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Students' Understanding of Velocity-Time Graphs and the Sources of Conceptual Difficulties

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Abstract

We investigated the students' ability to translate the velocity-time graph into a real physical situation and the possible sources of conceptual difficulties. The study involved the sample of n=324 high-school/university students, and physics teachers in Croatia. The students were presented the original non-traditional open-ended graph problem. Their written answers were classified and used to form a closed-ended prediction questionnaire for teachers. Comparing the teachers' expectations with the actual students' responses, we found out that teachers significantly overestimate students' understanding. Our results also show that the observed difficulties are common in all groups of participants and that they do not depend significantly on the educational level and curriculum. We also found that the physics textbooks are also potential sources of difficulties, giving unfinished or incomplete statements regarding the sign of physical quantities in kinematic expressions. We feel that non-traditional graph problems of this kind could help teachers in establishing an active learning process in the classroom to become aware of students' way of thinking and to overcome the difficulties.

Key words: *conceptual understanding; curriculum; graphical representation; physics textbooks; students' problem solving.*

Introduction

Drawing and interpreting graphs are the basic skills of a scientist (McKenzie, & Padilla, 1986). Unlike tables, graphs provide a quick overview of the major trends and details of irregularities among the numerous data describing an event (Mokros, & Tinker, 1987). Chambers et al. (1983) found that there is no better statistical tool than the graphs, to manage complex information.

When compared to scientists and other experts, students' ability to construct and use graph representations is significantly reduced (Larkin, 1981). They usually do not understand the formal language that teachers use when working with graphs (Beichner, 1994). Therefore, research aiming to develop these skills can be an important step towards the better solving of physical problems and understanding of physical concepts (Larkin, 1981). This is confirmed by the research results of Rosenquist and McDermott (1987) according to which many students' difficulties in solving kinematics problems arise from the inability of the interpretation of motion graphs.

The ability to work with graphs successfully is not developed spontaneously or by memorizing simple procedures such as drawing points in a coordinate system, reading the coordinates of points and calculating the slopes of the lines. Such a superficial approach to problem solving is common to the traditional, mathematically-oriented teaching and does not result in conceptual understanding (Rosenquist, & McDermott, 1987).

In order to be able to apply graphical analysis in their future jobs the students should learn how to interpret graphs in a more detailed way. This means, among other things, that they should learn: (i) to select and determine the feature of the graph that contains the relevant information, i.e. to establish an appropriate connection between the feature of the graph and the physical concept (for example, information may be contained in the coordinates of points, in the difference of coordinates of two points or in the slope); (ii) to identify the relationship between different graphs; (iii) to show graphically the real physical systems; (iv) to visualize the system based on its graphical representation. The studies have shown that many students have difficulties in each of these segments (Barclay, 1986; Beichner, 1994; Chambers et al., 1983; Larkin, 1981; McDermott, 1991; McDermott et al., 1987; Mokros, & Tinker, 1987; Zee, & McDermott, 1987). Therefore, they need professional help in learning and mastering the problem.

The difficulties cannot simply be attributed to a lack of mathematical knowledge (McDermott et al., 1987). Most students upon finishing high school have the necessary mathematical skills for simple procedures such as drawing points in a coordinate system or calculating the slope. However, when it comes to the graphs in physics, students often do not know how to apply mathematical knowledge. At the same time they show a fairly good understanding of concepts when solving problems that do not include graphs. Price et al. (1974) found that students have greater difficulty in calculating the slope than in determining the coordinates of points.

Among the responsible factors, with no mathematical background, is an inability to interpret graphs, i.e. inability to connect graphs with the physical concepts and the real world (McDermott et al., 1987). Difficulties of this nature occur regardless of age and educational level (McDermott et al., 1987; Peters, 1982). This confirms that the traditional emphasis on algebra formalism does not lead to a qualitative understanding of physical concepts (Rosenquist, & McDermott, 1987).

McDermott et al. (1987) detected the specific difficulties which arise when students connect graphs with physical concepts. The students most frequently do not know whether they can get the necessary information from the slope or from the graph height (Barclay, 1986; Mokros, & Tinker, 1987), they misinterpret the changes in slope and height, they are rarely able to connect different types of graphs describing the same situation, they hardly solve problems with physical quantities indirectly given by the graph and they misinterpret the area under the graph.

The second group of difficulties arises from the inability to connect the graph with the corresponding object and event in the real world. In carrying out the study where students were asked to present the laboratory situation by the graph and vice versa, McDermott et al. (1987) concluded that most students make the mistakes which will be listed below. They do not represent continuous motion by a continuous line. Instead, they plot a number of unconnected points. This indicates they do not differentiate between the physical quantity at a single instant and the continuous variation in physical quantity over a period of time. Also, in situations where they should fit a smooth curve to the data points, they connect plotted points in a zigzag line. Hence, they seem unaware that the measured values are only approximations. They believe that the shape of the graph should resemble the shape of the motion path (Barclay, 1986; Mokros, & Tinker, 1987). They do not connect a change in the direction of a motion with negative velocity. They misinterpret the sign of acceleration. For example, they associate a negative acceleration only with an object slowing down and having a positive velocity. They are not aware of the fact that an object with negative acceleration and negative velocity accelerates. They do not accept the fact that the same motion can be illustrated with graphs of different shapes (Halloun, & Hestenes, 1985).

The questions that naturally arise are: *What are the main causes of the detected students' difficulties?* and *How can they be overcome?* The aim of our research is to search for the answers to these questions and to search for new, more effective methods of teaching kinematics graphs.

Textbooks play an important role in the process of teaching and learning (Blanton, 2009; Lee, 2007; Podolefsky, & Finkelstein, 2006). Therefore, by analysing the content of physics textbooks it is possible to identify the generally accepted methods and trends in physics teaching and identify the possible sources of students' difficulties. Analysing the lessons on kinematics in the physics textbooks and other physics-teaching materials for high schools in Croatia, we found by far the largest presence of the traditional problems in the majority of tasks. In a traditional graph problem students are asked to represent the motion of a given object by a graph. However, the ability of creating a graph does not necessarily mean the student has the ability of reverse thinking, i.e. the ability of understanding the motion from the graph (Arons, 1984). The traditional approach to physics teaching is generally considered to be insufficient to develop a deeper level of conceptual understanding. For example, students are often asked to plot points to make a graph. Such algorithmic procedure enables students to get the right answer without understanding what the graph represents (Laverty, 2012). In this paper we started from the assumption that non-traditional approach with the encouragement of the reverse process can lead to improving the understanding of the graph representations. Unlike the study by Testa et al. (2002) in which students were called upon to read and interpret kinematics graphs of real-time experiments, we investigated students' ability to interpret the graph of the real-world motion and we use the open-ended graph problem.

Our analysis also showed that the textbooks contain many incomplete statements. They are also potential sources of students' misconceptions as they often provide the syllabus for courses and are used as a principal source of information or explanation (Dall'Alba, 1993). In this paper we highlighted some examples from the textbooks and discussed the possible background of the mistakes. A similar study was conducted by Dall' Alba et al. (1993).

Another source of difficulties is teachers' lack of knowledge of students' problem solving strategies and their inability to adapt their teaching strategy to students' thinking (Mayon, & Knutton, 1997; McDermott, 1993). We investigated this important issue by using the appropriate questionnaire for teachers. The closed-ended questionnaire for teachers was built on the basis of students' responses. We investigated their ability to predict students' strategies of solving the graph problem. Similar research was done by Lightman and Sadler (1993), and Viiri (2003).

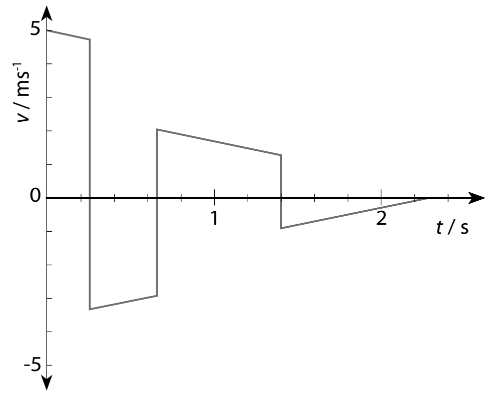
Our study involved high-school students, university students and physics teachers. The students were presented the original velocity-time graph of a billiard ball bouncing back from the edges of a pool table. The given graph problem was sufficiently complex to contain most of the elements needed for the graph representation of a motion, while some of its parts were simple enough to test the basic concepts of motion. It implicitly contained all three "graphicacy" categories of questions that a graph can answer (Bertin, 1973): (1) elementary questions that involve a simple extraction of data, (2) intermediate questions that involve identifying trends and (3) comprehensive questions that ask students to compare the whole structures of the graph. These categories have been refined over the years, as can be seen from the list of references (Friel, 2001). By using the open-ended questions (Johnson, & Christensen, 2004) we enabled the freedom of students' answers. Based on their written answers we were able to analyse their ability to extract the relevant information from the graph, find the physical concepts in the graph and to connect the graph to the real world.

In the paper we firstly described the graph problem and its solution. After that we described the sample in the context of the Croatian education system and research methods for the students, the teachers, and the physics textbooks. We presented the results of the research and finally we discussed the results in the context of the different types of conceptual difficulties and their possible sources.

Problem

Graph representations are a powerful tool to manage the complex information but students do not have sufficiently developed skills to use them. The skills of construction and interpretation of graphs obviously cannot be sufficiently developed by the traditional emphasis on the algebraic formalism, often presented in problems in which the student is asked to construct the graph of an imagined motion, usually not based on a real-life situation or an experiment, as recommended e.g. by Kariž Merhar et al. (2009) and McDermott et al. (1987). Therefore, to improve the methods of teaching graphs, we were motivated to explore the reverse process, students' ability to translate the graph to a real-life situation.

The respondents were presented the following original problem[1]:



1) Describe the changes in the velocity of the ball during the first second. What was happening to the acceleration at the same time?

2) Why is the velocity change stepwise at the time t = 1.4 s? What is the acceleration at that point?

Figure 1. The velocity-time graph of a billiard ball during its linear motion

From Figure 1 it can be concluded that the billiard ball is moving across the width of the pool table between the opposite rails. Note that the width of the billiard table is $d = 1.25$ m. Initially, the ball was placed close to the long rail. After being hit, at the instant $t_0 = 0$, the ball crosses the table perpendicularly to the opposite rail, bounces and moves back, and then bounces again. The bouncing is repeated three times until the ball stops. Rebounds occur at the moments $t_1 = 0.26$ s, $t_2 = 0.66$ s and $t_3 = 1.4$ s, and the ball stops at $t_4 = 2.3$ s, as calculated in Table 1. The time intervals between the rebounds increase and the height of the graph decreases while the absolute value of the slope does not change. This shows that the motion is uniformly decelerated[2] for each segment of the graph. Slowing down occurs due to the friction. The magnitude of the velocity decreases, while the magnitude of acceleration is constant. After each bounce, in a very short time, the directions of velocity and acceleration change, which is seen as a change of sign of both quantities. This stepwise, almost instantaneous velocity

[1] For comparison, the corresponding traditional problem would read:

A billiard ball is moving across the width of the pool table between the opposite rails slowing down uniformly due to the friction. Initially the ball is placed close to the long rail of the pool table. After being hit, the ball crosses the table perpendicularly to the opposite rail, bounces and moves back, and then bounces again. Energy loss occurs at each rebound. The motion continues until, after the three rebounds, the ball stops somewhere in the middle of the table. What would the corresponding velocity-time graph be like?

[2] When the velocity of an object changes, the object is generally said to be undergoing acceleration, or that it accelerates. Especially the slowing down of an object is commonly called deceleration.

change is shown by a vertical line for the actual t-axis resolution. At the reflection point the acceleration assumes a very large magnitude. In addition, the energy loss in the rebound (with the coefficient of restitution $k = 0.7$)[3] leads to the stepwise decrease of the magnitude of velocity. From the equality of the areas under the graph in the first three segments we can see that the ball travels the same distance d three times, i.e. the ball crosses the table three times. Because in the last segment the area under the graph is less than d we can conclude that the ball stops somewhere in the middle of the table.

The motion of the ball is divided into four time intervals (segments) separated by the instants of the rebounds. The values of kinematic physical quantities are calculated for each segment and listed in Table 1. The values for the i-th segment are obtained from the expressions in the first row for: v_{0i} - initial velocity, a_i - acceleration, Dt_i - time interval for crossing the distance d (with the exception of the last interval), v_i – final velocity, t_i - time at the end of the segment, Dx_i - displacement. The maximum displacement d_i changes the sign so that $d_i = d$ for $i = 1, 3$ and $d_i = - d$ for $i = 2$. The negative values denote the direction which coincides with the negative direction of the x axis. The initial velocity is $v_{01} = 5$ m/s, and the initial acceleration $a_1 = -1$ m/s^2. The acceleration is constant in magnitude and directed always opposite to the velocity.

