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ATTRIBUTE REDUCTION ALGORITHM BASED ON COGNITIVE MODEL OF GRANULAR COMPUTING IN INCONSISTENT DECISION INFORMATION SYSTEMS

Xiao Tang, Lan Shu

Original scientific paper
This article aims to explore a new method of attribute reduction in inconsistent decision information systems. By analyzing the connection of attribute reduction theory and cognitive science, an attribute reduction algorithm based on cognitive model of granular computing is proposed in this paper. Algorithm analysis and numerical experiment show the validity of the proposed attribute reductions algorithm. The method can be applied to both consistent and inconsistent information systems. The proposed model also provides a new model and thinking to study the connection of human's cognition and notion. It is useful to the development of cognitive model.

Keywords: *attribute reduction, cognitive model, granular computing, inconsistent decision table*

Algoritam redukcije atributa zasnovan na spoznajnom modelu granularnog računanja u informacijskim sustavima nedosljedne odluke

Izvorni znanstveni članak
Cilj je ovoga rada istražiti novu metodu redukcije atributa u informacijskim sustavima nekonzistentne odluke. Analizirajući povezanost teorije redukcije atributa i kognitivne znanosti, u radu se predlaže algoritam redukcije atributa zasnovan na kognitivnom modelu granularnog računanja. Analiza algoritma i numerički eksperiment pokazuju vrijednost predloženog algoritma redukcije atributa. Ta se metoda može primijeniti i na konzistentne i nekonzistentne sustave. Predloženi model također daje i novi model i način razmišljanja za proučavanje povezanosti ljudske spoznaje i poimanja. Koristan je za razvoj kognitivnog modela.

Ključne riječi: *kognitivni model, granularno računanje, redukcija atributa, tabela nekonzistentne odluke*

1 Introduction

Attribute reduction is an important issue of data mining. It is generally regarded as a reprocessing phase that alleviates the curse of dimensionality and keeps the same classification capability. Attribute reduction based on the theory of rough sets has been proved to be a very useful approach for knowledge discovery [18]. Many types of knowledge reductions have been proposed in the area of rough sets [3, 5, 6, 8÷15, 17, 26, 27]. In the real world, most decision information systems are inconsistent because of various factors such as noise in data, compact representation, prediction capability, etc. Inconsistent rules are defined as the rules with the same conditions but different decisions. In recent years, more attention has been paid to knowledge reduction in inconsistent systems [6, 8, 11, 14, 16, 17].

Granular Computing (GrC) is a new tool and computing paradigm of information processing. It was mainly used to handle the uncertain, fuzzy, incomplete, and mass information. Granular computing represents information in the form of some aggregates (called "information granules") such as subsets, classes, and clusters of a universe and then solves the targeted problem in each information granule. A granule may be interpreted as one of the numerous small particles forming a larger unit. Granular Computing is a kind of new concept and new method in the artificial intelligence field to simulate the human problem-solving in cognition process.

Cognitive models are appearing in all fields of cognition at a rapidly increasing rate, ranging from perception to memory to problem solving and decision-making. Applications of cognitive modeling are beginning to spill over into other fields including human factors, clinical psychology, cognitive neuroscience, machine learning. At present, there are four typical cognition models [7], "Psychology cognition model", "Cognitive informatics Models of the Brain", "Ontology cognition model" and "Concept cognition model". Cognitive psychology [1] is the branch of psychology that studies all our mental abilities including how people perceive, learn, remember, think, reason, and understand. As part of the larger field of cognitive science, cognitive psychology is an interdiscipline. It is closely related to the other disciplines including artificial intelligence, computer science, philosophy, anthropology, linguistics, biology, physics, and neuroscience. Prof. Ying-xu Wang [19 ÷ 22] has been studying cognitive informatics in connection with his fundamental research in software engineering many years. Cognitive informatics, founded by Prof. Wang and his colleagues during 2000 ÷ 2002, is a new discipline that studies the natural intelligence and internal information processing mechanisms of brain, as well as the processes involved in perception and cognition. It involves many subjects, such as software engineering, cognitive science, neuropsychology, life science, and philosophy, etc. It is recognized that many fundamental issues in computation and software engineering are related to a deeper understanding of the mechanisms of human information processing and cognition processes and models.

