ISSN 1330-3651(Print), ISSN 1848-6339 (Online)
UDC/UDK 537.84:530.145.7

MHD FLOW IN A CHANNEL USING NEW COMBINATION OF ORDER OF MAGNITUDE TECHNIQUE AND HPM

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Original scientific paper

The present work is concerned with the steady incompressible flow through a parallel plate channel with stretching walls under an externally applied magnetic field. The governing continuity and Navier-Stokes equations are reduced to a fourth order nonlinear differential equation by using vorticity definition and similarity solution transformation. The obtained equations are solved by applying the analytical homotopy perturbation method (HPM). The method is called order of magnitude suggested for simplifying series solution to finite expression that is useful in engineering problems. The results are verified by comparing with numerical solutions and demonstrate a good accuracy of the obtained analytical solutions. Profiles for velocity are presented for a range of plate velocity and magnetic field. The study shows that a back flow occurs near the center line of the channel and with the increase in strength of magnetic field, the back flow decreases.

Keywords: *channel, HPM, magnetohydrodynamic, stretching walls*

MHD tok u kanalu uporabom novih kombinacija tehnika grubog opisa vrijednosti i HPM

Izvorni znanstveni članak

Sadašnji rad se bavi ravnomjernim nestišljivim tokom kroz paralelne ploče kanala s rasteznim zidovima sukladno izvana narinutim magnetskim poljem. Jednadžbe kontinuiteta i Navier-Stokesove jednadžbe svedene su na nelinearne diferencijalne jednadžbe četvrtog reda pomoću definicije vrtloženja i transformacije sličnosti rješenja. Dobivene jednadžbe riješene su primjenom analitičke metode homotopijske perturbacije (MHP). Metoda se zove grubi opis vrijednosti, predložena za pojednostavljenje rješenja serija u konačni izraz, koristan u inženjerskim problemima. Rezultati su potvrđeni usporedbom s numeričkim rješenjima i pokazuju dobru točnost dobivenih analitičkih rješenja. Profili za brzine prikazani su za područje brzine ploče i magnetsko polje. Studija pokazuje da se povratni tok javlja blizu središnje linije kanala, a povećanjem snage magnetskog polja povratni tok se smanjuje.

Ključne riječi: *kanal, magnetohidrodinamika (MHD), metoda homotopijske perturbacije (MHP), rastezni zidovi*

1 Introduction

The magnetohyrodynamcs (MHD) phenomenon is characterized by an interaction between the hydrodynamic boundary layer and the electromagnetic field. Recently there has been an increasing interest in fluid flow through MHD channel. The study of MHD flow in a channel has applications in many engineering problems. An extensive theoretical work has been carried out on the hydromagnetic fluid flow in a channel under various situations by Hartmann [1]. Joneidi [2], Pourmahmoud [3], Morely et al. [4] studied the fully developed liquid metal in MHD duct flow. Theoretical investigation of the applicability of magnetic fields for controlling hydrodynamic separation in Jeffrey-Hamel flows of viscoelastic fluids has been studied by K. Sadeghy et al. [5]. The MHD viscoelastic fluid flow in a channel with stretching walls was numerically studied by Misra et al. [6]. Uwanta et al. [7] studied the convective heat transfer in a MHD Channel in a Porous Medium by perturbation method.

The main objective of the present paper is to study the problem of a steady laminar flow through a MHD parallel plate channel with stretching walls analytically. The governing partial differential equations (PDE) are coupled. Applying vorticity definition and similarity solution to these equations results in a fourth order nonlinear differential equation. The equation is solved by applying the analytical homotopy perturbation method (HPM) and velocity distributions are derived. It provides an efficient explicit solution with high accuracy, minimal calculations, and without physically unrealistic assumptions and linearization. Unlike analytical perturbation methods, the significant feature of this method is that it does not depend on a small parameter. The HPM was first proposed by He in 1998, then was developed and improved by him [8 ÷ 10].

