

Flexible manufacturing system with automatic control of product quality

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Keywords

Integral transform
Fourier transform
Fourier-Mellin transform
Phase correlation
Production

Ključne riječi

Integralna transformacija
Fourierova transformacija
Fourier-Mellinova transformacija
Fazna korelacija
Proizvodnja

Primljeno (Received): 2012-10-15

Prihvaćeno (Accepted): 2013-01-29

1. Introduction

Production is in each manufacturing enterprise, in which the production process takes place. The originality and quality of products is now a major element of competition between manufacturing companies. This requires the introduction of new technologies and methods in the process of manufacturing these products.

The manufacturing process is a set of human activities, means of production and physical processes in which the input changes (intermediate form) to the finished product. The manufacturing process takes place in the production system, the production system is a cluster of machines to manufacture the product (parts). The role of the production system is the implementation of the technological process, so that the product was created from semi (part) with prescribed geometric and

Original scientific paper

Abstract: This paper describes the verification method proposal for the detection of defective castings using the registration (reconciliation) of images by application of Fourier-Mellin transform. The paper also describes subsequent comparison of images using Fourier transform and phase correlation. Registration consists in finding a suitable geometric transform between images.

Fleksibilni proizvodni sustav sa automatskom kontrolom kvalitete proizvoda

Izvorno znanstveni članak

Sažetak: Ovaj rad predlaže i opisuje metodu provjere za otkrivanje neispravnih odljevaka koristeći registraciju slika primjenom Fourier-Mellinove transformacije. U radu se također opisuje naknadne usporedbe slika pomoću Fourierove transformacije i fazne korelacije. Registracija se sastoji u pronalaženju odgovarajuće geometrijske transformacije između slika.

qualitative properties, which are the technical documentation.

System for diagnosing faults in production is devoted to work [1]. Since 1975, the automated process control is booming through the deployment of Microchip. Automated process control in the production consists of two parts: the lower sequence control, preliminary and follow-up, at higher levels of coordination and optimization. Good progress shows in particular the use of digital sequential and continuous control; particularly systems achieve major improvements based on models that include observation of the state and parameter estimation of the product.

Symbols/Oznake			
$a(x, y)$	- reference image - referentna slika		
$b(x, y)$	- input image - ulazna slika		
(x, y)	- Cartesian coordinates - Kartezijeve koordinate		
x_0, y_0	- translation - translacija		
$A(u, v)$	- Fourier transform of image $a(x, y)$ - Fourierova transformacija slike $a(x, y)$		
$B(u, v)$	- Fourier transform of image $b(x, y)$ - Fourierova transformacija slike $b(x, y)$		
$A_{lp}(v, \omega)$	- Fourier transform for log-polar representation - Fourierova transformacija za log-polarnog prikaza		
$a_{ip}(\theta, \lambda)$	- Fourier-Mellin Invariant of $a(x, y)$ - Fourier-Mellin invarianta od $a(x, y)$		
$Q_0(v, \omega)$	- phase-only matched filter - filter usklađen samo sa fazom		
F^*	- complex composite function - kompleksna kompozitna funkcija		
\mathcal{F}	- Fourier transform - Fourierova transformacija		
\mathcal{F}^{-1}	- inverse Fourier transform - inverzna Fourierova transformacija		
C	- phase only correlation - korelacija samo faze		
$W(u, v)$	- spectral weighting function - težinska funkcija spectra		
$\tilde{q}_{a,b}(x, y)$	- modified phase only correlation - korelacija samo modificirane faze		
		<u>Greek letters/Grčka slova</u>	
		α	- angle of rotation - kut rotacije
		δ	- scaling factor - factor skaliranja
		$\phi_b(u, v)$	- phase of image $b(x, y)$ - faza slike $b(x, y)$
		(θ, r)	- polar coordinates - polarne coordinate
		(θ, λ)	- log-polar coordinates - log-polarne koordinate
		α	- parameter - parametar
		β	- Parameter - Parameter
		<u>Subscripts/Indeksi</u>	
		p	- polar representation - polarni prikaz
		lp	- log-polar representation - log-polarni prikaz

[2] is devoted to the pursuit of product quality in the manufacturing process of evaluating the data through data-mining, which provides tools for rapid detection of relationships, patterns and knowledge through large databases.

Theory using classification models is used in [3]. In the manufacturing process the increasing pressure is to produce better products and reduce spoilage. The aim of the system of classification of samples is to minimize misclassification error rate, in pursuit of performance and features of products. The collected data is categorized into classes, with each sample comparison of standards based on the theory of classification

models. This is followed by extraction properties, discriminant analysis and the correct choice of products.