Cognitive models describe human information processing at a more abstract and mathematical level of analysis. So we have reason to believe that attribute reduction and cognitive model has some internal relationship. Prof. Zhang [23] builds a novel cognition model based on formal concept analysis. Through analyzing the sufficiency and necessity of attributes and objects, they derive the description of cognition process and establish the mathematical model. The model provides a new and convenient tool for the research of artificial intelligence. By analyzing the sufficiency and necessity of attributes and objects, we may obtain that the necessary and sufficient granular may be the optimal attribute of objects.

This paper is an extended version of a paper [24]. In Section 2, we recall the basic concepts of information systems and rough set. In Section 3, the algorithm of attribute reduction based on cognitive model of granular computing will be described. A numerical experiment is presented to validate the efficiency of the proposed algorithm in inconsistent decision information systems in Section 4. In Section 5, we conclude this paper and discuss the outlook for further work.

2 Basic concepts

2.1 Basic concepts of reduction theory [2, 23, 25]

Definition 1: The information systems (IS) is a pair (U, A), where $U = \{x_1, x_2, \cdots, x_n\}$ is a non-empty finite set of objects; $A = \{a_1, a_2, \cdots, a_m\}$ is a non-empty finite set of attributes and it is composed of *C* and *D*. Each subset of attributes $p \subseteq A$ determines a binary indistinguishable relation $IND(p)$ as follows: $IND(p) = \{(u,v) \in U \times U \,|\, \forall a \in p, a(u) = a(v)\}$. A set $X \subseteq U$ represents a concept and partition included by *IND*(*p*) is called a knowledge base and denoted by *U*/*IND*(*p*). In particular, *U*/*IND*(*p*) $= \{Y_1, Y_2, \cdots, Y_k\}$ is the knowledge base of decision classes.

Definition 2: A quintuple (U, C, F, D, V) is called a decision information system (DIS), where (U, A, F) is an IS. *A* is a non-empty finite set of attributes with $C \bigcup D$. *C* is a condition attribute set. *D* is a decision attribute set with $C \bigcap D = \phi$ and *V* is attribute values 1 or 0. For every $a \in A$, there is a mapping $F, F = \{f_j : j \le m\}$ is a set of relationship between *U* and *A*, $f_j : U \to V_j (j \le m)$, V_j is the value set of attribute a_j.

Definition 3: If $IND(R) = IND(R - |a|)$, a is dispensable in the set *R*, otherwise indispensable. If $p = R - |a|$ is independent, *p* is a reduction in the set *R*. The indispensable relation in the set *R* is called a core. All of the cores compose a set called core set which is denoted as *core*(*R*). $core(p) = \bigcap red(p)$ in which $red(p)$ is all the reduction sets of p.

Definition 4: If the universe being discussed is *U*, for each subset $X \in U$ and an equivalence relation $p \subseteq R$, the set *X* can be divided according to the basic sets of the *R*, namely lower approximation set and upper approximation set.

$$\underline{R}(x) = \{x \in U \,|\, [x]_R \subseteq X\}, \tag{1}$$

$$\overline{R}(x) = \{x \in U \,|\, [x]_R \bigcap X \ne \phi\}, \tag{2}$$

where $[x]_R$ refers to an equivalence class of $IND(p) = \bigcap IND(R)$ determined by element *x*.

$R - positive$, $R - negative$ and $R - boundry$ regions of *X* are respectively defined in the following.

$$POS_R(X) = \underline{R}(x), \tag{3}$$

$$NEG_R(X) = U - \overline{R}(x), \tag{4}$$

$$BN_R(X) = \overline{R}(x) - \underline{R}(x). \tag{5}$$

If $\underline{R}(x) = \overline{R}(x)$, then *X* is called a definable set on *U* about *R*, If $\underline{R}(x) \ne \overline{R}(x)$, then *X* is called a rough set on *U* about *R*.

Definition 5: Information systems (*U*, *C*, *D*), $p \subseteq C, x_1, x_2 \in U$.

If $\forall a \in P, a(x_1) = a(x_2), \quad \exists d \in D, d(x_1) \ne d(x_2)$ then we say x_1 and x_2 are inconsistent about attribute *a*. If $P = C$, *U* is called an inconsistent system.

Theorem 1: Let $S = (U, C, D)$ be an information system (IS),

$$r_C(D) = \frac{card(POS_C(D))}{card(U)} \tag{6}$$

is the degree of dependency of conditional attributes in classifying the data set examples into the classes in *D*.