This method has been successfully applied to solve many types of nonlinear problems [13 ÷ 18].

The order of magnitude method is defined for simplifying solutions expression, which is useful in engineering problem. The results are verified by comparing them to numerical results. This comparison revealed that the obtained analytical solutions are reasonably accurate.

2 Problem statement

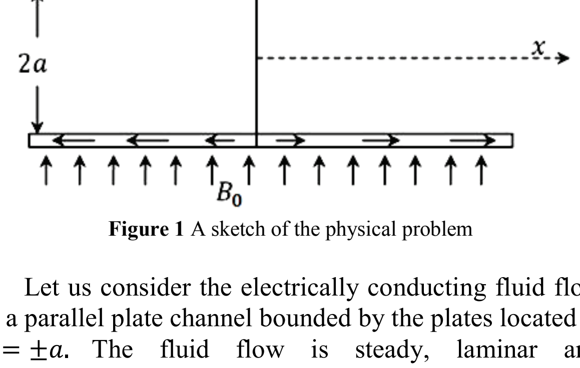

Figure 1 A sketch of the physical problem

Let us consider the electrically conducting fluid flow in a parallel plate channel bounded by the plates located at $y = \pm a$. The fluid flow is steady, laminar and incompressible. By applying two equal and opposite direction forces along the x-axis, the wall is being stretched with a speed proportional to the distance from the fixed origin, $x = 0$. The uniform magnetic field, B_0, is imposed along the y-axis as show in Fig. 1.

The induced magnetic field due to the motion of the electrically conducting fluid is negligible. This assumption is valid for small magnetic Reynolds Number.

It is also assumed that the electrical conductivity of fluid, σ, is constant and the external electric field is zero.

Under these assumptions, the governing continuity and momentum equations for the motion can be written as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma {B_0}^2}{\rho}u\,, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right). \tag{3}$$

Where (u, v) are the fluid velocity components along the x and y directions, respectively. ρ, υ, B_0 and σ are the density, kinematic viscosity, magnetic field strength and electrical conductivity respectively.

The flow is symmetric about the center line of the channel, $y=0$, and we only focus our attention on the flow in the region $0\le y\le a$. The boundary conditions for this problem can be written as

$$\frac{\partial u}{\partial y}=0\,,\ v=0\,,\ \text{at}\ y=0\quad (\text{i.e. symmetry}),$$

$$u=bx\,,\qquad v=0\,,\qquad \text{at}\qquad y=a\,, \tag{4}$$

where b is the stretching rate constant. The following dimensionless variables are introduced.

$$x'=\frac{x}{a}\,,\quad y'=\frac{y}{a}\,,\quad u'=\frac{u}{ab}\,,$$

$$v'=\frac{v}{ab}\,,\quad p'=\frac{p}{a^2b^2}\,, \tag{5}$$

$$M=\frac{\sigma {B_0}^2}{\rho b}\,,\quad \gamma=\frac{\upsilon}{ba^2}.$$

By substituting these changed variables, which were introduced in Eq. (5), into Eqs. (1) ÷ (4) we obtain (neglecting the prime for clarity).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\gamma\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-Mu \tag{7}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\gamma\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{8}$$

$$\frac{\partial u}{\partial y}=0\,,\qquad v=0\,,\quad at\ y=0\,,$$
$$u=x\,,\qquad v=0\,,\quad at\ y=a\,. \tag{9}$$

Where M are the magnetic and viscosity parameters. The pressure is eliminated between Eq. (7) and Eq. (8) by differentiating Eq. (7) with respect to y and Eq. (8) with respect to x and subtracting from each other. We will have

$$u\left(\frac{\partial^2 v}{\partial x^2}-\frac{\partial^2 u}{\partial x\partial y}\right)+v\left(\frac{\partial^2 v}{\partial y\partial x}-\frac{\partial^2 u}{\partial y^2}\right)+\frac{\partial v}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$
$$-\frac{\partial u}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\gamma\left(\frac{\partial^3 v}{\partial x^3}-\frac{\partial^3 u}{\partial x^2\partial y}\right)$$
$$+\gamma\left(\frac{\partial^3 v}{\partial\ y^2\partial x}-\frac{\partial^3 u}{\partial y^3}\right)\ +M\frac{\partial u}{\partial y}\,. \tag{10}$$