[4] describes the impact of multiple testing procedures for quality output products using an automated generator of test samples – ATPG (Automatic Test Pattern Generator), which produces samples with the greatest possible error detected.

[5] is devoted to the detection of mechanical faults of electric motors by laser vibration meter and accelerometer. It focuses on quality control made online in washing machines.

One possibility of verifying product quality is the quality control and detection of defective products in real-world production process. Our goal was to design a method designed to automatically detect errors that arise in the process of casting or completion. One possible solution to the problem is to use the Fourier spectrum and compare images using the appropriate metrics and comparative scores.

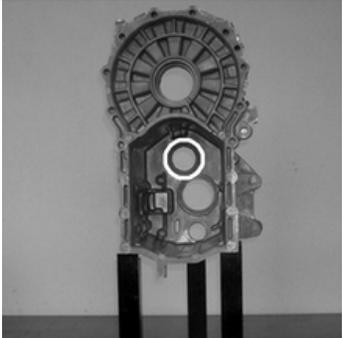
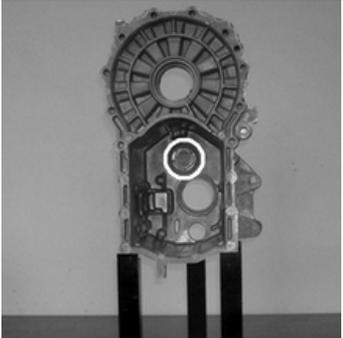
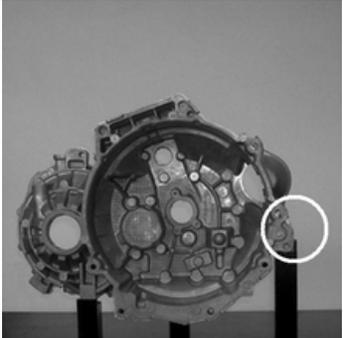
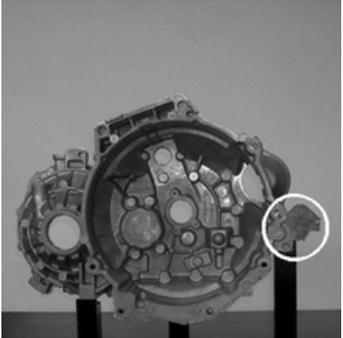
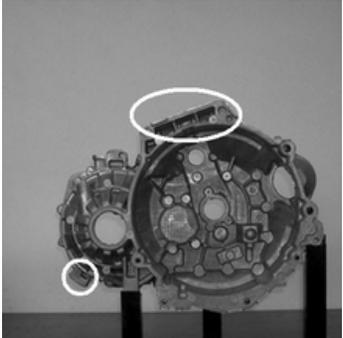
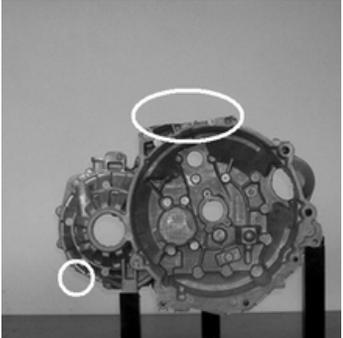
The company Fagor Ederlan Slovakia a.s., Žiar nad Hronom, is engaged in manufacturing aluminum casting, high pressure casting technology. In the experiment, castings for the automotive industry were used. During production or completion, different types of errors may be produced (see Table 1).

2. Registration of images using Fourier-Mellin transform

Image registration is used for geometric alignment of two or more digital images that represent the view of the same object, which can be obtained by various methods, sensors, or from different view directions, or by the same sensor but at different times. Comparative images can have different intensity and geometry. Registration of images using Fourier-Mellin transform allows fit images that are shifted, rotated and scaled.

Table 1. Demonstration of defective products

Tablica 1. Prikaz defektivnih proizvoda

The right product	Defective product	Failure
		undischarged hole
		so called "exhaust"
		missing parts

Space domain of the image function is a real function expressed by the colorfulness of pixels in the point given (256 shades of grey color were used). Via Fourier transform this real function is transferred into a complex function (spectrum) characterizing the

frequencies layout in the image. Figure 1 shows an amplitude spectrum and a phase spectrum of a Fourier transform image and a logarithm-polar transform.

