with $w(t)$ and $v_i(t)$.
Assumption 3 The system is completely observable and completely controllable.

3 The local steady-state Kalman filter

For the target tracking system (1)-(2) with the assumptions 1-3, based on the classical Kalman filtering theory, the local steady-state Kalman filter is given as [4,5]:

$$\hat{x}_i(t\,|\,t)=\Psi_i\hat{x}_i(t-1\,|\,t-1)+K_i y_i(t) \tag{4}$$

$$\Psi_i=\left[I_n-K_iH_i\right]\Phi \qquad K_i=\Sigma_iH_i^{\mathrm{T}}\left(H_i\Sigma_iH_i^{\mathrm{T}}+R_i\right)^{-1} \tag{5}$$

n is the dimension of state, Σ_i satisfies the steady-state Riccati equation

$$\Sigma_i=\Phi\left[\Sigma_i-\Sigma_iH_i^{\mathrm{T}}\left(H_i\Sigma_iH_i^{\mathrm{T}}+R_i\right)^{-1}H_i\Sigma_i\right]\Phi^{\mathrm{T}}+\Gamma Q\Gamma^{\mathrm{T}} \tag{6}$$

The actual filtering error variance matrix P_i is denoted as

$$P_i=\left[I_n-K_iH_i\right]\Sigma_i, \tag{7}$$

and P_i also satisfies the Lyapunov equation

$$P_i=\Psi_iP_i\Psi_i^{\mathrm{T}}+\left[I_n-K_iH_i\right]\Gamma Q\Gamma^{\mathrm{T}}\left[I_n-K_iH_i\right]^{\mathrm{T}}+K_iR_iK_i^{\mathrm{T}} \tag{8}$$

The actual steady-state filtering error cross-covariance P_{ij} satisfies the Lyapunov equation

$$P_{ij}=\Psi_iP_{ij}\Psi_j^{\mathrm{T}}+\left[I_n-K_iH_i\right]\Gamma Q\Gamma^{\mathrm{T}}\left[I_n-K_jH_j\right]^{\mathrm{T}} \tag{9}$$

4 The optimal distributed fusion steady-state Kalman filter

According to the above local steady-state Kalman filter when the local error variances $P_i\left(i=1,\cdots,L\right)$ and the error cross-covariances $P_{ij}\left(i,j=1,\cdots,L\right)$ are exactly known, based on the linear unbiased minimum variance rule, the form of the fused steady-state Kalman filter is as follows:

$$\hat{x}_0\left(t\,|\,t\right)=\sum_{i=1}^{L}A_i\hat{x}_i\left(t\,|\,t\right) \tag{10}$$

where A_i denotes the weights.

4.1 The steady-state fusion Kalman filter weighted by matrix [7,9]

The optimal Kalman filter weighted by matrix is given as

$$\hat{x}_m\left(t\,|\,t\right)=\sum_{i=1}^{L}\Omega_i\hat{x}_i\left(t\,|\,t\right) \qquad \sum_{i=1}^{L}\Omega_i=I_n \tag{11}$$

The optimal weights are computed as

$$\left[\Omega_1,\cdots,\Omega_L\right]=\left(e^{\mathrm{T}}P^{-1}e\right)^{-1}e^{\mathrm{T}}P^{-1} \tag{12}$$

where $e^{\mathrm{T}}=\left[I_n,\cdots,I_n\right]$, $P=\left(P_{ij}\right)_{nL\times nL}$.
The optimal fused error variance matrix is obtained by

$$P_m=\left(e^{\mathrm{T}}P^{-1}e\right)^{-1} \tag{13}$$

4.2 The steady-state fusion Kalman filter weighted by scalar [7]

The optimal Kalman filter weighted by scalar is given as

$$\hat{x}_s\left(t\,|\,t\right)=\sum_{i=1}^{L}\omega_i\hat{x}_i\left(t\,|\,t\right),\ \sum_{i=1}^{L}\omega_i=1 \tag{14}$$