Table 1.

The calculated values of physical quantities for the billiard ball moving across the pool table that are used for the velocity versus time graph in Figure 1. The expressions in the first row are valid for each i-th segment unless indicated otherwise.

	$v_{0i} = -kv_{i-1}$, $i > 1$	$a_i = -a_{i-}$ $i >$	$\Delta t_i = \frac{1}{a_i}\left(\sqrt{v_{0i}^2 - 2a_i d_i} - v_{0i}\right), i = 1, 3$ $\Delta t_i = \frac{1}{a_i}\left(-\sqrt{v_{0i}^2 - 2a_i d_i} - v_{0i}\right), i = 2$ $\Delta t_i = \frac{-v_{0i}}{a_i}, i = 4$	$v_i = v_{\ i} + a_i \Delta t_i$	$t_i = t_{i-} + \Delta t_i$ $t\ =$	$\Delta x_i = \frac{1}{2}(v_{oi} + v_i)\Delta t_i$
i	v_{0i} / m s^{-1}	a_i / m s^{-2}	Δt_i / s	vi / m s^{-1}	t_i / s	Δx_i / m
1	5	-1	0.26	4.74	0.26	1.25
2	- 3.32	1	0.40	- 2.92	0.66	- 1.25
3	2.04	-1	0.75	1.30	1.41	1.25
4	- 0.91	1	0.91	0	2.31	- 0.41

Research and Results

Student Questionnaire

A total of 276 students were involved in the study. The non-random convenience sampling technique (Johnson, & Christensen, 2004) was used. The sample included 72 Science Gymnasium (SG) students and Information-Technology Gymnasium (ITG) students from Rijeka and Zagreb (Croatia), 139 General Gymnasium (GG)

[3] The billiards and pool physics resources page and the typical coefficients of restitution for billiard ball can be found at http://billiards.colostate.edu/threads/physics.html.

students from Rijeka and Zagreb, 24 Vocational School (VS) students from Rijeka, 41 Physics Teacher (PT) students from Rijeka and Zagreb Universities. The students were at different learning levels, but they were all taught the physical concepts needed to solve the problem situation.

Note that gymnasium is a type of secondary school comparable to English grammar schools or U.S. high schools. Gymnasium is intended to prepare students for the university. It lasts for four years and ends with the "matura" exam, a state level examination, which is a requirement for admission into further levels of education. Different types of gymnasiums have different subjects taught at a higher level, e.g. natural sciences and mathematics in SG and information-technology in ITG. On the other hand, vocational schools last for 3-5 years and prepare students for a certain job. The students who will become physics teacher upon completing their studies study in the five-year university program at the faculty/department of science. They graduate with masters' degree in science education and are fully qualified for elementary and high-school teaching of physics and another subject major they studied (usually mathematics, chemistry or information technology).

Students were presented the graph problem described above in the open-ended question form and they were asked to answer the questions and to express their ideas freely. They were given up to 15 minutes to complete the task. After collecting their written responses we found it convenient to classify their responses into seven different groups – in the form of the answers to the seven hypothetical sub-questions; five for question 1 (1.1-1.5) and two for question 2 (2.1-2.2) of the problem, as listed in Table 2. We classified and evaluated the responses of each student by assigning one of three values (correct answer, incorrect answer and no response) to each sub-question. The results of this analysis are also shown in Table 2.

Table 2.
Classification and percentage distribution of the students' responses to the problem.

questions and responses	percentages
1.1 How does the velocity change from 0 s to 0.26 s?	
Correct answer: The velocity decreases.	75%
Incorrect answers:	15%
The velocity increases. (7.05%)	
The velocity is constant. (5.55%)	
The velocity changes non-uniformly. (1.05%)	
The velocity is positive. (1.05%)	
The motion is linear. (0.3%)	
N**o response:**	10%
1.2 How does the magnitude of the velocity change from 0.26 s to 0.66 s?	
Correct answer: The magnitude of the velocity decreases.	30%
Incorrect answers:	52%
The magnitude of the velocity increases. (45.24%)	
The magnitude of the velocity is constant. (3.64%)	
The velocity is negative. (2.08%)	
The magnitude of the velocity changes non-uniformly. (1.04%)	
No response	18%

1.3 What is the physical (not mathematical) meaning of the stepwise velocity change at t = 0.26s and t = 0.66s?

Correct answer: The change of the velocity direction, i.e. the direction of motion.	33%
Incorrect answers:	31%
Sudden deceleration or acceleration. (22.63%)	
Rest. (4.34%)	
Uniform motion. (1.86%)	
The stepwise velocity change. (1.55%)	
The velocity was in a plane. (0.31%)	
The velocity changes sign. (0.31%)	
No response	36%

1.4 What happens to the magnitude of the acceleration in this motion?

Correct answer: The magnitude of acceleration is constant.	7%
Incorrect answers:	27%
The acceleration decreases. (6.48%)	
The acceleration changes. (6.21%)	
The acceleration and deceleration alternate. (4.86%)	
The acceleration decreases and increases. (3.24%)	
The acceleration is negative. (2.16%)	
The acceleration is constant and positive / 10 m/s^2 / changes from deceleration to acceleration. (1.08%)	
The acceleration exists. (1.08%)	
The acceleration does not exist. (1.08%)	
The acceleration increases. (0.81%)	
No response	66%

1.5 What happens to the direction of the acceleration in this motion?

Correct answer:	4%
It changes so that it is always opposite to the direction of motion. (1.32%)	
It changes. (2.68%)	
No response	96%

2.1 Why does the velocity change stepwise?

Correct answer:	28%
The ball collides with the edge of the table. (16.8%)	
Direction of the ball motion changes. (11.2 %)	
Incorrect answers:	48%
The ball collides with another ball, with a cue or with an object/obstacle/ground. (26.4%)	
Answers which mention different types of motion (decelerated, accelerated, non-uniform, falling into a hole, vertical jump). (17.76%)	
Answers which mention various physical quantities (velocity, force, distance, time). For example, *Because the distance changes rapidly; Because the time does not change; The velocity is* 10 m/s; *Due to the friction.* (3.84%)	
No response	24%

2.2 What is the acceleration at t = 1.4 s?

Correct answer: The acceleration assumes a large (infinite) value.	8%
Incorrect answers:	52%
The results obtained by inserting numbers into the expression for acceleration or by some unknown calculation. (27.04%)	
The acceleration is (approximately) equal to zero. (18.2%)	
The acceleration is uniform (negative, equal to g, constant) or it changes (decreases, increases). (6.76%)	
No response	37%

The distribution of the students' answers is shown in Figure 2 in the form of column charts for each of the four groups of respondents: general gymnasium (GG), science gymnasium and information-technology gymnasium (SG&ITG), vocational school (VS), physics teacher (PT) students (Figures 2a - 2d). Each column corresponds to one of the questions from Table 2 and is marked by the same number as in the Table. Furthermore, each column is divided into three segments, marked with different shades, each corresponding to one of the three possible responses: correct answer, incorrect answer and no response. The heights of the segments are proportional to the percentage of the obtained responses.

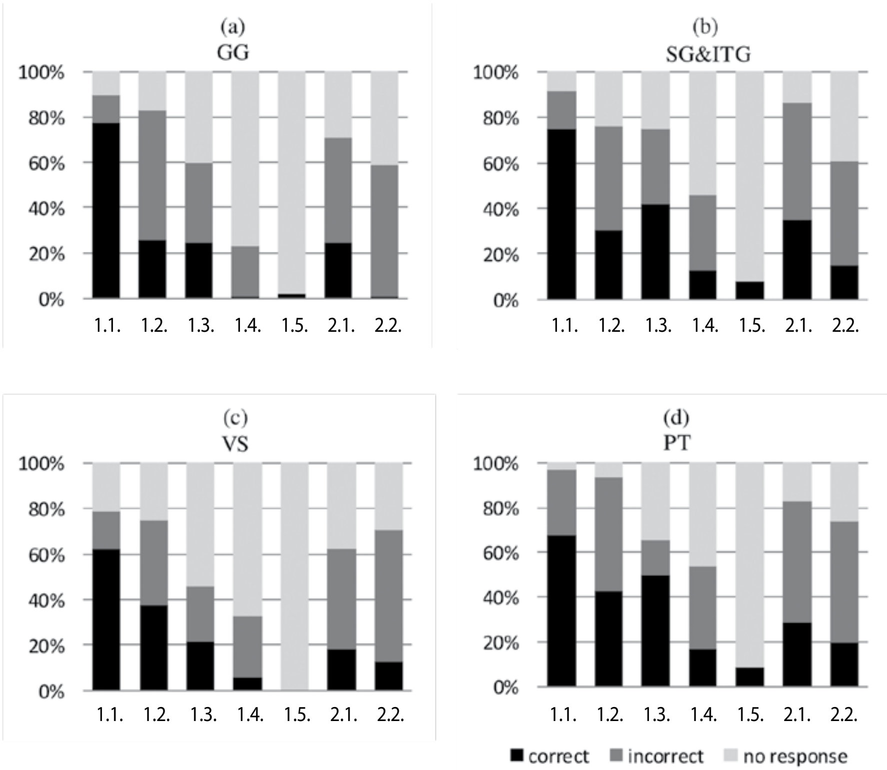


Figure 2. Distribution of the students' answers to the questions from Table 2 for: (a) general gymnasium (GG), (b) science gymnasium and information-technology gymnasium (SG&ITG), (c) vocational school (VS), and (d) physics teacher (PT) students. The result for each question is presented by the column. The number below each column denotes the particular question from the Table. Columns are divided into three segments with the heights proportional to the percentage of the obtained responses (correct answers, incorrect answers and no response).

Question 1.1 is the only question to which the majority of respondents (75%) gave the correct answer. Students chose correctly the height of the graph as the feature leading them towards the conclusion about the velocity. Since the height of the graph in the first segment decreases, the students concluded correctly that the velocity also decreases.

The fact that only 30% of students provided correct answers to question 1.2, where the respondents were supposed to discuss the velocity in the second segment, between

0.26 s and 0.66 s, indicates the difficulty in the interpretation of the negative values of the velocity. In the majority of 52% of incorrect responses students concluded that the magnitude of velocity increases. They considered this part of the graph to be also in the upper part of the coordinate half-plane, like the previous one. They estimated the change in the magnitude of the velocity from the change in graph height rather than from the distance to the x-axis, which led to errors.

Most of the respondents either did not discuss (36%) or they discussed incorrectly (31%) the stepwise change of the velocity at the instants $t = 0.26$ s and $t = 0.66$ s (question 1.3). Even 23% of them considered that the ball suddenly slows down at the first instant, and suddenly accelerates at the second one. Some participants argued that in these instants the ball is at rest or is moving uniformly. In neither of these cases did they relate the change in the sign of the velocity with the change in the direction of motion, which is in accordance with the results of Shaffer and McDermott (2005).

In order to provide the correct response to question 1.4, students were supposed to consider the slope as the relevant feature. It could be inferred from the fact that slopes have the same value for all the segments, that the magnitude of the acceleration is constant. However, even 93% participants did not give the correct response, so that 27% answered incorrectly and 66% did not give any response. Most of the incorrect responses arose from the misconception that the information about the acceleration can be obtained from the height of the graph. This is supported by the answers which claim that the acceleration changes, i.e. decreases and/or increases.

Question 1.5 examines the knowledge about the direction of the acceleration. In our case, the acceleration is always directed opposite to the velocity. This means it is always a deceleration having a negative sign when the velocity is positive and a positive sign when the velocity is negative. The direction of the acceleration was correctly interpreted by only 4% of the respondents, while the remaining 96% did not discuss this issue. The examinees obviously did not take into account the vector nature of the acceleration, which is, besides the amount, also defined by the direction, indicated by a sign. Obviously there are conceptual difficulties associated with the sign of the acceleration.

It is known that most difficulties related to the graphics arise from the inability to visualize the motion (McDermott et al., 1987). With the help of question 2.1 we examined the students' ability to visualize what is happening to the ball in reality. Based on the analysis of time changes of the velocity and the acceleration, and the interpretation of area under the graph as the travelled distance, it was possible to conclude that the motion of the billiard ball is a uniformly decelerated linear motion between the pool table rails, until the moment the ball stops. 28% of the respondents gave the correct answer, while the others were not able to visualize the motion. The explanations given by 48% examinees were incorrect. They did not notice that the area under the graph for the first three segments is the same, so they thought that the ball collides with another ball, with a cue or with some other object.