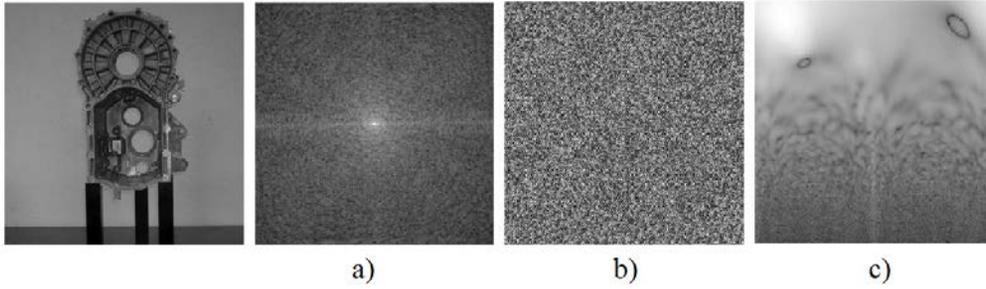


Figure 1. Image of a cast and its Fourier transform a) amplitude spectrum, b) phase spectrum, c) after logarithm-polar transform

Slika 1. Slika odljevka i Fourierovih transformacija: a) amplitudni spektar, b) fazni spektar, c) nakon log-polarne transformacije.

2.1. Fourier-Mellin descriptor

Fourier-Mellin descriptor according by [6]:

There is the reference image $a(x, y)$ and input image $b(x, y)$, which is the copy of image $a(x, y)$ and it is rotated, translated and has changed scale.

$$b(x, y) = a[\delta(x \cos \alpha + y \sin \alpha) - x_0, \delta(-x \sin \alpha + y \cos \alpha) - y_0] \quad (1)$$

$-\infty < x, y < \infty$

where α is the angle of rotation, δ is the scaling factor and x_0, y_0 is translation.

If $A(u, v) = \mathcal{F}\{a(x, y)\}$ is Fourier transform of image $a(x, y)$, then for Fourier transform of image $b(x, y)$ it applies:

$$B(u, v) = e^{-j\phi_b(u, v)} \delta^{-2} \left| A[\delta^{-1}(u \cos \alpha + v \sin \alpha), \delta^{-1}(-u \sin \alpha + v \cos \alpha)] \right| \quad (2)$$

$-\infty < u, v < \infty$

where $\phi_b(u, v)$ is the phase of image $b(x, y)$. This phase depends on the translation, change of scale and rotation, but the amplitude is resistant to translation:

$$|B(u, v)| = \delta^{-2} \left| A[\delta^{-1}(u \cos \alpha + v \sin \alpha), \delta^{-1}(-u \sin \alpha + v \cos \alpha)] \right| \quad (3)$$

$-\infty < u, v < \infty$

Equation (3) shows, that rotation of image $a(x, y)$ rotates the amplitude according the same angle and that the change of scale according to factor δ changes the amplitude scale according to δ^{-1} . However the center of the spectra $u = v = 0$, is resistant to translation, rotation and the image scale. Rotation and scale can be concentrated around this spectral center according to the amplitude definition from a and b in polar coordinates (θ, r) .

$$a_p(\theta, r) = |A(r \cos \theta, r \sin \theta)|; 0 \leq \theta < 2\pi, 0 \leq r < \infty \quad (4)$$

$$b_p(\theta, r) = |B(r \cos \theta, r \sin \theta)|; 0 \leq \theta < 2\pi, 0 \leq r < \infty \quad (5)$$

As the amplitude is periodical function of the polar angle

$$a_p(\theta \pm n\pi, r) = a_p(\theta, r); \text{ pre } n = 0, 1, 2, \dots \quad (6)$$

the half of the amplitude field is enough to estimate the function:

$$\begin{cases} \delta^{-1}(u \cos \alpha + v \sin \alpha) = \frac{r}{\delta} \cos(\theta - \alpha) \\ \delta^{-1}(-u \sin \alpha + v \cos \alpha) = \frac{r}{\delta} \sin(\theta - \alpha) \end{cases}; 0 \leq \theta < \pi \quad (7)$$

$$b_p(\theta, r) = \delta^2 a_p(\theta - \alpha, r / \delta). \quad (8)$$

Rotation of image will result in phase shift θ . The change of scale is reduced to the scale of radial coordinate and increases proportionately with the value of δ^2 . This scale can be reduced to the linear shift using logarithmic scale for radial coordinate.

We define:

$$a_{lp}(\theta, \lambda) = a_p(\theta, r); \quad -\infty < \lambda < \infty \quad (9)$$

$$b_{lp}(\theta, \lambda) = b_p(\theta, r) = \delta^2 a_{lp}(\theta - \alpha, \lambda - \rho); \quad -\infty < \lambda < \infty \quad (10)$$

where $\lambda = \log(r)$ and $\rho = \log(\delta)$. In this log-polar representation symbolized by subscript lp , the rotation and scale are reduced to translation along axes.