The optimal scalar weights are computed as

$$\left[\omega_1,\cdots\omega_L\right]=\left(e^{\mathrm{T}}P_{\mathrm{tr}}^{-1}e\right)^{-1}e^{\mathrm{T}}P_{\mathrm{tr}}^{-1} \tag{15}$$

where $e^{\mathrm{T}}=\left[1,\cdots,1\right]$, $P_{\mathrm{tr}}=\left(\mathrm{tr}\,P_{ij}\right)_{L\times L}$, and the symbol tr denotes the trace of matrix.
The optimal fused error variance matrix weighted by scalar is obtained by

$$P_s=\sum_{i=1}^{L}\sum_{j=1}^{L}\omega_i\omega_jP_{ij} \tag{16}$$

4.3 The steady-state fusion Kalman filter weighted by a diagonal matrix [7]

The optimal Kalman filter weighted by a diagonal matrix is given as

$$\hat{x}_d\left(t\,|\,t\right)=\sum_{i=1}^{L}A_i\hat{x}_i\left(t\,|\,t\right) \tag{17}$$

where A_i satisfies the constraints

$$A_i=diag\left(a_{i1},\cdots,a_{in}\right),\sum_{i=1}^{L}a_{ij}=1,\ j=1,\cdots,n \tag{18}$$

The optimal diagonal matrix weights are computed as

$$\left[a_{1j},\cdots,a_{Lj}\right]=\left[e^{\mathrm{T}}\left(P^{ii}\right)^{-1}e\right]^{-1}e^{\mathrm{T}}\left(P^{ii}\right)^{-1} \tag{19}$$

where $e^{\mathrm{T}}=\left[1,\cdots,1\right]$, $P^{ii}=\left(P_{sk}^{ii}\right)_{L\times L}$, $s,k=1,\cdots,L$, $P_{sk}^{(ii)}$ is the $\left(i,i\right)$ diagonal element of P_{sk}.
The fused error variance weighted by a diagonal matrix is given by

$$P_d=\sum_{i=1}^{L}\sum_{j=1}^{L}A_iP_{ij}A_j^{\mathrm{T}} \tag{20}$$

Apparently, for the above three distributed weighted fusion algorithms, computing the cross-covariance of the local errors is necessary. With an increasing number of the sensors, the difficulty and complexity of computation is greater. Further, in many practical applications, the cross-covariance is either unknownor the computational burden of the cross-covariance is very heavy. So, this paper presents a CI fusion Kalman filter that can avoid the computation of the cross-covariance $P_{ij}\left(i\neq j\right)$.

5 The covariance intersection fusion steady-state Kalman filter

Usually, the covariance intersection algorithm is the convex combination of mean and covariance estimates. Assume that there are two random variables a, b, whose covariance values are P_a, P_b respectively. The measurement errors are $\tilde{a} = a - \bar{a}$, $\tilde{b} = b - \bar{b}$, where $\bar{a}$, $\bar{b}$ are the mean of the a and b respectively. The actual variances and cross-covariance are $\tilde{P}_a = \mathrm{E}\left(\tilde{a}\tilde{a}^{\mathrm{T}}\right)$, $\tilde{P}_b = \mathrm{E}\left(\tilde{b}\tilde{b}^{\mathrm{T}}\right)$, $\tilde{P}_{ab} = \mathrm{E}\left(\tilde{a}\tilde{b}^{\mathrm{T}}\right)$ respectively. The cross-covariance $\tilde{P}_{ab}$ is unknown or is difficult to be obtained, and is not generally zero. If the cross-covariance is ignored, the filtering will diverge.
By means of fusing the local sensor information $\{a, P_a\}$ and $\{b, P_b\}$, we get a new estimate $\{c, P_c\}$, with the constraints that the local estimates are consistent, i.e., $P_a - \tilde{P}_a \geq 0$, $P_b - \tilde{P}_b \geq 0$. The new estimate is consistent too, i.e., $P_c - \tilde{P}_c \geq 0$ [11-13]. The computation is given as