The vorticity ω is defined as

$$\omega=\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right). \tag{11}$$

By using the Eq. (6) and Eq. (11), Eq. (10) can be written as follows

$$\left(u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y}\right)=\gamma\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right)+M\frac{\partial u}{\partial y}\,. \tag{12}$$

Introducing the similarity transformation [19]

$$u=xf'(y)\,. \tag{13}$$

Using Eq. (6), Eq. (11) and Eq. (13), the ω, v can be written in the following from

$$v=-f(y)\,, \tag{14}$$
$$\omega=-xf''(y)\,. \tag{15}$$

Substituting Eqs. (13) ÷ (15) into Eq. (12) give us

$$\gamma f^{(4)}-f''f'+ff'''-Mf''=0\,. \tag{16}$$

Transformation of boundary conditions described in Eqs. (9) are as follows

$$f''(0)=0\,,\quad f(0)=0\,,$$
$$f'(1)=1\,,\quad f(1)=0\,. \tag{17}$$

3 Analytical solutions for $f(y)$

In this section, we employ HPM to solve Eq. (16) subject to boundary conditions Eq. (17). We can construct homotopy function of Eq. (16) as described in [8]

$$H(f,p)=(1-p)\left(\gamma f^{(4)}(y)-g_0(y)\right)$$
$$+p\left(\gamma f^{(4)}-f''f'+ff'''-Mf''\right)=0\,. \tag{18}$$

Where $p\in[0\,,1\,]$ is an embedding parameter. For $p=0$ and $p=1$ we have

$$f(y,0)=f_0(y)\,,\qquad f(y,1)=f(y). \tag{19}$$

Note that when p increases from 0 to 1, $f(y,p)$ varies from $f_0(y)$ to $f(y)$. By substituting

$$f=f_0+pf_1+p^2f_2+\cdots\,,\quad g_0=0\,, \tag{20}$$

Into Eq. (18) and rearranging the result based on powers of p-terms, we have ($M=10$ and $\gamma=0,5$ for example)

$$P^0: \quad \frac{d^4}{dy^4} f_0(y) = 0 ,$$
$$f(0)=0, f''(0)=0, f(1)=0, f'(1)=1 ,$$
$$f_0(y) = 0,5000000000\, y^3 - 0,500000000\, y . \tag{21}$$

$$P^1: \quad \frac{d^4}{dy^4} f_1(y) - 0,5\left(\frac{d^4}{dy^4} f_0(y)\right) - 10\left(\frac{d^2}{dy^2} f_0(y)\right) - \left(\frac{d}{dy} f_0(y)\right)\left(\frac{d^2}{dy^2} f_0(y)\right) + f_0(y)\left(\frac{d^3}{dy^3} f_0(y)\right) = 0 ,$$
$$f(0)=0, f''(0)=0, f(1)=0, f'(1)=0 ,$$
$$f_1(y) = 0,003571428571 y^7 + 0,2500000000 y^5 - 0,5107142857 y^3 + 0,2571428574 y . \tag{22}$$

$$P^2: \quad -10\left(\frac{d^2}{dy^2} f_1(y)\right) - \left(\frac{d}{dy} f_0(y)\right)\left(\frac{d^2}{dy^2} f_1(y)\right) - 0,5\left(\frac{d^4}{dy^4} f_1(y)\right) - \left(\frac{d}{dy} f_1(y)\right)\left(\frac{d^2}{dy^2} f_0(y)\right) + f_1(y) \times \left(\frac{d^3}{dy^3} f_0(y)\right) + \frac{d^4}{dy^4} f_2(y) + f_0(y)\left(\frac{d^3}{dy^3} f_1(y)\right) = 0 ,$$
$$f(0)=0, f''(0)=0, f(1)=0, f'(1)=0 ,$$
$$f_2(y) = -0,00001082251082 + 0,001587301587 y^9 + 0,05996598640 y^7 - 0,1303571428 y^5 + 0,07452123258 y^3 - 0,005706555370 y . \tag{23}$$