Then (1) r = 1 Information systems (*U*, *C*, *D*) is consistent, (2) $r \ne 1$ Information systems (*U*, *C*, *D*) is inconsistent.

2.2 Cognitive modeling

Cognitive science is a new field which studies the natural intelligence and internal information processing mechanisms of the brain. The processes are involved in perception and cognition. Cognitive models are appearing in many fields of cognition at a rapid increase. Many articles appearing in major theoretical journals of Cognitive Science involve cognitive modeling. Furthermore, applications of cognitive modeling are beginning to spill over into other fields including clinical psychology, cognitive neuroscience, biomedical science, human computer interaction theory, and many more. Cognitive science is concerned with understanding the processes of the brain, especially the human brain. Cognition process is a complex task including perceiving, learning, remembering, thinking, predicting, inference, problem solving, decision-making. The goal of a cognitive model is to scientifically illustrate these basic cognitive processes, or explain how these processes interact.

A conceptualized cognitive model [23] is that it is described in formal, mathematical or computer languages. In the cognition, reasoning and decision making process of people, a large amount of complex information is divided into a number of simple blocks, classes, groups or sets according to their characteristics and performance. Such block, class, group or set is called an information granule.

Definition 5: A cognitive system is defined as a 4-tuple (L_1, L_2, L, H), such that $H \circ L(a) \ge a$; $L \circ H(b) \ge b$, where $H \circ L(a)$ and $L \circ H(b)$ means $H(L(a))$ and $L(H(b))$.

Proposition 1: Suppose (L_1, L_2, L, H) is a cognitive system, we may obtain:

1) $\forall a_1, a_2 \in L_1$, if $a_1 \le a_2$, $L(a_2) \le L(a_1)$.

2) $\forall b_1, b_2 \in L_2$, if $b_1 \le b_2$, $H(b_2) \le H(b_1)$

3) $\forall a_1, a_2 \in L_1$, then $L(a_1) \vee L(a_2) \le L(a_1 \wedge a_2)$.

4) $\forall b_1, b_2 \in L_2$, then $H(a_1) \vee H(a_2) \le H(b_1 \wedge b_2)$.

5) $\forall a \in L_1$, then $L \circ H \circ L(a) = L(a)$.

6) $\forall b \in L_2$, then $H \circ L \circ H(b) = H(b)$.

Prof:

1) Let L be an extension-intension operator.

For $a_1 \le a_2$, we have $L(a_2) = L(a_1 \vee a_2) = L(a_1) \wedge L(a_2)$, then $L(a_2) \le L(a_1)$.

2) The same to 1).

3) For $(a_1 \wedge a_2) \le a_1$, $(a_1 \wedge a_2) \le a_2$ and 1), we have

$L(a_1) \le L(a_1 \wedge a_2)$,

$L(a_2) \le L(a_1 \wedge a_2)$.

Then $L(a_1) \vee L(a_2) \le L(a_1 \wedge a_2)$.

4) The same to 3).

5) For $H \circ L(a) \ge a$ and 1),

we have $L(a) \ge L \circ H \circ L(a)$.

For $L \circ H(b) \ge b$, if $b = L(a)$,

we have $L \circ H \circ L(a) \ge L(a)$.

Then $L \circ H \circ L(a) = L(a)$.

6) The same to 5).

Proposition 2: For a formal context (U, A, F), we note $L_1 = p(U), L_2 = p(A)$,

where $U = \{x_1, x_2, \cdots, x_n\}$, $A = \{a_1, a_2, \cdots, a_m\}$. Then X^* is the intension operators and B^* is the extension operators of (U, A, F).

Proposition 3: For a fuzzy formal context $(U, \widetilde{A}, \widetilde{F})$, we note $L_1 = L^U, L_2 = L^A$,

where $U = \{x_1, x_2, \cdots, x_n\}$, $A = \{a_1, a_2, \cdots, a_m\}$. Then $X^*(a)$ is the intension operators and $B^*(a)$ is the extension operators of $(U, \widetilde{A}, \widetilde{F})$.