Using Fourier transform for log-polar representation from (9) to (10) we gain:

$$A_{lp}(v, \varpi) = \int_0^\pi \int_{-\infty}^\infty a_{lp}(\theta, \lambda) e^{-2\pi j(\theta v + \lambda \varpi)} d\lambda d\theta; \quad -\infty < v, \varpi < \infty \quad (11)$$

$$B_{lp}(v, \varpi) = \delta^2 e^{-2\pi j(v\alpha + \varpi\rho)} A_{lp}(v, \varpi); \quad -\infty < v, \varpi < \infty, \quad (12)$$

where the rotation and the change of scale manifest by phase shift.

This technique cancels the image rotation, the change of scale and the translation and therefore enables the image comparison.

Processing of log-polar coordinates from the amplitude corresponds to Fourier-Mellin transform and the function $a_{lp}(\theta, \lambda)$ is called Fourier-Mellin Invariant (FMI) of the image $a(x, y)$ by [6].

FMI invariant of the image $a(x, y)$ can be gained by resampling of the spectral amplitude of this image from the Cartesian coordinates to rectangular polar coordinates and then resampled to radial coordinates by the logarithmic function, or by resampling of the spectral amplitude in one step to rectangular log-polar coordinates.

2.2. Symmetric phase filter

Given the reference image $a(x, y)$ and the input image $b(x, y)$, FMI *phase-only matched filtering* (FMI-SPOMF) is defined [6]:

$$Q_0(\nu, \varpi) = \frac{A_{lp}^*(\nu, \varpi) \cdot B_{lp}(\nu, \varpi)}{|A_{lp}(\nu, \varpi)| \cdot |B_{lp}(\nu, \varpi)|}, \quad (13)$$

where * denotes complex conjugate function.

For the evaluation we will use:

$$q_0(\theta, \lambda) = \mathcal{F}^{-1}\{Q_0(\nu, \varpi)\}, \quad (14)$$

where \mathcal{F}^{-1} is the inverse Fourier transform.

If the two identical images are compared, the result of this method will be the correlation matrix with a strong peak. The position of FMI-SPOMF vertex will determine the image rotation and scale. The translation of the input image is determined by the position of the maxima of the correlation vertex from the comparison of the reference image with modified input image according to the observed rotation and scaling.

2.3. Transformation parameters estimate

1. Finding maxima $q_{0\max}(\theta, \lambda) = q_0(\theta_M, \lambda_M)$, of which the rotational angle $\alpha = \theta_M$ and the scale factor $\delta = e^{\lambda_M}$.
2. Since for the derivation of FMI invariants from the real image was used only a half of spectra (see equations (6) and (7)), estimated rotational angle α is in the interval $[0, \pi]$, but the correction of rotation can be α or $\alpha + \pi$: modification of image scale $b(x, y)$ according to δ^{-1} and creating of two copies of the image rotated by the angle α and $\alpha + \pi$.
3. Calculation of phase correlation between $a(x, y)$ and the two rotated images with a modified scale.

4. Finding the maxima of both phase comparison outputs.
5. Determine the correct rotational angle by higher value of phase correlation value.
6. Calculation of displacement (x_0, y_0) as the maximum deviation (item 4) from the center of the correlation matrix.
7. Output of image transformation parameters $(x_0, y_0, \alpha, \delta)$.

More information on the registration of images using Fourier-Mellin transform are in publications [7], [8].

3. Images comparison

In the next stage after the registration of images it was necessary to compare the two registered images (back turned, moved and with modified scale) and see if they are identical or not (there's an error).

In practice, there is usually the input image and the reference image intact, for example noisy. Therefore, it is not possible to find the absolute compliance with the reference image, but only partial – appropriate criteria maximum – level of agreement. Different criteria are used for assessment – functions that return a value from a range.

Comparative score that quantifies the similarity between the input and the reference image was calculated as:

Phase Only Correlation (POC) by [9]:

$$C = \mathcal{F}^{-1} \left(\frac{F_a F_b^*}{|F_a| |F_b|} \right), \quad (15)$$

where F – Fourier transform of images a, b ; F^{-1} – inverse Fourier transform, F^* – complex composite image.

When two images are similar, their phase correlation gives a clear maximum. When two images are not similar, there will be more minor peaks. Maximum size is used as a measure of similarity between two images.

Modified Phase Only Correlation (MPOC)

Whereas signal energy is lower in a high frequency domain, the phase components are not reliable in the domain of high-frequency [10]. The effect of unreliable phase components at high frequencies can be reduced by using filters or modifying the POC function using spectral weight function.

To improve recognition by removing insignificant high frequency components that have low reliability we use spectral weighting $W(u, v)$ [11]:

$$W(u, v) = \left(\frac{u^2 + v^2}{\alpha} \right) e^{-\frac{u^2 + v^2}{2\beta^2}}, \quad (16)$$

where u, v are 2D coordinates, β is parameter, which verifies the width of function and α is used only for normalization.

