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THE POSITIONING METHOD OF BDS RECEIVER WITH AUXILIARY CLOCK UNDER THE WEAK SIGNAL CIRCUMSTANCE

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Original scientific paper
The traditional time-of-transmission recovery algorithm cannot be applied to practice due to a drastic calculation increase when the approximate position of receiver is unknown. This paper puts forward a receiver positioning method based on auxiliary clock, which can be used when the receiver's approximate position is unknown. To determine search scope, the paper makes use of double GEO satellites to build custom polar coordinates, and reduces unknown searching numbers on the basis of elevation range information implied in the receiver. Furthermore, according to the restraints of satellite signal coverage scope, it is advocated to choose two farthest GEO satellites to build the custom coordinate system, and add any validation satellites for verification and selection, so as to realize the restoration of millisecond integer of signal transmission time. This method is verified through simulation analysis. To be specific, statistical analysis of the algorithm's precision requirements of auxiliary clock is made under the application circumstance of only GEO constellation in China and neighbouring areas. The conclusion shows that, when the auxiliary clock error is below 100 us and the number of validation satellites is enough, the receiver can be rapidly positioned under the weak signal circumstance even when the approximate position is unknown.

Keywords: *BeiDou Navigation Satellite System (BDS), GEO constellation, Global Navigation Satellite System (GNSS), time-of-transmission recovery, weak signal*

Metoda pozicioniranja BDS prijemnika pomoćnom urom u uvjetima slabog signala

Izvorni znanstveni članak
Tradicionalni algoritam za pronalaženje vremena odašiljanja ne može se primijeniti u praksi zbog drastičnog povećanja izračuna kada je nepoznat približan položaj prijemnika. U ovom se radu predlaže metoda za određivanje položaja prijemnika temeljena na pomoćnom satu, koja se može primijeniti kada je približan položaj prijemnika nepoznat. Kako bi se odredilo područje traženja, u radu se koriste dva GEO satelita za dobivanje polarnih koordinata, i reduciraju se nepoznati brojevi pretraživanja na osnovu informacija o dometu elevacije uključenih u prijemniku. Nadalje, prema ograničenjima područja pokrivenog satelitskim signalom, preporučuje se izabrati dva najudaljenija GEO satelita za stvaranje uobičajenog koordinatnog sustava te dodati bilo koji broj satelita zbog verifikacije i selekcije, da bi se vrijeme prijenosa signala uspjelo vratiti do tisućinke sekunde. Ta se metoda provjerava simulacijskom analizom, točnije, statistička analiza točnosti algoritma pomoćnog sata provodi se jedino u uvjetima GEO konstelacije u Kini i susjednim područjima. U zaključku se pokazuje da kada je greška na pomoćnom satu ispod 100 us, a broj satelita za provjeru je dovoljan, prijemnik se može brzo pozicionirati u uvjetima slabog signala čak i kada mu je približni položaj nepoznat.

Ključne riječi: *BeiDou navigacijski satelitski sustav (BDS), GEO konstelacija, globalni navigacijski satelitski sustav (GNSS), pronalaženje vremena odašiljanja, slabi signal*

1 Introduction

The Assisted-GPS (A-GPS) receiver positioning algorithm1-3 refers to suppose the receiver's millisecond integer time to be unknown, which awaits solution by introducing the fifth observation equation. Global System for Mobile Communications (GSM) and other auxiliary methods are used to acquire the user's approximate position, time and satellite message and so on. Positioning is done as soon as the receiver's millisecond integer time is worked out. However, when the receiver's approximate position error is over 150 km, amount of searching calculation will drastically increase and searching scope of receiver's millisecond time will be broadened [4 ÷ 6], especially when the receiver's approximate position is totally unknown, this method can be regarded as a failure.

The auxiliary high precision clock method can be adopted to acquire relatively accurate clock information [7, 8]. After auxiliary precise time information is achieved, the next issue is turned to make a rapid estimate of the receiver's approximate position. In fact, elevation information range (−1 ÷ 20 km) is implied when receiver is on the surface of the Earth. Although the range is rough, the millisecond integer vagueness of the relative signal transmission time can be ignored when it is projected to the vector direction of the satellite. Therefore, the unknown searching numbers can be reduced from the three-dimensional position and four time indicators to two horizontal positions, which can facilitate a rapid and simple search calculation after the approximate processing. As a result of search, the complete satellite signal transmission time can be acquired. Namely, positioning and the basic navigation is accomplished.