Definition 6: For a cognitive system (L_1, L_2, L, H), Zhang defined the information granule in [23]. Where, $L_1 = p(U), L_2 = p(A)$ is complete lattice and L, H is operator of extension-intension and intension-extension respectively. The power set of U and A is $p(U)$ and $p(A)$. We note

$$g_1 = \{(a,b)\big|\}b \le L(a), a \le H(b)\}; \tag{7}$$

$$g_2 = \{(a,b)\big|\}L(a) \le b, H(b) \le a\}. \tag{8}$$

If $(a,b) \in g_1$, (a,b) is called as necessary information granule and b is called as the necessary attribution of a. And g_1 is called as necessary information granule set.

If $(a,b) \in g_2$, (a,b) is called as sufficient information granule and b is called as the sufficient attribution of a. And g_2 is called as sufficient information granule set.

If $(a,\mathrm{b}) \in g_1 \bigcup g_2$, (a,b) is called as information granule of the cognitive system. $g_1 \bigcup g_2$ is called as information granule set of the cognitive system.

If $(a,b) \in g_1 \cap g_2$, such that $b = L(a)$ and $a = H(b)$, (a,b) is called as necessary and sufficient information granule of the cognitive system. b is called as the necessary and sufficient attribution of a.

3 Attribute reduction algorithm based on cognitive model of granular computing

Attribute reduction aims to find minimal attribute set. It has the same classification power as all the attributes, i.e., if $POS_C(S) = POS_{C-a_j}(S)$, a_j is dispensable in the set C, otherwise indispensable. In consistent information system, it is equal to computing the degree of dependency $r_{C-a_j}(D) = \dfrac{card(POS_{C-a_j}(D))}{card(U)}$ of conditional attributes is in classifying the data set examples into the classes in D. If $r_{C-a_j}(D) = 1$, a_j is dispensable in the set C and information systems (U, C, D) is consistent; if $r_{C-a_j}(D) \ne 1$, a_j is indispensable in the set C and Information systems (U, C, D) is inconsistent.

Definition 7: Let S = (U, C, D) be an information systems, for $\forall a_j \in C, (1 \le j \le m)$,

If $r_{C-a_j}(D) \ne 1$, then a_j is called as the necessary attribution granule of C, the set $B = \bigcup\{a_j \big| j \le m\}$ is called as necessary information granule set.

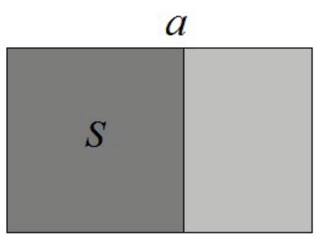

Figure 1 Sufficient attribution

If $r_{C-a_j}(D) = 1$, then a_k is dispensable in C, the set C is called as sufficient information granule set. We note $B' = C - a_j$.

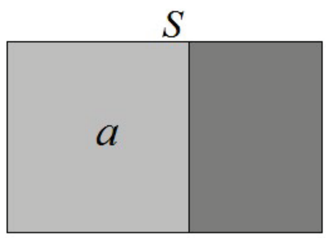

Figure 2 Necessary attribution

Theorem 2: If $B = B'$, then B is called as necessary and sufficient information granule set of the information system and $b_i = \{b \mid \forall b \in B\}$ is called as necessary and sufficient attribution of C, then $red(C) = B$. If $B \neq B'$, then $red(C) = B \cap B'$.

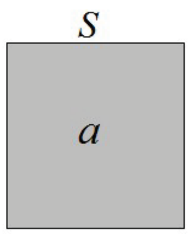

Figure 3 Necessary and sufficient attribution

Algorithm I: Attribute Reduction Algorithm in Consistent Decision System

Input: decision table $S = (U, C, F, D, V)$

Output: the attribute reduction result $red(C)$ of the decision table *S*.

Step 1: (Initiating)

Calculate the degree of dependency of conditional attributes $r_C(D) = \dfrac{card(POS_C(D))}{card(U)}$, if $r_C(D) = 1$, the decision table is consistent and we go to the next step.

Step 2: (Eliminating redundant attribute)

Compute every degree of dependency of $C - a_j$ respectively, $a_j \in \{a_1, a_2, \cdots, a_m\}$. For each $a_j \in A$, if $r_{C-a_j}(D) \neq 1$, end. Then a_j is indispensable in the set C, we could obtain the reduction set $red(C) = C$.