Such modified function of phase correlation of images a and b is given by [11].

$$\tilde{q}_{a,b}(x, y) = F^{-1} \left\{ W(u, v) \frac{F_a(u, v) F_b^*(u, v)}{|F_a(u, v)| |F_b(u, v)|} \right\}. \quad (17)$$

The value of the peak of function $\tilde{q}_{a,b}(x, y)$ is stable during the change of translation and brightness. This value was used to measure the images similarity: if the two pictures are similar, their function MPOC gives

clear sharp peak, if different, then the peak decreases considerably.

Graphs are shown to scale 1– N for coordinates x, y and functional values MPOC normalized to scale 0–1 (see Figure 2).

The design of the detection of defective products is based on the principles of biometric comparisons of images, where the reference model is compared with the test image. The aim of the comparison is to find enough consistency (the degree of similarity or correlation) of verified model with a registered reference template stored in the system.

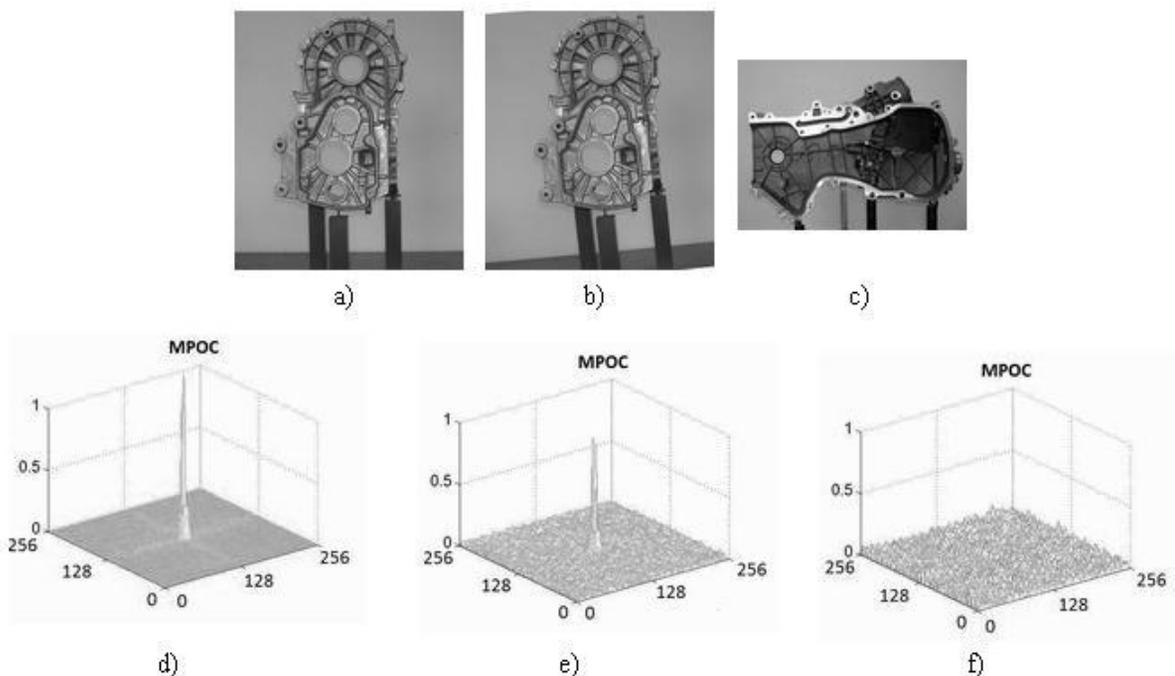


Figure 2. Using of Modified Phase Only Correlation (MPOC) a) reference image, b–c) tested images, d) MPOC between identical images (a–a), e) MPOC between similar images (a–b), f) MPOC between various images (a–c)

Slika 2. Korištenje korelacija samo modificirane faze (MPOC): a) referentna slika, b–c) testne slike, d) MPOC između identičnih slika (a–a), e) MPOC između sličnih slika (a–b), f) MPOC između različitih slika (a–c).

A comparative score is a number that quantifies the degree of similarity between the input and template stored in the database. The calculated score is compared with the verification **threshold**, which is selected by the system administrator. The threshold value determines the degree of correlation necessary for comparison to be taken as a coincidence. If comparative score is less than the threshold, the result is a mismatch (product is not identical to the template, it is wrong). If the comparative score exceeds the threshold, the comparison is a match (the product is identical to the template).

Depending on the threshold, the system can either be highly secure (will not release any defective product) or it is not at all safe.