This paper advocates a method based on receiver's auxiliary clock, takes the characteristics of GEO constellation [9] and information of satellite time within milliseconds into consideration. The principle of simplification is also put forward, indicating that the satellite signals' track points which share the same time delays, these track points' elevations on the surface of Earth are also similar. Thus, the millisecond integer time of signals can be rapidly worked out and the receiver's approximate position can also be determined.

2 Search algorithm
2.1 Custom polar coordinates

When supplementary information exists in the user's clock error, the search for user's unknown information can be transformed into the search for the three-dimensional position. During the searching process, there is some information implied in terms of elevation: there is an elevation limit of 20 km near the surface. Therefore, searching for user's unknown information equals to twodimensional search on the horizontal level. For the sake of convenience, two unknown numbers on the horizontal level are redefined by custom polar coordinates.

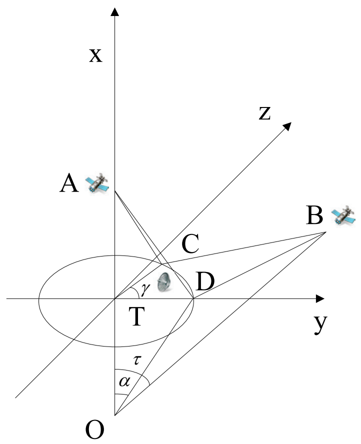


Figure 1 The diagram of the custom coordinate system

In the space rectangular coordinate system, suppose the coordinate of the satellite A as (x_A, y_A, z_A), the satellite B is (x_B, y_B, z_B), the receiver C is (x_C, y_C, z_C), and the Earth's core O is (0, 0, 0). Then, the respective vectors are as follows: $\overrightarrow{\mathrm{OA}}=\overrightarrow{(x_A, y_A, z_A)}$, $\overrightarrow{\mathrm{OB}}=\overrightarrow{(x_B, y_B, z_B)}$. O is defined as the origin of this coordinate, $\overrightarrow{\mathrm{OA}}$ shares the same direction with the axes Z. The axis Y is perpendicular to the plane of vector $\overrightarrow{\mathrm{OAB}}$ and the axis X is perpendicular to the plane of vector $\overrightarrow{\mathrm{O}YZ}$ and is right to the two other axes. Thus, the unit vectors of three axes in this custom rectangular coordinate system $\left(\vec{e}_x, \vec{e}_y, \vec{e}_z\right)$ are respectively expressed as $\vec{e}_x=\dfrac{\vec{e}_y \times \vec{e}_z}{\left|\vec{e}_y \times \vec{e}_z\right|}$, $\vec{e}_y=\dfrac{\overrightarrow{\mathrm{OA}} \times \overrightarrow{\mathrm{OB}}}{\left|\overrightarrow{\mathrm{OA}} \times \overrightarrow{\mathrm{OB}}\right|}$, $\vec{e}_z=\dfrac{\overrightarrow{\mathrm{OA}}}{\left|\overrightarrow{\mathrm{OA}}\right|}$.

The symbol × represents exterior product. Therefore the coordinate of an arbitrary point C in the custom rectangular coordinate system is $(\overrightarrow{\mathrm{OC}}, \vec{e}_x; \overrightarrow{\mathrm{OC}}, \vec{e}_y; \overrightarrow{\mathrm{OC}}, \vec{e}_z)$.

Let the point where the first satellite signal $\overline{\mathrm{AC}}$ arriving at the receiver, cutting the plane of axes XY be called D. The physical connotation of the circle $\overline{\mathrm{CTD}}$ is a collection of satellite signals arriving at surface with the same transmission delays. $\overline{\mathrm{BD}}$ refers to the shortest distance between the satellite B and the receiver when the time delay of the first satellite is certain. Thus, the delay vagueness of satellite B is equivalent to the length of the line $\overline{\mathrm{BD}}$. $\overline{\mathrm{OA}}$ stands for the distance between the satellites to the Earth's core whereas τ refers to the included angle between satellite A and B with the core as the joint. When the satellite is in the same plane with the equator, τ represents the longitude difference between two satellites. $\overline{\mathrm{OD}}$ and $\overline{\mathrm{OC}}$ refer to R, represent the sum of the Earth's radius R_a and the receiver's elevation estimate h which may contain errors.