Insufficient knowledge of the concept of acceleration was confirmed in question 2.2. The participants were expected to conclude that the acceleration acquires a very large value ($a \to \infty$) if the velocity changes almost instantly, i.e. within a short time interval (D$t \to 0$). Most respondents (55%) either gave a wrong answer or gave no answer (37%) to this question. Among the incorrect answers the most common ones were those obtained with the help of formulas for acceleration or by using incorrect calculations.

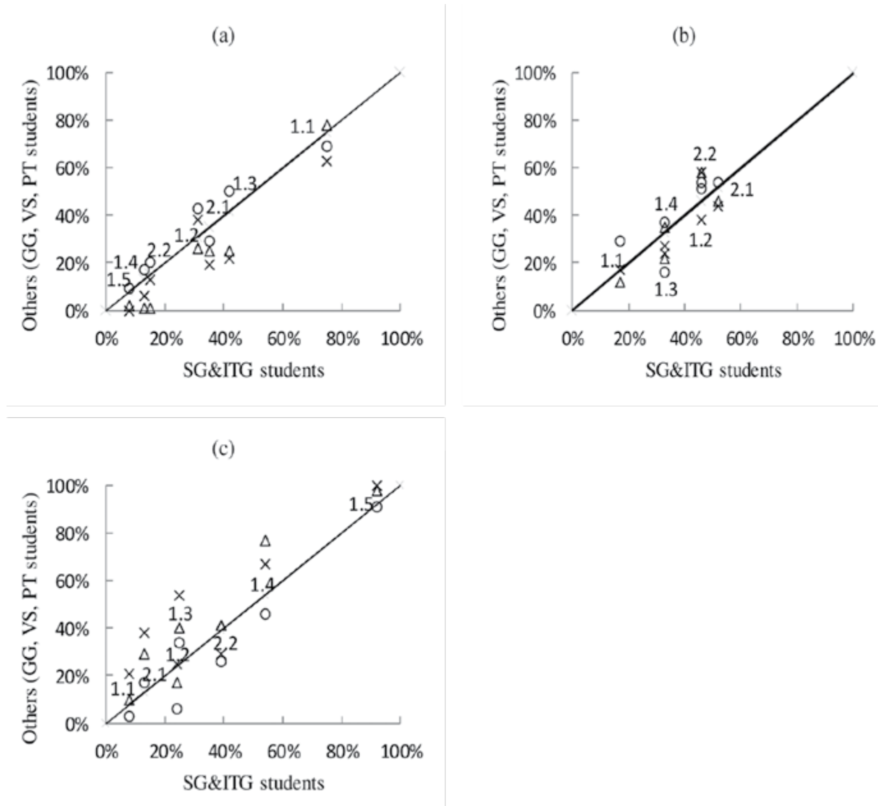


Figure 3. The symbols are the correlations of the percentages of correct answers (a), incorrect answers (b), and no answer responses (c) for GG (D), VS (X), and PT (O) students with respect to SG&ITG students. The numbers denote the groups of symbols that belong to the same question from Table 2. The line of proportionality shows the autocorrelation.

Figure 3 shows the correlations between the percentages of the responses of GG, VS and PT students, and responses of SG&ITG students. The correlations for the correct answers (Figure 3a), incorrect answers (Figure 3b) and no responses (Figure 3c) are shown in the separate graphs. For example, the symbols in Figure 3a are drawn from the heights of the black column segments of Figure 2 with the abscissa values obtained from Figure 2b and the ordinate values from the three graphs (Figures 2a, 2c, 2d), for each of the seven questions. The numbers denote the groups of symbols which correspond to the same question and are consistent with the numeration in Table 2 and Figure 2. The line of proportionality is the autocorrelation, i.e. the maximum possible correlation. The data above the line corresponds to better and the data below the line to lower problem-solving results in relation to SG&ITG students' results. The data closer to the origin denote more difficult questions and the data further from the origin denote easier questions. Figures 3b and 3c have been drawn by analogy.

Teachers' Questionnaire

In the second part of the study we examined a group of 48 high-school physics teachers in Split-Dalmatia County who attended a seminar for physics teachers. Based on the sub-questions (1.1-1.5 and 2.1-2.2) and the students' responses, a closed-ended questionnaire for teachers (see Table 3) was designed. The teachers were asked to mark one of the answers. They were not supposed to mark the answer they thought was correct but the answer they expected their students would probably give.

Table 3.
The closed-ended questionnaire for teachers. The numbers of questions are the same as in Table 2 and Figure 2.

Figure 1 shows the velocity - time graph for the linear motion of the billiard ball.

1.1. How does the velocity change from 0 s to 0.26 s?
a) It is constant. **b)** It increases. **c)** It decreases. **d)** It changes non-uniformly.

1.2. How does the magnitude of the velocity change from 0.26 s to 0.66 s?
a) It is constant. **b)** It increases. **c)** It decreases. **d)** It changes non-uniformly.

1.3. What is the physical meaning of the stepwise velocity change at t = 0.26 s and t = 0.66 s?
- **a)** Rest.
- **b)** Sudden deceleration or acceleration.
- **c)** Uniform motion.
- **d)** The change of the direction of motion.

1.4. What happens to the magnitude of the acceleration in this motion?
- **a)** The acceleration decreases.
- **b)** The magnitude of acceleration is constant.
- **c)** The acceleration increases.
- **d)** The acceleration and deceleration alternate.

1.5. What happens to the direction of the acceleration in this motion?
- **a)** It changes so that it is always opposite to the direction of motion.
- **b)** It does not change.
- **c)** We cannot conclude about the direction of acceleration.

2.1. Why does the velocity change stepwise?
- **a)** The ball falls into a hole.
- **b)** The ball collides with the edge of the table.
- **c)** The ball collides with another ball.
- **d)** The ball bounces.

2.2. What is the acceleration at the moment $t = 1.4$ s?
a) The acceleration is equal to zero.
b) The acceleration is infinitely large.
c) The acceleration is approximately equal to 0.8 m/s^2.
d) The acceleration is the same as before.

For comparison, the results are shown in Figure 4, as well as the actual percentage of the students' correct answers to each question in the problem. It can be seen that the teachers expected better answers from their students to all the questions than were the actual results obtained.

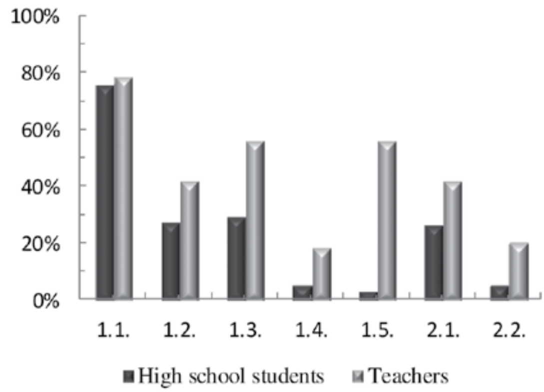


Figure 4. Percentage of the expected correct students' answers given by the teachers, and the actual correct students' answers.

Examination of Textbooks

The textbooks which we studied are physics textbooks for students in the first grade of gymnasiums and vocational schools with a four-year program in physics, which have been approved by the Ministry of Science, Education and Sport of the Republic of Croatia for the school year 2010-2011. The reasonable assumption was that most of our respondents have used these textbooks for learning and teaching. The list of the approved textbooks can be found in *The catalogues of compulsory textbooks and accompanying supplementary teaching materials for gymnasiums and vocational schools in the school years 2010-2011 and 2011-2012* (MZOS RH, 2010). English translations of the quotations of some of the incomplete statements found in the Croatian textbooks concerning the uniformly accelerated linear motion are listed in Table 4.

The most common incomplete statements found in the Croatian high-school physics textbooks are the following:

a) The expressions for the displacement and velocity of uniformly decelerated motion are obtained by putting the minus sign before the acceleration.

b) Acceleration is positive when the velocity increases, and negative when the velocity decreases.
c) Acceleration is positive if it is directed in the direction of motion, and negative if it is directed oppositely.

Table 4.

The quotations of some of the incomplete statements from the high-school physics textbooks concerning the uniformly decelerated linear motion (translated from Croatian).

"The equation $v = v_0 + at$ is the same as in the case of uniformly accelerated linear motion, but because $v < v_0$ it is clear that $a < 0$. This is what we expected due to the opposite direction of the acceleration vector with respect to the direction of the velocity vector." (Andreis et al., 2007, p. 48)
"Since the increase of the velocity is negative (because the final velocity is lower than the initial velocity, i.e. $\Delta v < 0$), the acceleration a is negative."(Negovec, & Pavlović, 2009, p. 24)
"During a uniformly decelerated motion the acceleration has a negative sign. The expressions for the velocity and the displacement of uniformly decelerated motion are: $v = v_0 - at$, $s = s_0 + v_0t - (at^2)/2$." (Paar, 2007, p. 16)
"If an object moves along a straight line, the acceleration is positive when the velocity increases, and negative when the velocity decreases. Positive acceleration has the direction of motion, and negative acceleration is opposite to the direction of motion." (Lopac, 2007, p. 23)
"When the acceleration is negative, i.e. when the velocity decreases, then the valid expressions are: $v(t) = v_0 - at$, $s(t) = v_0t - (at^2)/2$." (Horvat, & Hrupec, 2010, p. 29)
"*When the velocity of an object which moves along a straight line decreases by equal amounts in equal intervals of time, we are talking about uniformly decelerated motion. In this case the final velocity is lower than the initial velocity, so that is $a < 0$, and the acceleration is negative. The expressions for the velocity and the displacement, that we have derived for uniformly accelerated motion with initial velocity, are valid here too, except that a negative value for the acceleration should be inserted: $v = v_0 - at$...*"(Roginić, 2010a, pp. 42-43; Roginić, 2010b, pp. 42-43)
"When an object moves along a straight line, acceleration is positive if the velocity increases, and negative if the velocity decreases. Positive acceleration has the direction of the motion, and negative acceleration is opposite to the direction of the motion." (Kulišić, & Pavlović, 2010, p. 19)
"*When the velocity of an object which moves along a straight line decreases by equal amounts in equal time intervals, we are talking about uniformly decelerated linear motion. Acceleration is constant, but it has a negative sign because the final velocity is lower than the initial velocity."* (Labor, 2007a, p. 28; Labor, 2007b, p. 23)

Discussion

Firstly we discussed the students' ability to understand the kinematics of an object and their ability to determine the magnitude and the direction of the velocity and acceleration from the given velocity-time graph. In Figure 3a we can see that the percentages of the answers are almost uniformly distributed throughout the entire scale, except for the area of around 60%. Therefore, we conclude that the distribution of difficulty of the problems posed is appropriate for testing the students' knowledge. We also find a strong correlation among the data, showing that the distribution of the difficulty of the questions is almost the same for all groups of respondents. The questions which were easier for one group were also easier for other groups, and the questions that were difficult for one group were also difficult for other groups. However, if we compare the groups according to the difficulty of the questions, we may notice differences. Most of the data for PT students fall a bit above the autocorrelation line, which means that PT students have slightly better results than the SG & ITG students. On the other hand, the data for GG and VS students fall significantly below the line, because the problem was considerably more difficult for them. Along with the highest percentage of the correct answers, PT students also have the highest percentage of incorrect answers, as seen in Figure 3b. Since they have much greater experience with kinematics graphs gained in high school and the university, this result shows that the traditional teaching methods do not significantly improve the understanding of kinematics graphs. On the other hand, the curriculum develops the skills for a more detailed interpretation. This is supported by the result in Figure 3c showing the lowest percentage of the "no answer" response for PT students. They describe the problem in more detail and discuss more aspects of the motion.

The highest percentage of correct answers and the lowest percentage of "no answer" responses were found for question 1.1. For the correct interpretation of the first segment of the graph, intuitive thinking was sufficient. Students correctly concluded that the velocity decreases from the decrease of the height of the graph. Most difficulties were found in the description of the acceleration. This is clearly seen from the positions of the symbols 1.4, 1.5 and 2.2 for the questions regarding the magnitude and direction of the acceleration. In Figure 3a these symbols are found closest to, and in Figure 3c farthest from the origin.