Step 3: If $a_i \in A, r_{C-a_i}(D) = 1$, then a_i is dispensable in the set C,we note $B_i = C - a_i$.

Step 4: (Redefine the attribute set C)

Redefine $C = B_i = C - a_i$ and return step 3. Repeat step 3 (from $i = 1$ to m) until the decision is inconsistent, i.e., $r_{C-a_j}(D) \neq 1$.

Step 5: (Return and output reduction) $red(C) = \cup B_i$.

4 Inconsistent information granulation and harmonized

In an Information systems (U, C, D), $P \subseteq C$, $x_1, x_2 \in U$, if $\forall a \in P, a(x_1) = a(x_2)$, $d(x_1) \neq d(x_2)$, $\exists d \in D$, then we say x_1 and x_2 are inconsistent about attribute *a*. We called it as inconsistent information granule. The algorithm can be developed by four steps: (1) By calculating the degree of dependency of conditional attributes

$r_C(D) = \dfrac{card(POS_C(D))}{card(U)}$, judge the consistency

of the information system. (2) If the system is inconsistent, it will be divided into several subsystems. (3) For every subsystem call the algorithm I. (4) After $red_1(C)$ and $red_2(C)$ are gained, the attribute reduction $red(C)$ of S is computed. Next, we will discuss the specific procedure.

Algorithm II: Attribute Reduction Algorithm in Inconsistent Decision System

Input: decision table $S = (U, C, F, D, V)$

Output: the attribute reduction result $red(C)$ of the decision table *S*.

Step 1: (Initiating)

Calculate the degree of dependency of conditional attributes $r_C(D) = \dfrac{card(POS_C(D))}{card(U)}$, if $r_C(D) \neq 1$, the decision table is consistent and we go to the next step.

Step 2: (Decompose the inconsistent decision information system S)

The inconsistent decision information system S is divided into several consistent subsystems by combination method. For example, an inconsistent information decision tale S = (U, C, D), $U = (x_1, x_2, x_3)$, x_2 and x_3 are inconsistent about attribute *a* ($C(x_2) = C(x_3)$, $d(x_2) \neq d(x_3)$). S is decomposed to two subsystems $S_1 = (U_1, C, F, D, V)$ and $S_2 = (U_2, C, F, D, V)$, where, $U_1 = \{x_1, x_2\}$, $U_2 = \{x_1, x_3\}$.

Step 3: (Eliminating redundant attribute)

For every subsystem, call Algorithm I. The reduction sets of the decision table $S_1 = (U_1, C, F, D, V)$ and $S_2 = (U_2, C, F, D, V)$ are $red_1(C)$ and $red_2(C)$.

Step 4: (output reduction)

Output the reduction $red(C) = red_1(C) \cap red_2(C)$.

5 Experiments

To evaluate its performance, the proposed algorithm was implemented on a simulated data and a UCI machine learning data. The simulated data was chosen because Zeng's reduction algorithm [4] may lose useful information. For comparison, we used the same data from his algorithm and then we tested the algorithm with a UCI machine learning data.

Experiment 1: An inconsistent decision information table $S = (U, C, F, D, V)$.

Table 1 An inconsistent decision information table

U	Condition attribute			Decision attribute	
	a	*b*	*c*	*d*	*e*
x_1	1	0	2	2	0
x_2	0	1	1	1	2
x_3	2	0	0	1	1
x_4	1	1	0	2	2
x_5	1	0	2	0	1
x_6	2	2	0	1	1
x_7	2	1	1	1	2
x_8	0	1	1	0	1

Algorithm:

Step 1: Calculate $U|C, U|D, POS_C(D), r_C(D)$

$U|C = \{\{x_1, x_5\}, \{x_2, x_8\}, x_3, x_4, x_6, x_7\}$,

$U|D = \{\{x_1\}, \{x_2, x_7\}, \{x_3, x_6\}, x_4, \{x_5, x_8\}\}$

$POS_C(D) = \{x_3, x_4, x_6, x_7\}$,

$r_C(D) = \dfrac{card(POS_C(D))}{card(U)} = \dfrac{4}{8} \neq 1$.

The decision rules:

$DesC(x_1) = \{1,0,2\} \Rightarrow DesD(x_1) = \{2,0\}$ and

$DesC(x_5) = \{1,0,2\} \Rightarrow DesD(x_5) = \{0,1\}$;

$DesC(x_2) = \{0,1,1\} \Rightarrow DesD(x_2) = \{1,2\}$ and

$DesC(x_8) = \{0,1,1\} \Rightarrow DesD(x_8) = \{0,1\}$ are inconsistent.