Suppose the included angle where the plane of $\overline{\mathrm{CTD}}$ cuts the circular cross section plane is γ and the included angle of $\overline{\mathrm{AOD}}$ is α. For the sake of convenience, three-dimensional polar coordinates are introduced: the radius is the length from the origin O, the first reference angle is the included angle of $\overline{\mathrm{AOB}}$ plane and the second is the included angle of $\overline{\mathrm{CTD}}$ plane.

Then the coordinate of C is (r, α, γ), B is (R, τ, 0), and A is (R, 0, 0). Their respective space coordinates come as follows: ($r\cdot\sin\alpha\cdot\cos\gamma$, $r\cdot\sin\alpha\cdot\sin\gamma$, $r\cdot\cos\alpha$), ($R\cdot\cos\tau$, $R\cdot\sin\tau$, 0), (R, 0, 0). A conclusion can be drawn here:

$$\overline{\mathrm{AC}}=\sqrt{R^2+r^2-2rR\cos\alpha},$$

$$\overline{\mathrm{BC}}=\sqrt{R^2+r^2-2rR\cos\alpha\cos\tau-2rR\sin\alpha\sin\tau\cos\gamma}.$$

In reality, ever if the GEO satellite is dynamic and not lingering on the equator plane. Many factors especially the mechanical adjustment can alter the position marked by ICD. Through statistical calculations of measured ephemeris data, the maximum positioning error scope can reach as much as 160 km and error is most likely to occur in the direction of axis Z10.

Just as the chart has illustrated, when two reference satellites diverge from their original positions A, B to A' and B', a new custom coordinate system should be built according to actual positions. Here, because satellites are not in the equator plane, building new coordinate axes will have an impact on the altitude compensation formula. Therefore, two GEO satellites should be picked up to redefine the coordinate system. α can be calculated through $\overline{AC}$, then the search scope of $\overline{BC}$ can be determined. As long as search space is established, other satellites can be added for verification.

It should be pointed out that due to the fact that the custom coordinate system is established on the basis of the combination of two GEO satellites and validation satellites will not bring about errors, any types of satellites like GEO, IGSO, MEO can be adopted.

2.3 Research based on hypothesis testing

From the above formula, we can see that if $\overline{AC}$ is defined, α can be identified. As α is uniquely determined by the length of $\overline{\mathrm{AC}}$ and that $\overline{\mathrm{BC}}$ will form a corresponding relationship with γ. Then with the propagation of satellite signal ranging delay of 19 ms, we assume that $\overline{\mathrm{AC}}$ and $\overline{\mathrm{BC}}$ can constitute a 19×19-dimensional search space. In total, there are 361 groups of corresponding relationships between α and γ, which can be tested using hypothesis by adding the remaining satellites. For example, we take the third satellite as the testing satellite and represent its location information with a group of corresponding value of α and γ. If the calculated pseudo-range value is close to the whole milliseconds after the measured milliseconds decimal of that satellite is subtracted, then we can prove that α and γ are true values, so as to determine the milliseconds integer of the time of the satellite signal as well as the receiver position through α and γ. When we insert all groups of α and γ to obtain multitude results close to whole milliseconds, it is necessary to add more redundant satellites for verification. The maximum calculation amount of the added satellites is 361 × n. In this formula, n represents the number of satellites to be verified.

By adding the redundant satellites for verification, we can calculate the whole milliseconds of their propagation delay. Suppose the propagation delay arising from the insertion of α and γ by the n satellite as $d_n^{(\alpha,\gamma)}$, then the residual of its integer part can be presented as:

$$D_n^{(\alpha,\gamma)} = d_n^{(\alpha,\gamma)} - t_n^{chip} .$$

In the formula, t_n^{chip} represents the decimal within milliseconds of the time delay of the known satellite signal. Then the integer residual of a single satellite is

$$\Delta D_n^{(\alpha,\gamma)} = \left| D_n^{(\alpha,\gamma)} - \left[D_n^{(\alpha,\gamma)} \right] \right| .$$