Only 4% of the respondents correctly determined the direction of the acceleration vector, while the others did not comment on this issue at all. That is why in Figure 3b the label 1.5 is not found. This result is consistent with the results of similar studies (Labudde et al., 1988; Reif, & Allen, 1992) which found that students ignore the vector nature of acceleration. They also have difficulties in understanding the acceleration as the change in velocity over time. The most common misinterpretations of our respondents were: (i) the magnitude of acceleration changes in the time intervals in which the velocity changes, (ii) if the velocity change is stepwise, the acceleration magnitude assumes zero or some other finite value (see Table 2). Misinterpretation (i) indicates that our respondents do not distinguish the concepts of velocity and acceleration. They erroneously associate the change in acceleration with the change in velocity. Confusion of the concepts of velocity and acceleration is known from similar studies conducted by Rosenblatt and Heckler (2011), and Trowbridge and McDermott (1980; 1981). According to Shaffer and McDermott (2005), respondents who incorrectly believe that the acceleration is equal to zero at the reflection point (ii), also come to this conclusion because they do not make a distinction between the concepts of velocity and acceleration. Since the velocity is zero at the reflection point, they believe that the acceleration must also be zero. Those students, who calculated the acceleration in the reflection point, using formulas and inserting some values, did that by analogy to the solutions of similar problems they remembered, and in the absence of conceptual understanding. In doing so, they obviously did not take care how and under which conditions these solutions were obtained (Labudde et al., 1988; Reif, & Allen, 1992).

According to McDermott (1993), the difference between what is taught and what students learn, is often greater than the teachers realize, at all levels of education. For example, teachers often assume that the graph which they find especially clear and comprehensible will also be clear to the average student (Meltzer, 2005), but experience shows that this assumption is often not true (McDermot, 1990). Therefore, we considered it important to examine how teachers assess students' understanding of kinematics concepts because there are many preconceptions (e.g. about characteristics of vectors) that physics teachers consider so obvious that they do not discuss them in the class. The consideration of these characteristics is crucial to effective instruction (Aguirre, 1988), so this study also helped us to perceive the potential of non-traditional problem (Erceg, & Aviani, 2013; Erceg et al., 2011; Erceg et al., 2013; Erceg et al., 2014; Kariž Merhar, 2001; Marušić et al., 2011) in physics teaching. In Figure 4 we see that teachers significantly overestimate the students' understanding of kinematics concepts which is consistent with the results of McDermott (1993). This particularly refers to the concepts of directions of the acceleration and velocity (questions 1.3 and 1.5), where the largest discrepancies were found. For example, regarding the students' ability to interpret the direction of the acceleration, the teachers' expectations of students were 14 times higher than the actual results achieved by their students, with the ratio 56% to 4%. The slightest difference in the percentage of correct answers was found in the answer to question 1.1, relating to the velocity in the first segment of the graph, with 76% of correct answers.

The traditional teaching approach is considered to be the general cause of the conceptual difficulties associated with graph representations of the kinematics (Labudde et al., 1988; Reif, & Allen, 1992). However, by studying the content of the high-school textbooks in Croatia, we found that the authors of the textbooks also carry a large part of the responsibility for the conceptual problems. Many textbooks contain the incomplete or unfinished statements regarding the description of the uniformly accelerated linear motion. It is not possible to understand the kinematic graph representation properly if the vector character of the physical quantities (displacement, velocity and acceleration), and in this respect the meaning of the negative values, are not clarified. The incomplete statements found in the Croatian high-school physics textbooks are mainly due to the definition of the direction of the acceleration which depends on the direction of motion or on the type of motion, but also due to the unfinished statements related to the type of variables in the kinematic expressions. We did not examine the physics textbooks from other countries, but the recent discussion (Hayes, & Wittmann, 2010b; Mungan, 2010; Paetkau, 2010) makes it obvious that this is a common problem. We feel that inconsistency and disagreement about sign conventions in kinematic expressions are a frequent cause of misunderstanding and misconceptions among the students.

The important issue to clarify is what one-dimensional kinematic scalar equations actually present. The common unfinished statement found in the textbooks is that these are magnitude equations, i.e. that variables (s, v, a) are the magnitudes of the corresponding vectors - the positive real numbers. Thus, to describe the uniformly decelerated motion they put a minus sign before the acceleration: $v = v_0 - at$, $s = s_0 + v_0t - (at^2)/2$. In these expressions the set of numbers for the acceleration a is usually not defined, so that it remains unclear whether this is a set of real numbers or a set of positive real numbers. However, one-dimensional scalar equations are not magnitude equations and should be treated as component equations with variables assuming the negative values as well. This concept is crucial for the interpretation of kinematic graphs where the negative values appear. The correct expressions are the same for both the accelerated and decelerated motion, with no outer minus signs, and with the real numbers denoting vector variables (*s, v, a*). Hayes and Wittmann (2010a) found that college physics students use physical and mathematical reasoning inconsistently when determining the signs of terms in equations and they believe the problem lies in how a vector equation is interpreted into a scalar equation.

It does not necessarily mean that the object accelerates if it has positive acceleration, and that it decelerates if it has negative acceleration (Serway, 2006). In the case of linear motion, there are the four possible combinations for directions of the velocity and acceleration vectors, as shown in Table 5. If the velocity and acceleration vectors have the same directions (both positive or both negative), then the magnitude of the velocity increases with time, i.e. the object speeds up. If the velocity and acceleration vectors have the opposite directions (one positive and other negative), then the magnitude of the velocity decreases with time; i.e. the object slows down. From the Table it can be easily seen that the statement which appears in the textbooks is only valid if the velocity is positive, i.e. if the body moves in the direction of the positive orientation of the coordinate axes.

Table 5.

Different combinations of the directions of the velocity and acceleration vectors cause the accelerated or decelerated linear motion.

Direction (sign) of the velocity *v*	Direction (sign) of the acceleration *a*	Type of motion
positive (+)	positive (+)	accelerated
negative (–)	negative (–)	accelerated
positive (+)	negative (–)	decelerated
negative (–)	positive (+)	decelerated

For example, if an object moves in the negative x direction with the velocity that changes from v_1 = -10 m/s to v_2 = -20 m/s within Dt = 2 s, then the acceleration is

a = Dv / Dt = (v_2 - v_1) / Dt = -5 m/s^2.

Although the acceleration is negative, the object accelerates, i.e. its velocity (its speed) increases in time. Talking about positive and negative acceleration makes sense only with respect to the frame of reference, and not with respect to the direction of motion. The concepts of accelerated and decelerated motions are associated with speeding up or slowing down, i.e. with an increase or decrease in the magnitude of the velocity vector. Thus, to determine the direction of acceleration, the kinematics analysis is necessary (McDermott et al., 1994).

The correct determination of the sign of the acceleration should be made with respect to the direction of the corresponding coordinate axes in the reference system, not with respect to the direction of motion which changes. Since there is no absolute reference system, i.e. the absolute motion has no physical meaning, it is necessary to introduce a reference system to describe the relative kinematics motion (Benenson et al., 2002). The physical quantities used to describe motion (displacement, velocity and acceleration) are vectors. Vectors are geometric objects characterized by a magnitude and a direction and usually denoted as a bold letter or a letter with an arrow above it. Linear motion of a body is usually described in a one-dimensional positively oriented laboratory coordinate system, e.g. along x-axis with positive values right to and negative values left to the origin. In that case the vectors in the kinematic expressions are replaced by their components - the real numbers. Another interpretation is that vectors are replaced by the real numbers in such a way that the vector quantities which are directed to the right are represented with positive real numbers and vectors directed to the left with negative real numbers. The vector magnitude is represented by the absolute value of the real number and the vector direction by the (+) or (–) sign (Halliday et al., 2005). The arrow above the letter or the bold formatting of the letter is removed and vector algebra, which is based on geometrical considerations, is replaced by simple algebra of real numbers. With such representation of physical quantities it is much easier to work, but because there is no visible difference between the real numbers and the vectors, this representation becomes the main source of incomplete statements and misinterpretations.

Our results point to the disagreement between the teaching methods and the way of learning, which is expressed in the traditional approach to teaching (McDermott, 1993). Traditional education is based on the teachers' perception of the content which is taught. Trying to transfer knowledge and enthusiasm to their students, teachers often rely on their own perceptions of students, i.e. they ignore the possibility that students' perception is significantly different from theirs. They teach students to solve the traditional problems, hoping that students will become able to apply this knowledge in new situations. However, in this way they exclude students from the active-learning process, which produces the opposite effect.

Applying the non-traditional problem, used in our research, provides some new opportunities in teaching. The problem describes a real situation with almost all elements needed for teaching the kinematics. It can encourage discussions on kinematics concepts in the classroom. Through the discussion students are given the opportunity to consider their own ideas as well as encouragement which help them modify those ideas when necessary, in accordance with the recommendation of Beichner (1994). Teachers have the opportunity to discover students' problem solving strategies and to adapt to the students' way of thinking during the active learning process.

Conclusion

We investigated the students' ability to recognize the necessary information from the graph, to understand the kinematics concepts in the graph, and to translate the graph representation into a real-life situation. The study included a survey of 235 high-school students, 41 physics teacher students, and 48 high-school physics teachers. The students were presented the original velocity-time graph problem in the form of open-ended questions, to allow them the freedom to express their ideas in written form. Based on their responses, the closed-ended questionnaire for the teachers was prepared in order to investigate the teachers' ability to predict the students' way of thinking during the problem solving process.

The results show that the distribution of item difficulties is similar for all groups of students, i.e. that the difficulties in connecting the graph with the kinematic concepts and the real world are almost the same regardless of educational level and curriculum. The main difficulties are related to: (i) connecting the basic features of the graph (height, slope, area under the graph) to the corresponding physical quantity (velocity, acceleration, displacement), (ii) understanding the velocity and acceleration as vector quantities having direction as well as magnitude, (iii) inability to visualize a real motion on the basis of the graph representation. The lowest percentage of "no answer" responses was found among the students who are the future physics teachers. They described the motion in more detail, and analysed it from different points of view.

Training and education obviously improve the abilities for a detailed interpretation of the graphs. The students - future physics teachers had the highest percentage of correct answers to most of the questions, but also the highest percentage of incorrect answers. This suggests that the traditional teaching methods, even after an extended training, do not lead to significantly better understanding of the graphs.

Comparing the teachers' predictions, i.e. answers that the teachers expected to obtain from their students with the actual students' responses we found that the teachers overestimate the students' understanding of kinematic concepts in most of the elements. This is particularly apparent for the concept of the direction of the velocity and acceleration.

Inconsistency and disagreement about sign conventions in kinematic expressions describing only examples of a straight line motion in one direction could be an important source of misunderstanding and misconceptions among the students. Exploring the content of our textbooks for high-school physics, we found that authors carry a large share of responsibility for this situation. We have found a number of inconsistencies and incomplete statements regarding the texts and expressions for the uniformly decelerated linear motion. Therefore, apart from the traditional approach to the kinematic problems as a common source of conceptual difficulties, we think that textbooks which support such an approach are also partially responsible for the difficulties of our respondents.

The problem that we used in our study describes the real situation, and contains the essential elements of kinematics. This type of problem could be helpful in further investigations and curriculum development at all levels of physics education. We recommend it as a basis and stimulus for discussion on the concepts of kinematics in the active learning process in the classroom. Teachers thus may discover the students' problem solving strategies and adapt their teaching strategy to the students' way of thinking.

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Učeničko i studentsko razumijevanje grafova vremenske promjene brzine i izvori konceptualnih poteškoća

Sažetak

Ispitivali smo učeničke i studentske sposobnosti prevođenja grafičkog prikaza vremenske promjene brzine u realnu fizikalnu situaciju i moguće izvore konceptualnih poteškoća. Istraživanjem je obuhvaćen uzorak od n = 324 srednjoškolska učenika, studenta fizike i srednjoškolska nastavnika fizike iz Republike Hrvatske. Učenicima i studentima zadali smoo originalan netradicionalni grafički problem s pitanjima otvorenog tipa. Njihove pisane odgovore smo klasificirali i na temelju odgovora sastavili upitnik predviđanja zatvorenog tipa za nastavnike. Uspoređujući odgovore koje nastavnici unaprijed očekuju od učenika sa stvarnim odgovorima učenika uočili smo da nastavnici značajno precjenjuju učeničko razumijevanje. Naši rezultati također pokazuju da su opažene poteškoće zajedničke za sve skupine ispitanika i da ne ovise u značajnijoj mjeri o stupnju obrazovanja i kurikulu. Ustanovili smo da hrvatski udžbenici također predstavljaju mogući izvor poteškoća jer sadrže nedorečene i necjelovite tvrdnje koje se odnose na predznake fizičkih veličina u kinematičkim jednadžbama. Smatramo da netradicionalni grafički problemi tog tipa potiču aktivan proces učenja i nastavnicima omogućuju prilagodbu učeničkom načinu razmišljanja s ciljem svladavanja opaženih poteškoća.