In the same manner, the rest of equations up to p^{10} were obtained using the MAPLE package. The function $f(y)$ can be obtained by using Eq. (20) with $p=1$.

$$\begin{aligned} f(y) = &-0,2530001030y + 0,09381734305y^3 \\ &+0,09379897042y^5 + 0,04714111563y^7 \\ &+0,01461756693y^9 + 0,003110880511y^{11} \\ &+0,0004650798206y^{13} + 0,00004675968152y^{15} \\ &+0,00000254233626y^{17} - 1,13512089\times10^{-7}y^{19} \\ &-4,317824487\times10^{-8}y^{21} - 5,57683819\times10^{-10}y^{23} \\ &+1,730766352\times10^{-9}y^{25} + 2,15887971\times10^{-10}y^{27} \\ &-4,629144954\times10^{-11}y^{29} - 4,7832265\times10^{-12}y^{31} \\ &+1,138973516\times10^{-12}y^{33} - 6,8251361\times10^{-14}y^{35} \\ &+1,642594534\times10^{-15}y^{37} 1,43452140\times10^{-17}y^{39} \\ &+7,804290113\times10^{-21}y^{41} + 2,01469972210^{-22}y^{43}. \end{aligned} \tag{24}$$

For simplifying, let us consider $f(y)$ in the following from

$$f(y) = \sum_{i=1}^{43} g_i(y) \qquad \text{for odd } i . \tag{25}$$

We define the average values of functions $g_i(y)$ on interval [0 ,1] as

$$\bar{g}_i = \int_0^1 g_i(y)\, dy . \tag{26}$$

For $i=1$ to 43, Let $\bar{g}_{i_{\max}}$ be maximum magnitude of values $\bar{g}_i$ and the order of magnitude for $g_i(y)$ is defined as

$$OR_i = \frac{\bar{g}_i}{\bar{g}_{i_{max}}} . \tag{27}$$

By neglecting the $g_i(y)$ which their OR_i is less than a base value OR_b, the expression (24) is simplified. For $OR_b = 0,0001$ we have following function.

$$\begin{aligned} f(y) = &-0,2530001030y + 0,09381734305\, y^3 \\ &+0,09379897042\, y^5 + 0,04714111563\, y^7 \\ &+0,01461756693\, y^9 + 0,003110880511\, y^{11} \\ &+0,0004650798206\, y^{13} + 0,00004675968152\, y^{15} . \end{aligned} \tag{28}$$

4 Results and discussion

The partial differential Eqs. (1) ÷ (3) with boundary conditions Eq. (4) are transformed to nonlinear ordinary differential Eq. (16) and boundary conditions Eq. (17).

The HPM and finite difference method (FDM) are used for solving resultant equation and the validity of results is verified by Figs. 2 ÷ 5. We could produce analytical solution for forth order nonlinear Eq. (16) for different values of γ and M. The values of the M are ranging from 0 to 15 and γ from 0,1 to 1, respectively.