Calculation of comparative score has been validated in various combinations of application/ applications without hamming window, application/application without lower-permeable filter application/application without preprocessing. To check the results of the entire series of tests it was performed the comparison of each image with every other image.

The comparison result of the comparison scores and the threshold is a **decision**, in which the identification

conclusion is determined. The decision can be: consensus, disagreement and without result, although varying degrees of possible hits and strong disagreements.

In Figure 3 there is an example of comparing products.

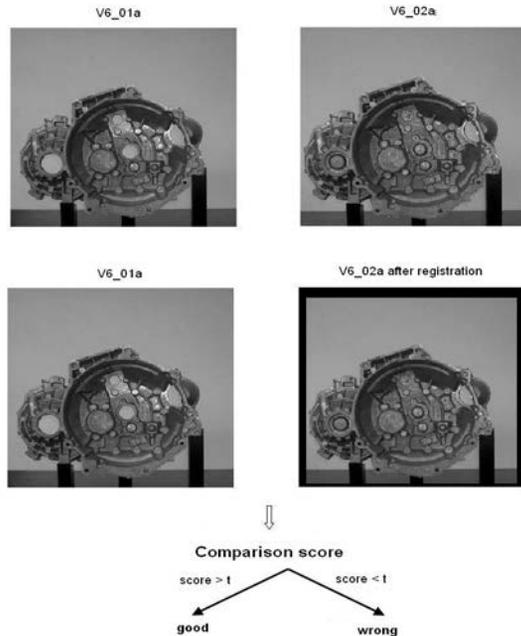


Figure 3. The example of registration and products comparison

To evaluate the quality of the authentication method we used the coefficients of performance used in biometrics:

False Acceptance Rate (FAR) in our case reflects the probability with which the system incorrectly identifies the defective product as good.

$$FAR = \frac{NFA}{NPNT} \times 100 \% \tag{18}$$

where *NFA* – number of false acceptances, *NPNT* – number of products that should not be taken.

False Rejection Rate (FRR) in our case reflects the probability with which the system incorrectly rejects the correct product as defective.

$$FRR = \frac{NFR}{NPT} \times 100 \% \tag{19}$$

where *NFR* – number of false rejections, *NPT* – number of products that should be taken.

4. Achievements

It was followed eight kinds of items: tested 36 pairs that were right and 588 were wrong.

In Figure 4, Figure 5 and Figure 6 there are examples of some of the products after registration.

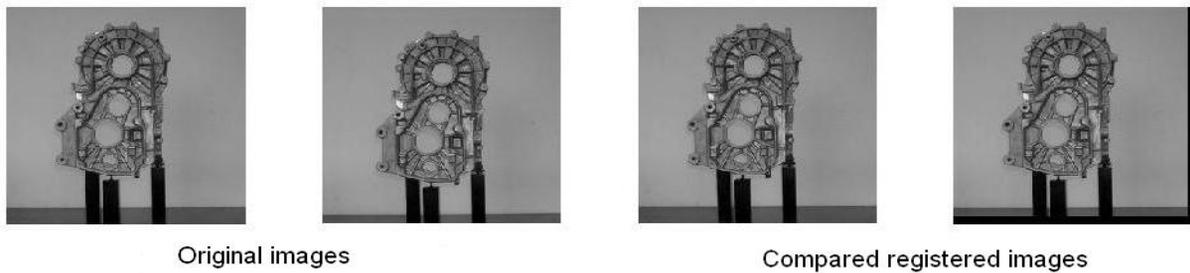
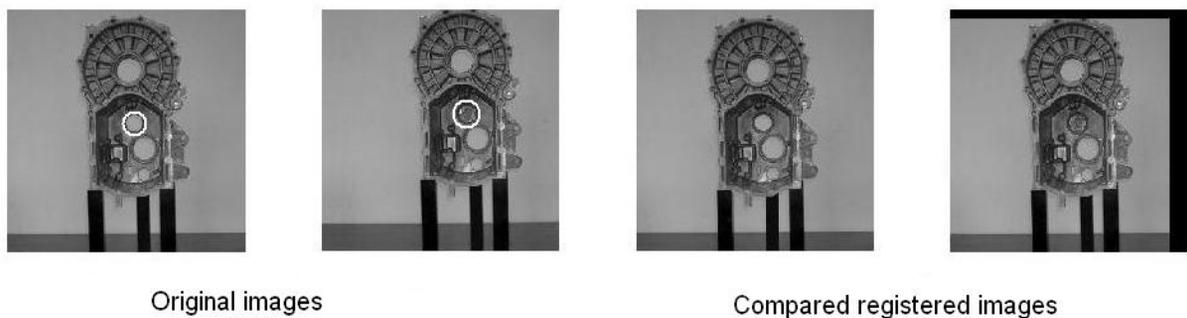