Ključne riječi: *grafički prikaz; konceptualno razumijevanje; kurikul; učeničko/studentsko rješavanje problema; udžbenici iz fizike.*

Uvod

Crtanje i interpretacija grafičkih prikaza ubrajaju se u temeljne sposobnosti znanstvenika (McKenzie i Padilla, 1986). Za razliku od tabličnih prikaza oni omogućuju brz uvid u glavne trendove, kao i u detalje nepravilnosti među mnogobrojnim podacima koji opisuju neki događaj (Mokros i Tinker, 1987). Chambers i suradnici (1983) smatraju da ne postoji bolji statistički alat od grafova za snalaženje među složenim informacijama.

U odnosu na znanstvenike i druge stručnjake učenici i studenti imaju znatno smanjenu sposobnost konstruiranja i upotrebe grafičkih prikaza (Larkin, 1981). Oni najčešće ne razumiju formalni jezik kojim se nastavnici služe u radu s grafovima (Beichner, 1994). Stoga istraživanja s ciljem razvoja tih sposobnosti mogu biti važan korak prema boljem rješavanju fizikalnih problema i razumijevanju fizikalnih koncepata (Larkin, 1981). Tome u prilog govore i rezultati istraživanja (Rosenquist i McDermott, 1987) prema kojima mnoge studentske poteškoće prilikom rješavanja problema iz kinematike proizlaze uglavnom iz nemogućnosti interpretacije grafičkih prikaza gibanja.

Sposobnost uspješnog rada s grafovima ne razvija se spontano ili zapamćivanjem jednostavnih postupaka poput crtanja točaka u koordinatnom sustavu, očitavanja koordinata točaka i računanja nagiba pravaca. Takav površan pristup problemskom rješavanju primjenjuje se u okviru tradicionalne, matematički orijentirane nastave i ne rezultira konceptualnim razumijevanjem (Rosenquist i McDermott, 1987).

Da bi mogli samostalno primjenjivati grafičku analizu u budućim zanimanjima, učenici/studenti bi trebali znati detaljnije interpretirati graf funkcije. To znači da bi, između ostalog, trebali naučiti: (i) odabrati i odrediti svojstvo grafičkog prikaza koje sadrži relevantne informacije, tj. uspostaviti vezu između odgovarajućeg svojstva grafa funkcije i fizikalnog koncepta (primjerice informacija može biti sadržana u koordinatama točke, u razlici odgovarajućih koordinata dviju točaka ili u nagibu pravca), (ii) prepoznati vezu među različitim grafičkim prikazima, (iii) grafički prikazati realne fizikalne sustave, (iv) vizualizirati sustav na temelju njegova grafičkog prikaza. Istraživanja pokazuju da mnogi studenti imaju poteškoća u svakom od tih segmenata (Barclay, 1986; Beichner, 1994; Chambers i suradnici, 1983; Larkin, 1981; McDermott, 1991; McDermott i suradnici, 1987; Mokros i Tinker, 1987; Zee i McDermott, 1987). Stoga im je potrebna stručna pomoć u učenju i svladavanju toga problema.

Poteškoće se ne mogu jednostavno pripisati nedostatnom znanju iz matematike (McDermott i suradnici, 1987). Većina studenata nakon srednje škole ima potrebne matematičke vještine za jednostavne postupke poput crtanja točaka u koordinatnom sustavu ili računanja nagiba pravaca. Međutim, kada je riječ o grafovima funkcija, često se događa da postojeće znanje iz matematike studenti ne znaju primijeniti u fizici. Istodobno pokazuju prilično dobro poznavanje koncepata prilikom rješavanja problema koji ne uključuju grafove funkcija. Price i suradnici (1974) ustanovili su da studenti imaju većih poteškoća prilikom određivanja nagiba nego prilikom određivanja koordinata točaka.

Među odgovorne faktore, koji nemaju matematičku pozadinu, ubraja se nesposobnost interpretacije grafičkih prikaza, tj. nesposobnost njihova povezivanja s fizikalnim konceptima i s realnim svijetom (McDermott i suradnici, 1987). Takva priroda poteškoća pojavljuje se neovisno o učeničkoj dobi i razini obrazovanja (McDermott i suradnici, 1987; Peters, 1982), što potvrđuje činjenicu da tradicionalni naglasak na algebarskom formalizmu ne vodi do kvalitativnog razumijevanja fizikalnih koncepata (Rosenquist i McDermott, 1987).

McDermott i suradnici (1987) smatraju da postoje specifične poteškoće koje se javljaju prilikom povezivanja grafova funkcija s fizikalnim konceptima. Studenti najčešće ne znaju treba li potrebnu informaciju izvući iz nagiba ili iz visine grafa funkcije (Barclay, 1986; Mokros i Tinker, 1987), pogrešno interpretiraju promjene nagiba i visine, teško povezuju različite tipove grafova funkcija koji opisuju istu situaciju, teško rješavaju probleme u kojima se fizičke veličine zadaju posredno s pomoću grafičkog prikaza i pogrešno interpretiraju površine ispod grafa funkcije (u odnosu na apscisnu os).

Druga skupina poteškoća proizlazi iz nemogućnosti povezivanja grafa funkcije s odgovarajućim objektom i događajem u stvarnom svijetu. Provodeći istraživanje u kojemu su od studenata zahtijevali da prevedu laboratorijsku situaciju u grafički prikaz i obrnuto, McDermott i suradnici (1987) došli su do zaključka da studenti najčešće rade sljedeće pogreške. Kontinuirano gibanje tijela ne prikazuju kontinuiranom crtom. Umjesto toga crtaju niz diskretnih točaka što ukazuje na to da ne razlikuju trenutačnu vrijednost fizičke veličine od kontinuirane promjene fizičke veličine tijekom vremenskog intervala. Također, u situacijama u kojima bi trebali interpolirati krivulju između točaka, oni crtaju razlomljene crte povezujući eksperimentalno dobivene točke. Time pokazuju da nisu svjesni činjenice o približnoj točnosti izmjerenih vrijednosti fizičkih veličina. Oni smatraju da grafički prikaz oblikom treba nalikovati putanji gibanja (Barclay, 1986; Mokros i Tinker, 1987). Ne povezuju promjenu smjera gibanja s negativnom brzinom. Pogrešno interpretiraju predznak akceleracije. Primjerice, negativnu akceleraciju povezuju isključivo s objektom koji usporava i kojemu je brzina pozitivna. Time isključuju mogućnost ubrzavanja objekta s negativnom akceleracijom ako mu je brzina negativna. Ne prihvaćaju činjenicu da isto gibanje može biti prikazano grafovima funkcija različitih oblika (Halloun i Hestenes, 1985).

Pitanja koja se prirodno nameću glase: *Koji su glavni uzroci spomenutih učeničkih/studentskih poteškoća?* i *Kako ih prevladati?* Naše istraživanje rađeno je s ciljem pronalaženja odgovora na ta pitanja, odnosno s ciljem otkrivanja novih, učinkovitijih pristupa u nastavnom procesu poučavanja kinematičkih grafova funkcija.

Udžbenici imaju važnu ulogu u procesu poučavanja i učenja (Blanton, 2009; Lee, 2007; Podolefsky i Finkelstein, 2006). Stoga, analizirajući sadržaj udžbenika iz fizike, moguće je ustanoviti općenito prihvaćene metode i trendove u nastavi fizike, kao i moguće izvore učeničkih/studentskih poteškoća. Analizom nastavnih tema iz kinematike u udžbenicima i ostalim nastavnim materijalima iz fizike za srednje škole u Republici Hrvatskoj, ustanovili smo da među zadacima daleko najveću zastupljenost imaju tradicionalni problemi. U njima se, kada je riječ o grafičkim prikazima, od učenika zahtijeva grafički prikaz opisanog gibanja. Međutim, sposobnost kreiranja grafičkog prikaza ne znači nužno sposobnost obrnutog razmišljanja, odnosno razumijevanja gibanja na temelju grafa funkcije (Arons, 1984). Tradicionalni pristup u nastavi fizike općenito se smatra nedovoljnim za razvoj dubljeg konceptualnog razumijevanja. Primjerice, od studenata se često zahtijeva da nacrtaju točke kako bi nacrtali graf funkcije. Takav algoritamski pristup omogućava studentima dolazak do ispravnog rješenja bez razumijevanja grafičkog prikaza (Laverty, 2012). U ovom smo radu krenuli od pretpostavke da netradicionalni pristup i poticanje obrnutog procesa razmišljanja može doprinijeti poboljšanju razumijevanja grafičke reprezentacije. Za razliku od istraživanja (Testa i suradnici, 2002) u kojemu se od studenata zahtijevalo čitanje i interpretacija kinematičkih grafičkih prikaza realnih eksperimenata, mi smo istraživali učeničku/studentsku sposobnost interpretacije grafičkog prikaza gibanja iz svakodnevnog života, postavljajući pred ispitanike problem otvorenog tipa.

Naša je analiza pokazala da udžbenici sadrže mnoge necjelovite tvrdnje. Stoga oni također predstavljaju potencijalne izvore učeničkih/studentskih miskoncepcija jer se često pišu prema nastavnom planu i programu određenog predmeta pa se koriste kao glavni izvor informacija i objašnjenja (Dall'Alba, 1993). U ovom smo radu izdvojili neke primjere iz udžbenika i diskutirali o mogućim uzrocima pogrešaka. Slično istraživanje napravili su Dall' Alba i suradnici (1993).

Drugi je izvor poteškoća nastavničko nedovoljno poznavanje učeničkih/studentskih strategija problemskog rješavanja i neprilagođivanje učeničkom/studentskom načinu razmišljanja (Mayon i Knutton, 1997; McDermott, 1993). Tu smo važnu problematiku i mi istraživali, ispitujući nastavničku sposobnost predviđanja učeničkih strategija rješavanja zadanog problema. Pritom smo se koristili upitnikom zatvorenog tipa s pitanjima koja se temelje na učeničkim/studentskim odgovorima. Slična istraživanja proveli su Lightman i Sadler (1993), i Viiri (2003).

Uzorkom ispitanika obuhvatili smo srednjoškolske učenike, sveučilišne studente i nastavnike fizike. Pred učenike i studente postavili smo originalan problem u obliku realnog *v-t* grafičkog prikaza gibanja bilijarske kugle koja se odbija od rubova bilijarskog stola. Zadani grafički prikaz dovoljno je složen da sadrži gotovo sve potrebne elemente grafičkog prikaza gibanja. Istodobno je po dijelovima dovoljno jednostavan da je njime moguće ispitati i najjednostavnije koncepte gibanja. Problem implicitno sadrži sve tri „grafičke" kategorije pitanja na koja se može odgovoriti s pomoću grafa funkcije (Bertin, 1973): (1) osnovna pitanja koja zahtijevaju jednostavno izvlačenje podataka, (2) pitanja koja zahtijevaju utvrđivanje trendova i (3) sveobuhvatna pitanja koja zahtijevaju uspoređivanje cjelovitih struktura grafičkog prikaza. Te kategorije godinama se usavršavaju, što se vidi iz referencije (Friel, 2001) koja daje pregled. Postavljanjem pitanja otvorenog tipa (Johnson i Christensen, 2004) učenicima i studentima omogućili smo slobodu u davanju pisanih odgovora. Na temelju tih odgovora istraživali smo njihove sposobnosti izvlačenja potrebnih informacija iz grafičkog prikaza, otkrivanja fizikalnih koncepata u grafičkom prikazu i prevođenja grafičkog prikaza u realnu situaciju.

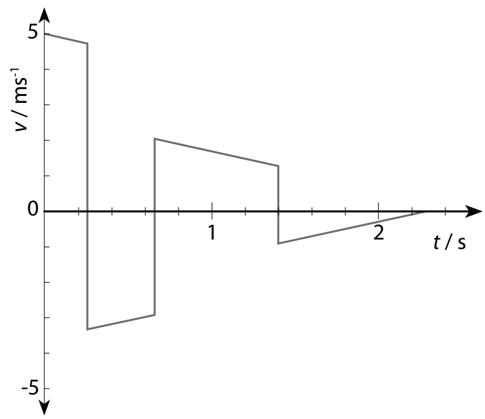
U radu smo najprije opisali grafički problem i njegovo rješenje. Nakon toga smo opisali uzorak u kontekstu hrvatskoga obrazovnog sustava, a zatim metode istraživanja za učenike/studente, nastavnike i udžbenike iz fizike. Prikazali smo rezultate istraživanja i diskutirali o tim rezultatima u kontekstu različitih vrsta konceptualnih poteškoća i njihovih mogućih izvora.