$$P_c^{-1} = wP_a^{-1} + (1-w)P_b^{-1} \tag{21}$$

$$c = P_c\left[wP_a^{-1}a + (1-w)P_b^{-1}b\right] \tag{22}$$

where $0 \leq w \leq 1$. The coefficient w determines the weights of the random scalar a, b. According to different criteria, we can select different optimization methods to improve the weight w. w has the only optimal value in the interval $0 \leq w \leq 1$.
The intersection domain of the covariance is shown in Fig. 1.

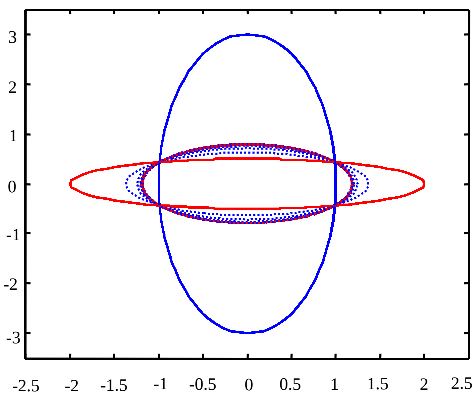

Figure 1. The covariance ellipses.

In Fig. 1, the dotted lines represent the covariance intersection fusion covariance matrix ellipses. Obviously, the CI fusion ellipse encloses the intersection region of covariance ellipses for P_a and P_b, and passes through the four intersections of covariance ellipses for P_a and P_b. The smaller the CI fusion ellipse is, the higher the accuracy of the fusion algorithm is. The dashed red ellipse is the optimal covariance ellipse by optimizing the coefficient w.
In this paper, for the target tracking system, applying the multi-sensor CI fused algorithm, the multi-sensor CI steady-state Kalman fuser is presented as follows:

$$\hat{x}_c(t\,|\,t) = P_c\sum_{i=1}^{L}\omega_i P_i^{-1}\hat{x}_i(t\,|\,t) \tag{23}$$

$$P_c = \left[\sum_{i=1}^{L}\omega_i P_i^{-1}\right]^{-1}, \sum_{i=1}^{L}\omega_i = 1, \omega_i \geq 0 \tag{24}$$

The weighting coefficients ω_i can be obtained by minimizing the performance function as

$$\min_{\omega_i} \operatorname{tr} P_c = \min_{\substack{\omega_i \in [0,1] \\ \omega_1 + \cdots + \omega_L = 1}} \operatorname{tr}\left\{\left[\sum_{i=1}^{L}\omega_i P_i^{-1}\right]^{-1}\right\} \tag{25}$$

For the nonlinear optimization formula (25), the optimal weighting coefficients ω_i can be obtained by means of Matlab fmincon function. From (25), we can know that when the number of the sensors L increases, the computation burden of the nonlinear equation (25) is heavier.
From (13), (16) and (20), we can know that the complexity of the fuser with matrix weights is $nL \times nL$, and they all need to compute the cross-covariances P_{ij}, while from (24) and (25) we can know the complexity of the CI fuser is a nonlinear L-dimensional function and the computation of the cross-covariances is completely avoided, so the CI fuser can reduce the computation burden.

6 Simulation example

Consider a three- sensor two- channel tracking system

$$x(t+1) = \Phi x(t) + \Gamma w(t) \tag{26}$$

$$y_i(t) = H_i x(t) + v_i(t), i = 1, 2, 3 \qquad (27)$$

$$\Phi = \begin{bmatrix} 1 & T_0 \\ 0 & 1 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.5T_0^2 \\ T_0 \end{bmatrix},$$