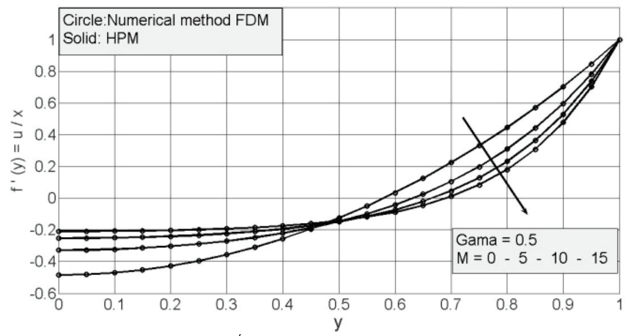

Figure 2 $u/x = f'(y)$ for γ =0,5 and different M values

Figs. 2 ÷ 5, illustrate the fluid velocity profiles. The study shows that a back flow occurs near the center line of the channel due to the stretching walls. As it is shown in Fig. 2, for a constant γ with the increases in the strength of the magnetic field the back flow velocity in the center line decreases but the region of back flow increases. The area between curves and axis of $f'(y)=0$ in Fig. 2 which is proportional to mass flow rate, shows that back mass flow rate decreases when the M increases. The viscosity parameter γ is proportional to kinematic viscosity. Fig. 4 exhibits that when M is constant and γ is decreasing, the back flow velocity in the center line as well as back mass flow rate decreases but the region of back flow increases.

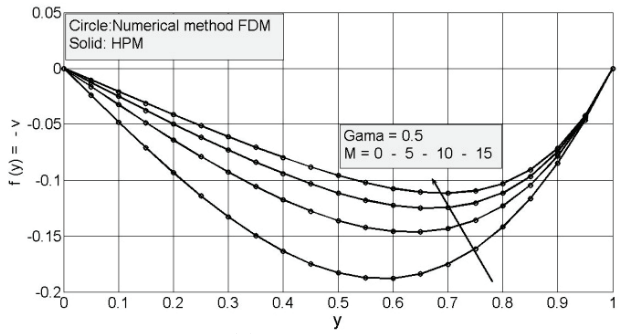

Figure 3 $-v = f(y)$ for γ =0,5 and different M values

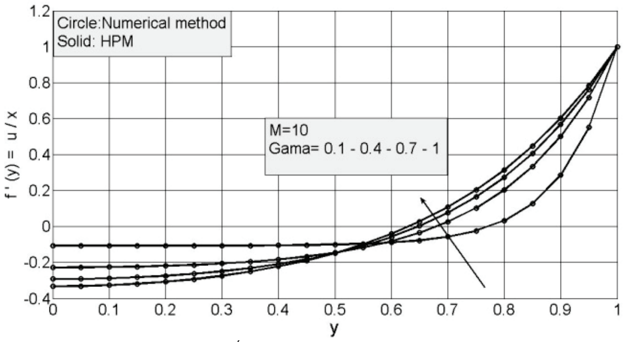

Figure 4 $u/x = f'(y)$ with M=10 and different γ values

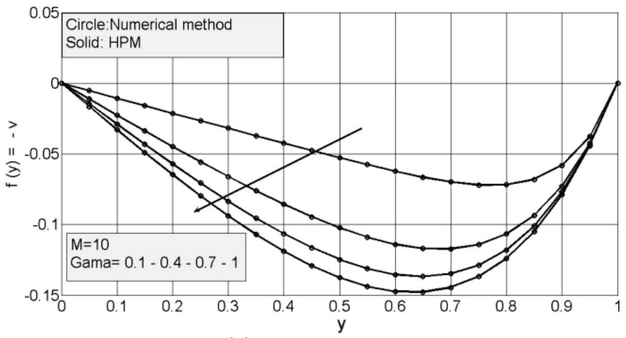

Figure 5 $-v = f(y)$ with M=10 and different γ values

5 Conclusion

The MHD flow in a stretching walls channel has been considered. The partial differential equations which govern the flow were first transformed to a forth order nonlinear differential equation and then solved by employing HPM.

It has been demonstrated that the series solution in HPM can be replaced by a finite expression by using the order of magnitude method. This method can be useful in other analytical methods that use the series solution. The influence of the parameters of interest has been discussed through graphs. It has been noted that the graph shows quite reasonable results.

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