Problem

Grafički prikazi predstavljaju snažan alat za snalaženje među mnogobrojnim informacijama. Međutim, učenici i studenti nemaju u dovoljnoj mjeri razvijene sposobnosti njihove primjene. Očito je da se vještine crtanja i interpretacije grafova ne mogu razviti tradicionalnim naglaskom na algebarskom formalizmu koji je često zastupljen u problemima u kojima se od učenika zahtijeva jednosmjerno prevođenje gibanja u grafički prikaz. Ta gibanja moraju se zamišljati i uglavnom se ne temelje na stvarnoj situaciji ili eksperimentu, kao što to preporučaju primjerice Kariž Merhar i suradnici (2009), McDermott i suradnici (1987). Stoga smo bili motivirani provesti istraživanje u kojemu smo proučavali obrnuti proces razmišljanja, od grafičkog prikaza prema realnoj fizikalnoj situaciji, s ciljem poboljšanja metodike poučavanja kinematike iz grafičkih prikaza.

Pred ispitanike smo postavili originalni problem[4] koji glasi:

Na slici 1 je grafički prikazana ovisnost brzine o vremenu tijekom pravocrtnog gibanja bilijarske kugle.



1) *Opišite kako se mijenjala brzina kugle tijekom prve sekunde. Što se pritom događalo s ubrzanjem?*

2) *Zašto se brzina skokovito mijenja u trenutku t = 1,4 s? Koliko je ubrzanje u tom trenutku?*

Sa slike 1 možemo zaključiti da se bilijarska kugla giba pravocrtno po širini bilijarskog stola, između njegovih suprotnih rubova. Širina bilijarskog stola iznosi d = 1,25 m. Kugla je na početku smještena uz duži rub stola. Nakon udarca bilijarskim

[4] Odgovarajući tradicionalni problem glasio bi primjerice:

Bilijarska se kugla giba pravocrtno po širini bilijarskog stola, između njegovih suprotnih rubova i jednoliko usporava zbog trenja. Kugla je na početku smještena uz duži rub stola. Nakon udarca bilijarskim štapom, ona se kreće okomito prema suprotnom rubu, odbija se i na isti način nastavlja gibanje. Pri svakom odbijanju gubi energiju. Nakon trostrukog odbijanja kugla se zaustavi negdje na sredini stola. Kako izgleda pripadajući grafički prikaz ovisnosti brzine o vremenu?

štapom u trenutku $t_0 = 0$, ona se kreće okomito prema suprotnom rubu, odbija se i na isti način nastavlja gibanje odbijajući se do zaustavljanja. Sudari kugle s rubom stola dogode se u trenucima $t_1 = 0,26$ s, $t_2 = 0,66$ s i $t_3 = 1,4$ s, a u trenutku $t_4 = 2,3$ s dolazi do njezina zaustavljanja, kao što je izračunato u tablici 1. Vremenski intervali između sudara se povećavaju, visina grafa funkcije u odnosu na apscisnu os se smanjuje, a nagibi dijelova grafa funkcije u svim segmentima su jednaki. Te činjenice ukazuju na to da je unutar svakog segmenta riječ o jednoliko usporenom gibanju[5] kugle. Usporavanje je uzrokovano trenjem. Pritom se iznos brzine $|v|$ smanjuje, a iznos akceleracije $|a|$ je konstantan. Prilikom svakog sudara, u vrlo kratkom vremenskom intervalu, promijeni se smjer[6] brzine i ubrzanja, što se očituje promjenom predznaka tih veličina. Ta skokovita, gotovo trenutna, promjena brzine označena je vertikalnom crtom jer je t-os baždarena tako da se na njoj ne mogu prikazati tako kratki vremenski intervali. U točki odbijanja ubrzanje poprima vrlo velik iznos. Osim toga, gubitak energije prilikom sudara (uz koeficijent restitucije k = 0,7)[7] dovodi do skokovitog pada iznosa brzine. Na temelju jednakih površina ispod grafa, u prva tri vremenska intervala, moguće je zaključiti da se iznos pomaka kugle ne mijenja i da je u sva tri intervala maksimalan, odnosno da kugla prelazi cijelu širinu stola *d*. Međutim, zaključujemo da se u posljednjem vremenskom intervalu kugla zaustavi prije nego što stigne do ruba stola jer je brojčana vrijednost površine ispod toga dijela grafa manja od brojčane vrijednosti širine stola *d*.

Gibanje kugle podijeljeno je u četiri vremenska intervala (segmenta), odvojena trenucima u kojima se dogode sudari s rubom stola. Vrijednosti kinematičkih fizičkih veličina važnih za raspravu o ovom problemu izračunate su i pregledno prikazane u tablici 1. Vrijednosti za *i*-ti segment dobivene su s pomoću formula prikazanih u prvom retku za: v_{0i} - početnu brzinu, a_i - akceleraciju, Dt_i - vremenski interval u kojem kugla prelazi udaljenost *d* (osim u posljednjem intervalu), v_i - konačnu brzinu, t_i - vrijeme na kraju segmenta, Dx_i - pomak. Maksimalan pomak d_i mijenja predznak tako da je $d_i = d$ za $i = 1, 3$, odnosno $d_i = - d$ za $i = 2$. Negativne vrijednosti označavaju smjer koji se podudara s negativnim smjerom *x*-osi. Početna brzina je $v_{01} = 5$ m/s, a početna akceleracija $a_1 = -1$ m/s^2. Ubrzanje ima stalan iznos i smjer uvijek suprotan smjeru brzine.

Tablica 1.

Istraživanje i rezultati

Ispitivanje učenika i studenata

U ovom je istraživanju sudjelovalo ukupno 276 učenika/studenata koji su odabrani tehnikom nenasumičnog praktičnog uzorkovanja (Johnson i Christensen, 2004). Među

[5] Kada se tijelu tijekom gibanja brzina mijenja, općenito kažemo da se ono giba ubrzano ili akcelerirano. U posebnom slučaju, kada se brzina tijela smanjuje, umjesto pojmova ubrzanja ili akceleracije često se koriste pojmovi usporenja ili deceleracije.

[6] Pojmom *smjer vektora* objedinjuju se pojmovi *pravca nosioca* i *orijentacije*.

[7] Izvori literature za tipične koeficijente restitucije bilijarske kugle, kao i za cjelokupnu fiziku bilijara, mogu se pronaći na stranici http://billiards.colostate.edu/threads/physics.html.

njima su bila 72 učenika iz Gimnazija prirodoslovno-matematičkog i informatičkog smjera u Rijeci i Zagrebu (PMI), 139 učenika iz Gimnazija općeg smjera u Rijeci i Zagrebu (OG), 24 učenika iz Strukovne škole u Rijeci (SŠ) i 41 student nastavničkog smjera sveučilišnih studija fizike na fakultetima u Rijeci i Zagrebu (SF). Obuhvaćeni su učenici i studenti koji su se nalazili na različitim razinama obrazovanja, a svi su u okviru nastave fizike obradili koncepte potrebne za rješenje istraživačke problemske situacije.

Gimnazija je općeobrazovna srednja škola koja se može usporediti s „English grammar school" ili „U. S. high school". Ona osposobljava učenike za daljnje školovanje na sveučilištima. Traje četiri godine i završava pismenim ispitom, državnom maturom, koja predstavlja ulaznu kvalifikaciju za daljnje obrazovanje. Različite gimnazije podrazumijevaju povećane programe za različite predmete. Primjerice, predmeti prirodnih znanosti, matematike i informatike imaju veći fond sati u PMI gimnazijama. S druge strane, stručne škole traju 3-5 godina i osposobljavaju učenike za odgovarajuće zanimanje. Studenti nastavničkog smjera sveučilišnih studija fizike pohađaju petogodišnje programe na prirodoslovno-matematičkim fakultetima/odjelima. Na kraju studija dobivaju diplome, stječu zvanje magistra u području edukacije i osposobljeni su za izvođenje nastave fizike i dodatnog predmeta koji su studirali (npr. matematike, kemije, informatike) u osnovnim i srednjim školama.

Učenicima i studentima postavili smo opisani grafički problem s pitanjima otvorenog tipa, koja su im omogućavala slobodu u davanju odgovora. Na raspolaganju su imali 15 minuta za rješavanje. Nakon sakupljanja pisanih odgovora, uočili smo da ih možemo razvrstati u sedam različitih skupina, kao odgovore na sedam hipotetskih potpitanja; pet za pitanje 1 (1.1-1.5) i dva za pitanje 2 (2.1-2.2) u istraživačkom problemu, kao što je prikazano u tablici 2. Uz pomoć tih potpitanja klasificirali smo i vrednovali odgovore svakog ispitanika tako da smo svakom potpitanju pridružili jednu od tri vrijednosti: točno, netočno i bez odgovora. Rezultati te analize također su prikazani u tablici 2.

Tablica 2.

Raspodjela postotaka učeničkih/studentskih odgovora po pitanjima prikazana je na slici 2, grafikonima za sve četiri skupine ispitanika: gimnazijalce općeg smjera (OG), gimnazijalce prirodoslovno-matematičkog i informatičkog smjera (PMI), učenike strukovne škole (SŠ) i studente fizike nastavničkog smjera (SF) (Slika 2a - 2d). Svaki stupić unutar grafikona odgovara jednom pitanju iz tablice 2 i označen je istim brojem kao i u tablici. Nadalje, svaki je stupić podijeljen u tri segmenta, osjenčana različitim nijansama. Svaki stupić odgovara jednom od tri moguća odgovora: točnom odgovoru, netočnom odgovoru ili bez odgovora. Visine segmenata proporcionalne su postocima pojedinih odgovora.

Slika 2.

Pitanje 1.1 jedino je pitanje na koje je većina ispitanika (75 %) dala točan odgovor. Oni su ispravno odabrali visinu kao značajku grafičkog prikaza da bi donijeli zaključak o brzini. Budući da se u prvom vremenskom intervalu visina grafa funkcije smanjuje, ispravno su zaključili da se i brzina smanjuje.

Rezultat od samo 30 % točnih odgovora na pitanje 1.2, u okviru kojeg je trebalo diskutirati o promjenama brzine u drugom vremenskom intervalu, od 0,26 s do 0,66 s, ukazuje na poteškoće pri interpretaciji negativnih vrijednosti brzine. Većina od 52 % ispitanika koji su odgovorili pogrešno zaključila je da se iznos brzine povećava. Razmišljali su kao da se taj dio grafa funkcije, poput prethodnog, nalazi u prvom kvadrantu koordinatne poluravnine. O promjeni iznosa brzine ponovno su sudili na temelju promjene visine grafa funkcije, a ne na temelju promjene njegove udaljenosti od apscisne osi, što je dovelo do pogreške.

Većina ispitanika ili nije interpretirala (36 %) ili je pogrešno interpretirala (31 %) skokovitu promjenu brzine u trenucima 0,26 s i 0,66 s (pitanje 1.3). Čak je 23 % ispitanika smatralo da se u jednom trenutku kugla naglo usporava, a u drugom trenutku naglo ubrzava. Bilo je i onih koji su tvrdili da kugla u oba spomenuta trenutka miruje ili se pak jednoliko giba. U svakom slučaju, nisu povezali promjenu predznaka brzine s promjenom smjera gibanja, što je u skladu s rezultatima istraživanja Shaffer i McDermott (2005).

Za ispravan odgovor na pitanje 1.4, trebalo se usredotočiti na nagib kao značajku grafa funkcije. Iz činjenice da su nagibi dijelova grafa vremenske promjene brzine u svim segmentima po iznosu jednaki, mogao se izvesti zaključak o konstantnom iznosu ubrzanja. Međutim, čak 93 % ispitanika nije dalo ispravan odgovor. Pritom je njih 27 % odgovorilo pogrešno, a 66 % nije dalo nikakvu interpretaciju. Većina neispravnih odgovora proizlazi iz miskoncepcije da se informacija o ubrzanju može dobiti iz visine grafa te funkcije. Tome u prilog idu odgovori u kojima se tvrdi da se ubrzanje mijenja, odnosno da se smanjuje i/ili povećava.