$$H_1 = [1 \quad 0],\ H_2 = I_2,\ H_3 = [1 \quad 0] \qquad (28)$$

where $T_0 = 0.5$ second is the sample period, $x(t) = [x_1(t), x_2(t)]^{\mathrm{T}}$ is the state vector, $x_1(t)$, $x_2(t)$ and $w(t)$ are the position, velocity and acceleration of the target at time t, $y_i(t)$ is the measurement of the *ith* sensor, $w(t)$ is the input white noise, $v_i(t)$ is the measurement noise of the *ith* sensor. $w(t)$ and $v_i(t)$ are independent Gaussion white noises with zero mean and variances Q and R_i respectively. In the simulation, we take $Q = 2.5$, $R_1 = 1.8$, $R_2 = diag(12, 0.25)$, and $R_3 = 1.64$.
The simulation graphics of $x_1(t)$ are shown in Fig. 2 – Fig. 8. The simulation graphics of $x_2(t)$ are shown in Fig. 9 – Fig. 15, where the solid lines represent the true values of the state and the dotted lines represent the estimated values.

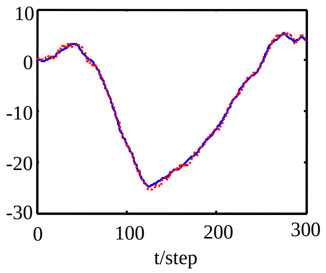

Figure 2. The simulation graphics of $x_1(t)$ in the local sensor 1.

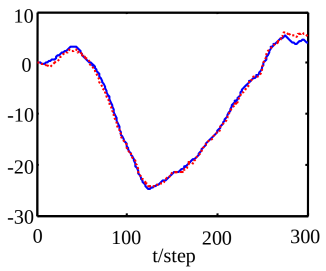

Figure 3. The simulation graphics of $x_1(t)$ in the local sensor 2.

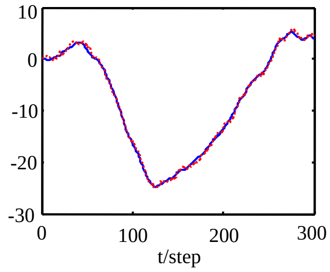

Figure 4. The simulation graphics of $x_1(t)$ in the local sensor 3.

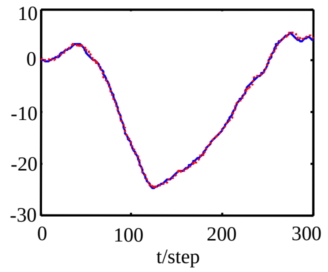

Figure 5. The simulation graphics of $x_1(t)$ in the filter weighted by the/a matrix.

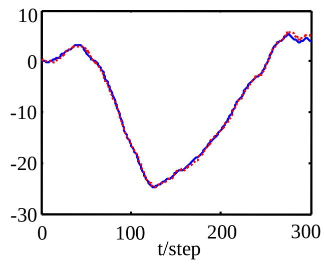

Figure 6. The simulation graphics of $x_1(t)$ in the filter weighted by scalar.

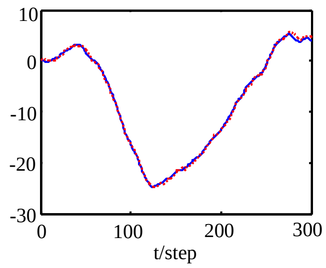

Figure 7. The simulation graphics of $x_1(t)$ in the filter weighted by a diagonal matrix.

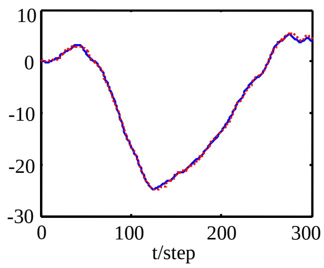

Figure 8. The simulation graphics of $x_1(t)$ in the CI fusion filter.

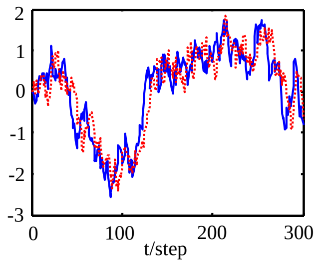

Figure 9. The simulation graphics of $x_2(t)$ in the local sensor 1.