Figure 4. The right products – correctly recognized
Slika 4. Pravilni proizvodi – pravilno prepoznati



Original images

Compared registered images

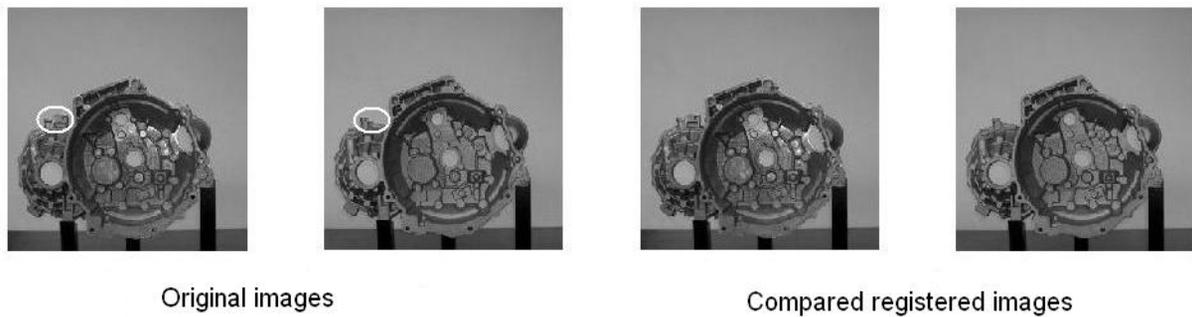


Figure 5. The wrong products – correctly rejected
Slika 5. Pogrešni proizvodi – pravilno odbačeni.

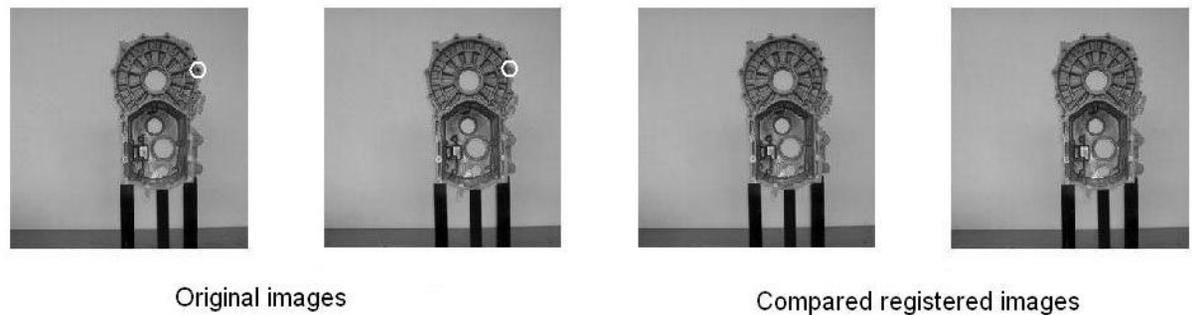


Figure 6. The wrong products – not recognized
Slika 6. Pogrešni proizvodi – nisu prepoznati.

The number of correctly rejected and correctly received products depends on the set threshold. Suppose the requirement, so that we completely reject the adoption of a defective product, that $FAR = 0$.

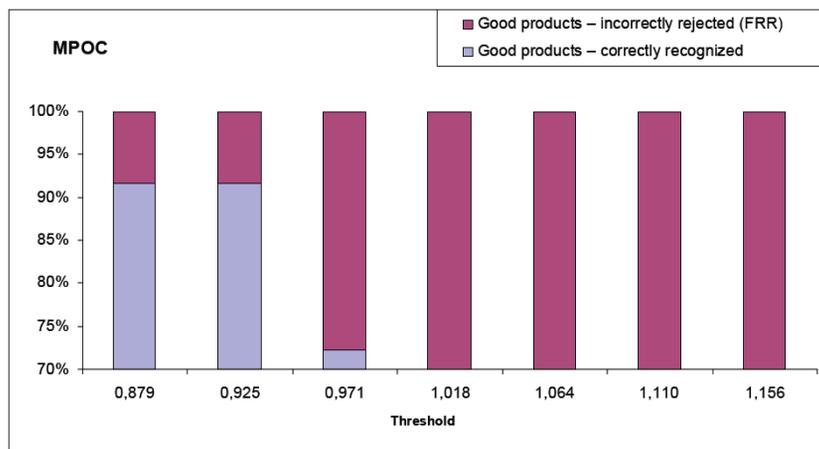
Table 2 gives an overview of the results of correct rejection of defective products, correct recognition of the right products and results of improper rejection of the right products (FRR) and false acceptance of incorrect product (FAR), depending on the set threshold criteria MPOC. Highlighted in color value $FAR = 0$ is

the threshold above which there is no longer incorrectly received any defective product. At this threshold value (0.925) there was 91.7 % of correctly detected and 8.3 % of incorrectly rejected products.