Pitanjem 1.5 ispitivali smo znanje o smjeru akceleracije. U zadanom problemu on se mijenja tako da je uvijek suprotan smjeru brzine. Dakle, riječ je o deceleraciji koja ima negativan predznak kada je brzina pozitivna, a pozitivan predznak kada je brzina negativna. Samo 4 % ispitanika ispravno je interpretiralo smjer akceleracije, a preostalih 96 % nije diskutiralo o smjeru ubrzanja. Ispitanici nisu uzeli u obzir činjenicu da je ubrzanje vektorska veličina i da uz iznos ima i smjer ili orijentaciju na pravcu nosiocu, koja se određuje predznakom. Dakle, postoje konceptualne poteškoće povezane s predznakom akceleracije.

Smatra se da većina poteškoća vezanih uz grafičke prikaze proizlazi iz nemogućnosti vizualizacije gibanja (McDermott i suradnici, 1987). U okviru pitanja 2.1 ispitivali smo učeničku/studentsku sposobnost oblikovanja predodžbe o tome što se događa s kuglom u stvarnosti. Na temelju analize vremenske promjene brzine i ubrzanja, kao i na osnovi interpretacije površina ispod grafa te funkcije kao prijeđenog puta, može se zaključiti da je riječ o pravocrtnom jednoliko usporenom gibanju kugle između dva ruba bilijarskog stola do njezina zaustavljanja. Dvadeset i osam posto (28 %) ispitanika dalo je točan odgovor, dok ostali ispitanici nisu mogli vizualizirati gibanje. Njih 48 % stvorilo je pogrešne predodžbe, jer nisu uočili jednake površine ispod dijelova grafa funkcije u prva tri vremenska intervala. Oni su smatrali da se kugla sudara s drugom kuglom, štapom ili nekim drugim tijelom.

Nedovoljno poznavanje koncepta ubrzanja potvrđeno je i pitanjem 2.2. Trebalo je zaključiti da kada se brzina promijeni gotovo trenutačno, tj. u kratkom vremenskom intervalu ($\Delta t \rightarrow 0$), da tada akceleracija poprima vrlo veliku vrijednost ($a \rightarrow \infty$). Većina ispitanika dala je ili pogrešan odgovor (55 %) ili nisu odgovorili na to pitanje (37 %). Među neispravnim odgovorima najučestaliji su bili oni u kojima su ispitanici dobivali netočne rezultate iz formula za ubrzanje ili koristeći se neprikladnim računima.

Slika 3.

Na slici 3 prikazana je korelacija postotaka odgovora na pitanja ispitanika OG, SŠ i SF u odnosu na postotke te iste vrste odgovora ispitanika PMI. Grafički prikazi spomenutih korelacija redom se odnose na korelacije postotaka točnih odgovora (sl. 3a), netočnih odgovora (sl. 3b) i bez odgovora (sl. 3c). Primjerice, simboli na slici 3a nacrtani su tako da su za vrijednosti apscise uzete visine crnih dijelova stupića iz grafa 2b, a za ordinate su uzete visine crnih dijelova stupića iz triju ostalih grafova (2a, 2c, 2d), za svako od sedam pitanja. Brojevi označavaju grupe simbola koji pripadaju istom pitanju i u skladu su s numeracijom u tablici 2, kao i na sl. 2. Linija proporcionalnosti prikazuje autokorelaciju, odnosno maksimalnu moguću korelaciju. Podaci iznad pravca odgovaraju boljoj rješivosti, a podaci ispod njega slabijoj rješivosti zadatka u odnosu na gimnazijalce PMI. Podaci bliže ishodištu odnose se na teža pitanja, tj. na pitanja čija je rješivost slabija, a podaci udaljeniji od ishodišta odnose se na lakša pitanja. Na analogan način nastale su slike 3b i 3c.

Ispitivanje nastavnika

U drugom dijelu istraživanja ispitali smo 48 sudionika Stručnog skupa srednjoškolskih nastavnika fizike Splitsko-dalmatinske županije. Na temelju potpitanja (1.1-1.5 i 2.1-2.2), zatim učeničkih i studentskih odgovora oblikovali smo upitnik zatvorenog tipa za nastavnike (tablica 3). Nastavnike smo zamolili da odgovore zaokruživanjem jednog od ponuđenih odgovora, ne na način kako oni misle da je ispravno, već onako kako to očekuju da bi učinila većina njihovih učenika kada bi se našla pred pojedinim pitanjem.

Tablica 3.

Rezultati su radi usporedbe prikazani na slici 4 zajedno sa stvarnim postocima točnih odgovora učenika na svako pojedino pitanje u problemu. Vidimo da nastavnici očekuju bolje učeničke odgovore na sva pitanja nego što su to u stvarnosti.

Slika 4.

Istraživanje udžbenika

Istražili smo udžbenike iz fizike za 1. razred gimnazija i srednjih strukovnih škola s četverogodišnjim programom fizike, koji su odobreni od Ministarstva znanosti, obrazovanja i sporta Republike Hrvatske za školsku godinu 2010./2011. Razumno smo pretpostavili da se većina naših ispitanika koristila upravo tim udžbenikom za učenje, odnosno poučavanje. Popis odobrenih udžbenika nalazi se u *Katalozima obveznih udžbenika i pripadajućih dopunskih nastavnih sredstava za gimnazije i srednje strukovne škole u školskoj godini 2010./2011. i 2011./2012.* (MZOS RH, 2010). Citati nekih necjelovitih tvrdnji koje smo pronašli za jednoliko akcelerirano pravocrtno gibanje dani su u tablici 4.

Izdvajamo tri najčešće nedorečene tvrdnje u našim srednjoškolskim udžbenicima iz fizike:

a) Formule za put i brzinu jednoliko usporenog gibanja dobivaju se stavljanjem negativnog predznaka ispred oznake za ubrzanje.
b) Akceleracija je pozitivna pri povećanju brzine, a negativna pri smanjenju brzine.
c) Akceleracija je pozitivna ako ima smjer jednak smjeru gibanja, a negativna ako ima smjer suprotan smjeru gibanja.

Tablica 4.

Diskusija

Najprije smo diskutirali o učeničkoj/studentskoj sposobnosti razumijevanja kinematike objekta i o sposobnosti određivanja iznosa i smjera brzine i akceleracije iz danog v-t grafičkog prikaza. Na slici 3a vidimo da su vrijednosti postotaka, s izuzetkom područja od oko 60 %, gotovo homogeno zastupljene kroz cijelu ljestvicu pa zaključujemo da postavljena pitanja imaju dobru raspodjelu težina potrebnu za testiranje znanja. Također, vidimo da postoji izražena korelacija među podacima, odnosno da je raspodjela težine pitanja približno jednaka za svaku pojedinu grupu ispitanika. Pitanja koja su lakša za jednu grupu ispitanika lakša su i za ostale grupe, a pitanja koja su teža za jednu grupu ispitanika teža su i za ostale grupe. Međutim, ako grupe međusobno usporedimo s obzirom na težinu pitanja, uočit ćemo razlike. Većina podataka za studente SF pada nešto iznad linije autokorelacije, što znači da su studenti nešto bolje riješili zadatak od gimnazijalaca PMI. S druge strane, podaci za učenike OG i SŠ padaju znatno ispod linije, što znači da je zadani problem za te skupine ispitanika znatno teži. Uz najveći postotak točnih odgovora studenti na većinu pitanja također imaju najveći postotak netočnih odgovora, što je vidljivo na slici 3b. S obzirom na njihovo znatno veće iskustvo u radu s kinematičkim grafovima funkcija, koje su stekli u srednjoj školi i na sveučilištu, taj rezultat pokazuje da tradicionalne metode poučavanja ne pridonose bitno razumijevanju kinematičkih grafova. S druge strane, kurikul utječe na razvijanje sposobnosti detaljnije interpretacije. Tome u prilog govore rezultati prikazani na slici 3c iz kojih je vidljivo da studenti u najmanjem postotku ostaju bez odgovora. Oni gibanje opisuju opširnije, promatrajući ga s više aspekata.

Na pitanje 1.1 otpada najveći postotak točnih odgovora i najmanji postotak bez odgovora. Za ispravnu interpretaciju prvoga dijela grafičkog prikaza bio je dovoljan intuitivan način razmišljanja. Ispitanici su ispravno zaključili da se brzina smanjuje na temelju smanjenja visine *v-t* grafičkog prikaza. Najviše poteškoća bilo je s opisom akceleracije. To se vidi iz položaja simbola 1.4, 1.5 i 2.2 koji se odnose na opis iznosa i orijentacije akceleracije. Spomenute točke najbliže su ishodištu na sl. 3a, a najudaljenije su od ishodišta na sl. 3c.

Samo je 4 % ispitanika ispravno odredilo orijentaciju ubrzanja, dok ostali nisu komentirali tu karakteristiku, zbog čega graf 3b ne sadrži oznaku 1.5. Taj je rezultat u skladu s rezultatima sličnih istraživanja (Labudde i suradnici, 1988; Reif i Allen, 1992) prema kojima učenici/studenti zanemaruju vektorsku prirodu ubrzanja. Osim toga oni pokazuju poteškoće u razumijevanju akceleracije kao vremenske promjene brzine. Najčešće pogrešne interpretacije naših ispitanika, koje govore tome u prilog, jesu: (i) iznos ubrzanja mijenja se u vremenskim intervalima u kojima se mijenja brzina, (ii) pri skokovitoj promjeni brzine smatraju da je iznos akceleracije jednak nuli ili nekoj drugoj konačnoj vrijednosti (tablica 2).

Pogrešna interpretacija (i) ukazuje na to da ispitanici ne razlikuju koncepte brzine i ubrzanja jer promjenu iznosa ubrzanja pogrešno opisuju promjenom iznosa brzine. Miješanje koncepata brzine i akceleracije poznato je iz sličnih istraživanja koje su proveli Rosenblatt i Heckler (2011), zatim Trowbridge i McDermott (1980; 1981). Prema rezultatima istraživanja Shaffer i McDermott (2005), ispitanici koji pogrešno smatraju da je akceleracija u točki odbijanja kugle jednaka nuli (ii), također dolaze do tog zaključka zbog nerazlikovanja koncepata brzine i akceleracije. Budući da je brzina u toj točki jednaka nuli, smatraju da i akceleracija mora biti jednaka nuli. Oni učenici/studenti koji su računali trenutačnu vrijednost akceleracije u točki odbijanja kugle koristeći se formulama i uvrštavajući u njih neke vrijednosti, očito je da su se u nedostatku konceptualnog razumijevanja prisjećali odgovora na slična pitanja. Pritom nisu vodili računa kako i pod kojim su uvjetima ti odgovori dobiveni (Labudde i suradnici, 1988; Reif i Allen, 1992).

Prema rezultatima istraživanja McDermott (1993), na svim razinama obrazovanja razlika između onoga što se poučava i onoga što učenici nauče često je veća nego što nastavnici uviđaju. Primjerice, nastavnici često pretpostavljaju da će grafički prikaz koji je njima jasan i razumljiv, biti jednako tako jasan i prosječnom studentu (Meltzer, 2005). Međutim, praksa pokazuje da ta pretpostavka često nije istinita (McDermot, 1990). Stoga smo smatrali važnim sagledati u kojoj mjeri nastavnici dobro ili loše procjenjuju učeničko razumijevanje kinematičkih koncepata, jer postoje mnoge pretkoncepcije (npr. o karakteristikama vektora) koje nastavnici fizike smatraju toliko očitim da o njima nije potrebno raspravljati u učionici. Razmatranje tih karakteristika ključno je za učinkovitost nastave (Aguirre, 1988), stoga nam je ovo istraživanje dodatno pomoglo u ispitivanju potencijala netradicionalnog problema (Erceg i Aviani, 2013; Erceg i suradnici, 2011; Erceg i suradnici, 2013; Erceg i suradnici, 2014; Kariž Merhar, 2001; Marušić i suradnici, 2011) u nastavi fizike. Iz grafikona na slici 4 vidi se da nastavnici značajno precjenjuju učeničko razumijevanje kinematičkih koncepata, što je u skladu s rezultatima sličnih istraživanja (McDermott, 1993). To se posebno odnosi na koncepte smjera ubrzanja i smjera brzine (u okviru pitanja 1.3 i 1.5), gdje su najveće razlike u postocima ispravnih odgovora. Primjerice, kada je riječ o ispravnoj interpretaciji smjera ubrzanja, očekivanja nastavnika su precijenjena čak 14 puta i iznose 56 % u odnosu na stvarnih 4 %. Najmanja razlika u postocima ispravnih odgovora (3 %) odnosi se na pitanje 1.1 na koje su učenici najbolje odgovorili (76 % ispravnih odgovora), a vezano je uz interpretaciju brzine u prvom vremenskom intervalu.