The symbol $[\ ]$ stands for the closest integer obtained. In a similar vein, with regard to the hypothesis combination (α, γ), the integer residual of n satellites can be represented as

$$g_n(\alpha,\gamma) = \sqrt{\sum_{i=1}^{n} \left(\Delta D_i^{(\alpha,\gamma)} \right)^2} .$$

2.3 Compression of the search space

Before verifying the hypothetical situation within the search space, we can exclude certain hypothetical situations according to the features of the satellite time delay, to reduce the amount of calculation. To begin with, when $\overline{\mathrm{AC}}$ is determined based on the range of time delay, α can get a corresponding set of value.

$$\alpha(\overline{\mathrm{BC}}) = \arccos\left(\sqrt{\frac{R^2 + r^2 - \overline{\mathrm{AC}}^2}{2Rr}} \right).$$

Then on the basis of α, we can define the value relationship between $\overline{\mathrm{BC}}$ and γ as

$$\overline{\mathrm{BC}}(\gamma) = \sqrt{R^2 + r^2 - 2rR\cos\alpha\cos\tau - 2rR\sin\alpha\sin\tau\cos\gamma} .$$

In the light of the formula, we can get the conclusion that as the value of cosγ ranges from −1 to 1, the value range of $\overline{\mathrm{BC}}$ can be determined. The physical meaning of the formula is that when the propagation delay of the first satellite signal $\overline{\mathrm{AC}}$ identifies the receiver trajectory $\overline{\mathrm{CTD}}$, the propagation delay of the secondary satellite $\overline{\mathrm{BC}}$ reaches the maximum and minimum value of $\overline{\mathrm{BCD}}$. Integrating the binding conditions of the value space of $\overline{\mathrm{BC}}$ itself, the range of $\overline{\mathrm{BC}}$ can be identified as the following:

$$\overline{\mathrm{BC}}_{\min} = \max\left[120, \overline{\mathrm{BC}}(\gamma)_{\min}\right],$$
$$\overline{\mathrm{BC}}_{\max} = \min\left[138, \overline{\mathrm{BC}}(\gamma)_{\max}\right].$$

Therefore, we will establish proper binding conditions by assuming the number of combinations could be less than 361. As for two GEO satellites, there are fewer combinations that meet the requirement when the angle between the satellite and the earth's core is larger.

3 Results and Discussion

To illustrate the impacts of the GEO satellite constellation on this algorithm, the example employs the pure GEO constellation combination. Take the latitude of the receiver position be 28,22 °N, longitude 112,99 °E and elevation 86,5 m. In accordance with the BDS ICD, the elevation of the satellite position is 35 786 km with the east longitude being 140 °E, 80 °E, 110,5 °E, 160 °E and 58,75 °E respectively. To show the influence of the satellite position in actual processing, we collect the message data of the B1 frequency point from five GEO satellites, on the basis of which we calculate its true position as the experiment data.

We calculate the complete delay of satellite distance through the real positions of receivers and satellites. Then taking the milliseconds part of complete time as the known quantity, we employ the algorithm put forward in this article to seek a solution to the ambiguity of satellites' transmitted time in millisecond scale.

3.1 Compression of the search space

When choosing the farthest satellites, namely the No. 5 satellite at 58,75 °E and No. 4 satellite at 160 °E as the preferred satellites, we can figure out the search range based on search space which are illustrated as the following:

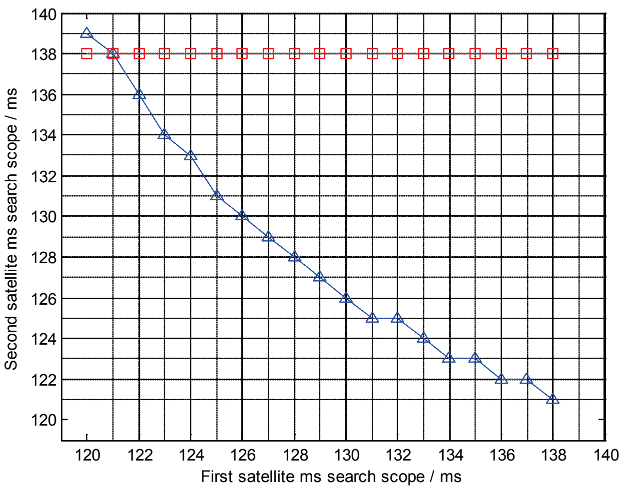
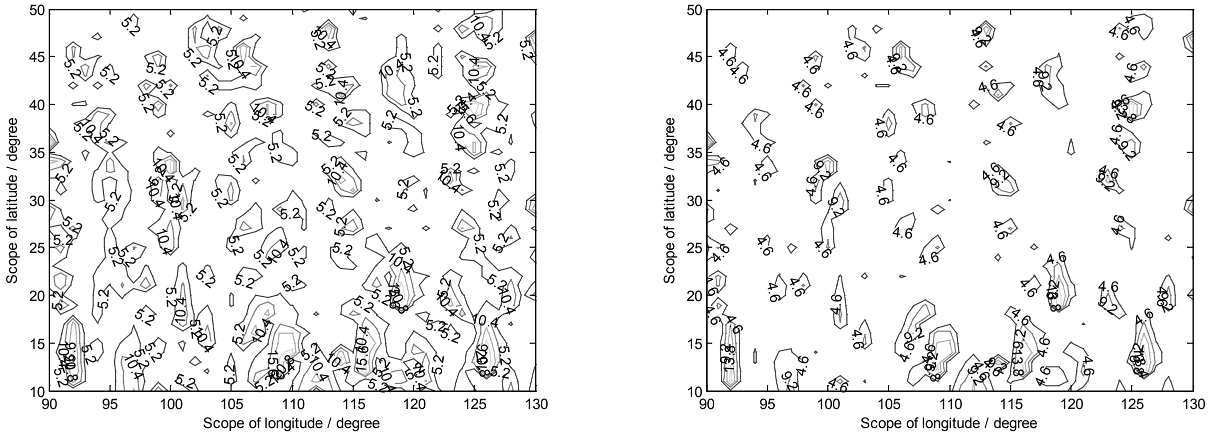


Figure 2 Time search range after compression

According to the result, we can see that when the time delay of the first satellites is 120 ms the secondary satellites have no possible value within reasonable geometric relationship. Under such circumstance, there is no need to verify the situation when $\overline{\mathrm{AC}}$ is 120 ms. With respect to other time delay, the verification range of secondary satellites has been compressed to some extent. Seen from the general distribution, we can conclude that the search scope decreases from 361 to 217 combinations, about 60 % compared to the original combinations.

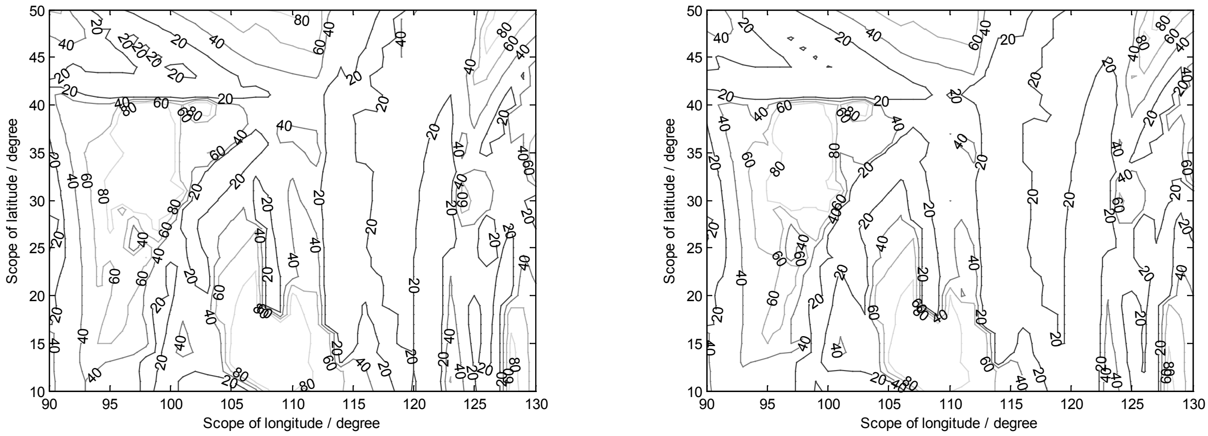
3.2 The impact of receivers' position

In order to analyze the performance of the algorithm with receivers at different positions, we will do the traversal experimental statistics in China and its bordering areas. From 90 °E to 130 °E and from 10 °N to 50 °N, each degree will be calculated as receivers' position to count the biggest clock error that can accommodate. Then we draw comparisons and do statistics between known and unknown input elevation. We set the real elevation to 10 km, and assumed it as 0 m when without elevation information.