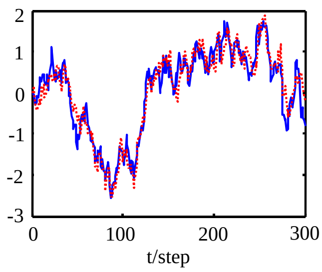

Figure 10. The simulation graphics of $x_2(t)$ in the local sensor 2.

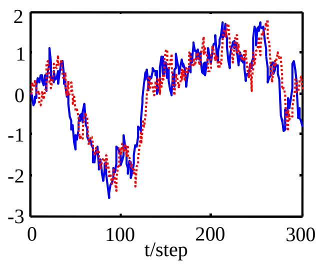

Figure 11. The simulation graphics of $x_2(t)$ in the local sensor 3.

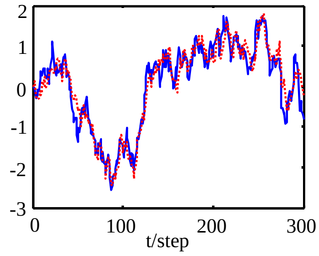

Figure 12. The simulation graphics of $x_2(t)$ in the filter weighted by the/a matrix.

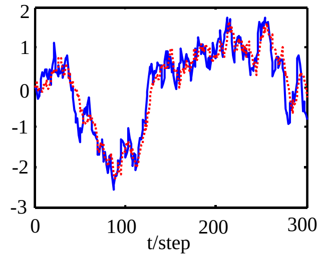

Figure 13. The simulation graphics of $x_2(t)$ in the filter weighted by scalar.

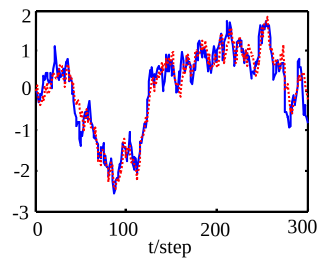

Figure 14. The simulation graphics of $x_2(t)$ in the filter weighted by a diagonal matrix.

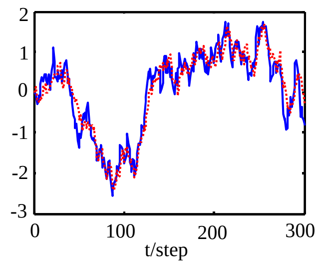

Figure 15. The simulation graphics of $x_2(t)$ in the CI fusion filter.

In order to further verify the accuracy relations, the values of mean square error (MSE) in the local and fusion Kalman filters are shown in Table 1.

Table 1. The values of MSE in the local filters and fusion Kalman filters

	MSE
The local sensor 1	0.0053
The local sensor 2	0.0099
The local sensor 3	0.032
The filter weighted by a matrix	0.0012
The filter weighted by scalar	0.0015
The filter weighted by a diagonal matrix	0.0014
The CI fusion filter	0.0036

From Table 1, we can see that the accuracy of the CI fusion filter is higher than the accuracy of the local estimates, and a little lower than the accuracy of the optimal distributed fusion Kalman filters.
The computation time of the proposed CI fusion Kalman filter and the three optimal distributed fusion Kalman filters is shown in Table 2. The unit of time is second.

Table 2. The computation time of error variances in the fusion Kalman filters

	Computation time
The filter weighted by a matrix	0.025s
The filter weighted by scalar	0.0030s
The filter weighted by a diagonal matrix	0.0032s
The CI fusion filter	0.0026s

From Table 2, we can see that the computation time of the CI fuser is shorter than other fusers.
In general, the CI fusion Kalman filter has enough filtering accuracy, light computation burden, which avoids computing the cross-covariance of the local filters.