Step 2: For $r_C(D) \neq 1$, the inconsistent decision information system S is divided into several consistent subsystems $\{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\}$.

$S_1 = \{S^*, x_1\}$, $S_2 = \{S^*, x_1, x_2\}$, $S_3 = \{S^*, x_1, x_8\}$, $S_4 = \{S^*, x_5\}$

$S_5 = \{S^*, x_2, x_5\}$, $S_6 = \{S^*, x_5, x_8\}$, $S_7 = \{S^*, x_2\}$,

$S_8 = \{S^*, x_8\}$, $S^* = \{x_3, x_4, x_6, x_7\}$

Step 3: For every subsystem $S_i, (i = 1, 2, \cdots, 8)$, call Algorithm I.

Step 4: $red(C) = \bigcap red_i(C) = \{\{a,b\},\{a,c\}\}$

Figure 4 is the flow chart of the algorithm. $red(C) = \bigcap red_i(C) = \{\{a,b\},\{a,c\}\}$

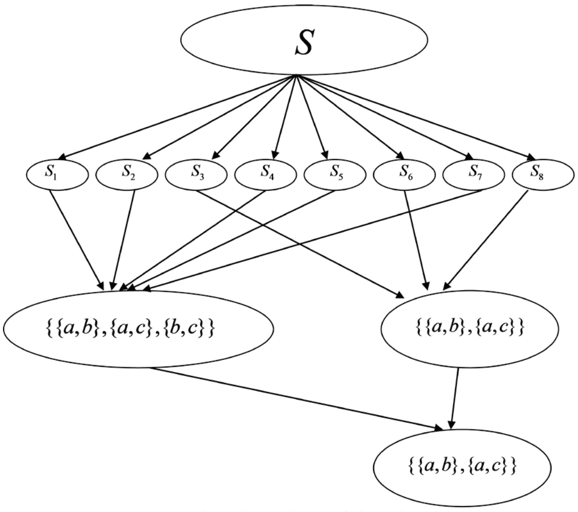

Figure 4 The Flow Chart of the Algorithm

Experiment 2: Tab. 2 is another example of inconsistent decision information system. The data comes from UCI Machine-Learning-Databases/chess/king-rook-vs-king: krkopt. data, 16-Nov-1994 23:24 /519K.

Table 2 An UCI Data

U	Condition attribute		Decision attribute	
	a	b	c	d
x_1	1	a	2	1
x_2	1	b	2	1
x_3	1	a	2	2
x_4	1	b	1	3
x_5	2	b	2	3
x_6	1	b	2	4
x_7	2	b	1	4

We easily obtained the reduction set $red(C) = \{a,c\}\, or\, \{a,b\}$, $core(C) = \{a\}$ by the algorithm II and algorithm I. With the above discussion, the proposed algorithm performs better than conventional approach, especially in small sample and inconsistent information system.

6 Conclusions

Cognitive models are appearing in many fields of cognition at a rapid increase, ranging from perception to memory to problem solving and decision-making. Applications of cognitive modeling are beginning to spill over into other fields including human factors, clinical psychology, cognitive neuroscience, machine learning. Attribute reduction algorithm based on cognitive model of granular computing is proposed in this paper. Algorithm analysis and numerical experiment show the validity of the proposed attribute reductions algorithm. And the proposed model provides a new model and thinking to study the connection of human's cognition and notion. It is also useful to the development of cognitive model.

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Authors' addresses

Xiao Tang, doctoral student
School of Mathematical Sciences, University of Electronic Science and Technology of China
Qingshuihe Campus of UESTC, No. 2006, Xiyuan Avenue, West Hi-tech Zone, Chengdu, Sichuan, P. R. China
Zip/Postal Code: 611731
E-mail: 80651177@163.com

Lan Shu, professor
School of Mathematical Sciences, University of Electronic Science and Technology of China
Qingshuihe Campus of UESTC, No. 2006, Xiyuan Avenue, West Hi-tech Zone, Chengdu, Sichuan, P. R. China
Zip/Postal Code: 611731
E-mail: lanshu@uestc.edu.cn