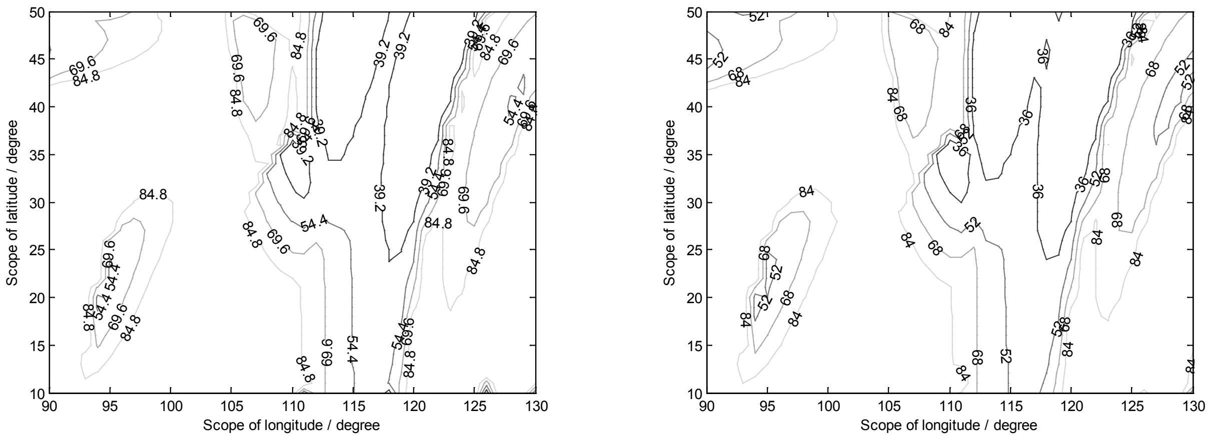
a) with elevation information b) without elevation information

Figure 3 The adaptation to the clock error within China when one satellite is added for verification



a) with elevation information b) without elevation information

Figure 4 The adaptation to the clock error within China when two satellites are added for verification

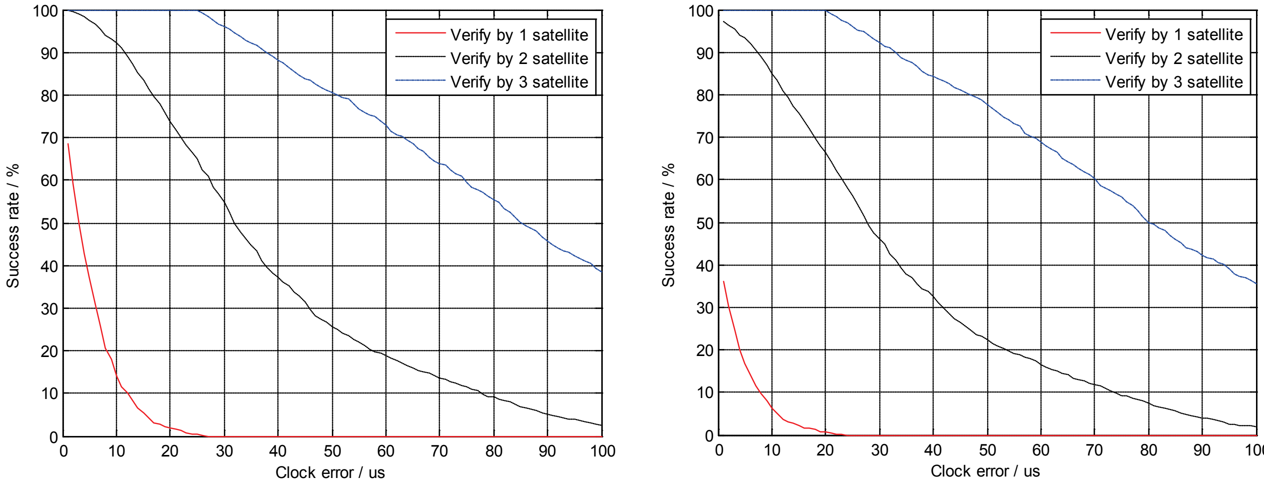


a) with elevation information b) without elevation information

Figure 5 The adaptation to the clock error within China when three satellites are added for verification