7 Conclusion

In this paper, for the target tracking system, the multi-sensor CI fusion Kalman filter is presented based on the covariance intersection algorithm. Its advantage is that it can avoid computing the cross-covariance of the local filtering errors, thus, it can decrease the computational complexity. Compared with the three distributed weighted fusion algorithms, the CI fusion filter shortens the computational time, eases the computational burden, and solves the problems with unknown correlations. The accuracy of the CI fusion filter is much higher than the accuracy of the local estimates, and is a little lower than the accuracy of the optimal distributed fusion Kalman filters. The simulation results of the three- sensor two - channel tracking system show that the CI fusion Kalman filter has a good fused performance. The proposed fusion Kalman filter can be extended into a time-varying system with relevant noise variances. It will be researched in the future.

References

[1] Shalom, Y. B., Li, X. R., Kirubarajan, T.: *Estimation with Application to Tracking and Navigation*, John Wiley & Sons Inc, New York, 2001.

[2] Mori, S., Barker, W., Chong, C., chang, K.: *Track association and track fusion with nondeterministic target dynamics*, IEEE Trans. on Aerospace and Electronic System, 38 (2002), 2, 659-668.

[3] Zhu, Y., Song, E., Zhou, J., You, Z.: *Optimal dimensionality reduction of sensor data in multisensor estimation fusion*, IEEE Trans. on Signal Processing, 53 (2005), 5, 1631-1639.

[4] Deng, Z. L., Gao, Y., Mao, L., Li, Y., Hao, G.: *New approach to information fusion steady-state Kalman filtering*, Automatica, 41 (2005), 10, 1695-1707.

[5] Zhu, Y. M., You, Z. S., Zhao, J.: *The optimality for the distributed Kalman filtering fusion with feedback*, Automatica, 37(2001), 9, 1489-1493.

[6] Marano, S., Matta, V., Willett, P.: *Distributed estimation in large wireless sensor networks via a locally optimum approach*, IEEE Trans. on Signal Processing, 56 (2008), 2, 748-756.

[7] Gao, Y., Ran, C. J., Sun, X. J., Deng, Z. L.: *Optimal and self-tuning weighted measurement fusion Kalman filters and their asymptotic global optimality*, International Journal of Adaptive Control and Signal Processing, 24 (2010), 11, 982-1004.

[8] Wang, Y. M., Li, X. R.: *Distributed estimation fusion with unavailable cross-correlation*, IEEE Trans. on Aerospace and Electronic Systems, 48 (2012), 1, 259-278.

[9] Cattivelli, F., Sayed, A. H.: *Diffusion distributed Kalman filtering with adaptive*

weights, Proceedings of the 2009 Asilomar Conference on Signals, Systems, Computers, Pacific Grove, California, USA, 2009, 908-912.

[10] Julier, S.J., Uhlmann, J.K.: Simultaneous localization and map building using split covariance intersection, Proceedings of the 2001 IEEE International Conference on Intelligent Robots and Systems, Maui, Hawaii, USA, 2001, 1257-1262.

[11] Julier, S. J.: *The stability of covariance inflation methods for SLAM*, Proceedings of the 2003 IEEE International Conference on Intelligent Robots and Systems, Las Vegas, Nevada, USA, 2003, 2749-2754.

[12] Uhlmann, J. K.: *Covariance consistency methods for fault-tolerant distributed data fusion*, Information Fusion, 4 (2003), 3, 201-215.

[13] Arambel, P. O., Rago, C., Mehra, R. K.: *Covariance intersection algorithm for distributed spacecraft state estimation*, Proceedings of the 2001 American Control Conference, Arlington, VA, USA, 2001, 4398-4403.

[14] Chong, C.Y., Mori, S.: *Convex combination and covariance intersection algorithm in distributed fusion*, Proceeding of the 2001 International Conference on Information Fusion, Montreal, Canada, 2001, 111-118.

[15] Hu, J. W., Xie, L. H., Zhang, C. S.: *Diffusion Kalman filtering based on covariance intersection*, IEEE Trans. on Signal Processing, 60(2012), 5, 891-902.