Tradicionalni pristup nastavi smatra se općenitim uzrokom konceptualnih poteškoća vezanih uz grafičke prikaze iz kinematike (Labudde i suradnici, 1988; Reif i Allen, 1992). Međutim, istraživanjem sadržaja naših srednjoškolskih udžbenika ustanovili smo da autori udžbenika također nose velik dio odgovornosti za takvo stanje. U mnogim smoudžbenicima pronašli necjelovite navode vezane uz jednoliko akcelerirano pravocrtno gibanje. Kinematički grafički prikaz nije moguće razumjeti ako nisu razjašnjene vektorske karakteristike fizičkih veličina (pomaka, brzine i akceleracije), odnosno značenja njihovih negativnih vrijednosti. Necjelovite tvrdnje pronađene u hrvatskim udžbenicima pojavljuju se uglavnom zbog definicije smjera akceleracije koji ovisi o smjeru gibanja ili o vrsti gibanja, ali i zbog nedorečenih interpretacija odgovarajuće vrste varijabli u kinematičkim jednadžbama. Iako nismo istraživali udžbenike fizike iz drugih zemalja, očito je iz nedavne rasprave (Hayes i Wittmann, 2010b; Mungan, 2010; Paetkau, 2010) da je problem općenite prirode. Smatramo da nedosljednost i neslaganje u konvencijama o predznaku u kinematičkim jednadžbama često uzrokuju nerazumijevanje i miskoncepcije kod učenika i studenata.

Važno je pojasniti što zapravo predstavljaju jednodimenzijske kinematičke skalarne jednadžbe. Uobičajena nedorečena interpretacija koja se može pronaći u udžbenicima jest da su to jednadžbe koje sadrže iznose fizičkih veličina. Drugim riječima, varijable (s, v, a) su iznosi odgovarajućih vektora, tj. pozitivni realni brojevi. U tom slučaju se u jednadžbama koje opisuju jednoliko usporeno gibanje stavlja negativan predznak ispred oznake za akceleraciju ($v = v_0 - at$, $s = s_0 + v_0t - (at^2)/2$), a skup brojčanih vrijednosti akceleracije a obično se ne definira. Stoga ostaje nejasno je li riječ o skupu realnih brojeva ili o skupu pozitivnih realnih brojeva. Međutim, jednodimenzijske skalarne jednadžbe nisu jednadžbe iznosa fizičkih veličina pa bi se trebale tretirati kao komponentne jednadžbe čije varijable mogu poprimiti, osim pozitivnih, i negativne vrijednosti. Takav je koncept ključan za interpretaciju kinematičkih grafičkih prikaza u kojima se pojavljuju negativne vrijednosti fizičkih veličina. Ispravno bi bilo zadržati dosljednost u formulama za ubrzano i usporeno gibanje, tako da se u njima zapisuju samo pozitivni predznaci, a da se brojčane vrijednosti vektorskih fizičkih veličina (s, v, a) definiraju kao realni brojevi. Hayes i Wittmann (2010a) su ustanovili da se studenti fizike nedosljedno koriste fizičkim i matematičkim znanjima prilikom određivanja predznaka članova u jednadžbama te smatraju da je problem u njihovu nastojanju da se vektorska jednadžba interpretira kao skalarna jednadžba.

Ne znači nužno da se objekt ubrzava ako ima pozitivnu akceleraciju te da se usporava ako ima negativnu akceleraciju (Serway, 2006). U slučaju pravocrtnog gibanja moguće su četiri kombinacije smjerova vektora brzine i ubrzanja, kao što je prikazano u tablici 5. Kada vektori brzine i akceleracije imaju jednake smjerove (oba pozitivna ili oba negativna), tada se iznos brzine objekta povećava s vremenom, odnosno govorimo o njegovu ubrzavanju. Kada su vektori brzine i akceleracije suprotnih smjerova (jedan pozitivan, a drugi negativan), tada se iznos brzine smanjuje s vremenom, odnosno objekt usporava. Iz tablice je također vidljivo da tvrdnja koja se navodi u udžbenicima vrijedi samo u slučaju kada je brzina pozitivna, tj. kada se tijelo giba u pozitivnom smjeru odgovarajuće koordinatne osi.

Tablica 5.

Primjerice, ako se tijelo giba u negativnom smjeru *x*-osi i ima negativnu brzinu koja se promijeni od v_1 = -10 m/s na v_2 = -20 m/s u vremenu Dt = 2 s, tada akceleracija iznosi

$$a = \mathrm{D}v \,/\, \mathrm{D}t = (v_2 - v_1) \,/\, \mathrm{D}t = -5 \text{ m/s}^2.$$

Iako ubrzanje ima negativnu vrijednost, tijelo ubrzava, tj. iznos njegove brzine se tijekom vremena povećava. O pozitivnoj i negativnoj akceleraciji ima smisla govoriti samo s obzirom na referentni sustav, a ne s obzirom na smjer gibanja. Koncepti ubrzanog i usporenog gibanja povezani su s pojmovima ubrzavanja odnosno usporavanja, tj. s povećanjem odnosno smanjenjem iznosa vektora brzine. Stoga je za određivanje smjera akceleracije neophodna kinematička analiza (McDermott i suradnici, 1994).

Ispravno bi bilo smjer akceleracije određivati s obzirom na smjer odgovarajućih koordinatnih osi u referentnom sustavu, a ne s obzirom na promjenljiv smjer gibanja. Budući da ne postoji apsolutni referentni sustav, tj. da apsolutno gibanje nema fizikalno značenje, neophodno je uvođenje referentnog sustava za opisivanje relativnih kinematičkih gibanja (Benenson i suradnici, 2002). Fizičke veličine koje se koriste za opis gibanja (pomak, brzina i ubrzanje) su vektori. Vektori su geometrijski objekti koji imaju iznos i smjer, uglavnom se označavaju debelo otisnutim slovom ili slovom iznad kojeg se nalazi strelica. Pravocrtno gibanje tijela uglavnom se opisuje u 1-dimenzijskom pozitivno orijentiranom koordinatnom laboratorijskom sustavu, npr. duž apscisne osi na kojoj se pozitivne vrijednosti nalaze desno, a negativne vrijednosti lijevo od ishodišta. U tom slučaju vektori u kinematičkim jednadžbama zamjenjuju se njihovim komponentama – realnim brojevima. Druga je interpretacija da se vektori zamjenjuju realnim brojevima tako da se vektorske veličine koje imaju smjer udesno prikazuju pozitivnim realnim brojevima i mjernom jedinicom, a vektori koji imaju smjer ulijevo, prikazuju se negativnim realnim brojevima i mjernom jedinicom. Iznos vektora označava se apsolutnom vrijednošću realnog broja, a smjer se označava predznacima (+) ili (-) (Halliday i suradnici, 2005). Strelica iznad slova ili podebljani tisak slova se izostavljaju, a vektorska algebra, koja se temelji na geometrijskim razmatranjima, zamjenjuje se jednostavnom algebrom realnih brojeva. S takvim prikazom fizičkih veličina je puno lakše raditi, ali budući da ne postoji vidljiva razlika između realnih brojeva i vektora, takav prikaz postaje glavni izvor nedorečenih izjava i interpretacija.

Očito je i iz naših rezultata istraživanja da postoji neslaganje između načina poučavanja i načina učenja koje je izraženo u tradicionalnom pristupu nastavi (McDermott, 1993). Tradicionalna nastava temelji se na nastavničkom viđenju sadržaja koji se obrađuju. Osim toga, u nastojanju da učenicima prenesu znanje i entuzijazam, nastavnici se često oslanjaju na vlastitu percepciju učenika, tj. ignoriraju mogućnost da učenička percepcija bude značajno različita od njihove. Poučavaju ih o načinu rješavanja tradicionalnih problema, nadajući se da će to znanje moći primijeniti u novim situacijama. Međutim, na taj način ih isključuju iz aktivnog procesa učenja i postižu suprotan učinak.

Primjena netradicionalnog tipa problema u istraživanju otvara nove mogućnosti u nastavi. On opisuje situaciju iz stvarnog svijeta te sadrži gotovo sve elemente za poučavanje kinematike. Stoga može biti poticaj i temelj za brojne razredne diskusije o kinematičkim konceptima. Na taj način učenici i studenti imaju priliku raspravljati o svojim vlastitim idejama te ih modificirati kada se ukaže potreba, u skladu s preporukom Beichnera (1994). Nastavnici imaju mogućnost otkriti učeničke i studentske strategije rješavanja problema i prilagoditi se njihovu načinu razmišljanja tijekom procesa aktivnog učenja.

Zaključak

Ispitivali smo učeničke i studentske sposobnosti izvlačenja potrebnih informacija iz grafičkog prikaza, razumijevanja kinematičkih koncepata u grafičkom prikazu i prevođenja grafičkog prikaza u stvarnu situaciju. U istraživanju je sudjelovalo 235 srednjoškolskih učenika, 41 student fizike i 48 srednjoškolskih nastavnika fizike. Pred učenike i studente postavili smo originalan problem koji se temelji na v-t grafičkom prikazu, s pitanjima otvorenog tipa na koja su slobodno odgovarali u pisanoj formi. Na temelju njihovih odgovora sastavili smo pitanja zatvorenog tipa za nastavnike, kako bismo istražili u kojoj mjeri nastavnici mogu predvidjeti učeničke strategije rješavanja takvog problema.

Rezultati pokazuju da je raspodjela težine pitanja približno jednaka za sve skupine ispitanika, odnosno da se poteškoće povezivanja grafičkog prikaza s kinematičkim konceptima i realnim svijetom javljaju neovisno o stupnju obrazovanja i kurikulu. Glavne poteškoće su: (i) uspostavljanje veze određenog svojstva grafa određene funkcije (visine, nagiba, površine ispod krivulje) s odgovarajućim fizikalnim konceptom (brzinom, ubrzanjem, pomakom), (ii) razumijevanje brzine i ubrzanja kao vektorskih veličina koje osim iznosa imaju i smjer, (iii) nemogućnost vizualizacije stvarnog gibanja na temelju grafičkog prikaza. Studenti u najmanjem postotku ostaju bez odgovora, što znači da gibanje opisuju opširnije, promatrajući ga s više aspekata.

Očito je da kurikul i stupanj obrazovanja utječu na sposobnost detaljnog pristupa interpretaciji grafičkog prikaza. Studenti imaju najveći postotak ispravnih odgovora na većinu pitanja, ali istodobno i najveći postotak neispravnih odgovora. Taj rezultat može značiti da dodatna izloženost tradicionalnim metodama poučavanja ne utječe značajnije na bolje razumijevanje grafičkih prikaza.

Uspoređujući predviđanja nastavnika, odnosno odgovore koje nastavnici unaprijed očekuju od učenika, sa stvarnim odgovorima učenika, uočili smo da nastavnici precjenjuju sve elemente učeničkog razumijevanja kinematičkih koncepata koje smo istraživali. To se posno odnosi na koncepte smjera ubrzanja i smjera brzine.

Nedosljednost i neslaganje u konvencijama u vezi s predznakom fizičkih veličina u kinematičkim jednadžbama koje opisuju isključivo primjere pravocrtnog gibanja u jednom smjeru može biti značajan izvor učeničkih/studentskih miskoncepcija i nerazumijevanja. Istraživanjem sadržaja naših srednjoškolskih udžbenika iz fizike ustanovili smo da autori udžbenika nose velik dio odgovornosti za takvo stanje. Pronašli smo niz nedorečenosti i nekonzistentnosti u tekstovima i formulama koje se odnose na jednoliko usporeno pravocrtno gibanje. Stoga, osim tradicionalnog pristupa kinematičkim problemima, koji je najvažniji uzrok konceptualnih poteškoća, smatramo da su za poteškoće naših ispitanika odgovorni i udžbenici koji potiču takav pristup.

Problem koji smo imali u istraživanju opisuje situaciju iz stvarnog svijeta te sadrži sve bitne elemente kinematike. Takav tip problema daje poticaj za daljnja istraživanja i razvoj kurikula nastave fizike na svim razinama obrazovanja. Predlažemo ga za primjenu u nastavi tijekom procesa aktivnog učenja kao poticaj i temelj za diskusije o kinematičkim konceptima. Nastavnici tako mogu otkrivati učeničke strategije rješavanja problema te prilagoditi nastavu njihovu načinu razmišljanja.

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