The above graph illustrates that success rate is mainly subject to the clock error when only one satellite is added for verification. The added satellites enormously bolster the success rate, and the 20 us adaptation range of clock error can be obtained 100 % by 3 verified satellites, some of which can reach 100 us. Meanwhile, according to comparative experiments, the clock error's impact on the success rate can be neglected when the elevation information is unknown.

To summarize the clock error's impact on the success rate for different receiver's position range above, the results are illustrated as the figures below.



a) with elevation information b) without elevation information

Figure 6 Success rate

We can discover from the figure that the success rate is low when verified by only one satellite, whether the elevation information is known or not. By adding verified satellite, it can significantly improve the performance. When clock error is less than 20 us, three verified satellites receive a 100 % success rate, in the simulated receiver's position range.

To describe the basic properties of the method, we select GEO satellite which is visible throughout 24 hours in China. In a real world application, the performance could be improved by adding more visible satellites.

4 Conclusion

This paper puts forward a time auxiliary BDS positioning method under the weak signal circumstances. The key procedure of the method lies in restoring the transmitting time of the satellite signal. By taking advantage of GEO's satellite orbital characteristics and concealing the elevation information, and the auxiliary clock, we streamline four unknown numbers of the position solution to two. Customized polar coordinates transform reach the earth's surface as satellites signals to two opening angles, which make it easier to calculate. In addition, reasonable compression of the search space has a physical meaning of finding a solution within the common coverage for multiple satellites. As this method merely compresses the range of two GEO satellites, the scope may not be the smallest but with a modest amount of calculation.

Of course, this method also has obvious drawbacks in that it needs auxiliary timing and the degree of reliance is higher especially when there are fewer verification satellites. Furthermore, after error search occurs with few verification satellites, there is no way to test the error simply through the observation information of satellites as the result of the shortage of information.

Compared to traditional methods of the searching on space rectangular coordinates and clock error, this method can effectively reduce the amount of search and calculation, and make the positioning solution when approximate location is unknown be possible under the weak signal circumstance.

5 References

[1] Agarwal, N.; Basch, J.; Beckmann, P. Algorithms for GPS operation indoors and downtown. // GPS Solutions, 6, (2002), pp. 149-160.
[2] Diggelen, F. V. Indoor GPS theory & implementation. // IEEE Position, Location & Navigation Symposium, 2002, pp. 240-247.
[3] Sirola, N.; Syrjärinne, J. GPS Position Can be Computed without the Navigation Data. // ION GPS, 2002, pp. 2741-2744.
[4] Sirola, N. Exhaustive Global grid search in computing receiver position from modular satellite range measurements. // Journal of Physics: Conference Series, 2006, pp. 73-82.
[5] Smith, P. D.; Rowe, R. Comparative A-GPS and 3G-Matrix testing in a dense urban environment. // Proc ION GNSS 19th International Technical Meeting of the Satellite Division, 2006, pp. 3006-3011.
[6] Kaplan, E. D.; Hegarty, C. J. Understanding GPS: principles and applications. // 2nd ed. Norwood, MA, Artech House, 2006.
[7] John Kitching; Svenja Knappe; Li-Anne Liew et al. Chip-Scale Atomic Frequency References. // ION GNSS 18th International Technical Meeting of the Satellite Division, Long Beach, CA, September 2005, pp. 1662-1669.
[8] SA.45s Chip Scale Atomic Clock Data Sheet. Symmetricom 2013. http://www.symmetricom.com/products/
[9] BeiDou Navigation Satellite System Signal In Space Interface Control Document Open Service Signal B1I (Version 1.0). // China Satellite Navigation Office December 2012.
[10] Zhenghang Li; Weixing Zhang; Xiaoying Gong; Xiaochuan Qu. Solution of orbit maneuver problem in autonomous orbit. // Geomatics and Information Science of Wuhan University, 36, 11(2011).

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