In the effort to detect also minor insignificant errors such as kinked thorn in Figure 6, it would be necessary to modify the program and not to test the whole picture, but divide it into several smaller parts, by which we could detect the slight variations.

Table 2. The dependence of FRR and FAR from the set threshold of MPOC**Tablica 2.** Ovisnost FRR i FAR sa postavljenim pragom MPOC-a.

Threshold	Wrong products – correctly rejected	Good products – correctly recognized	Good products – incorrectly rejected (FRR)	Wrong products – incorrectly accepted (FAR)
0,601	95,3 %	100,0 %	0,0 %	4,7 %
0,648	96,9 %	100,0 %	0,0 %	3,1 %
0,694	97,3 %	100,0 %	0,0 %	2,7 %
0,740	97,8 %	100,0 %	0,0 %	2,2 %
0,786	97,8 %	97,2 %	2,8 %	2,2 %
0,833	98,2 %	97,2 %	2,8 %	1,8 %
0,879	99,1 %	91,7 %	8,3 %	0,9 %
0,925	100,0 %	91,7 %	8,3 %	0,0 %
0,971	100,0 %	72,2 %	27,8 %	0,0 %
1,018	100,0 %	27,8 %	72,2 %	0,0 %
1,064	100,0 %	0,0 %	100,0 %	0,0 %
1,110	100,0 %	0,0 %	100,0 %	0,0 %
1,156	100,0 %	0,0 %	100,0 %	0,0 %

**Figure 7.** Dependence of FRR from the set threshold of comparing criteria MPOC**Slika 7.** Ovisnost FRR i postavljenog praga kriterija usporedbe MPOC.

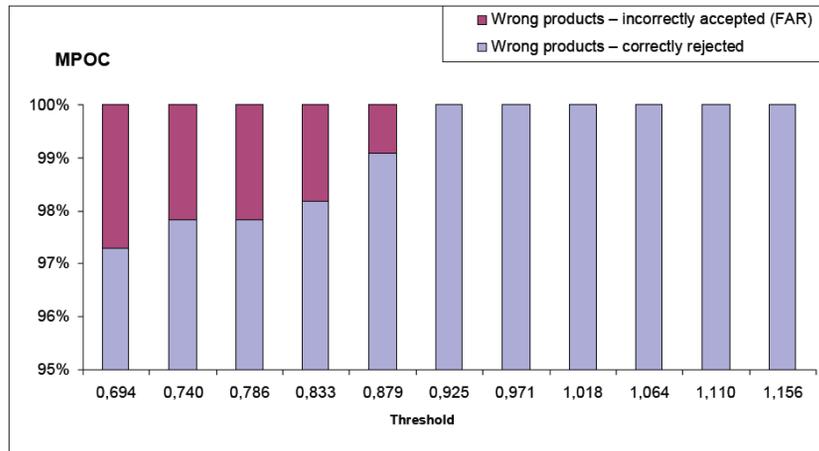


Figure 8. Dependence of FAR from the set threshold of comparing criteria MPOC

Slika 8. Ovisnost FAR od postavljenog praga kriterija usporedbe MPOC

5. Conclusion

An important attribute of the manufacturing process, the functional characteristics of the product are a set of properties defining the meaning, purpose, nature and use of the product. Based on meeting the needs of customers and the broad basically determine the identity of the product, for example performance, speed, reliability, availability, security, and ultimately, the real utility.

Trying to solve a particular problem, automated inspection of defective products in real-world production process, led to the research that led to the design of software designed for evaluating the captured images of the products to determine the quantitative indicators of the quality and reliability of manufactured products.

Solving the problem lies in transforming the position of the test image (image registration), whose task is to remove the mutual displacement, rotation and scaling of images and prepare them both for subsequent comparison.

The contribution does not provide a satisfactory answer to all the problems associated with automated tracking of defective products. However, it is part of other tasks that need to be addressed in the applied research in the field of production engineering.

In the next phase of the research we can proceed in accordance with the previous results. When analyzing the possibility of real applications deployed in the production process it is necessary to examine possible options for the inclusion of the sensing element to capture the image of the product in the production cycle. Custom automated scanning process should be included as an integral part of the manufacturing process. To obtain accurate results, it is necessary for scanning product to ensure stability (elimination of vibration, resonance, motion, the way of lighting). It is also

important to automate the process of registered images and then evaluate by the proposed software.

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